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# Modified null space strategy to solve consensus problem

Aman Sharma[1], Naga G. Kurapati[2], Ravi Prasad K. Jagannath[3], Patrice Wira[4], Shyam Lal[5,*], Adapa V. Narasimhadhan[5]

[1]*JSW Jaigarh Port Ltd, JSW Infrastructure Ltd, Jaigad, Ratnagiri, Maharashtra, India*

[2]*Dept. of Electrical and Computer Engineering, University of Massachusetts, Lowell, USA*

[3]*Dept. of Mathematics, National Institute of Technology Goa, Goa, India*

[4]*MIPS Laboratory, Universite de Haute Alsace, France*

[5]*Dept. of Electronics and Communication Engineering, National Institute of Technology Karnataka, India*

[*]*Corresponding author: shyam.mtec@gmail.com*

## Abstract

In the domain of multi robot systems, several applications necessitate agreement of all the individual robots at consensus/rendezvous point. Such an agreement can only be achieved by means of a control strategy. However, presence of obstacles in the navigation-environment makes the achievement of control objective floundering. This paper accentuates the failure of extant null space based control strategy to circumvent rectangular obstacles by means of mathematical proofs and extensive simulation studies. To over-come these short-comings, a modified null space based control strategy is proposed to solve the consensus problem. Proposed control strategy is tested in a complex environment consisting of rectangular and concave obstacles by means of computer simulations. Finally, a qualitative comparative analysis is presented to contrast the differences between conventional null space based strategy and the proposed modified null space strategy.

**Keywords:** Multi robot system; follow wall behaviour; null space control strategy; modified null space control strategy.

## 1. Introduction

The domain of multi robot system has been attracting researchers to develop various algorithms to accomplish flocking, formation keeping, foraging etc. Central idea in all these was to create an adept group of mobile robots, who can proficiently effectuate a predefined objective. Earlier works have addressed different aspects like, formation control (Fax & Murray (2004); Olfati-Saber & Murray (2002); Eren *et al*. (2002); Vidal *et al.* (2003)), flocking (Reynolds (1987); Vicsek *et al*. (1995); Toner & Tu (1998), Memet Kule (2016)), etc. Moreover, earlier oeuvre by Parker (1998) has posited a fault tolerant and adaptive multi robot system (MRS) framework. Behaviour based strategies control the velocity of a mobile robot to accomplish the objective of corresponding behaviour. For instance, Go To Goal behaviour drives a robot from any initial position to a specified location, whereas Obstacle Avoidance behaviour drives a robot away from any intermittent obstacle. Presence of obstacles in the environment make agreement of all the robots at consensus point challenging and thus there is a need of generating appropriate control signals to drive all the robots towards the goal (rendezvous point), while avoiding all the intermittent obstacles. Also, the control strategy should restrain a robot from colliding with other robots in it's neighbourhood. For a single mobile robot, navigating in an environment consisting of circular obstacles, aforementioned behaviours can be combined to yield Null Space Behaviour (NSB) which can guarantee fulfilment of above stated objective, as elucidated in Antonelli *et al*. (2005). NSB framework has also been proposed for multi-robot patrolling using centralised/decentralised architecture by Marino (2004) and flocking in presence of rendezvous point and obstacle by Antonelli *et al*. (2010).

Presented paper theoretically assays the NSB approach, discussed by Antonelli *et al*. (2005), for a single mobile robot in an environment consisting of rectangular obstacle. Further, by means of mathematical reasoning and computer simulations, it will be shown that NSB (proposed by Antonelli *et al*. (2010)) founders to solve the consensus problem in environments containing rectangular and concave shaped obstacles. A non hierarchical projection based null space strategy is also proposed to accomplish the navigation objectives: 1. Flocking of robots in the vicinity of rendezvous point. 2. Avoiding intermittent obstacles. 3. Avoiding collision with neighbouring robot. The generalised concept of adding the control signal vector of higher priority task to a projected vector (which is obtained by projecting a lower priority task control vector on to the null space of higher priority task control vector) is not followed and yet all objectives are achieved. The proposed null space based strategy is additionally equipped with another existing behaviour referred to as Follow Wall behaviour[1] which makes a robot follow the boundary of obstacle.

This paper is organised as follows: Section 2 elucidates navigation objectives in the language of mathematics. Section 3 discusses about NSB and mentions the objective functions associated with Go To Goal and Obstacle Avoidance behaviours. Section 4 mathematically evaluates the performance of NSB strategy for a single mobile robot navigating in an environment containing rectangular obstacle. Section 5 is devoted to the proposed modified NSB strategy to solve consensus problem in an environment consisting rectangular obstacles. Section 6 and 7 tests the proposed modified NSB strategy in an environment consisting of rectangular and concave obstacles by means of computer simulations and finally in section 8 concluding remarks will be made.

## 2. Navigation objective

Consider a multi-robot system (MRS) consisting of $N$ mobile robots and $\mathbf{x} = \left[\mathbf{x}_1^T \; \mathbf{x}_2^T \cdots \mathbf{x}_N^T\right]^T$ be the state vector where $\mathbf{x}_i \in \mathbb{R}^2$ denotes position coordinates of the $i^{th}$ robot. The objective is to drive the state of MRS from an initial state $\mathbf{x}_0$ to a final state $\mathbf{x}_f = [\mathbf{c}^T \; \mathbf{c}^T \cdots \mathbf{c}^T]^T$, under the action of appropriate control signal:

$$\mathbf{u}_i = \dot{\mathbf{x}}_i = k_p(\mathbf{c} - \mathbf{x}_i), \tag{1}$$

where $\mathbf{c} \in \mathbb{R}^2$ denotes the coordinates of consensus point and $k_p \in \mathbb{R}^+$. MRS can thus be modelled as a linear time invariant (LTI) control system:

$$\dot{\mathbf{x}} = k_p\left(\mathbf{A}\mathbf{x} + \mathbf{b}\right), \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & 0 & -1 & 0 \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} \mathbf{c} \\ \mathbf{c} \\ \vdots \\ \mathbf{c} \end{bmatrix} \tag{3}$$

Also note that $\mathbf{x}, \mathbf{b} \in \mathbb{R}^{2N\text{X}1}$ and $\mathbf{A} \in \mathbb{R}^{2N\text{X}2N}$. Above modelled LTI system is stable as all the eigen values of $\mathbf{A}$ are negative ($-1$ in this case) and hence the system stabilises at steady state:

$$\lim_{t\to\infty} \mathbf{x} = \mathbf{x}_f, \tag{4}$$

where t denotes time; in words, MRS converges at the rendezvous point. This analysis quixotically assumes that navigation environment does not contain any obstacle and path of robots can intersect simultaneously in time and space, which is not the case.

The navigation objectives are explicitly stated below:

1. $\mathbf{x}_f = [\mathbf{c}^T \; \mathbf{c}^T \cdots \mathbf{c}^T]^T$
2. $||\mathbf{x}_i - \mathbf{x}_{ob_i}|| > d$, where $\mathbf{x}_{ob_i}$ denote the coordinate of obstacle and $d \in \mathbb{R}^+$ denotes safe distance.
3. $||\mathbf{x}_i(t^*) - \mathbf{x}_j(t^*)|| > d \;\; \forall \; i \neq j$, where $t^*$ denotes any instant of time.

Condition 1 and 3 are antithetical to each other as the former necessitates convergence of all the robots at a single point, whereas the latter entails a minimum safe distance between any two robots. Therefore, objective is achieved when the constituent robots of MRS flock around the consensus point, without colliding with each other and intermittent obstacles.

## 3. Null space behavioural control

In a behaviour based control strategy, global task is decomposed into elementary behaviours (which have to be simultaneously managed), to accomplish a global objective. Challenge in such a situation is to appropriately compose elementary behaviours in a robust framework to simultaneously accomplish the objective of each behaviour, which may be conflicting in nature, to control the motion of a robot. Idea of task priority inverse kinematics, introduced in Maciejewski & Klein (1988); Nakamura *et al*. (1987) for ground fixed redundant manipulators was used in Bishop (2003); Bishop & Stilwell (2001) to assign relative priority to behaviours, but with reference to Chiaverini (1997), singularity robust

[1]This behaviour has been taken up from video lectures on control of mobile robots course offered by Magnus Egerstedt on coursera.org; the video can also be viewed on https://www.youtube.com/watch?v=_HBFm4ky0hw

algorithms need to be devised to manage conflicting behaviours. In the proceeding paragraph, mathematical framework of NSB (Antonelli *et al*. (2005); Antonelli *et al*. (2010)) is discussed.

Elementary behaviours are represented by means of a task function:

$$\sigma_i = f\left(\mathbf{x}_i\right) \tag{5}$$

where $\sigma_i \in \mathbb{R}^m$ denotes the task to be controlled for $i^{th}$ robot. Equation 5 is called direct kinematics equation which is used to compute the value of task function based on given parameters. Considering neighbouring robots static and $\sigma_i$ to be differentiable,

$$\dot{\sigma}_i = \mathbf{J}_i(\mathbf{x}_i)\dot{\mathbf{x}}_i, \tag{6}$$

where $\mathbf{J}_i \in \mathbb{R}^{m\text{X}2}$ denotes configuration dependent task Jacobian matrix for $i^{th}$ robot. By inverting the locally linear mapping in Equation 6, motion references $\mathbf{x}_{d_i}(t)$ for $i^{th}$ robot starting from desired value $\sigma_{d_i}(t)$ can be obtained

$$\dot{\mathbf{x}}_{d_i} = \mathbf{v}_{d_i} = \mathbf{J}_i^{+}\dot{\sigma}_{d_i}, \tag{7}$$

where $\mathbf{J}_i^{+} = \mathbf{J}_i^{T}\left(\mathbf{J}_i\mathbf{J}_i^{T}\right)^{-1}$ denotes Moore–Penrose pseudo-inverse of $\mathbf{J}_i$. Discrete time integration of robot's reference velocity results in numerical drift of reconstructed position of robot and hence Closed Loop Inverse Kinematics version of the algorithm is used to counteract the undesired numerical drift to yield

$$\dot{\mathbf{x}}_{d_i} = \mathbf{v}_{d_i} = \mathbf{J}_i^{+}\left(\dot{\sigma}_{d_i} + \mathbf{\Lambda}\tilde{\sigma}_i\right), \tag{8}$$

where $\mathbf{\Lambda}$ is a positive definite matrix and

$$\tilde{\sigma}_i = \sigma_{d_i} - \sigma_i. \tag{9}$$

For multiple behaviours, a priority index is assigned to each behaviour denoted by $q$ ($q = 1$ would mean highest priority task); control signal associated with $q^{th}$ priority task for $i^{th}$ robot is given by

$$\mathbf{v}_i^{(q)} = \mathbf{J}_i^{+(q)}\left(\dot{\sigma}_{d_i}^{(q)} + \mathbf{\Lambda}^{(q)}\tilde{\sigma}_i^{(q)}\right). \tag{10}$$

With reference to Chiaverini (1997) and Antonelli *et al*. (2005) , for two behaviours

$$\mathbf{v}_{d_i} = \mathbf{v}_i^{(1)} + \left(\mathbf{I} - \mathbf{J}_i^{+(1)}\mathbf{J}_i^{(1)}\right)\cdot\mathbf{v}_i^{(2)} \tag{11}$$

Therefore motion of $i^{th}$ robot is governed in accordance to Equation 11.

Now task function $\sigma_i^{(q)}$ for *Obstacle Avoidance* behaviour, with priority index 1, and *Go To Goal* behaviour, with priority index 2, will be explicitly defined.

**Obstacle Avoidance behaviour:** Let $\sigma_i^{(1)}$ represent the task function, for $i^{th}$ robot, to maintain a safe distance $d$ from obstacle :

$$\sigma_i^{(1)} = ||\mathbf{x}_i - \mathbf{x}_{ob}|| \text{ and } \sigma_{d_i}^{(1)} = d, \tag{12}$$

where $\mathbf{x}_{ob} \in \mathbb{R}^2$ denotes coordinates of obstacle. The associated Jacobian and it's Penrose pseudo inverse is given by

$$\mathbf{J}_i^{(1)} = \left[\frac{\mathbf{x}_i - \mathbf{x}_{ob}}{||\mathbf{x}_i - \mathbf{x}_{ob}||}\right]^T = \hat{\mathbf{r}}_i^T \tag{13}$$

$$\mathbf{J}_i^{+(1)} = \hat{\mathbf{r}}_i. \tag{14}$$

With reference to Equation 8, 12, 13, 14

$$\mathbf{v}_i^{(1)} = \mathbf{J}_i^{+(1)}\Lambda_{ob}\left(d - ||\mathbf{x}_i - \mathbf{x}_{ob}||\right) = \mathbf{u}_{OA_i}, \tag{15}$$

where $\Lambda_{ob} \in \mathbb{R}^{+}$. The null space projector matrix associated with $\mathbf{J}_i^{(1)}$ is given by

$$\mathbf{N}_{1_i} = \mathbf{I} - \hat{\mathbf{r}}_i \cdot \hat{\mathbf{r}}_i^T \tag{16}$$

**Go To Goal behaviour:** Let $\sigma_i^{(2)}$ represent the task function, for $i^{th}$ robot, to reach the goal:

$$\sigma_i^{(2)} = \mathbf{x}_i \text{ and } \sigma_{d_i}^{(2)} = \mathbf{x}_g, \tag{17}$$

where $\mathbf{x}_g \in \mathbb{R}^2$ denotes coordinates of the goal. The associated Jacobian and it's Penrose pseudo inverse is given by

$$\mathbf{J}_i^{(2)} = \mathbf{I} = \mathbf{J}_i^{+(2)} \tag{18}$$

With reference to Equation 8, 17 and 18

$$\mathbf{v}_i^{(2)} = \mathbf{\Lambda}_{GTG}\cdot(\mathbf{x}_g - \mathbf{x}_i) = \mathbf{u}_{GTG_i}, \tag{19}$$

where $\mathbf{\Lambda}_{GTG}$ is a positive definite matrix. Velocity of $i^{th}$ robot would thus be:

$$\mathbf{v}_{d_i} = \mathbf{u}_{OA_i} + \mathbf{N}_{1_i}\cdot\mathbf{u}_{GTG_i} \tag{20}$$

## 4. Navigation of single robot in presence of rectangular obstacle

In this section, conventional NSB strategy will be mathematically examined for a single mobile robot navigating in an environment consisting a rectangular obstacle.

**Proposition 1** *For a given initial position of the robot,* $\mathbf{p}_0 = [x_0\ y_0]^T$*, and goal* $\mathbf{p}_g = [x_g\ y_g]^T$ *in an*

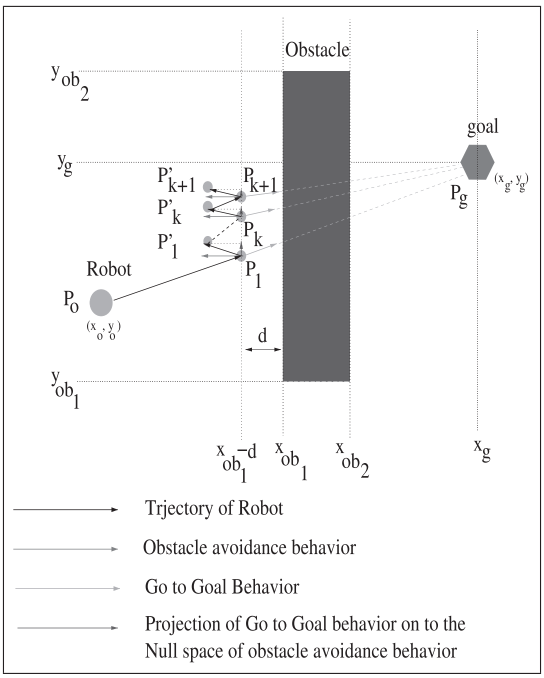


**Fig. 1.** Modelling the trajectory of encounter point

*environment, if the rectangular obstacle lies such that line segment joining* $\mathbf{p}_0$ *and* $\mathbf{p}_g$ *intersects any two parallel edges of the rectangle (Figure 1), then combining Go To Goal and Obstacle Avoidance behaviours in NSB framework results in exhibition of perpetual to and fro motion by the robot, along a line segment, thereby failing to reach the goal.*

**Proof 1** *Let* $\mathbf{p}_1$ *be the point of first encounter as shown in Figure 1, where direction of velocity vector of robot changes for the first time due to switching of behaviour in accordance to Figure 2. The state NSB, in Figure 2, refers to combining Go to Goal and Obstacle Avoidance behaviours in NSB framework to control the motion of robot. Therefore*

$$\mathbf{p}_1 = [x_1\ y_1]^T = \begin{bmatrix} x_{ob_1} - d \\ y_g + m_1(x_{ob_1} - d - x_g) \end{bmatrix}, \quad (21)$$

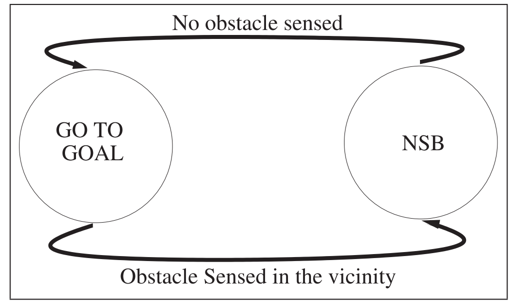


**Fig. 2.** Hybrid system to accomplish navigation objective for single mobile robot.

*where*

$$m_1 = \frac{y_g - y_0}{x_g - x_0},$$

$d$ *represents desired safe distance to be maintained from the obstacle and* $x = x_{ob_1}$ *represents abscissa of considered edge of the obstacle. Let* $\Delta t_1$ *be the time for which robot's dynamics are governed by NSB and hence it reaches a point*

$$\mathbf{p}_1' = \mathbf{p}_1 + \mathbf{v}_1 \Delta t_1, \quad (22)$$

*where* $\mathbf{v}_1$ *is velocity of the robot during interval* $\Delta t_1$

$$\mathbf{v}_1 = \begin{bmatrix} -\beta & \alpha(y_g - y_1) \end{bmatrix}^T \text{ and}$$

$$\begin{cases} \beta = -k_b(x_{ob_1} - (d - x_{ob_1})) \\ k_b \in \mathbb{R}^+ \\ \alpha \in \mathbb{R}^+ \end{cases}.$$

*Velocity due to* Obstacle Avoidance *behaviour will be always along negative direction of x-axis with a constant magnitude:* $[-\beta \quad 0]^T$ *at all encounter points and velocity due to* Go To Goal *behaviour, projected along null space of* Obstacle Avoidance *behaviour will always be along y-axis.* $\mathbf{p}_1'$ *lies at a distance greater than* $d$ *from the obstacle and hence dynamics of robot will now be governed, for an interval* $\Delta t_2$*, by* Go To Goal *behaviour till it reaches second encounter point*

$$\mathbf{p}_2 = \mathbf{p}_1' + \mathbf{v}_1' \Delta t_2, \quad (23)$$

*where*

$$\mathbf{v}_1' = \alpha \begin{bmatrix} x_g - x_1' & y_g - y_1' \end{bmatrix}^T$$

*Both* $\Delta t_1$ *and* $\Delta t_2$ *are small and hence each translation of the robot from* $\mathbf{p}_k$ *to* $\mathbf{p}_k'$ *can be assumed to occur in* $\Delta t_1$ *and similarly each translation from* $\mathbf{p}_k'$ *to* $\mathbf{p}_{k+1}$ *can* *be assumed to occur in $\Delta t_2$. Since both $\Delta t_1$ and $\Delta t_2$ are comparable, they can be assumed to be approximately equal to each other*

$$\Delta t_1 = \Delta t_2 = \Delta t.$$

*Based on the analysis so far, coordinates of $k^{th}$ encounter point can be computed if the initial position of the robot is known. Let the current position of robot be $\mathbf{p}_k = [x_k \quad y_k]^T$ therefore due to NSB the robot reaches*

$$\mathbf{p}'_k = [x'_k \;\; y'_k]^T = \begin{bmatrix} x_{ob_1} - d - \beta\Delta t \\ y_k + (y_g - y_k)\alpha\Delta t \end{bmatrix}. \quad (24)$$

*Now, $\mathbf{p}_{k+1}$ can be written as a function of $\mathbf{p}_k$*

$$\begin{aligned} \mathbf{p}_{k+1} &= [x_{k+1} \;\; y_{k+1}]^T \\ &= \begin{bmatrix} x_{ob_1} - d \\ y_g + m'_k(x_{ob_1} - d - x_g) \end{bmatrix}, \end{aligned} \quad (25)$$

*where*

$$m'_k = \frac{y_g - y'_k}{x_g - x'_k} \quad (26)$$

*Using Equation 24,25,26,*

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_{ob_1} - d \\ ay_k + b \end{bmatrix}, \quad (27)$$

*where*

$$\begin{aligned} a &= \tfrac{\alpha\Delta t - 1}{M} \\ b &= y_g\left(\tfrac{M + 1 - \alpha\Delta t}{M}\right) \\ M &= x_g - x_{ob_1} + d - \beta\Delta t \end{aligned} \quad (28)$$

*With reference to Equation 28, $|b| < H < \infty$ as $M,\ y_g,\ \alpha,\ \Delta t$ are strictly bounded. Also*

$$x_g - x_{ob_1} + d + 1 > (\alpha + \beta)\Delta t \quad (29)$$

*should be true as $\Delta t$ is very very small and $\alpha$, $\beta$ are also small proportional gains. Choosing a very large value of $\beta$ will drive the robot too far from the goal while avoiding obstacles whereas a very high value of $\alpha$ may lead a robot extremely close to the obstacle and in worst case may also result in collision with the obstacle. Choosing large $\Delta t$ will make the robot extremely vulnerable to collision. Inequality depicted by Equation 29 must be satisfied for the proposition to hold good and it is indeed legitimate, by virtue of above arguments, to choose $\alpha,\ \beta,\ \Delta t$ such that above stated inequality is satisfied. The fulfilment of Equation 29 points to $|a| < 1$, therefore the system modelled by Equation 27 satisfies BIBO stability criterion and thus*

$$\left|\lim_{k\to\infty} y_k\right| < H < \infty. \quad (30)$$

*and*

$$\lim_{k\to\infty} y_k = y^*. \quad (31)$$

*Computing steady state of $y_k$, using Equation 27:*

$$\lim_{k\to\infty} y_{k+1} = a \lim_{k\to\infty} y_k + \lim_{k\to\infty} b. \quad (32)$$

*Using Equation 31, Equation 32 reduces to*

$$y^* = \frac{b}{1-a} = y_g \quad (33)$$

*thus,*

$$\lim_{k\to\infty} \mathbf{p}_k = \begin{bmatrix} x_{ob_1} - d \\ y_g \end{bmatrix} \quad (34)$$

*Note that $\lim_{k\to\infty} \mathbf{p}_k$ will be referred to as point of complete conflict (PCC) in the subsequent discussion. Now, $\lim_{k\to\infty} \mathbf{p}'_k$ will be computed*

$$\mathbf{p}'_{k+1} = [x'_{k+1} \;\; y'_{k+1}]^T = \begin{bmatrix} x_{k+1} - \beta\Delta t \\ y_{k+1} + \alpha(y_g - y_{k+1})\Delta t \end{bmatrix}; \quad (35)$$

*using Equation 34:*

$$\lim_{k\to\infty} \mathbf{p}'_{k+1} = \begin{bmatrix} x_{ob_1} - d - \beta\Delta t \\ y_g \end{bmatrix} \quad (36)$$

*From Equation 34 and 36, it can be concluded that robot exhibits perpetual to and fro motion along a line segment, whose endpoints are given by Equation 34 and 36. A subtlety: it may so happen, according to chosen parameters, that $y_k$ or $y'_k$ becomes greater than $y_g$ for some value of $k$ but as $k \to \infty$ they finally converge in accordance to Equation 34 and 36.*

**Follow Wall behaviour:** This section illustrates the Follow Wall behaviour[2], which guides a robot to escape the PCC and reach the goal in presence of rectangular obstacles.

[2]This behaviour has been taken up from video lectures on control of mobile robots course offered by Magnus Egerstedt on coursera.org; the video can also be viewed on https://www.youtube.com/watch?v=_HBFm4ky0hw

Trajectory of the robot, controlled in accordance to below mentioned modes is shown in Figure 3:

1. **No obstacle sensed :** Robot heads towards the goal in accordance to *Go To Goal* behaviour.
2. **Obstacle sensed :** Robot follows boundary of the obstacle till certain conditions are satisfied.

Illustrated trajectory can be obtained by generating appropriate control signal

$$\mathbf{u}_{FW} = k_a \mathcal{R}(\pm\pi/2) \cdot \mathbf{u}_{OA}, \tag{37}$$

where $k_a \in \mathbb{R}^+$ and

$$\mathcal{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

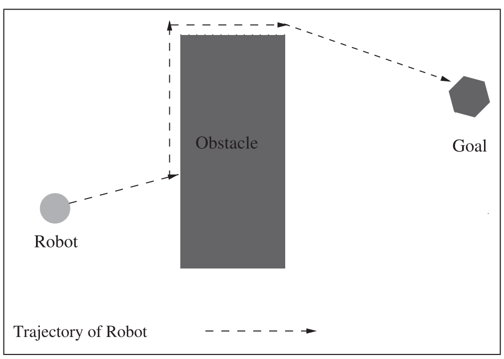


**Fig. 3.** Illustration of follow wall behaviour

There are two ways of following boundary or wall of obstacle i.e. clockwise or counter clockwise with respect to *Obstacle Avoidance* behaviour; direction of following wall can be decided by using a simple inner product test

$$\mathbf{u}_{FW} = \begin{cases} \mathbf{u}_{FW}^{C} \text{ if } \mathbf{u}_{FW}^{C} \cdot \mathbf{u}_{GTG} > 0 \\ \mathbf{u}_{FW}^{CC} \text{ if } \mathbf{u}_{FW}^{CC} \cdot \mathbf{u}_{GTG} > 0 \end{cases}, \tag{38}$$

where $\mathbf{u}_{FW}^{C}$ and $\mathbf{u}_{FW}^{CC}$ denote clockwise and counter clockwise *Follow Wall* behaviour

$$\begin{aligned} \mathbf{u}_{FW}^{C} &= k_a \mathcal{R}(-\pi/2) \cdot \mathbf{u}_{OA} \\ \mathbf{u}_{FW}^{CC} &= k_a \mathcal{R}(+\pi/2) \cdot \mathbf{u}_{OA}. \end{aligned} \tag{39}$$

A robot can stop following walls and switch to *Go To Goal* behaviour if

$$\begin{aligned} ||\mathbf{p} - \mathbf{p}_g|| < \chi(\tau) \text{ and} \\ \mathbf{u}_{OA} \cdot \mathbf{u}_{GTG} > 0, \end{aligned} \tag{40}$$

where $\chi(\tau)$ denotes the distance of robot from the goal at the instant it starts following the wall. Aforementioned conditions are quite intuitive: first condition basically takes into consideration the progress that a robot has made towards goal from the time instant it started following walls; second condition accounts for the fact that if the robot goes towards the goal, its path will not be obstructed by any obstacle.

Both aforementioned conditions have to be satisfied for the robot to stop following the boundary of obstacle.

## 5. Modified null space based control strategy

This section is a denouement of all the formulations developed so far in this paper. Necessary condition for the convergence of all the robots of MRS, navigating in an environment consisting of rectangular obstacle, is stated below as an extension of proposition 1:

***A robot may reach the neighbourhood of rendezvous point if there exists atleast one point $\hat{\mathrm{x}}$ in the trajectory of the robot such that line segment joining $\hat{\mathrm{x}}$ and the coordinates of rendezvous point $\mathrm{x}_g$ doesn't intersect any two parallel edges of the rectangle.***

This will become lucid after the behaviour of avoiding neighbouring robots is introduced which indeed drives a robot away from the vertex of rectangular obstacle, in some situations, such that line joining the position of robot to the rendezvous point no more intersects parallel edges of the rectangle and hence the robot is able to find it's way to the rendezvous point. In this section, a non- hierarchical projection based null space strategy is proposed to enable a robot to reach the neighbourhood of rendezvous point. In proceeding analysis it is assumed that a robot is able to distinguish between an obstacle and a robot lying in it's range of sensing. Each robot in MRS can operate in either of the four different modes described below to accomplish navigation objective:

1. **No obstacle and no other robot sensed:** Robot heads towards the goal and its dynamics are purely based on *Go To Goal* behaviour:

$$\mathbf{v}_{d_i} = \mathbf{u}_{GTG_i}.$$

The mode of operation can switch to any of the other three modes depending on what is sensed by the robot.

2. **Obstacle sensed and no other robot sensed:** In this situation, robot's dynamics are based on pure *Follow Wall* behaviour and direction to follow the wall is governed by:

$$\mathbf{v}_{d_i} = \begin{cases} \mathbf{u}_{FW_i}^{C} \text{ if } \mathbf{u}_{FW_i}^{C} \cdot \mathbf{u}_{GTG_i} > 0 \\ \mathbf{u}_{FW_i}^{CC} \text{ if } \mathbf{u}_{FW_i}^{CC} \cdot \mathbf{u}_{GTG_i} > 0 \end{cases}.$$

Note that while following the wall, mode of operation can switch to mode 3 if a robot senses other robot in it's neighbourhood but switching to mode 1 would require Equation 40 to be satisfied.

3. **Obstacle and other robot sensed:** In this situation, robot follows wall of the obstacle while avoiding collision with the other robot. Let $\mathbf{u}_{RA_{ij}}$ be the behaviour which controls the motion of $i^{th}$ robot so as to prevent it's collision with $j^{th}$ robot, if it lies at a distance less than safe distance. The $j^{th}$ robot can be considered as an obstacle and using *Obstacle Avoidance* behaviour can help the $i^{th}$ robot to avoid $j^{th}$ robot located at $\mathbf{x}_j$. It is assumed that $i^{th}$ robot avoids nearest neighbour whose position in denoted by $\mathbf{x}_j$ in this section. The dynamics are governed by

$$\mathbf{v}_{d_i} = \mathbf{u}_{FW_i} + \mathbf{N}_{OA_i} \cdot \mathbf{u}_{RA_{ij}},$$

where $\mathbf{N}_{OA_i}$ is the Null space associated with Obstacle Avoidance behaviour. Also note that once robot starts following the boundary of obstacle, it will continue to do so unless conditions mentioned in Equation 40 are satisfied. In other words, mode of operation can switch to mode 2, if no robot is sensed in it's neighbourhood but to switch to mode 1, Equation 40 must be satisfied.

4. **No obstacle sensed but other robot sensed:** In this situation, conventional blending of *Go To Goal* and *Obstacle Avoidance* behaviours in NSB framework is employed and thus dynamics is governed by:

$$\mathbf{v}_{d_i} = \mathbf{u}_{RA_{ij}} + \mathbf{N}_{ij} \cdot \mathbf{u}_{GTG_i},$$

where $\mathbf{N}_{ij}$ is the null space projector matrix associated with Jacobian of the task function of obstacle avoidance behaviour, with position of obstacle as position of closest neighbouring robot. The robot can switch to any of the other three modes depending on what the robot senses.

Primary reason for calling the proposed control strategy as modified Null space based control strategy is embodied in mode 3, wherein lower priority task control signal, i.e. avoiding neighbour, is projected on to the null space of Jacobian of the task function associated with *Obstacle Avoidance* behaviour and not *Follow Wall* behaviour. This basically speeds or slows following of the wall of obstacle by the robot. There is no hierarchy observed in the final control signal obtained due to null space projection in mode 3 as compared to Antonelli *et al*. (2005); Antonelli *et al*. (2010) and hence it would be legitimate to call this strategy as modified NSB approach. One can also observe that three conflicting behaviours are simultaneously handled in this strategy without following the conventional hierarchical based projection.

## 6. Simulation results and discussion

This section is a compendium of obtained simulation results, pertaining to modified null space based strategy, performed using MASON library (Luke *et al*. (2004, May); Luke *et al*. (2003)). This library has also been used previously to simulate control algorithms pertaining to MRS (Hrolenok *et al*. (2010); Luke *et al*. (2005); Panait & Luke (2004, January), & Mohammad, S. *et al*. (2014)). Snapshots of simulations have been taken up and presented in the section and videos can be found on https://sites.google.com/site/nitgoaeeedepartmentprojects/home.

A 100 X 100 grid is considered, where robots depicted by small gray squares are initialised at known position coordinates; the objective is to flock in the neighbourhood of consensus point depicted by a violet circle. General parameters common to all simulations are portrayed in Table 1.

**Table 1.** General simulation parameters

| | |
|---|---|
| Sampling time | 0.01 units |
| Safe distance | 2 units |
| Grid size | 100 X 100 |
| Robot | Gray |
| Obstacle | Light green |
| Rendezvous point | Violet |

### Observations

1. The trajectory of a single mobile robot, initialised at $[50 \;\; 80]^T$, heading towards the goal, $[40 \;\; 20]^T$, is simulated in an environment containing square shaped obstacle. Snapshots of obtained results are presented in Figure 4. Dynamics of the robot are governed by the conventional blending of Go To Goal and Obstacle Avoidance behaviours in null space based framework, as presented by Antonelli *et al*. (2005) (hybrid system depicted in Figure 2). It is observed that the robot gets struck and perpetually oscillates in the neighbourhood of point of complete conflict, thereby unable to reach the goal.
2. A multi robot system consisting of 10 randomly initialised robots is simulated in an environment containing a square shaped obstacle. The aim of all the robots is to reach the rendezvous point $[40 \;\; 10]^T$. The motion of each individual robot is governed by combining Go To Goal, Obstacle Avoidance and the behaviour associated with avoiding nearest

neighbouring robot in NSB framework Antonelli *et al.*. Snapshots of obtained results are presented in Figure 5. It is observed that four out of ten robots reach the goal without colliding with each other or obstacle and remaining six get struck (but they do not collide with their neighbours or obstacle). Also, out of the total four robots reaching the goal, one robot takes more time than other three.

3. Follow Wall behaviour is simulated for a single mobile robot, initialised at $[42\ 80]^T$, heading towards the goal - $[40\ 20]^T$ in an environment containing square shaped obstacle. Snapshots of obtained result are presented in Figure 6. It is observed that the robot on reaching close to the obstacle, follows it's boundary till it has a clear shot at the goal.

4. On simulating a MRS consisting of ten randomly initialised mobile robots heading towards rendezvous point $[40\ \ 10]^T$ using proposed control strategy described in section 5. It is observed that all the robots successfully reach the rendezvous point, while avoiding collisions among themselves or with square shaped obstacle. Snapshots of simulation are presented in Figure 7.

5. A MRS consisting of four robots heading towards the rendezvous point $[70\ \ 40]^T$ is simulated, using the proposed modified NSB strategy, in an environment containing a concave obstacle. Obtained results are portrayed in Figure 8. It is observed that all the robots successfully reach the rendezvous point without colliding with each other or the concave obstacle.

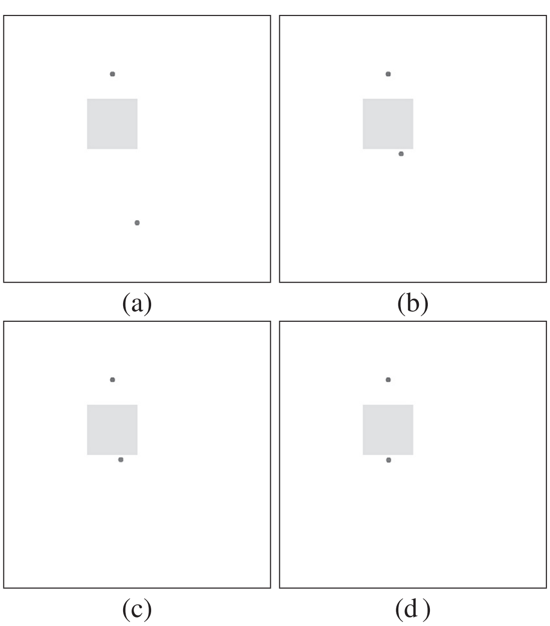


**Fig. 4.** Failure of single mobile robot to reach the goal in presence of a square shaped obstacle: (a), (b), (c), (d) depict the positions of the robot during simulation

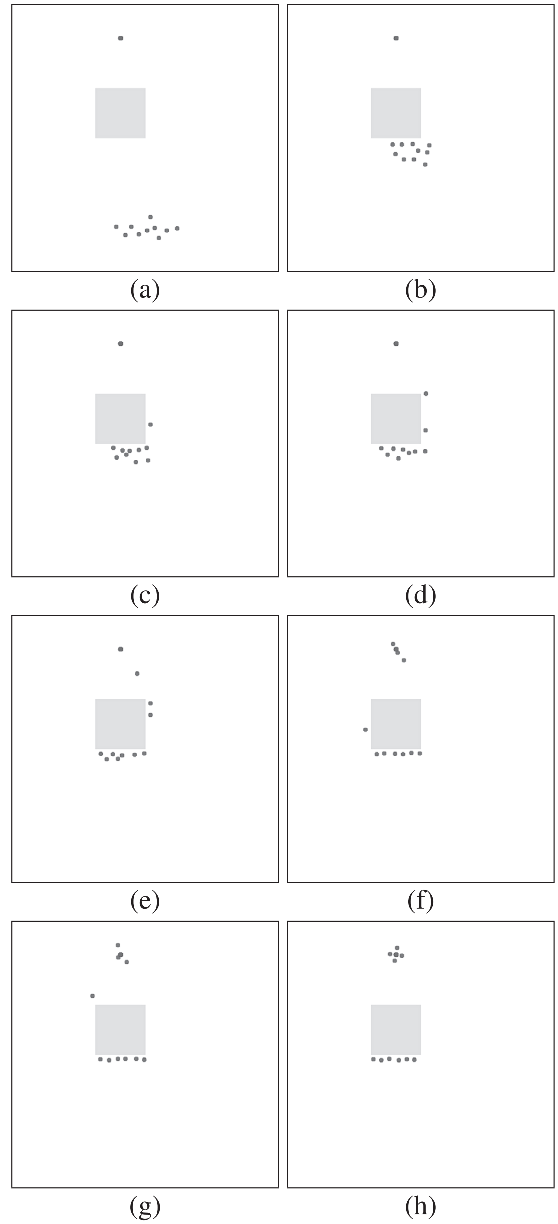


**Fig. 5.** Failure of MRS to reach the rendezvous point in presence of square shaped obstacle: (a), (b), (c), (d), (e), (f), (g), (h) depict the positions of the robots during simulation

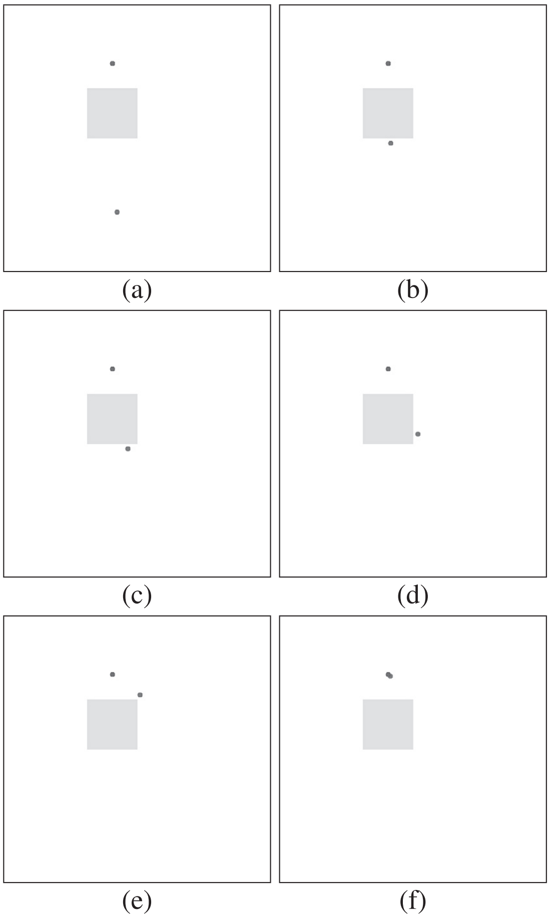


**Fig. 6.** Simulation of follow wall behaviour, for single mobile robot, to avoid the square shaped obstacle: (a), (b), (c), (d), (e), (f) depict the positions of the robot during simulation

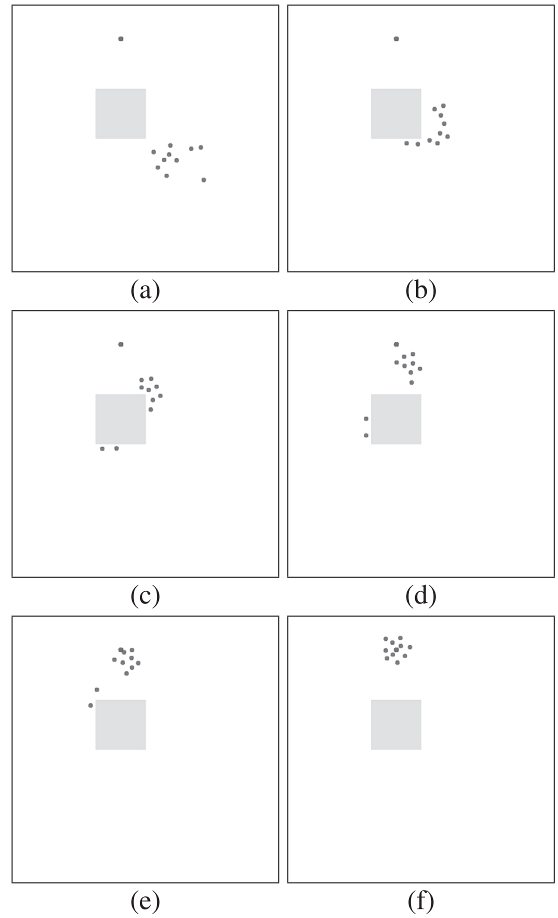


**Fig. 7.** Simulation of the proposed control strategy to reach rendezvous point in presence of square shaped obstacle: (a), (b), (c), (d), (e), (f) depict the positions of the robots constituting the MRS during simulation

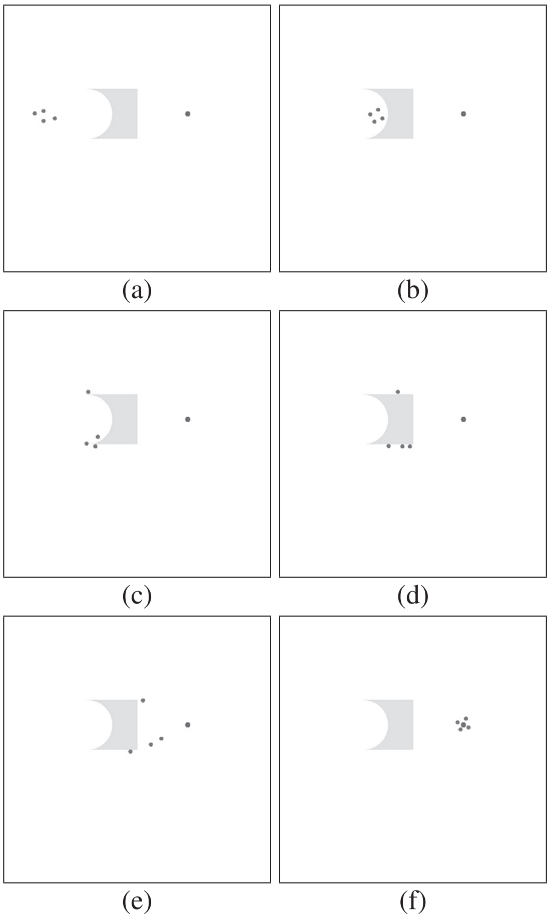

**Fig. 8.** Navigation of multiple robots, in an environment containing concave obstacle, based on proposed control strategy: (a), (b), (c), (d), (e) and (f) depict the positions of the robots constituting the MRS during simulation

## Qualitative evaluation of results

1. With reference to Figure 4, it is very evident from initial position of the robot and coordinates of the goal that line segment joining these two points would definitely intersect two horizontal edges of the obstacle and thus combining *Go To Goal* and *Obstacle Avoidance* behaviours in NSB framework, with reference to **Proposition 1**, would result in perpetual exhibition of to and fro motion by the robot along a line segment in the neighbourhood of point of complete conflict. Observed results depicted in Figure 4 are in complete agreement to presented proposition.

2. In case of multiple robots, apart from reaching the goal and avoiding intermittent obstacle, a robot has to avoid collisions with it's neighbours and thus path of a robot, heading towards the rendezvous point, is not a straight line even in the absence of obstacles if it's neighbour is close to it. It may so happen that few of the robot's, near vertex of the square, may escape the obstacle in such a way that they no more abide to the conditions mentioned in **Proposition 1** and hence find a path to the goal. In the simulation presented in Figure 5, three robots very quickly avoid the vertex of the obstacle to prevent collision with a neighbouring robot in it's vicinity but one robot avoids vertex of the obstacle after some delay; but majority of robots get struck and thereby do not reach the goal.

3. With reference to **Proposition 1**, a single mobile robot cannot reach the goal, if line segment joining it's initial position and coordinates of the goal intersect any two parallel sides of the square. To avoid the exhibition of perpetual to and fro motion along a line segment, a robot follows boundary of the obstacle till it has a clear shot at the goal and has made enough progress towards it. In this way a robot can clearly pass through neighbourhood of point of complete conflict and reach the goal as shown in Figure 6. The observed results completely abide to theoretical formulations.

4. The proposed control strategy accomplishes all the navigation objectives i.e. reaching the goal, while avoiding collision with obstacles and neighbouring robots. Dynamics of each robot of MRS is controlled using a hybrid system consisting of four states or modes which are described in detail in section 5. The observed results, presented in Figure 7 and Figure 8 testify the validity of the proposed control strategy in environments containing square and concave obstacles.

## 7. Comparative analysis

In this section, simulation results of control strategy proposed in section 5 is juxtaposed with the one proposed in Antonelli *et al*. (2010). To qualitatively compare the control strategy, simulation parameters depicted in Table 1 are used with 10 robots initialised at known coordinates:

$$[06\ 34]^T \quad [12\ 36]^T \quad [14\ 38]^T \quad [46\ 06]^T \quad [48\ 08]^T$$

$$[50\ 04]^T \quad [15\ 70]^T \quad [15\ 72]^T \quad [10\ 74]^T \quad [10\ 76]^T$$

heading towards rendezvous point $[90\ \ 65]^T$ with two circular, one square and one concave shaped obstacle present in the navigation environment. Snapshots of obtained results are portrayed in Figure 9 and Figure 10. It is observed from obtained results that for both the control strategies, mobile robots do not collide among themselves or with any of the obstacle. The robots successfully avoid circular obstacles in both the simulations but main difference between the two control strategies is observed when robot encounters a square or concave shaped obstacle. It is observed that six robots, controlled using strategy developed in Antonelli, G (2010), get struck and are unable to reach near rendezvous point but proposed control strategy ensures each and every robot of MRS to flock in the neighbourhood of rendezvous point at steady state. Salient features of the proposed control strategy are summarised in Table 2.

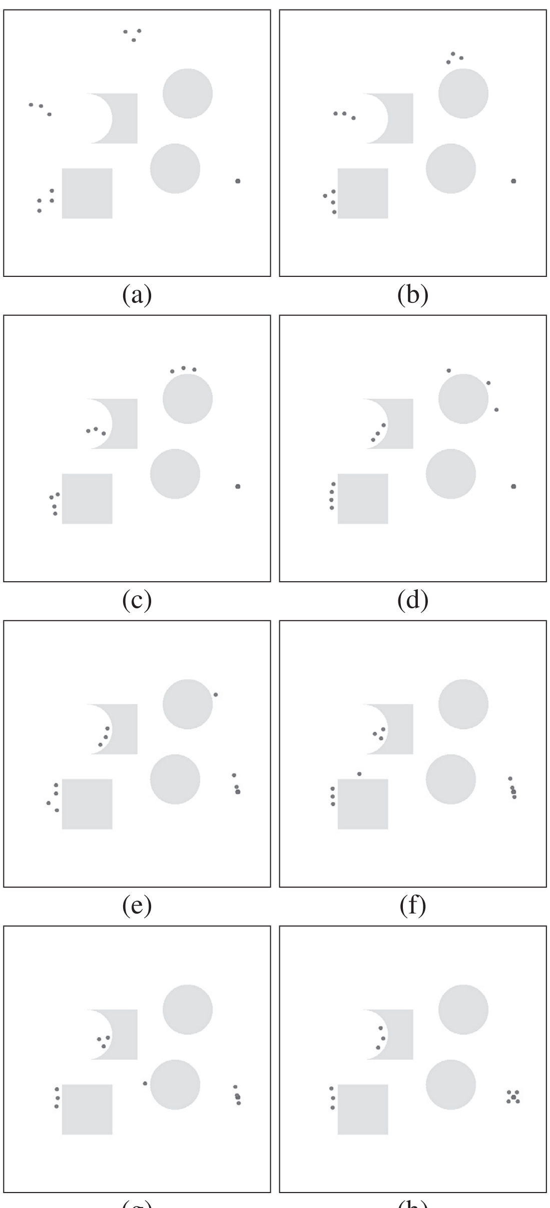


**Fig. 9.** Navigation of multiple robots based on control strategy developed by Antonelli, G., *et al*. (2010): (a), (b), (c), (d), (e), (f), (g) and (h) depict the positions of the robots constituting the MRS during simulation

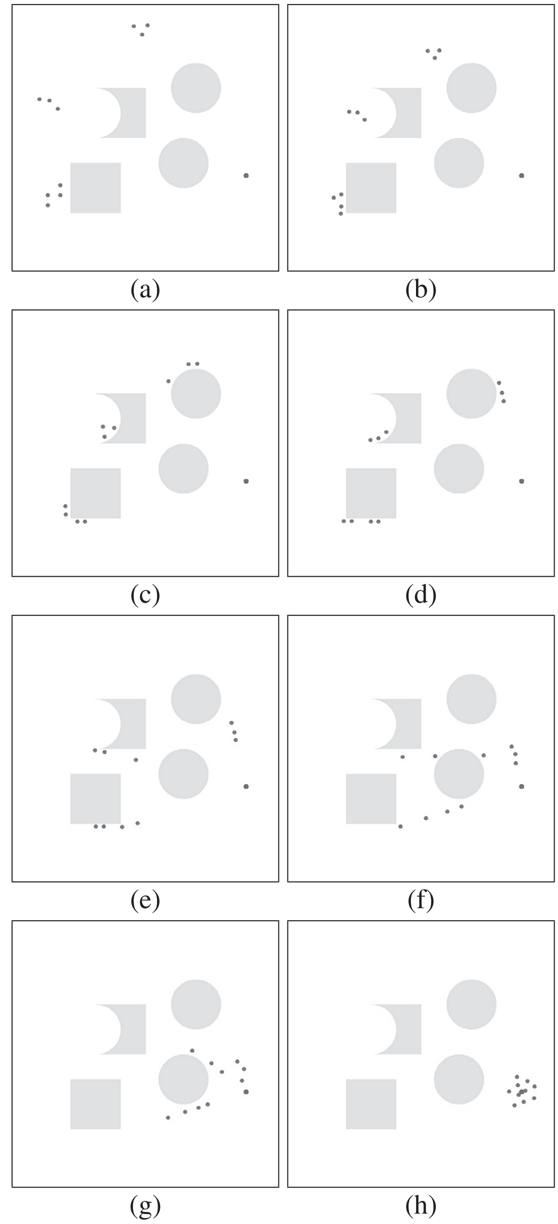


**Fig. 10.** Navigation of multiple robots based on proposed modified control strategy developed in this paper: (a), (b), (c), (d), (e), (f), (g) and (h) depict the positions of the robots constituting the MRS during simulation

**Table 2.** Salient features of the proposed control strategy in comparison to existing

| **Control strategy by Antonelli, G., *et al*. (2010)** | **Modified NSB Control Strategy** |
|---|---|
| No collision with neighbouring robots | No collision with neighbouring robots |
| No collision with any obstacle | No collision with any obstacle |
| Circumvents circular obstacles | Circumvents circular obstacles |
| No guarantee of circumventing-rectangular obstacles | Guaranteed avoidance of-rectangular obstacles |
| No guarantee of outmanoeuvring-concave obstacles | Guaranteed avoidance of-concave obstacles |

## 8. Conclusion

Proposed work has mathematically postulated the plausible failure of conventional NSB, for a single mobile robot, to circumvent a rectangular obstacle, thereby failing to reach the goal. This telltale failure has been also observed to be existing in the case of a multi robot system, consequently engendering the failure of entire system to flock in the neighbourhood of rendezvous point. To avert from the aforesaid setbacks convoyed with the extant NSB strategy, this paper has highlighted a non-hierarchical projection based null space control strategy to solve the consensus problem for a multi robot system. The performance of proposed strategy has been qualitatively scrutinised in an environment consisting of rectangular and concave shaped obstacles. The strategy is found to have successfully effectuated flocking of all the individual robots in the vicinity of consensus point. The video simulations presented in the paper corroborated the success of purported control strategy and revealed that modified null space based control strategy performs better as compared to conventional null space control strategy for a multi robot systems.

## 9. Acknowledgment

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# إستراتيجية الفضاء الفارغ المعدل لحل مشكلة التوافق

**[1]أمان شارما، [2]ناجا كوراباتي، [3]رافي براساد جاجاناث، [4]باتريس ويرا، [5,*]شيام لال، [5]أدابا ناراسيمهادهان**

[1]شركة ميناء JSW Jaigarh المحدودة، JSW للبنية التحتية المحدودة، جيجاد ، راتناجيري، ماهاراشترا، الهند.

[2]قسم الهندسة الكهربائية وهندسة الحاسوب، جامعة ماساتشوستس، لويل، الولايات المتحدة الأمريكية.

[3]مختبر MIPS، جامعة دي هوت، الألزاس، فرنسا.

[4]قسم الرياضيات، المعهد الوطني للتكنولوجيا غوا، غوا، الهند.

[5]قسم الإلكترونيات وهندسة الاتصالات، المعهد الوطني للتكنولوجيا ولاية كارناتاكا، الهند.

[*]المؤلف: shyam.mtec@gmail.com

## خلاصة

في مجال أنظمة الروبوتات المتعددة، يتطلب العديد من التطبيقات التوافق بين جميع الروبوتات عند نقطة التقاء. مثل هذا الاتفاق لا يمكن أن يتحقق إلا عن طريق استراتيجية السيطرة. ومع ذلك، وجود عقبات في البيئة الملاحية يجعل تحقيق السيطرة نوعا من التخبط. يبرز هذا البحث فشل استراتيجية التحكم القائمة على الفضاء الفارغ في التغلب على العقبات المستطيلة بواسطة إثبات رياضي ودراسات المحاكاة واسعة النطاق. وللتغلب على هذه المشكلة يقترح استراتيجية للسيطرة على أساس الفضاء الفارغ المعدل وذلك لحل مشكلة التوافق في الآراء. وتم اختبار استراتيجية السيطرة المقترحة في بيئة معقدة تتألف من العقبات مستطيلة الشكل ومقعرة عن طريق المحاكاة الحاسوبية. وأخيرا، تم إجراء تحليل مقارن نوعي للإختلافات بين الاستراتيجية التقليدية المبنية على الفضاء الفارغ والإستراتيجية المعدلة المقترحة.

**الكلمات المفتاحية:** نظام متعدد الروبوتات؛ سلوك تتبع الجدار؛ استراتيجية الفضاء الفارغ للسيطرة؛ استراتيجية الفضاء الفارغ المعدل للسيطرة.