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A septic B-spline collocation method for solving the generalized equal width wave equation

Seydi B.G. Karakoç[1,*], Halil Zeybek[2]

[1]*Dept. of Mathematics, Faculty of Science and Art, Nevşehir Hacı Bektaş Veli University, Turkey*

[2]*Dept. of Applied Mathematics, Faculty of Computer Science, Abdullah Gül University, Turkey*

[*]*Corresponding author: sbgk44@gmail.com*

Abstract

In this work, a septic B-spline collocation method is implemented to find the numerical solution of the generalized equal width (GEW) wave equation by using two different linearization techniques. Test problems including single soliton, interaction of solitons and Maxwellian initial condition are solved to verify the proposed method by calculating the error norms L_2 and L_∞ and the invariants I_1, I_2 and I_3. Applying the Von-Neumann stability analysis, the proposed method is shown to be unconditionally stable. As a result, the obtained results are found in good agreement with the some recent results.

Keywords: Collocation method; GEW equation; septic B-spline; solitary waves; soliton.

AMS Classification: 41A15, 65L60, 76B25.

1. Introduction

The generalized equal width (GEW) equation's form is given by,

$$U_t + \varepsilon U^p U_x - \delta U_{xxt} = 0, \tag{1}$$

with physical boundary conditions $U \to 0$ as $x \to \pm\infty$, where p is a positive integer, ε and δ positive constant, t is time and x is the space coordinate. In this study, boundary and initial conditions are chosen

$$\begin{array}{ll} U(a,t)=0, & U(b,t)=0, \\ U_x(a,t)=0, & U_x(b,t)=0, \\ U_{xxx}(a,t)=0, & U_{xxx}(b,t)=0, \\ U(x,0)=f(x), & a \le x \le b, \end{array} \tag{2}$$

where $f(x)$ is a localized disturbance inside the considered interval and will be determined later. In the fluid problems, U is related to the wave amplitude of the water surface or similar physical quantity. In the plasma applications, U is the negative of the electrostatic potential.

The GEW equation was presented firstly as a model for small-amplitude long waves on the surface of water in a channel by Peregrine (1967) and Benjamin *et al.* (1972). GEW equation derived for long waves propagating in the positive x-direction is related to the generalized long wave (GRLW) equation and the generalized Korteweg-de Vries (GKdV) equation and is based upon the equal width wave (EW) equation. These general equations are nonlinear wave equations with $(p+1)$th nonlinearity and have solitary solutions, which are pulse-like Raslan (2006). Equation (1) is an alternative model to the generalized RLW equation and GKdV equation. So, the solitary wave solution of the GEW equation has an important role in understanding the many physical phenomena.

GEW equation has been solved with various methods. Hamdi *et al.* (2003) presented the analytic solution technique for this problem. Evans & Raslan (2005) solved the equation numerically by using quadratic B-spline collocation method. Raslan (2006) obtained the numerical solutions of the equation with collocation method using cubic B-spline. Roshan (2011) studied the equation numerically using linear hat function by Petrov-Galerkin method. A RBF collocation method has been presented by Panahipour (2012). Exact solution of the GEW equation has been obtained by Taghizadeh *et al.* (2013) using the homogeneous balance method.

If p = 1 in Equation (1), we get the equal width (EW) wave equation. As the EW equation define the many physical phenomena like no shallow water waves and ionacoustic plasma waves, it has an important role in nonlinear wave propagation. The EW equation has been solved by using various numerical methods. For example, the EW equation was solved with cubic, quartic and septic

B-spline collocation method by Dağ & Saka (2004); Raslan (2005) and Fazal-i-Haq *et al.* (2013). Gardner *et al.* (1997) and Zaki (2001) presented the quadratic B-spline Petrov-Galerkin method to find the numarical solution of the EW equation. A cubic B-spline Galerkin method was implemented to the EW equation by Gardner & Gardner (1991). A spectral method for the EW equation was given by Garcia-Archilla (1996). Numerical solution of the EW equation was investigated by using an adaptive method of lines Hamdi *et al.* (2001). For $p = 2$, we get the modified equal width (MEW) wave equation. The MEW equation was solved numerically finite element methods by Esen (2006); Saka (2007); Geyikli & Karakoç (2011); Geyikli & Karakoç (2012); Karakoç & Geyikli (2012) and Islam *et al.* (2010). The tanh and sine-cosine method was investigated for solving the ZK-MEW equation byWazwaz (2006). He's variational iteration method was used for solving the MEW equation by Lu (2009). Also, spline functions were used for different solving techniques by Prenter (1975); Rubin & Graves (1975); Dogan (2005); Esen (2005); Karakoç *et al.* (2014); Karakoç *et al.* (2014); Karakoç *et al.* (2015) and Başhan *et al.* (2015).

In the present paper, GEW equation has been solved numerically by using the septic B-spline collocation method with two different linearization techniques.

2. Septic B-spline collocation method

To be able to apply the numerical method, the solution region of the problem is restricted over an interval $a \leq x \leq b$. The interval $[a,b]$ is partitioned into uniformly sized finite elements of length h by the knots x_m such that $a = x_0 < x_1 < ... < x_N = b$ and $h = \frac{b-a}{N}$. The set of septic B-spline functions $\{\phi_{-3}(x), \phi_{-2}(x), \ldots, \phi_{N+3}(x)\}$ forms a basis over the solution region $[a,b]$. The numerical solution $U_N(x,t)$ is expressed in terms of the septic B-splines as

$$U_N(x,t) = \sum_{m=-3}^{N+3} \phi_m(x)\delta_m(t) \tag{3}$$

where $\delta_m(t)$ are time dependent parameters and will be determined from the boundary and collocation conditions. Septic B-splines $\phi_m(x)$, $(m = -3, -2, ..., N+3)$ at the knots x_m are defined over the interval $[a,b]$ by Prenter (1975)

$$\phi_m(x) = \frac{1}{h^7}\begin{cases} (x-x_{m-4})^7 & [x_{m-4},x_{m-3}] \\ (x-x_{m-4})^7 - 8(x-x_{m-3})^7 & [x_{m-3},x_{m-2}] \\ (x-x_{m-4})^7 - 8(x-x_{m-3})^7 + 28(x-x_{m-2})^7 & [x_{m-2},x_{m-1}] \\ (x-x_{m-4})^7 - 8(x-x_{m-3})^7 + 28(x-x_{m-2})^7 - 56(x-x_{m-1})^7 & [x_{m-1},x_m] \\ (x_{m+4}-x)^7 - 8(x_{m+3}-x)^7 + 28(x_{m+2}-x)^7 - 56(x_{m+1}-x)^7 & [x_m,x_{m+1}] \\ (x_{m+4}-x)^7 - 8(x_{m+3}-x)^7 + 28(x_{m+2}-x)^7 & [x_{m+1},x_{m+2}] \\ (x_{m+4}-x)^7 - 8(x_{m+3}-x)^7 & [x_{m+2},x_{m+3}] \\ (x_{m+4}-x)^7 & [x_{m+3},x_{m+4}] \\ 0 & otherwise. \end{cases} \tag{4}$$

Each septic B-spline covers 8 elements, thus each element $[x_m,x_{m+1}]$ is covered by 8 splines. A typical finite interval $[x_m,x_{m+1}]$ is mapped to the interval $[0,1]$ by a local coordinate transformation defined by $h\xi = x - x_m$, $0 \leq \xi \leq 1$. So septic B-splines (4) in terms of ξ over $[0,1]$ can be given as follows:

$$\begin{aligned} \phi_{m-3} &= 1 - 7\xi + 21\xi^2 - 35\xi^3 + 35\xi^4 - 21\xi^5 + 7\xi^6 - \xi^7, \\ \phi_{m-2} &= 120 - 392\xi + 504\xi^2 - 280\xi^3 + 84\xi^5 \\ &\quad - 42\xi^6 + 7\xi^7, \\ \phi_{m-1} &= 1191 - 1715\xi + 315\xi^2 + 665\xi^3 - 315\xi^4 \\ &\quad - 105\xi^5 + 105\xi^6 - 21\xi^7, \\ \phi_m &= 2416 - 1680\xi + 560\xi^4 - 140\xi^6 + 35\xi^7, \\ \phi_{m+1} &= 1191 + 1715\xi + 315\xi^2 - 665\xi^3 - 315\xi^4 \\ &\quad + 105\xi^5 + 105\xi^6 - 35\xi^7, \\ \phi_{m+2} &= 120 + 392\xi + 504\xi^2 + 280\xi^3 - 84\xi^5 \\ &\quad - 42\xi^6 + 21\xi^7, \\ \phi_{m+3} &= 1 + 7\xi + 21\xi^2 + 35\xi^3 + 35\xi^4 + 21\xi^5 + 7\xi^6 - \xi^7, \\ \phi_{m+4} &= \xi^7. \end{aligned} \tag{5}$$

For the problem, the finite elements are identified with the interval $[x_m,x_{m+1}]$. Using Equation (4) and Equation (3), the nodal values of U_m, U'_m, U''_m, U'''_m are given in terms of the element parameters δ_m by

$$\begin{aligned} U_N(x_m,t) = U_m &= \delta_{m-3} + 120\delta_{m-2} + 1191\delta_{m-1} + 2416\delta_m \\ &\quad + 1191\delta_{m+1} + 120\delta_{m+2} + \delta_{m+3}, \\ U'_m &= \tfrac{7}{h}(-\delta_{m-3} - 56\delta_{m-2} - 245\delta_{m-1} + 245\delta_{m+1} \\ &\quad + 56\delta_{m+2} + \delta_{m+3}), \\ U''_m &= \tfrac{42}{h^2}(\delta_{m-3} + 24\delta_{m-2} + 15\delta_{m-1} - 80\delta_m \\ &\quad + 15\delta_{m+1} + 24\delta_{m+2} + \delta_{m+3}), \\ U'''_m &= \tfrac{210}{h^3}(-\delta_{m-3} - 8\delta_{m-2} + 19\delta_{m-1} - 19\delta_{m+1} \\ &\quad + 8\delta_{m+2} + \delta_{m+3}), \end{aligned} \tag{6}$$

and the variation of U over the element $[x_m,x_{m+1}]$ is given by

$$U = \sum_{m=-3}^{N+3} \phi_m\delta_m. \tag{7}$$

Now, we identify the collocation points with the knots and use Equation (6) to evaluate U_m, its space derivatives and substitute into Equation (1) to obtain the set of the coupled ordinary differential equations: For the first linearization technique, we get the following equation:

$$\begin{aligned}&\dot{\delta}_{m-3}+120\dot{\delta}_{m-2}+1191\dot{\delta}_{m-1}+2416\dot{\delta}_m+1191\dot{\delta}_{m+1}\\&+120\dot{\delta}_{m+2}+\dot{\delta}_{m+3}+\tfrac{7\varepsilon Z_m}{h}(-\delta_{m-3}-56\delta_{m-2}\\&-245\delta_{m-1}+245\delta_{m+1}+56\delta_{m+2}+\delta_{m+3})\\&-\tfrac{42\delta}{h^2}(\dot{\delta}_{m-3}+24\dot{\delta}_{m-2}+15\dot{\delta}_{m-1}-80\dot{\delta}_m+15\dot{\delta}_{m+1}\\&+24\dot{\delta}_{m+2}+\dot{\delta}_{m+3})=0,\end{aligned}\tag{8}$$

where

$$Z_m=(U_m)^p=(\delta_{m-3}+120\delta_{m-2}+1191\delta_{m-1}+2416\delta_m+1191\delta_{m+1}+120\delta_{m+2}+\delta_{m+3})^p.$$

For the second (Rubin & Graves (1975)) linearization technique, we obtain the following general form of the solution method:

$$\begin{aligned}&\dot{\delta}_{m-3}+120\dot{\delta}_{m-2}+1191\dot{\delta}_{m-1}+2416\dot{\delta}_m+1191\dot{\delta}_{m+1}\\&+120\dot{\delta}_{m+2}+\dot{\delta}_{m+3}+\varepsilon Z_m(\delta_{m-3}+120\delta_{m-2}\\&+1191\delta_{m-1}+2416\delta_m+1191\delta_{m+1}+120\delta_{m+2}+\delta_{m+3})\\&-\tfrac{42\delta}{h^2}(\dot{\delta}_{m-3}+24\dot{\delta}_{m-2}+15\dot{\delta}_{m-1}-80\dot{\delta}_m+15\dot{\delta}_{m+1}\\&+24\dot{\delta}_{m+2}+\dot{\delta}_{m+3})=0,\end{aligned}\tag{9}$$

where

$$Z_m=(U_m)^{p-1}(U_m)x$$

and · denotes derivative with respect to time. If time parameters δ_i and its time derivatives $\dot{\delta}_i$ in Equation (8) and Equation (9) are discretized by the Crank-Nicolson formula and usual finite difference aproximation, respectively,

$$\delta_m=\frac{1}{2}(\delta_m^n+\delta_m^{n+1}),\qquad \dot{\delta}_m=\frac{\delta_m^{n+1}-\delta_m^n}{\Delta t}\tag{10}$$

for the first linearization, we obtain a recurrence relationship between two time levels n and $n+1$ relating two unknown parameters δ_i^{n+1}, δ_i^n for $i=m-3, m-2, ..., m+2, m+3$

$$\begin{aligned}&\gamma_1\delta_{m-3}^{n+1}+\gamma_2\delta_{m-2}^{n+1}+\gamma_3\delta_{m-1}^{n+1}+\gamma_4\delta_m^{n+1}+\gamma_5\delta_{m+1}^{n+1}\\&+\gamma_6\delta_{m+2}^{n+1}+\gamma_7\delta_{m+3}^{n+1}=\gamma_7\delta_{m-3}^n+\gamma_6\delta_{m-2}^n+\gamma_5\delta_{m-1}^n\\&+\gamma_4\delta_m^n+\gamma_3\delta_{m+1}^n+\gamma_2\delta_{m+2}^n+\gamma_1\delta_{m+3}^n,\end{aligned}\tag{11}$$

where

$$\begin{aligned}&\gamma_1=(1-EZ_m-M), && \gamma_2=(120-56EZ_m-24M),\\&\gamma_3=(1191-245EZ_m-15M), && \gamma_4=(2416+80M),\\&\gamma_5=(1191+245EZ_m-15M), && \gamma_6=(120+56EZ_m-24M),\\&\gamma_7=(1+EZ_m-M),\\&m=0,1,\ldots,N,\quad E=\tfrac{7\varepsilon}{2h}\Delta t,\quad M=\tfrac{42\delta}{h^2}.\end{aligned}\tag{12}$$

For the second (Rubin & Graves (1975)) linearization technique, the recurrence relationship has been obtained as follows

$$\begin{aligned}&\gamma_1\delta_{m-3}^{n+1}+\gamma_2\delta_{m-2}^{n+1}+\gamma_3\delta_{m-1}^{n+1}+\gamma_4\delta_m^{n+1}+\gamma_3\delta_{m+1}^{n+1}\\&+\gamma_2\delta_{m+2}^{n+1}+\gamma_1\delta_{m+3}^{n+1}=\gamma_5\delta_{m-3}^n+\gamma_6\delta_{m-2}^n+\gamma_7\delta_{m-1}^n\\&+\gamma_8\delta_m^n+\gamma_7\delta_{m+1}^n+\gamma_6\delta_{m+2}^n+\gamma_5\delta_{m+3}^n,\end{aligned}\tag{13}$$

where

$$\begin{aligned}&\gamma_1=(1+EZ_m-M), && \gamma_2=(120+120EZ_m-24M),\\&\gamma_3=(1191+1191EZ_m-15M), && \gamma_4=(2416+2416EZ_m+80M),\\&\gamma_5=(1-EZ_m-M), && \gamma_6=(120-120EZ_m-24M),\\&\gamma_7=(1191-1191EZ_m-15M), && \gamma_8=(2416-2416EZ_m+80M),\\&m=0,1,\ldots,N,\quad E=\tfrac{\varepsilon\Delta t}{2},\quad M=\tfrac{42\delta}{h^2}.\end{aligned}\tag{14}$$

In the first linearization technique, the U^p term in nonlinear term U^pU_x is taken as

$$Z_m=(U_m)^p=(\delta_{m-3}+120\delta_{m-2}+1191\delta_{m-1}+2416\delta_m+1191\delta_{m+1}+120\delta_{m+2}+\delta_{m+3})^p.\tag{15}$$

In the second (Rubin & Graves (1975)) linearization technique, the $U^{p-1}U_x$ term in non-linear term U^pU_x is taken as

$$Z_m=(U_m)^{p-1}(U_m)x\cdot\tag{16}$$

When the Rubin & Graves (1975) linearization technique is applied to the $U^{p-1}U_x$ term, we get

$$(U^{p-1}U_x)^{n+1}=(U^{p-1})^n(U_x)^{n+1}+(U^{p-1})^{n+1}(U_x)^n-(U^{p-1})^n(U_x)^n.\tag{17}$$

The system (11) and (13) consist of $(N+1)$ linear equations including $(N+7)$ unknown parameters $(\delta_{-3},\delta_{-2},\delta_{-1},...,\delta_{N+1},\delta_{N+2},\delta_{N+3})^T$. To obtain a unique solution for this system, we need six additional constraints. These are obtained from the boundary conditions (2) and can be used to eliminate $\delta_{-3},\delta_{-2},\delta_{-1}$ and $\delta_{N+1},\delta_{N+2},\delta_{N+3}$ from the systems (11) and (13) which then becomes a matrix equation for the $N+1$ unknowns $d^n=(\delta_0,\delta_1,...,\delta_N)^T$ of the form

$$A\mathbf{d}^{\mathbf{n+1}}=B\mathbf{d}^{\mathbf{n}}.\tag{18}$$

The matrices A and B are $(N+1)\times(N+1)$ septa-diagonal matrices and this matrix equation can be solved by using the septa-diagonal algorithm.

Two or three inner iterations are applied to the term $\delta^{n*} = \delta^n + \frac{1}{2}(\delta^n - \delta^{n-1})$ at each time step to cope with the nonlinearity caused by Z_m. Before the commencement of the solution process, initial parameters d^0 must be determined by using the initial condition and following derivatives at the boundaries;

$$
\begin{aligned}
U_N(x,0) &= U(x_m,0); \quad & m=0,1,2,...,N \\
(U_N)_x(a,0) &= 0, & (U_N)_x(b,0)=0, \\
(U_N)_{xx}(a,0) &= 0, & (U_N)_{xx}(b,0)=0, \\
(U_N)_{xxx}(a,0) &= 0, & (U_N)_{xxx}(b,0)=0.
\end{aligned}
$$

So we have the following matrix form for the initial vector d^0;

$$Wd^0 = b,$$

$$
\text{where } W = \begin{bmatrix}
1536 & 2712 & 768 & 24 & & & & & & \\
\frac{82731}{81} & \frac{210568.5}{81} & \frac{104796}{81} & \frac{10063.5}{81} & 1 & & & & & \\
\frac{9600}{81} & \frac{96597}{81} & \frac{195768}{81} & \frac{96474}{81} & 120 & 1 & & & & \\
 & & \ddots & & & & & & & \\
 & & & 1 & 120 & 1191 & 2416 & 1191 & 120 & 1 \\
 & & & & 1 & 120 & \frac{96474}{81} & \frac{195768}{81} & \frac{96597}{81} & \frac{9600}{81} \\
 & & & & & 1 & \frac{10063.5}{81} & \frac{104796}{81} & \frac{210568.5}{81} & \frac{82731}{81} \\
 & & & & & & 24 & 768 & 2712 & 1536
\end{bmatrix}
$$

$d^0 = (\delta_0, \delta_1, \delta_2, ..., \delta_{N-2}, \delta_{N-1}, \delta_N)^T$ and
$b = (U(x_0,0), U(x_1,0), ..., U(x_{N-1},0), U(x_N,0))^T$.

3. A linear stability analysis

To apply the Von-Neumann stability analysis, the GEW equation can be linearized by assuming that the quantity U^p in the nonlinear term U^pU_x is locally constant. Substituting the Fourier mode $\delta_m^n = \xi^n e^{imkh}$ $(i=\sqrt{-1})$ in which k is a mode number and h is the element size, into the Equation (11) gives the growth factor ξ of the form

$$\xi = \frac{a-ib}{a+ib},$$

where

$$
\begin{aligned}
a &= \gamma_4 + (\gamma_5+\gamma_3)\cos[hk] + (\gamma_6+\gamma_2)\cos[2hk] \\
&\quad + (\gamma_7+\gamma_1)\cos[3hk], \\
b &= (\gamma_5-\gamma_3)\sin[hk] + (\gamma_6-\gamma_2)\sin[2hk] \\
&\quad + (\gamma_7-\gamma_1)\sin[3hk].
\end{aligned}
$$

The modulus of $|\xi|$ is 1, therefore the linearized scheme is unconditionally stable.

4. Numerical examples and results

To show the accuracy of the numerical scheme and to compare our results with both exact values and other results given in the literature, the L_2 and L_∞ error norms are calculated by using the analytical solution in (19). Three test problems including: motion of a single solitary wave, interaction of two solitary waves and the maxwellian initial condition are investigated. Also three invariants (20) are calculated to show the conservation properties of the numerical scheme. The error norms L_2 and L_∞ are given as follows:

$$L_2 = \left\|U^{exact} - U_N\right\|_2 \simeq \sqrt{h\sum_{J=0}^{N}\left|U_j^{exact} - (U_N)_j\right|^2},$$

and the error norm L_∞

$$L_\infty = \left\|U^{exact} - U_N\right\|_\infty \simeq \max_j \left|U_j^{exact} - (U_N)_j\right|.$$

The analytic solution of GEW equation (1) obtained by applying the transformation $U(x,t) = f(x-ct)$, given by Evans & Raslan (2005), Raslan (2006) is

$$U(x,t) = \sqrt[p]{\frac{c(p+1)(p+2)}{2\varepsilon}\sec h^2\left[\frac{p}{2\sqrt{\delta}}(x-ct-x_0)\right]} \quad (19)$$

where c is the the constant velocity of the wave travelling in the positive direction of the x-axis, x_0 is arbitrary constant. And the three invariants of motion which correspond to conservation of mass, momentum and energy given by Evan & Raslan (2005), Raslan (2006) are

$$I_1 = \int_a^b U dx, \quad I_2 = \int_a^b \left[U^2 + \delta U_x^2\right] dx, \quad I_3 = \int_a^b U^{p+2} dx. \quad (20)$$

4.1. The motion of single solitary wave

In this section, to apply our numerical method, we have considered the five sets of parameters for different values of p, c and $amplitude = \sqrt[p]{\frac{c(p+1)(p+2)}{2\varepsilon}}$. The other parameters for all of five sets are chosen to be $h = 0.1$, $\Delta t = 0.2$, $\varepsilon = 3$, $\delta = 1$, $x_0 = 30$, $0 \leq x \leq 80$ and the numerical computations are done up to $t = 20$.

Firstly, we take $p = 2$, $c = 1/32$, so the solitary wave has *amplitude* = 0.25. The invariants I_1, I_2, I_3 and the error norms L_2, L_∞ have been calculated by using our numerical method. The obtained results are reported in Table 1. As seen in Table 1, the changes of the invariants $I_1 \times 10^5$, $I_2 \times 10^5$ and $I_3 \times 10^5$ from their initial count are less than 0.0038, 0.0027 and 0.0002, respectively. Also, we observed that the quantity of the error norms L_2 and L_∞ obtained with second linearization technique are less than the obtained with first linearization technique.

Secondly, we consider the values $p = 2$, $c = 1/2$, hence the solitary wave has *amplitude* = 1. The invariants I_1, I_2, I_3 and the the error norms L_2, L_∞ have been calculated by using our numerical method. The obtained results are given in Table 2. Table 2 shows that the changes of the invariants $I_1 \times 10^3$, $I_2 \times 10^3$ and $I_3 \times 10^3$ from their initial state are less than 0.0005, 0.0017 and 0.0017, respectively. If the magnitude of the error norms L_2 and L_∞ calculated with first and second linearization technique compare, it is shown that the magnitude for the second linearization technique is smaller than the ones.

Thirdly, if it is taken the paremeters $p = 3$, $c = 0.001$, the solitary wave has *amplitude* = 0.15. The invariants I_1, I_2, I_3 and the the error norms L_2, L_∞ have been calculated by using our numerical method. The obtained results are tabulated in Table 3. It is observed from Table 3 that he changes of the invariants $I_1 \times 10^6$, $I_2 \times 10^6$ and $I_3 \times 10^6$ from their initial case are less than 0.0001, 0.0001 and 0.0001, respectively. When we evaluate the error norms L_2 and L_∞ obtained using the first and second linearization, it is seen that the second linearization is better for our numerical scheme.

And we choose the parameters $p = 3$, $c = 0.3$, that's why the solitary wave has *amplitude* =1. The invariants I_1, I_2, I_3 and the the error norms L_2, L_∞ have been calculated by using our numerical method. The obtained results are shown in Table 4. It is clearly seen from Table 4 that the changes of the invariants $I_1 \times 10^3$, $I_2 \times 10^3$ and $I_3 \times 10^3$ from their initial value are less than 0.0637, 0.1606 and 0.1607, respectively. And the values of the error norms L_2 and L_∞ in the second linearization are smaller than the first. Solitary wave profiles are depicted at different time levels in Figure 1. In this figure, the soliton moves to the right at a constant speed and nearly unchanged amplitude as time increases, as expected.

Finally, for the quantities $p = 4$, $c = 0.2$, the solitary wave has *amplitude* = 1. The invariants I_1, I_2, I_3 and the error norms L_2, L_∞ have been calculated by using our numerical method. The obtained results are listed in Table 5. It is detected from Table 5 that the changes of the invariants $I_1 \times 10^3$, $I_2 \times 10^3$ and $I_3 \times 10^3$ from their initial quantity are less than 0.1305, 0.2822 and 0.2823, respectively. By using the second linearization, we found that the quantity of the error norms L_2 and L_∞ is smaller than the ones. Figure 2 shows that our numerical scheme performs the soliton, which moves to the right at a constant speed and conserves its amplitude and shape with increasing time, as expected.

In Table 6, we compare the values of the invariants and error norms obtained by the present method with methods obtained by Evans & Raslan (2005), Raslan (2006), Roshan (2011) at $t = 20$. In this table, we observed that the error norms obtained by our method is smaller than the ones in previous studies for p = 2,3 values and nearly same the given before for p = 4. The values of the invariants are also found in good agreement with the others.

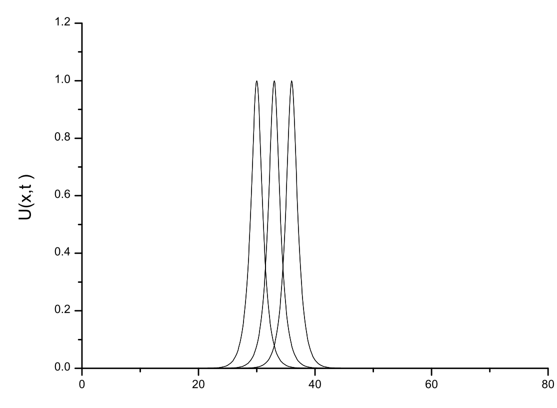


Fig. 1. Single solitary wave with $p = 3$, $c = 0.3$, $x_0 = 30$, $0 \leq x \leq 80$, $t = 0,10,20$.

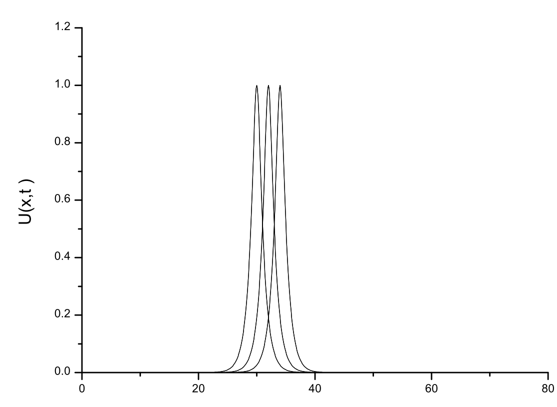


Fig. 2. Single solitary wave with $p = 4$, $c = 0.2$, $x_0 = 30$, $0 \leq x \leq 80$, $t = 0,10,20$.

Table 1. The invariants and the error norms for single solitary wave with $p = 2$, *amplitude* = 0.25, $\Delta t = 0.2$, $h = 0.1$, $\varepsilon = 3, \delta = 1$, $0 \le x \le 80$.

t		0	5	10	15	20
I_1	First	0.7853966	0.7853966	0.7853965	0.7853965	0.7853965
	Second	0.7853966	0.7853966	0.7853966	0.7853966	0.7853965
I_2	First	0.1666664	0.1666663	0.1666663	0.1666663	0.1666663
	Second	0.1666664	0.1666663	0.1666663	0.1666663	0.1666663
I_3	First	0.0052083	0.0052083	0.0052083	0.0052083	0.0052083
	Second	0.0052083	0.0052083	0.0052083	0.0052083	0.0052083
$L_2 \times 10^5$	First	0.00000000	0.03067279	0.06285007	0.09693233	0.13336822
	Second	0.00000000	0.02900012	0.05967250	0.09243870	0.12775844
$L_\infty \times 10^5$	First	0.00000000	0.01989833	0.04083748	0.06230627	0.08399884
	Second	0.00000000	0.01721060	0.03441138	0.05158717	0.06887276

Table 2. The invariants and the error norms for single solitary wave with $p = 2$, *amplitude* = 1, $\Delta t = 0.2$, $h = 0.1$, $\varepsilon = 3, \delta = 1$, $0 \le x \le 80$.

t		0	5	10	15	20
I_1	First	3.1415863	3.1415861	3.1415859	3.1415857	3.1415854
	Second	3.1415863	3.1415864	3.1415862	3.1415860	3.1415858
I_2	First	2.6666616	2.6666613	2.6666610	2.6666607	2.6666604
	Second	2.6666616	2.6666616	2.6666611	2.6666606	2.6666600
I_3	First	1.3333283	1.3333275	1.3333272	1.3333269	1.3333266
	Second	1.3333283	1.3333283	1.3333278	1.3333272	1.3333267
L_2	First	0.00000000	0.00438263	0.00853676	0.01262954	0.01671823
	Second	0.00000000	0.00421699	0.00849425	0.01279079	0.01708960
L_∞	First	0.00000000	0.00289068	0.00539302	0.00789694	0.01040121
	Second	0.00000000	0.00261076	0.00524102	0.00787126	0.01050088

Table 3. The invariants and the error norms for single solitary wave with $p = 3$, *amplitude* = 0.15, $\Delta t = 0.2$, $h = 0.1$, $\varepsilon = 3, \delta = 1$, $0 \le x \le 80$.

t		0	5	10	15	20
I_1	First	0.4189154	0.4189154	0.4189154	0.4189154	0.4189154
	Second	0.4189154	0.4189154	0.4189154	0.4189154	0.4189154
I_2	First	0.0549807	0.0549807	0.0549807	0.0549807	0.0549807
	Second	0.0549807	0.0549807	0.0549807	0.0549807	0.0549807
$I_3 \times 10^4$	First	0.7330748	0.7330748	0.7330748	0.7330748	0.7330748
	Second	0.7330748	0.7330748	0.7330748	0.7330748	0.7330748
$L_2 \times 10^7$	First	0.00000000	0.01575841	0.03157299	0.04744419	0.06337251
	Second	0.00000000	0.01574216	0.03154053	0.04739557	0.06330776
$L_\infty \times 10^7$	First	0.00000000	0.00855102	0.01715751	0.02582082	0.03454222
	Second	0.00000000	0.00855128	0.01715803	0.02582167	0.03454333

Table 4. The invariants and the error norms for single solitary wave with $p = 3$, *amplitude* = 1, $\Delta t = 0.2$, $h = 0.1$, $\varepsilon = 3, \delta = 1$, $0 \le x \le 80$.

t		0	5	10	15	20
I_1	First	2.8043580	2.8043577	2.8043575	2.8043572	2.8043570
	Second	2.8043580	2.8043425	2.8043265	2.8043104	2.8042943
I_2	First	2.4639101	2.4639097	2.4639094	2.4639090	2.4639086
	Second	2.4639101	2.4638709	2.4638305	2.4637900	2.4637496
I_3	First	0.9855618	0.9855613	0.9855610	0.9855606	0.9855602
	Second	0.9855618	0.9855225	0.9854821	0.9854416	0.9854012
L_2	First	0.00000000	0.00204205	0.00404586	0.00603031	0.00800997
	Second	0.00000000	0.00166798	0.00341195	0.00522557	0.00708099
L_∞	First	0.00000000	0.00144917	0.00275209	0.00406426	0.00537733
	Second	0.00000000	0.00114859	0.00234526	0.00356386	0.00480353

4.2. The interaction of two solitary waves

In this section, we have studied the interaction of two well seperated solitary waves by using the following initial condition

Table 5. The invariants and the error norms for single solitary wave with $p = 4$, *amplitude* = 1, $\Delta t = 0.2$, $h = 0.1$, $\varepsilon = 3, \delta = 1$, $0 \leq x \leq 80$.

t		0	5	10	15	20
I_1	First	2.6220516	2.6220514	2.6220512	2.6220510	2.6220508
	Second	2.6220516	2.6220193	2.6219866	2.6219539	2.6219211
I_2	First	2.3561915	2.3561912	2.3561909	2.3561905	2.3561902
	Second	2.3561915	2.3561216	2.3560509	2.3559801	2.3559093
I_3	First	0.7853952	0.7853948	0.7853945	0.7853942	0.7853939
	Second	0.7853952	0.7853252	0.7852545	0.7851837	0.7851130
L_2	First	0.00000000	0.00105910	0.00211286	0.00316045	0.00420836
	Second	0.00000000	0.00075057	0.00156686	0.00245793	0.00341485
L_∞	First	0.00000000	0.00078877	0.00151318	0.00223807	0.00296955
	Second	0.00000000	0.00055460	0.00116121	0.00180868	0.00249360

Table 6. For $p = 2,3$ and 4, Comprasions of result for the single solitary wave with $\Delta t = 0.2$, $h = 0.1$, $\varepsilon = 3, \delta = 1$, $0 \leq x \leq 80$.

	p	2	3	4
I_1	Collocation (quadratic)[Evans and Raslan (2005)]	0.78528640		
	Collocation (cubic)[Raslan (2006)]	0.78466760	0.65908330	
	Petrov-Galerkin (quadratic)[Roshan (2011)]	0.78539800	0.41891600	2.62206000
	Ours - Collocation (septic)	0.78539650	0.41891540	2.62192110
I_2	Collocation (quadratic)[Evans and Raslan (2005)]	0.16658180		
	Collocation (cubic)[Raslan (2006)]	0.16643400	0.05938137	
	Petrov-Galerkin (quadratic)[Roshan (2011)]	0.16666900	0.05497830	2.35615000
	Ours - Collocation (septic)	0.16666630	0.05498070	2.35590930
I_3	Collocation (quadratic)[Evans and Raslan (2005)]	0.00520600		
	Collocation (cubic)[Raslan (2006)]	0.00519380	0.00006871	
	Petrov-Galerkin (quadratic)[Roshan (2011)]	0.00520829	0.00007330	0.78534400
	Ours - Collocation (septic)	0.00520830	0.00007330	0.78511300
$L_2 \times 10^3$	Collocation (quadratic)[Evans and Raslan (2005)]	0.15695390		
	Collocation (cubic)[Raslan (2006)]	0.19588780	0.51496770	
	Petrov-Galerkin (quadratic)[Roshan (2011)]	0.00250172	0.00006407	2.30499000
	Ours - Collocation (septic)	0.00127758	0.00000633	3.41485000
$L_\infty \times 10^3$	Collocation (quadratic)[Evans and Raslan (2005)]	0.20214760		
	Collocation (cubic)[Raslan (2006)]	0.17443300	0.32060590	
	Petrov-Galerkin (quadratic)[Roshan (2011)]	0.00275164	0.00008206	1.88285000
	Ours - Collocation (septic)	0.00068872	0.00000345	2.49360000

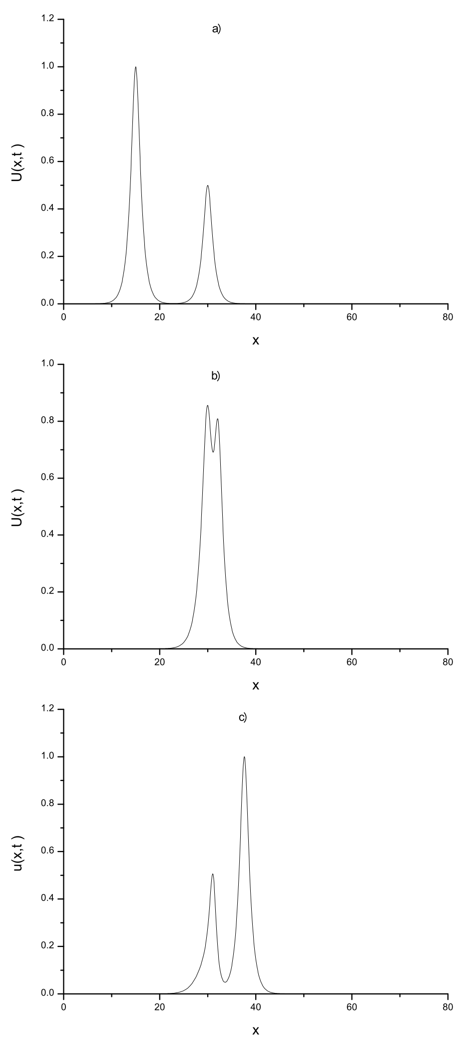
$$U(x,0) = \sum_{i=1}^{2} \sqrt[p]{\frac{c_i(p+1)(p+2)}{2\varepsilon}\,\mathrm{sec}\,h^2\left[\frac{p}{2\sqrt{\delta}}(x - x_i)\right]} \quad (21)$$

where c_i and x_i, $i = 1,2$ are arbitrary constants. Equation (21) represents two solitary waves having different amplitudes at the same direction. We have considered the three sets of parameters for different values of p, c_i. The other parameters for all of three sets are chosen to be $h = 0.1$, $\Delta t = 0.025$, $\varepsilon = 3$, $\delta = 1$, $x_1 = 15, x_2 = 30, 0 \leq x \leq 80$. The amplitudes of two well seperated solitary waves are 1, 0.5.

Firstly, we take $p = 2$, $c_1 = 0.5$ and $c_2 = 0.125$. The experiments are run from $t = 0$ to $t = 60$ and the values of the invariant quantities I_1, I_2 and I_3 are listed in Table 7. Table 7 shows that the changes of the invariant $I_1 \times 10^3$, $I_2 \times 10^3$ and $I_3 \times 10^3$ from their initial case are less than 0.0013, 0.0002 and 0.005, respectively. The invariants are also found to be very close with the obtained by using quadratic Petrov-Galerkin method.

Secondly, we take the parameters $p = 3$, $c_1 = 0.3$ and $c_2 = 0.0375$. The simulations are done up to time $t = 100$ to find the numerical invariants I_1, I_2 and I_3 at various time. The obtained results are reported in Table 8. From the Table 8 it is seen that the changes of the invariants $I_1 \times 10^3$, $I_2 \times 10^3$ and $I_3 \times 10^3$ from their initial case are less than 0.002, 0.0001 and 0.0005, respectively. It is observed that the numerical values of the invariants remain almost constant during the computer run and are found in good agreement with the quadratic Petrov-Galerkin method. Figure 3(a)-(d) illustrates the interaction of two solitary waves at different times. From this figure, we observed that at time $t = 0$ the wave with larger amplitude is to the left of the second wave with smaller amplitude. As the time increases, overlapping process occurres. After the time $t = 50$, waves start to resume their original shapes.

Finally, we have chosen the parameters $p = 4$, $c_1 = 0.2$ and $c_2 = 1/80$. The computer program was run to time $t = 120$. To record the conservate quantities of the invariants I_1, I_2 and I_3, the calculated values are given in Table 9. As shown in Table 9, the changes of the invariants $I_1 \times 10^4$, $I_2 \times 10^4$ and $I_3 \times 10^4$ from their initial case are less than 0.01, 0.001 and 0.005, respectively. The invariants are the almost same of the given by Roshan. The motion of two solitary waves using our method is plotted at different time levels in Figure 4(a)-(d). This figure shows that at time $t = 0$ the wave with larger amplitude is on the left of the second wave with smaller amplitude. In progress of time, interaction starts and overlapping process occurres. At the time $t = 100$, waves start to resume their original shapes.



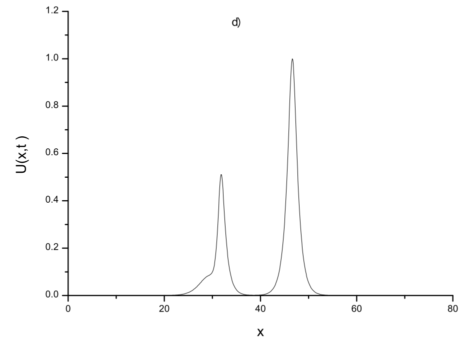


Fig. 3. Interaction of two solitary waves at $p = 3$; *a)* $t = 0$,*b)* $t = 50$,*c)* $t = 70$,*d)* $t = 100$.

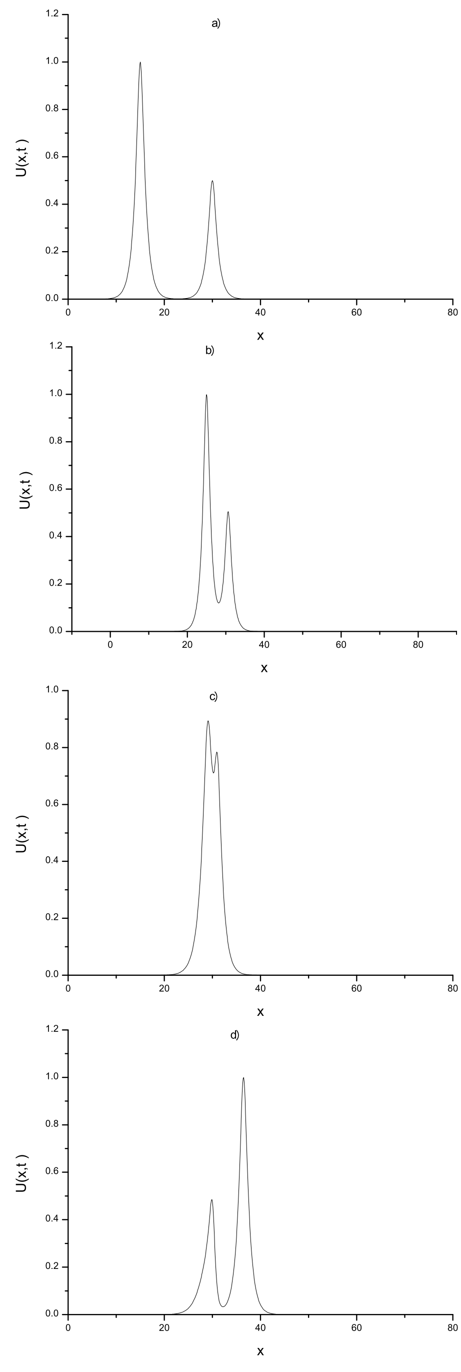


Fig. 4. Interaction of two solitary waves at $p = 4$; *a)* $t = 0$,b) $t = 50$,*c)* $t = 70$,*d)* $t = 100$.

Table 7. The invariants for interaction of two solitary waves with $p = 2$, $c_1 = 0.5$, $c_2 = 0.125$, $x_1 = 15$, $x_2 = 30$, $\Delta t = 0.025$, $h = 0.1$, $\varepsilon = 3$, $\delta = 1$, $0 \leq x \leq 80$.

	t	0	10	20	30	40	50	60
I_1	Ours - First	4.7123733	4.7123745	4.7123745	4.7123745	4.7123745	4.7123745	4.7123745
	Ours - Second	4.7123733	4.7123745	4.7123743	4.7123665	4.7123702	4.7123746	4.7123747
	QBSPG[Roshan (2011)]	4.7123900	4.7123900	4.7123900	4.7123900	4.7123900	4.7123900	4.7123900
I_2	Ours - First	3.3333294	3.3333294	3.3333294	3.3333295	3.3333295	3.3333295	3.3333295
	Ours - Second	3.3333294	3.3333294	3.3333290	3.3333139	3.3333214	3.3333296	3.3333296
	QBSPG[Roshan (2011)]	3.3332400	3.3332400	3.3332400	3.3332400	3.3333300	3.3333800	3.3333300
I_3	Ours - First	1.4166643	1.4166643	1.4166642	1.4166594	1.4166615	1.4166644	1.4166644
	Ours - Second	1.4166643	1.4166643	1.4166639	1.4166446	1.4166532	1.4166642	1.4166644
	QBSPG[Roshan (2011)]	1.1416660	1.1416660	1.1416660	1.1416640	1.1416650	1.1416660	1.1416660

Table 8. The invariants for interaction of two solitary waves with $p = 3$, $c_1 = 0.3$, $c_2 = 0.0375$, $x_1 = 15$, $x_2 = 30$, $\Delta t = 0.025$, $h = 0.1$, $\varepsilon = 3$, $\delta = 1$, $0 \leq x \leq 80$.

	t	0	10	20	40	60	80	90	100
I_1	Ours - First	4.2065320	4.2065329	4.2065330	4.2065330	4.2065330	4.2065330	4.2065330	4.2065330
	Ours - Second	4.2065320	4.2065328	4.2065328	4.2065303	4.2065314	4.2065325	4.2065324	4.2065323
	QBSPG[Roshan (2011)]	4.2065500	4.2065500	4.2065500	4.2065500	4.2065500	4.2065500	4.2065500	4.2065500
I_2	Ours - First	3.0798892	3.0798892	3.0798892	3.0798892	3.0798892	3.0798892	3.0798892	3.0798892
	Ours - Second	3.0798892	3.0798889	3.0798887	3.0798842	3.0798862	3.0798879	3.0798877	3.0798875
	QBSPG[Roshan (2011)]	3.9797700	2.0798600	3.0798200	3.0798600	3.0798700	3.0799100	3.0797400	3.0797200
I_3	Ours - First	1.0163623	1.0163623	1.0163623	1.0163619	1.0163620	1.0163624	1.0163625	1.0163625
	Ours - Second	1.0163623	1.0163621	1.0163619	1.0163573	1.0163585	1.0163606	1.0163604	1.0163602
	QBSPG[Roshan (2011)]	1.0163400	1.0163400	1.0163400	1.0163400	1.0163300	1.0163300	1.0163300	1.0163400

Table 9. The invariants for interaction of two solitary waves with $p = 4$, $c_1 = 0.2$, $c_2 = 1/80$, $x_1 = 15$, $x_2 = 30$, $\Delta t = 0.025$, $h = 0.1$, $\varepsilon = 3$, $\delta = 1$, $0 \leq x \leq 80$.

	t	0	10	20	40	60	80	100	120
I_1	Ours - First	3.9330730	3.9330737	3.9330738	3.9330738	3.9330738	3.9330738	3.9330738	3.9330739
	Ours - Second	3.9330730	3.9330736	3.9330735	3.9330732	3.9330702	3.9330709	3.9330728	3.9330725
	QBSPG[Roshan (2011)]	3.9330900	3.9330900	3.9330900	3.9330900	3.9330900	3.9330900	3.9330900	3.9330800
I_2	Ours - First	2.9452406	2.9452406	2.9452406	2.9452406	2.9452406	2.9452406	2.9452406	2.9452406
	Ours - Second	2.9452406	2.9452403	2.9452401	2.9452394	2.9452339	2.9452353	2.9452384	2.9452379
	QBSPG[Roshan (2011)]	2.9451200	2.9451800	2.9451700	2.9451500	2.9450500	2.9450600	2.9450800	2.9451100
I_3	Ours - First	0.7976683	0.7976683	0.7976683	0.7976683	0.7976680	0.7976679	0.7976684	0.7976684
	Ours - Second	0.7976683	0.7976680	0.7976677	0.7976671	0.7976617	0.7976622	0.7976655	0.7976649
	QBSPG[Roshan (2011)]	0.7976140	0.7976120	0.7976110	0.7976120	0.7976220	0.7976130	0.7976110	0.7976110

4.3. A Maxwellian initial condition

As a last problem, we consider the Equation (1) with the following Maxwellian initial condition

$$U(x,0) = Exp(-x^2), \quad -20 \leq x \leq 20. \qquad (22)$$

In this case, the behaviour of the solution depends on the values of δ. Therefore, we chose the values of $\delta = 0.01$, $\delta = 0.025$, $\delta = 0.05$, $\delta = 0.1$ for $p = 2,3,4$. The numerical computations are done up to $t = 12$. The values of the three invariants of motion for different δ are presented in Table 10. The changes of the invariants $I_1 \times 10^3$, $I_2 \times 10^3$ and $I_3 \times 10^3$ from their initial values are less than 0.03, 0.07 and 0.2 for $p = 2$; 0.05, 0.2 and 0.2 for $p = 3$; 0.08, 0.2 and 0.6 for $p = 4$, respectively. The difference of the invariants between our method and quadratic Petrov-Galerkin method is too little at the time $t = 12$.

Also Figure 5(a)-(d), Figure 6(a)-(d) illustrates the development of the Maxwellian initial condition into solitary waves. In Figure 5(a) and Figure 6(a), the solitary wave with larger one is on the right of the smaller one. For $\delta = 0.1$, only single stable solition appeared. When $\delta = 0.05$, two stable solitary wave appeared in Figure 5(b) and Figure 6(b). As seen in Figure 5(c), (d) and Figure 6(c), (d), three and five stable solitary wave occured at the $\delta = 0.025$ and $\delta = 0.01$, respectively. It is understood from these figures that as the value of δ is decrease, the number of the stable solitary wave is increase.

Table 10. The invariants for Maxwellian initial condition.

δ	t	$p = 2$			$p = 3$			$p = 4$		
		I_1	I_2	I_3	I_1	I_2	I_3	I_1	I_2	I_3
	0	1.772453	1.265847	0.886226	1.772453	1.265847	0.792665	1.772453	1.265847	0.723601
	4	1.773567	1.272162	0.913749	1.776431	1.280719	0.851731	1.803566	1.363095	1.083499
0.010	8	1.774354	1.273668	0.905325	1.782107	1.293911	0.847407	1.805571	1.392182	1.468426
	12	1.773219	1.267638	0.897781	1.788222	1.329233	1.014441	1.757360	1.218707	0.577822
QBSPG[Roshan (2011)]	12	1.772400	1.265800	0.886200	1.772400	1.266500	0.794700	1.772500	1.266900	0.725300
	0	1.772453	1.284646	0.886226	1.772453	1.284646	0.792665	1.772453	1.284646	0.723601
	4	1.772624	1.285168	0.887871	1.772841	1.285658	0.799622	1.776099	1.298322	0.787247
0.025	8	1.772635	1.285208	0.887926	1.772963	1.285086	0.792383	1.770003	1.274934	0.705119
	12	1.772636	1.285180	0.887737	1.772636	1.283938	0.793308	1.777013	1.302710	0.808295
QBSPG[Roshan (2011)]	12	1.772400	1.283500	0.885600	1.772300	1.283400	0.791000	1.772400	1.284900	0.724300
	0	1.772453	1.315979	0.886226	1.772453	1.315979	0.792665	1.772453	1.315979	0.723601
	4	1.772519	1.316150	0.886577	1.772578	1.316226	0.793414	1.772432	1.315294	0.722397
0.050	8	1.772520	1.316152	0.886582	1.772577	1.316198	0.793400	1.772717	1.316536	0.726374
	12	1.772520	1.316151	0.886579	1.772592	1.316254	0.793420	1.773333	1.318824	0.731885
QBSPG[Roshan (2011)]	12	1.772400	1.316000	0.886100	1.772400	1.315600	0.792200	1.772400	1.317700	0.724500
	0	1.772453	1.378645	0.886226	1.772453	1.378645	0.792665	1.772453	1.378645	0.723601
	4	1.772478	1.378707	0.886327	1.772501	1.378748	0.792856	1.772530	1.378826	0.724088
0.100	8	1.772479	1.378707	0.886327	1.772500	1.378745	0.792853	1.772531	1.378843	0.724131
	12	1.772479	1.378707	0.886327	1.772499	1.378742	0.792847	1.772524	1.378812	0.724054
QBSPG[Roshan (2011)]	12	1.772400	1.378500	0.886100	1.772400	1.378700	0.792600	1.773400	1.383600	0.722400

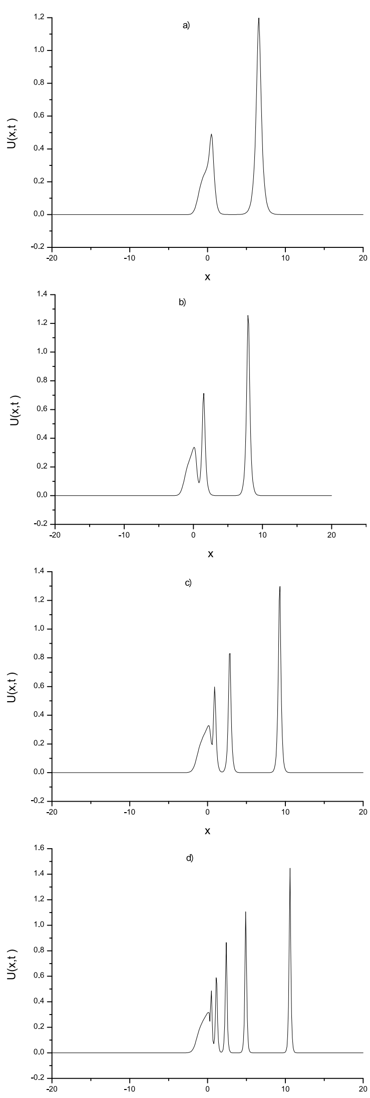


Fig. 5. Maxwellian initial conditon $p = 3$ at $t = 12$; *a*) δ = 0.1,*b*) δ = 0.05,*c*) δ = 0.025,*d*) δ = 0.01.

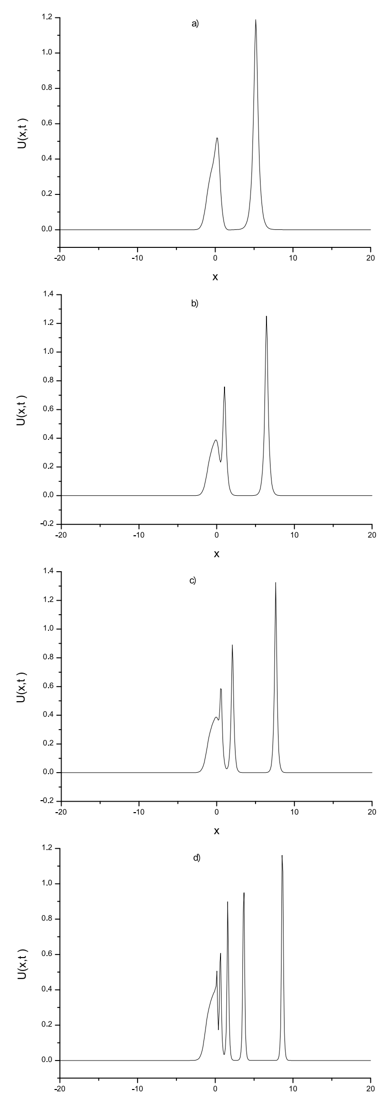


Fig. 6. Maxwellian initial conditon $p = 4$ at $t = 12$; *a*) δ = 0.1,*b*) δ = 0.05,*c*) δ = 0.025,*d*) δ = 0.01.

5. Conclusion

In this paper, a numerical scheme based on the septic B-spline collocation method have been implemented to find the numerical solution of the GEW equation by using two different linearization techniques. To show the accuracy of the method, we have solved the three test problems including single soliton, interaction of solitons and Maxwellian initial condition by calculating the error norms L_2, L_∞ and the invariants I_1, I_2, I_3. As seen from the tables, for each linearization technique, the changes of the invariants are adequately small and consistent with previous numerical results. The quantity of obtained error norms are less than the ones in existing collocation methods Evans & Raslan (2005), Raslan (2006) and Petrov-Galerkin method Roshan (2011) for each linearization technique. So, our numerical algorithm is efficient and reliable numerical technique for solving the GEW equation and can be efficiently applied to similiar types of non-linear partial differential equations.

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طريقة رصف محور B الخمجة معادلة موجية متساوية العرض معممة

[1]س بطال غازي كاراكوتش، [2]خليل زيبك
[1]قسم الرياضيات – كلية العلوم والفن – جامعة نفسهير حاجي بكتيس فيلي – تركيا.
[2]قسم تطبيق ورقة الرياضيات – قسم العلوم الرياضية – جامعة عبد الله جول – تركيا.
المؤلف البريد الالكتروني: sbgk44@gmail.com

خلاصة

نستخدم في هذا البحث طريقة رصف محور B الخمجة لإيجاد الحل العددي لمعادلة موجية متساوية العرض معممة وذلك بإستخدام تقنيتين مختلفتين للأخطاط. ونقوم بحل مسائل اختبار لتقنياتنا بما في ذلك حل مسائل السوليتون الفريد، تفاعل السوليتونات و شروط ماكسويل الابتدائية و ذلك للتحقق من طريقتنا المقترحة وذلك بحساب معياري الخطأ L_2 و L_∞ وكذلك اللامتغيرات I_1،I_2،I_3. ونستخدم تحليل فون – نويمان للاستقرار لنثبت بأن طريقتنا المقترحة مستقرة بشكل غير مشروط. كما يتبين أيضاً أن نتائجنا تتفق بشكل جيد مع بعض النتائج الحديثة.