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Sampling plans using extended EWMA statistic with and without auxiliary information

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Abstract

In this paper, we propose an Acceptance Sampling Plan (ASP) using the statistic suggested by Naveed *et al.* (2018) under the condition of known and unknown population standard deviation (SD) for the presence of with and without Auxiliary Information (AI). It is presumed that the study variable of quality trait and AI follow the bivariate normal distribution. The plan parameters of the recommended plan are discussed for all four cases under the constraint that specified producer and consumer risks are gratified. The suggested plan is compared in terms of sample size (SS) with numerous existing plans and showed that the presented plan has a smaller SS for any value of AQL, LQL. Various tables of plan parameters have been erected using various combinations of smoothing constants for industrial use. For the workable purpose, the industrial example has also been examined. At last, concluding remarks are discussed.

Keywords: Acceptance number; acceptance sampling plan; auxiliary information; extended EWMA; sample size.

1. Introduction

As buyer trust the specification mentioned by the manufacturer on their products. It's very obligatory for the manufacturer to produce the product according to the mentioned specification limits. Good experience of buyers enhances the reputation of the product. In practice, it is very difficult to inspect the peculiarity of every product because a lot of time is consumed in this exercise, and cost is also increased. So, to overwhelm this hurdle a device used in statistical quality control (SQC) is known as an acceptance sampling plan (ASP). ASPs ensure both the purchaser and seller a decision rule to acknowledge the lot or discard it based on sample data taken from the lot. ASPs are comprehensively practiced in assembling industries to scrutinize the crude material, in-process and finished items. There are so many types of plans which are practiced in inspection of the lot, like single, double, repetitive, multiple, etc. among them single sampling plan (SSP) is quite familiar owing to its ease of implementation. A lot of work can be seen in the literature using SSP like Lieberman & Resnikoff (1955) constructed various tables for parametric values of SSP using various acceptance quality level (AQL) under the condition of MIL-STD 414 scheme. After that Owen (1967) proposed SSP for normal distribution (ND) under the condition of unknown process standard deviation (SD). Hamaker (1979) suggested the method for finding the values of the parameter in SSP under the scenario of unknown SD. Two foremost types of ASPs are attribute and variable sampling plans. Collani (1991) had shown that attribute plans are more appropriate if the buyer has concerned with fraction non-conforming in lots. But later on Seidel (1997) has shown that a variable sampling plan (VSP) is always optimum. More work using VSP can be viewed in S Balamurali & Jun (2006), Saminathan Balamurali & Jun (2007), Klufa (2010), Aslam *et al*. (2011), Wang & Tsai (2012), Al-Omari & Al-Hadhrami (2018), Cha & Badía (2020), Chakrabarty *et al.* (2020), *Tripathi et al.* (2020), Yemshanov *et al*. (2020)

The utilization of memory type statistic in the field of ASPs has attained much more consideration in recent past owing to his eminent performance in determining the lesser values of its plan parameters. Aslam *et al.* (2015b) suggested an SSP based on exponentially weighted moving average (EWMA) statistic and calculate the parametric values of SSP. The results are likening to the memoryless plan which shows the far better evaluation in the form of lesser values of its parameters. The authors also proved that the recommended plan achieved much better results in comparison to the plans presented by

Saminathan Balamurali & Jun (2007) and Aslam *et al.* (2013). Yen *et al.* (2014) presented an SSP based on EWMA statistic using yield index and exhibit the lesser values of sample size (SS) as collating it to the memoryless yield index plan. After that Aslam *et al.* (2015a) improved the result of EWMA based SSP using repetitive sampling (RS). Khan *et al.* (2018) presented another memory-based ASP using modified EWMA statistic and showed a far better results when compared with two existing plans.

The use of auxiliary information (AI) in ASP is very beneficial in terms of reducing the time as well as the cost of the inspection. The concept of AI in ASP has been introduced by Aslam *et al.* (2017) and showed the lesser value of SS using AI. later on Aslam *et al.* (2018) used a memory type regression estimator in ASP to enhance the capability of the recommended plan in terms of smaller values of parameters. More work on ASPs using AI can be seen in Azam *et al.* (2016), Khan *et al.* (2019), motivated by the use of memory-based information as well as AI in ASPs, we present an ASP using memory-based statistic presented by Naveed *et al.* (2018) with and without AI. The remaining paper is described as: in Section 2, the designing of the recommended plan without AI has been discussed. In section 3, the construction of the suggested plan using AI has been examined. In section 4, comparability of the presented plan has been discussed with the existing plan. In Section 5, the industrial application has been examined. In the last Section concluding remarks are given.

2. Designing of the proposed plan without AI

Naveed *et al.* (2018) presented a memory-based statistic for the quality trait of Normal Distribution (ND) named as Extended EWMA statistic which is given as

$$\mathrm{W_i} = \tau_1 \bar{\mathrm{Z}}_\mathrm{i} - \tau_2 \bar{\mathrm{Z}}_{\mathrm{i}-1} + (1 - \tau_1 + \tau_2)\mathrm{W}_{\mathrm{i}-1} \quad (1)$$

Such that $0 < \tau_1 \leq 1 \; and \; 0 \leq \tau_2 < \tau_1$. Where τ_1 and τ_2 are smoothing constants. Like traditional EWMA statistic the performance of initiated statistic is quite well for smaller values of smoothing constants. The proposed statistic is the generalization of Roberts (1959) and converted to EWMA statistic when $\tau_2 = 0$. The intended statistic is an unbiased estimator of process mean μ with variance

$$var(\mathrm{W_i}) = \frac{\sigma^2}{m}\left[({\tau_1}^2 + {\tau_2}^2)\left\{\frac{1-r^{2i}}{1-r^2}\right\} - 2r\tau_1\tau_2\left\{\frac{1-r^{2i-2}}{1-r^2}\right\}\right] \quad (2)$$

Where $r = 1 - \tau_1 + \tau_2$.

The above expression of variance is reduced to Roberts (1959) the expression if we put $\tau_2 = 0$. It is to be remembered that $\tau_2 < \tau_1$. The variance expression of the proposed statistic will give a minimum result if we take the value of τ_2 is nearer to τ_1. For example if $\tau_1 = 0.1$ then for any value within the range of $\tau_2 (0 \le \tau_2 < \tau_1)$ will give the smaller variance result than EWMA variance, but if we take $\tau_2 = 0.09$ than the result of the proposed variance is smallest for any other values of τ_2.

Now we will construct a sampling plan based on recommended idea under two different scenarios

- When process variability is known
- When process variability is not known

2.1. When process variability is known

We present the succeeding recommended plan for variable suggested by Naveed *et al.* (2018):

Step 1: Firstly, we chose a sample of size $m$from the running lot at the time i and calculate $\bar{Z}_i$, the average value of the study variable. After that, we utilized it to find out the value of offered statistic using predefine values of τ_1 and $\tau_2$as

$$W_i = \tau_1 \bar{Z}_i - \tau_2 \bar{Z}_{i-1} + (1 - \tau_1 + \tau_2) W_{i-1} \qquad (3)$$

Step 2: After getting the value of W_i, calculate the statistic M_i as follows

$$M_i = \frac{USL - W_i}{\sigma} \qquad (4)$$

If the value of $M_i \ge L_a$, we accept the current lot, contrarily we discard the running lot. In the first step, while we calculate the value of the presented statistic, we set the value of $\bar{Z}_0$ and W_0 is zero at $i = 1$.The prospective plan is converted to Aslam *et al.* (2015b) when $\tau_2 = 0$, and reduced to a conventional variable single plan if $\tau_1 = 1$ and $\tau_2 = 0$. it is experienced that when the engineers apply this type of plan repeatedly, the effect of time index i is very low which is negligible. That's why we omit the time index factor invariance expression of the recommended statistic. The two parameters of the suggested plan named as the sample size (SS) denoted by m and acceptance number (AN) designated by L_a.

The operating characteristic (OC) function of the presented plan when p is the proportion of nonconformist is extracted as follows

$$P_a(p) = P(M_i \ge L_a | p) = P(W_i + L_a \sigma \le USL | p) \qquad (5)$$

According to Duncan (1986), we have

$$W_i + L_a \sigma \sim N\left(\mu + L_a \sigma, \frac{\sigma^2}{m}\left[\frac{\tau_1{}^2 + \tau_2{}^2 - 2r\tau_1\tau_2}{1 - r^2}\right]\right)$$

where $r = 1 - \tau_1 + \tau_2$

After simplification of the above equation (5), the final form of OC function under the condition of known standard deviation (SD) is given as

$$P_a(p) = \Phi\left(\left(Z_p - L_a\right)\sqrt{\frac{m}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]}}\right) \qquad (6)$$

Here $\Phi(.)$ is the cumulative distribution function (CDF) and Z_p is the pth percentile of standard normal distribution (SND).

Two risks are involved while we making a decision about the lot. One is the producer's risk designated by α is the probability of discarding the correct the lot and other is consumer risk expressed by β is the probability of accepting the defective lot. Every manufacture wishes that the probability of lot accepting (PLA) should be larger than $(1-\alpha)$ at an acceptable quality level (AQL). Perversely, every client wants that PLA should be smaller than β at the limiting quality level (LQL). The parametric values of the initiated plan will be determined under the condition that both risks are gratified concurrently. During the simulation experience, we noticed that there are so many combinations of SS and AN satisfy the conditions, from these combinations we selected that pair of combination which has the smallest value of SS. The parametric values of the offered plan are determined such that the OC curve must pass the points $(\alpha, AQL) and (\beta, LQL)$. we will utilize the subsequent non-linear optimization to find the parametric values of the initiated plan:

minimize m (7)

With the condition that

$$P_a(AQL) = \Phi\left(\left(Z_p - L_a\right)\sqrt{\frac{m}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]}}\right) \geq 1-\alpha \qquad (8)$$

And

$$P_a(LQL) = \Phi\left(\left(Z_p - L_a\right)\sqrt{\frac{m}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]}}\right) \leq \beta \quad (9)$$

The parametric values of the proposed plan when SD is known are placed in Table 1 for numerous levels of AQL, LQL, and distinct values of smoothing constants.

1. If the values of AQL and LQL are fixed, the SS m increases as the value of the smoothing constant is increased. For example, when $AQL = 0.001$ and $LQL = 0.002$, then the value of $m = 3$ using $\tau_1 = 0.1, \tau_2 = 0.09$. For $\tau_1 = 0.3, \tau_2 = 0.29$, $\mathrm{m} = 18$, For $\tau_1 = 0.5, \tau_2 = 0.49$, the value of SS is 51.
2. For fixed values of smoothing constants and AQL, we observe the diminishing trend in SS m as LQL increases. For example, when $\tau_1 = 0.5, \tau_2 = 0.49$ and $\mathrm{AQL} =$

0.005, then for $= 0.010, m = 37$, $= 0.015, m = 14$, $= 0.020, m = 9$, $= 0.030$, $= 5$, $= 0.040, m = 4$

2.2. When process variability is not known

In this section, we discuss the scenario when its process SD is not known and we estimate it with the help of sample data. The first step of the recommended plan is the same as for the known case in section 2.1, but in the second step we use sample standard deviation (SD) S instead of σ in statistic M_i i.e

M = ——— (10)

Table 1. The plan parameters of suggested plan when SD is known

AQL	LQL	= 0.1	= 0.09	= 0.3	= 0.29	= 0.5	= 0.49
0.001	0.002	3	2.9665	18	2.9704	51	2.9701
	0.004	2	2.8339	5	2.8367	12	2.8476
	0.006	2	2.8307	3	2.7514	7	2.7639
	0.008	2	2.6385	2	2.7127	5	2.7124
0.005	0.010	3	2.4543	14	2.4327	37	2.4325
	0.015	2	2.3532	5	2.3513	14	2.3484
	0.020	2	2.1688	4	2.2592	9	2.2770
	0.030	2	2.3776	2	2.1589	5	2.2048
0.03	0.06	2	1.7119	8	1.6934	22	1.6977
	0.09	2	1.4527	3	1.5763	8	1.5768
	0.12	2	1.3231	2	1.4630	5	1.4941
	0.15	2	1.2841	2	1.5079	4	1.4266
0.05	0.10	2	1.4271	7	1.4381	17	1.4396
	0.15	2	1.3050	3	1.2784	6	1.3028
	0.20	2	1.1856	2	1.1593	4	1.1792
	0.25	2	0.8364	2	1.0855	3	1.1417

Where $S = \sqrt{\frac{\sum_{i=1}^{m}(Z_i - \bar{Z}_i)}{}}$

We reject the lot if M $<$ L and accept it if M L

The OC function for the unknown case is derived as

$= P(\text{M} \quad) = P(\text{W}_\text{i} + \quad)$ (11)

According to (Duncan, 1986), we have

$\text{W}_\text{i} + \quad (\mu + \quad E(S), Var(\text{W}_\text{i}) + \quad ar(S))$

As we know that $E(S) =$ $ar(S) = (1 \quad)\sigma$

where $c_4 = \sqrt{[2/(m-1)]}\Gamma(m/2)/\Gamma[m-1/2]$ these results are used by various researches like Aslam *et al.* (2015b), Aslam *et al.* (2015a), Azam *et al.* (2016), Aslam *et al.* (2018),

So

$$W_i + L_a S \sim N\left(\mu + L_a \sigma c_4, \left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{\sigma^2}{m} + L_a^2(1-c_4^2)\sigma^2\right) \quad (12)$$

where $r = 1 - \tau_1 + \tau_2$

The final form of OC function is given as

$$P_a(p) = \Phi\left(\left(Z_p - L_a c_4\right)\sqrt{\frac{1}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{1}{m}+L_a^2(1-c_4^2)}}\right) \quad (13)$$

The optimization problem for the case of unknown SD is given as

$$minimize\ m \quad (14)$$

With respect to

$$P_a(AQL) = \Phi\left(\left(Z_p - L_a c_4\right)\sqrt{\frac{1}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{1}{m}+L_a^2(1-c_4^2)}}\right) \geq 1-\alpha \quad (15)$$

And

$$P_a(LQL) = \Phi\left(\left(Z_p - L_a c_4\right)\sqrt{\frac{1}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{1}{m}+L_a^2(1-c_4^2)}}\right) \leq \beta \quad (16)$$

We used grid search method to calculate the SS and AN. The parametric values $m\ and\ L_a$ of the proposed plan for unknown SD are reported in Table 2 using numerous values of AQL, LQL, and smoothing constants. From Table 2, we perceive the same pattern in the values of parameters as we noticed in the known case. However, we see that SS is much large for the unknown case. For example, when $AQL = 0.03\ and\ LQL = 0.06$ the value of m is 119 for an unknown case and it was just 2 for a known case when $\tau_1 = 0.1\ and\ \tau_2 = 0.09$.

3. Designing of the proposed plan using AI

The utilization of AI has always beneficial in enhancing the precision of decisions. So, in this section, we consider that some extra information that is correlated to the study variable is given. In this section, we will discuss the suggested plan for the case of known and unknown SD.

Table 2. The plan parameters of suggested plan when SD is unknown

		= 0.1	= 0.09	= 0.3	= 0.29	= 0.5	= 0.49
0.001	0.005	129	2.8064	131	2.8060	137	2.8057
	0.007	82	2.7429	83	2.7423	87	2.7418
	0.009	61	2.6922	61	2.6940	65	2.6947
	0.012	44	2.6366	45	2.6346	47	2.6375
	0.016	33	2.5787	34	2.5729	35	2.5779
0.006	0.015	200	2.3227	207	2.3223	219	2.3224
	0.017	150	2.2950	154	2.2950	164	2.2946
	0.020	107	2.2591	109	2.2599	117	2.2592
	0.025	71	2.2087	72	2.2094	77	2.2093
	0.030	52	2.1666	55	2.1644	57	2.1676
0.03	0.055	162	1.7246	171	1.7247	188	1.7243
	0.057	143	1.7155	150	1.7153	165	1.7146
	0.059	126	1.7054	133	1.7051	147	1.7056
	0.06	119	1.7013	126	1.7006	138	1.7006
	0.09	39	1.5887	41	1.5855	46	1.5883
0.05	0.08	174	1.5123	186	1.5123	210	1.5122
	0.09	104	1.4770	111	1.4777	126	1.4774
	0.10	70	1.4465	76	1.4472	85	1.4450
	0.11	51	1.4177	55	1.4154	62	1.4156
	0.15	22	1.3166	23	1.3175	27	1.3173

3.1 When population SD is known

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The initiated plan using AI for the case of known SD is stated as follows:

Step-1: Select a bivariate sample of size mmon (x_i, t_i) from running a lot and compute the

regression estimate as

$$= \bar{x} + \quad (\mu_t - \bar{t}) \qquad (17)$$

where $= -r,$ $,S$ are sample standard deviations of the main variable and AI respectively, and r is the sample correlation between two variables. It is presumed that these variables are linearly related and regression line does not pass through the

origin. It is also supposed that the population mean of t_i, μ_t is known. The mean and variance of the regression estimator are

$E(Reg_i) = \mu_x = \mu$ (18)

$var(Reg_i) = \frac{\sigma_x}{mm}(1-\rho^2)$ (19)

After obtaining the value of Reg_i we calculate the value of EEWMA statistic using smoothing constant τ_1 and τ_2 as

EEWMA = +

$(1-\tau_1+\tau_2)\text{EEWMA}_{i-1}$ (20)

Step-2: Calculate the following statistic as

= (21)

Accept the lot if $MReg_i \geq J_a$ and reject the lot if $MReg_i < J_a$. The presented plan has two parameters named as SS mm and AN J_a. We use the value of and EEWMA is zero at $i = 1$. The suggested plan is the generalization of numerous existing plans. When $\rho = 0$, this plan gives the same results as discussed in Section 2.1. If $\tau_2 = 0$, the recommended plan is converted to *Aslam et al.* (2018). If $= 0$ $= 0$,the suggested plan is transformed to Aslam *et al.*, (2015b), if $= 1$ $= 0$, the presented plan is converted to Aslam *et al.* (2017), when $= 1$, $= 0$ and $= 0$, the proposed plan is converted to traditional variable ASP. It is to be noted that for a large value of , EEWMA approximately purse the normal distribution with parameters:EEWMA

$(\mu, \quad (1 \quad))$

The OC function of the proposed plan can be derived as

$= P(MReg_i \geq J_a) = P(\text{EEWMA} + USL)$ (22)

According to Duncan (1986), we have

EEWMA + $(\mu + \quad , (1 \quad) [\quad])$ (23)

After simplification of the above equation, the final form of OC function under the condition of known standard deviation (SD) is given as

$(p) =$

$(Z \quad) \sqrt{[\quad](1 \quad ^2)}$ (24)

Here $\Phi(.)$ is the cumulative distribution function (CDF) and Z_p is the pth percentile of standard normal distribution (SND).

Now we will utilize the following two optimizations to find the parametric values of the initiated plan:

minimize mm (25)

With the condition that

$(AQL) =$

$(Z \quad) \sqrt{[\quad](1 \quad ^2)}$ 1

(26)

and

$$P_a(LQL) = \Phi\left(\left(Z_p - J_a\right)\sqrt{\frac{mm}{\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right](1-\rho^2)}}\right) \le \beta \quad (27)$$

3.2 When population SD is not known

Here, we discuss the working operation of the suggested plan with the presence of AI when population SD is not known. The proceeding of step-1 remains the same as mentioned in section 3.1 for known SD. Later on in step-2, we use sample SD in acceptance statistic instead of σ i.e

$$MReg_i = \frac{USL - \mathrm{EEWMA_i}}{S_x} \quad (28)$$

where $S = \sqrt{\frac{\sum_{i=1}^{m}(x_i - \bar{\mathrm{x}}_\mathrm{i})^2}{m-1}}$

The lot is rejected if $MReg_i < \mathrm{J_a}$ otherwise we accept it.

The OC function of the proposed plan for the unknown case using AI is derived as

$$P_a = P(MReg_i \ge J_a) = P(\mathrm{EEWMA_i} + J_a S_x \le USL) \quad (29)$$

According to Duncan (1986), we have

$$\mathrm{EEWMA_i} + J_a S_x \sim N\left(\mu + J_a E(S_x), Var(\mathrm{EEWMA_i}) + J_a^2 Var(S_x)\right)$$

As we know that $E(S_x) = \sigma c_4$ and $Var(S_x) = (\sigma^2 - \sigma^2 c_4^2)$ where $c_4 = \sqrt{[2/(m-1)]}\Gamma(m/2)/\Gamma[m-1/2]$

So

$$\mathrm{EEWMA_i} + L_a S \sim N\left(\mu + J_a\sigma c_4, (1-\rho^2)\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{\sigma^2}{mm} + J_a^2(\sigma^2 - \sigma^2 c_4^2)\right) \quad (30)$$

Where $r = 1 - \tau_1 + \tau_2$

The final form of OC function is given as

$$P_a(p) = \Phi\left(\left(Z_p - J_a c_4\right)\sqrt{\frac{1}{(1-\rho^2)\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{1}{mm} + J_a^2(1-c_4^2)}}\right) \quad (31)$$

The optimization problem for the case of unknown SD is given as

$minimize\ mm$ (32)

With respect to

$$P_a(LQL) = \Phi\left(\left(Z_p - L_a c_4\right)\sqrt{\frac{1}{(1-\rho^2)\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{1}{mm} + L_a^2(1-c_4^2)}}\right) \le \beta \quad (33)$$

And

$$P_a(AQL) = \Phi\left(\left(Z_p - J_a c_4\right)\sqrt{\frac{1}{(1-\rho^2)\left[\frac{\tau_1{}^2+\tau_2{}^2-2r\tau_1\tau_2}{1-r^2}\right]\frac{1}{mm}+J_a^2(1-c_4^2)}}\right) \geq 1-\alpha \quad (34)$$

The values of parameters mm and $J_a$are placed in Tables 3-6 using different values of $\tau_1, \tau_2$and ρ for various levels of AQL and LQL under the condition of known and unknown SD. The behavior of mm are alike as we have earlier discussed for the case of without AI. However, we notice the fast-declining behavior in mm for known and unknown SD as we increase the value ofρ. For example, when$\tau_1 = 0.3$, $\tau_2 = 0.29, \rho = 0.5$ the value of $mm = 12$ for $AQL = 0.0025$ and $LQL = 0.005$, for $\rho = 0.95$, the value of mm is just 2. Similarly, when $AQL = 0.05, LQL = 0.10, \tau_1 = 0.5$, $\tau_2 = 0.49, \rho = 0.5, mm = 13$, for $\rho = 0.95$ it is just 2 for known SD. For unknown SD, we observe $mm = 182$ when $\rho = 0.5$, $\tau_1 = 0.5$, $\tau_2 = 0.49, AQL = 0.03$ and $LQL = 0.055$ and for $\rho = 0.95$ the value of $mm = 164$ keeping the rest of the values is same

Table 3. The plan parameters of suggested plan when SD is known when $\rho = 0.50$

AQL	LQL	$\tau_1 = 0.1$	$\tau_2 = 0.09$	$\tau_1 = 0.3$	$\tau_2 = 0.29$	$\tau_1 = 0.5$	$\tau_2 = 0.49$
		mm	J_a	mm	J_a	mm	J_a
0.001	0.002	3	2.9576	14	2.9711	36	2.9711
	0.004	2	2.7951	4	2.8219	9	2.8495
	0.006	2	2.7208	2	2.7700	5	2.7659
	0.008	2	2.6455	2	2.7828	4	2.7142
0.0025	0.005	2	2.6748	12	2.6750	31	2.6766
	0.010	2	2.4574	3	2.5561	7	2.5368
	0.015	2	2.4523	2	2.4209	4	2.4498
	0.020	2	2.6597	2	2.4803	3	2.3940
0.03	0.06	2	1.7083	6	1.6966	16	1.6986
	0.09	2	1.5161	3	1.5510	6	1.5783
	0.12	2	1.5862	2	1.5357	4	1.4946
	0.15	2	1.5983	2	1.3724	3	1.4069
0.05	0.10	2	1.4836	5	1.4427	13	1.4385
	0.15	2	1.3165	2	1.2817	5	1.2928
	0.20	2	0.9433	2	1.1740	3	1.2038
	0.25	2	1.1033	2	1.3373	2	1.1383

Table 4. The plan parameters of suggested plan when SD is known when $\rho = 0.950$

AQL	LQL	$\tau_1 = 0.1$	$\tau_2 = 0.09$	$\tau_1 = 0.3$	$\tau_2 = 0.29$	$\tau_1 = 0.5$	$\tau_2 = 0.49$
		mm	J_a	mm	J_a	mm	J_a
0.001	0.002	2	2.9594	2	2.9786	5	2.9686
	0.004	2	2.7407	2	2.8442	2	2.8728
	0.006	2	2.7358	2	2.6854	2	2.7682
	0.008	2	2.7818	2	2.4912	2	2.6336
0.0025	0.005	2	2.6596	2	2.6639	4	2.6766
	0.010	2	2.4198	2	2.5658	2	2.6006
	0.015	2	2.5434	2	2.2787	2	2.3840
	0.020	2	2.2446	2	2.3399	2	2.5311
0.01	0.02	2	2.1096	2	2.1740	3	2.1723
	0.03	2	2.1737	2	2.1561	2	2.1143
	0.04	2	2.0983	2	1.9925	2	2.0675
	0.05	2	2.1783	2	2.0702	2	1.9574
0.05	0.10	2	1.3843	2	1.4268	2	1.4590
	0.15	2	1.4098	2	1.5150	2	1.1968
	0.20	2	1.4551	2	1.1339	2	1.3474
	0.25	2	0.7157	2	1.2959	2	1.2701

Table 5. The plan parameters of suggested plan when SD is unknown and $\rho = 0.50$

AQL	LQL	$\tau_1 = 0.1$	$\tau_2 = 0.09$	$\tau_1 = 0.3$	$\tau_2 = 0.29$	$\tau_1 = 0.5$	$\tau_2 = 0.49$
		mm	J_a	mm	J_a	mm	J_a
0.003	0.009	191	2.5365	194	2.5361	201	2.5360
	0.010	154	2.5151	157	2.5151	163	2.5143
	0.015	77	2.4313	79	2.4302	82	2.4313
	0.020	51	2.3694	52	2.3694	55	2.3716
0.015	0.030	211	2.0104	215	2.0100	227	2.0098
	0.035	132	1.9722	136	1.9726	144	1.9721
	0.040	93	1.9398	96	1.9393	102	1.9398
	0.050	57	1.8849	58	1.8830	62	1.8827
0.03	0.055	162	1.7244	168	1.7246	182	1.7240
	0.057	142	1.7149	148	1.7145	160	1.7140
	0.059	126	1.7058	131	1.7061	141	1.7051
	0.06	119	1.7012	125	1.7019	133	1.7007
	0.09	39	1.5858	40	1.5878	44	1.5850
0.05	0.08	173	1.5123	182	1.5124	200	1.5119
	0.09	103	1.4775	109	1.4773	120	1.4765
	0.10	70	1.4460	74	1.4453	81	1.4453
	0.11	51	1.4176	54	1.4168	60	1.4159

	0.15	22	1.3189	23	1.3210	26	1.3191

4. Advantages of proposed plan

In this section, we will examine the competence of the suggested plan with a various existing plan using with and without auxiliary information for the case of known and unknown SD. To save space, we will consider the case without AI only for $\tau_1 = 0.3, \tau_2 = 0.29$ for known and unknown SD. Similarly, for AI we consider the case only for $\rho = 0.95, \tau_1 = 0.3, \tau_2 = 0.29$ and $\tau_1 = 0.5, \tau_2 = 0.49$ for known SD, and for unknown SD we use $\rho = 0.5, \tau_1 = 0.3,$ $\tau_{2=}0.29$ and $\tau_1 = 0.5, \tau_2 = 0.49$.We tabulated all the values of plan parameters in Tables 7-10. From these Tables 7-10, we observe that the proposed plan is ameliorated than the several existing plans in the form of lesser value of plan parameter for all cases using any combination of AQL and LQL. For example for the case of without AI and known SD, using $AQL = 0.001, LQL = 0.0015, \tau_1 = 0.3, \tau_2 = 0.29$ the value of the proposed plan is 54, for the case Khan *et al.* (2018) it was 86, for Aslam *et al.* (2015b) it was 101 and for a single variable plan, it was 574. Similarly for unknown SD and without AI, when $AQL = 0.05, LQL = 0.08, \tau_1 = 0.3, \tau_2 = 0.29$ the value of suggested plan is 185, for the case Khan *et al.* (2018) the value of plan parameter was 194, for Aslam *et al.* (2015b) it was 198 and for a single variable plan, the value of the plan parameter do not exist. For the case of AI and known SD, using $AQL = 0.0025, LQL = 0.0030, \tau_1 = 0.5, \tau_2 = 0.49$ and $\rho = 0.95$, the parametric value of the proposed plan is 61, for Aslam *et al.* (2018) the value was 82 and for Aslam *et al.* (2017) it was 240. Similarly, for the case of AI using unknown SD, when $AQL = 0.03, LQL = 0.055, \tau_1 = 0.3, \tau_2 = 0.29$ and $\rho = 0.5$, the parametric value of the suggested plan is 168, for Aslam *et al.* (2018) the value was 176 and for Aslam *et al.* (2017) it was 241.

Table 6. The plan parameters of suggested plan when SD is unknown and $\rho = 0.950$

		$\tau_1 = 0.1$	$\tau_2 = 0.09$	$\tau_1 = 0.3$	$\tau_2 = 0.29$	$\tau_1 = 0.5$	$\tau_2 = 0.49$
AQL	LQL	mm	J_a	mm	J_a	mm	J_a
0.003	0.009	191	2.5360	192	2.5363	194	2.5365
	0.010	154	2.5159	156	2.5150	158	2.5153
	0.015	77	2.4315	78	2.4310	79	2.4305
	0.020	51	2.3690	51	2.3693	53	2.3661
0.015	0.030	208	2.0099	209	2.0097	211	2.0101
	0.035	131	1.9727	132	1.9721	133	1.9729
	0.040	93	1.9401	95	1.9395	97	1.9396
	0.050	56	1.8835	57	1.8819	59	1.8841
0.03	0.055	161	1.7246	162	1.7246	164	1.7248
	0.057	142	1.7158	143	1.7156	144	1.7152
	0.059	125	1.7059	126	1.7061	127	1.7055
	0.06	118	1.7009	119	1.7007	120	1.7009
	0.09	38	1.5883	39	1.5894	39	1.5882
0.05	0.08	173	1.512767	174	1.512327	175	1.5123
	0.09	103	1.478186	103	1.477579	105	1.4769
	0.10	69	1.445621	70	1.446434	71	1.4455
	0.11	51	1.417004	51	1.41655	52	1.4171
	0.15	21	1.319319	22	1.317124	22	1.3157

Table 7. Comparison of plan parameter for known SD

		$\tau_1 = 0.3$ $\tau_2 = 0.29$	$\tau = 0.3$	$\tau = 0.3$	$\tau = 1$
p_1	p_2	Proposed EEWMA	Khan *et al.* (2018)	Aslam *et al.* (2015b)	Single variable plan
0.001	0.0015	54	86	101	574
0.001	0.0020	18	30	34	193
0.001	0.0025	10	17	19	108
0.005	0.007	57	93	109	617
0.005	0.010	13	21	25	139
0.005	0.015	5	8	10	54

Table 8. Comparison of plan parameter for unknown SD

		$\lambda_1 = 0.3$	$\tau = 0.3$	$\tau = 0.3$	$\tau = 1$
		$\lambda_2 = 0.29$			
p_1	p_2	Proposed EEWMA	Khan *et al.*(2018)	Aslam e*t al.* (2015b)	Single variable plan
0.03	0.055	171	177	180	268
0.03	0.058	141	146	148	222
0.03	0.060	125	130	132	198
0.05	0.08	185	194	198	-----
0.05	0.10	75	79	80	133
0.05	0.015	23	25	25	44

5. Industrial application

In this section, we will evaluate the competency of the suggested plan using industry- related data of color STN presentations. These colors are formed by amalgamating the color filters and monochrome, where every color pixel is separated by G, B, R sub-pixels. This data set is used by Aslam *et al.* (2012), Aslam *et al.* (2015b). In this study, the quality trait is the layer density of pixel. We have given that $USL = 12500A^0$ and target density$12000A^0$. We presume that$\alpha = 0.05, \beta = 0.10, AQL = 0.05, LQL = 0.11, \tau_1 = 0.3, \tau_2 = 0.29$, then from Table 2 we have $L_a = 1.4154\ and\ m = 55$. The implementation of the proposed plan is as follow

Table 9. Comparison of plan parameter for known SD using AI when $\rho = 0.95$

		$\tau_1 = 0.3$	$\tau = 0.3$	$\tau_1 = 0.5$	$\tau = 0.5$	$\tau = 1$
		$\tau_2 = 0.29$		$\tau_2 = 0.49$		
p_1	p_2	Proposed EEWMA	Aslam *et al.* (2018)	Proposed EEWMA	Aslam *et al.* (2018)	Aslam *et al.* (2017)
0.0025	0.0030	23	43	61	82	240
	0.0035	7	13	18	23	70
	0.0040	4	7	9	12	36

0.01	0.011	61	115	164	220	----
	0.012	17	32	44	59	176
	0.013	8	15	21	28	85
0.05	0.055	36	69	98	130	388
	0.057	19	37	53	68	204
	0.060	10	19	27	35	105

Table 10. Comparison of plan parameter for unknown SD using AI when $\rho = 0.5$

		$\tau_1 = 0.3$	$\tau = 0.3$	$\tau_1 = 0.5$	$\tau = 0.5$	$\tau = 1$
		$\tau_2 = 0.29$		$\tau_2 = 0.49$		
p_1	p_2	Proposed EEWMA	Aslam *et al.* (2018)	Proposed EEWMA	Aslam *et al.* (2018)	Aslam *et al.* (2017)
0.03	0.055	168	176	182	188	241
0.03	0.057	147	153	159	165	213
0.03	0.059	131	138	141	146	188
0.03	0.060	123	128	134	138	179
0.05	0.08	182	192	201	209	----
0.05	0.09	109	115	120	126	171
0.05	0.10	74	78	81	85	118
0.05	0.11	54	57	59	62	87

Step-1: we take a sample of size 55 from the running lot at time $i$and compute$\bar{\bar{Z}}_i = 11715.2\ ans\ S = 49.21$. Now we suppose that $\bar{Z}_{i-1} = W_{i-1} = 1100, \tau_1 = 0.3, \tau_2 = 0.29,$ then $W_i = 11214.65$for the current lot. Therefore, the accepting statistic M_i is calculated as

$$M_i = \frac{12500-11214.65}{49.21} = 26.11$$

Step-2: Accept the lot as $26.11 > 1.1454$

6. Concluding remarks

We presented a memory type acceptance sampling plan using AI for known and unknown SD assuming that the main variable and AI of quality trait follow the bivariate normal distribution. We constructed the various Tables of plan parameters using different values of smoothing constants, different values of ρ for known and unknown SD. We likened the capability of the suggested plan with numerous existing plans and exhibited that the recommended plan has a lesser value of SS for the investigation of the product. We also noticed that results were improved by using the auxiliary information. In addition, for highly correlated data, a smaller sample is needed for the inspection of the product. As the SS becomes large, the time and cost of the inspection of the commodities have increased. So it is suggested to use the proposed plan in the industry for investigating the goods to save cost and time maintaining the producer's and consumer's risk protection

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