Efficient transformed ratio-type estimator using single auxiliary information

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Abstract

This paper provides an efficient transformed ratio-type estimator to estimate the study variable's population variance by utilizing information of a single auxiliary variable under simple random sampling without replacement. The bias and mean squared error of the proposed estimator are derived up-to 1st order approximation. In addition to this, the efficiency comparison of the proposed estimator has been done with traditional ratio-type variance estimator and some other widely used modified ratio-type variance estimators by taking real-life data. A simulation study has also been carried out to see the performance of the proposed estimator. It is worth noticing that our proposed estimator performs better than the competing estimators in real-life data applications as the mean squared error and root mean squared error of our proposed estimator are smaller than the competing estimators. Hence, our proposed estimator is better than existing variance estimators.

Keywords: Auxiliary information; bias; mean squared error; simple random sampling; study variable.

1. Introduction

The estimation of the population variance has significance in different life areas, e.g., industries, agriculture, and medical sciences. It is conventional that the population variance is unknown and has to be estimated using given data. In this paper, an attempt has been made to estimate this unknown characteristic reasonably and efficiently. The principle aim of this paper is to propose a method that can be used to estimate dispersion efficiently. Various estimators are available in the literature to estimate variation, including ratio, regression, transformed regression, and ratio-type variance estimators. Some of the authors used the above techniques in randomized response models (RTT) like Singh & Suman, 2019.

Consider a finite population of size N from which a sample of size n is to be drawn under simple random sampling (SRSWOR). Suppose

y denotes the study variable, and x is auxiliary information about which some information is available. Some necessary notations to be used in the paper are given as

$\bar{Y}, S_Y^2$: Population mean and variance of Y

$\bar{X}, S_X^2$: Population mean and variance of X

The sample counterparts of means and variances will be denoted by small case roman letters. ρ: The population correlation coefficient, r: The sample correlation coefficient, and S_{xy}: Covariance between X and Y. The coefficient of variations for Y and

X will be denoted by C_y and C_x. Also $\beta_2'(y) = \mu_{40}/\mu_{20}^2$ and $\beta_2'(x) = \mu_{04}/\mu_{02}^2$ be the coefficient of kurtosis for Y and X, respectively where,

$$\mu_{st} = \frac{1}{N}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^s \left(X_i - \bar{X}\right)^t$$

is (s, t) the central order moment. Some additional notations are $\lambda' = n^{-1}$, $\lambda = (Nn)^{-1}(N-n)$ and $a = \mu_{22}/\mu_{20}\mu_{02}$.

Some important expectations given by Yadav *et al*., 2013, which are useful in computing mean square error of an estimator, are given as

$$E(e_0) = 0, \quad E(e_1) = 0 \ , \quad E(e_0^2) = \lambda\beta_2^{\wedge}(y)$$

$$E(e_1^2) = \lambda\beta_2^{\wedge}(x) \quad , \quad E(e_0 e_1) = \lambda a'$$

$$\beta_2^{\wedge}(y) = \beta_2'(y) - 1 \ , \ \beta_2^{\wedge}(x) = \beta_2'(x) - 1$$

$$a' = a - 1 \quad , \quad \mu' = \frac{a'}{\beta_2^{\wedge}(x)}$$

For estimation of the population variance, Isaki, 1983, proposed a ratio-type estimator by using one auxiliary information (x). Some other references are Upadhyaya & Singh, 1999; Ahmed *et al*., 2000; Kadilar & Cingi, 2006; Subramani & Kumarapandiyan, 2012a, b; Subramani & Kumarapandiyan, 2013; Chami *et al*., 2012; Singh & Solanki, 2013a, b; Yadav & Kadilar, 2014; Ismail *et al*., 2018. and Akhlaq *et al*., 2019. Few biased variance estimators, as well as their bias and mean squared errors, are given below:

Isaki, 1983, ratio-type estimator

$$t_{Is} = s_y^2\left(\frac{S_x^2}{s_x^2}\right) \quad (1)$$

With bias and mean squared error given as

$$BIAS(t_{Is}) \cong \lambda S_y^2\{\beta_2^{\wedge}(x) - a'\}$$

$$MSE(t_{Is}) \cong \lambda S_y^4\{\beta_2^{\wedge}(y) + \beta_2^{\wedge}(x) - 2a'\} .$$

Upadhyaya & Singh, 1999, suggested following ratio-type estimator

$$t_{upsi} = s_y^2\left[\frac{S_x^2 + \beta_2'(x)}{s_x^2 + \beta_2'(x)}\right] \quad (2)$$

With bias and mean squared error given as

$$BIAS(t_{upsi}) \cong \left[\begin{array}{l} \lambda S_y^2 \beta_2^{\wedge}(x) \dfrac{S_x^2}{S_x^2 + \beta_2'(x)} \\ \left(\dfrac{S_x^2}{S_x^2 + \beta_2'(x)} - \mu'\right) \end{array}\right]$$

$$MSE(t_{upsi}) \cong \lambda S_y^4 \left[\begin{array}{l} \beta_2^{\wedge}(y) + \beta_2^{\wedge}(x) \dfrac{S_x^2}{S_x^2 + \beta_2'(x)} \\ \left(\dfrac{S_x^2}{S_x^2 + \beta_2'(x)} - 2\mu'\right) \end{array}\right]$$

Kadilar & Cingi, 2006, proposed following four modified ratio type estimators

$$t_{KC1} = s_y^2\left[\frac{S_x^2 - \beta_2'(x)}{s_x^2 - \beta_2'(x)}\right] \quad (3)$$

$$t_{KC2} = s_y^2\left[\frac{C_x S_x^2 - \beta_2'(x)}{C_x s_x^2 - \beta_2'(x)}\right] \quad (4)$$

$$t_{KC3} = s_y^2\left[\frac{S_x^2 - C_x}{s_x^2 - C_x}\right] \quad (5)$$

$$t_{kc4} = s_y^2\left[\frac{\beta_2'(x) S_x^2 - C_x}{\beta_2'(x) s_x^2 - C_x}\right] \quad (6)$$

with biases and mean squared errors given as

$$BIAS(t_{KCi}) \cong \lambda S_y^2 \beta_2^{\wedge}(x) B_i (B_i - \mu')$$

$$MSE(t_{KCi}) \cong \lambda S_y^4 \left[\beta_2^{\wedge}(y) + \beta_2^{\wedge}(x) B_i (B_i - 2\mu') \right]$$

$i = 1, 2, 3, 4$

$$B_1 = \frac{S_x^2}{S_x^2 - \beta_2'(x)} \quad , \quad B_2 = \frac{C_x S_x^2}{C_x S_x^2 - \beta_2'(x)}$$

$$B_3 = \frac{S_x^2}{S_x^2 - C_x} \quad , \quad B_4 = \frac{\beta_2'(x) S_x^2}{\beta_2'(x) S_x^2 - C_x}$$

Chami *et al*., 2012, estimator

$$t_{ch} = \bar{y} \left[\alpha \{ \frac{(1-\beta)\bar{x} + \beta \bar{X}}{\beta \bar{x} + (1-\beta)\bar{X}} \} + (1-\alpha) \{ \frac{\beta \bar{x} + (1-\beta)\bar{X}}{(1-\beta)\bar{x} + \beta \bar{X}} \} \right] \quad (7)$$

with bias and mean squared error

$$BIAS(t_{ch}) = \frac{1-f}{n} (1-2\beta) \left[1 - \alpha - \beta - (1-2\alpha) C \right] C_x^2 \bar{Y}$$

$$MSE(t_{ch}) = \frac{1-f}{n} S_Y^2 (1-\rho^2)$$

Ismail *et al*., 2018, estimator

$$t_{RPG} = (s_y^2 + a) \left[\frac{\lambda \left(\frac{S_g^2}{s_g^2} \right)^{\tau} + \left(1 - \lambda - \omega \right)}{\left(\frac{s_g^2}{S_g^2} \right)^{\chi} + \omega \left(\frac{S_n^2}{s_n^2} \right)^{\vartheta}} \right] - a \quad (8)$$

with bias and mean squared error

$$Bias(t_{RPG}) \cong \frac{1}{2} \lambda S_y^2 \vartheta_0 \left[\begin{array}{l} \beta_2^{\wedge}(x) h_1^2 \{ \lambda \tau^2 + (1-\lambda-\omega)\chi^2 \} + \\ \beta_2^{\wedge}(x) \{ \lambda \tau - (1-\lambda-\omega)\chi \} h_1 \{ h_1 - 2\mu' \} + \\ \beta_2^{\wedge}(z) \omega \theta^2 h_2^2 + \beta_2^{\wedge}(z) \omega \theta h_2 (h_2 - 2v') \end{array} \right]$$

$$MSE(t_{RPG}) \cong \lambda S_y^4 \left[\begin{array}{l} \beta_2^{\wedge}(y) + \beta_2^{\wedge}(x) N_1 (N_1 - 2\mu') + \\ \beta_2^{\wedge}(z) N_2 (N_2 - 2v') + 2 N_1 N_2 c' \end{array} \right]$$

where

$N_1 = (\beta_2^{\wedge}(z) a' - b'c') / E$

$N_2 = (\beta_2^{\wedge}(x) b' - a'c') / E$

and

$E = \beta_2^{\wedge}(x) \beta_2^{\wedge}(z) - c'^2$

2. Proposed efficient transformed ratio type estimator using single auxiliary information

By taking motivation from Chami *et al*., 2012, we have suggested a new estimator based on the single auxiliary variate information.

$$t_{sa} = s_y^2 \left[\left\{ \frac{S_x^2 - \beta_{xy}(S_x^2 - s_x^2)}{s_x^2 + \beta_{xy}(S_x^2 - s_x^2)} \right\}^{\alpha} \left\{ \frac{S_x^2 + (S_x^2 - s_x^2)}{s_x^2 - (S_x^2 - s_x^2)} \right\}^{1-\alpha} \right] \quad (9)$$

We aim to derive the value of constant α such that the mean squared error (MSE) and root mean squared error (RMSE) of the proposed estimator t_{sa} are minimum. In order to study the large sample properties of the proposed estimator t_{sa}, we define $s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_1)$.

Now replacing s_y^2 and s_x^2 in equation (9), we have

$$t_{sa} = S_y^2(1+e_0) \left[\begin{array}{l} \left\{ \frac{S_x^2 - \beta_{xy}(S_x^2 - S_x^2(1+e_1))}{S_x^2(1+e_1) + \beta_{xy}(S_x^2 - S_x^2(1+e_1))} \right\}^{\alpha} \\ \left\{ \frac{S_x^2 + (S_x^2 - S_x^2(1+e_1))}{S_x^2(1+e_1) - (S_x^2 - S_x^2(1+e_1))} \right\}^{1-\alpha} \end{array} \right]$$

By retaining powers up to e_1^2 we get

$$t_{sa} - S_y^2 \cong S_y^2 \left[\begin{array}{l} e_0 - 3e_1 - 3e_0 e_1 + 2\alpha e_1 + \\ 2\alpha e_0 e_1 + 2e_1^2 - \alpha e_1^2 - \alpha^2 e_1^2 + \\ 2\alpha \beta_{xy} e_1 + 2\alpha \beta_{xy} e_0 e_1 - 6\alpha e_1^2 \beta_{xy} + \\ 6\alpha^2 e_1^2 \beta_{xy} - \alpha^2 \beta_{xy} e_1^2 + \alpha^2 e_1^2 \beta_{xy}^2 \end{array} \right]$$

Bias of new estimator t_{sa} up-to 1st-degree approximation is

$$BIAS(t_{sa}) \cong \lambda S_y^2 \left[\begin{array}{l} -3a' + 2\alpha a' + 2\beta_2^{\wedge}(x) - \alpha \beta_2^{\wedge}(x) - \\ \alpha^2 \beta_2^{\wedge}(x) + 2\alpha \beta_{xy} a' - 6\alpha \beta_{xy} \beta_2^{\wedge}(x) + \\ 5\alpha^2 \beta_{xy} \beta_2^{\wedge}(x) + \alpha^2 \beta_{xy}^2 \beta_2^{\wedge}(x) \end{array} \right]$$

The mean squared error (MSE) of the new estimator t_{sa} is

$$MSE(t_{sa}) \cong \lambda S_y^4 \begin{bmatrix} \beta_2^{\wedge}(y) + 9\beta_2^{\wedge}(x) + 4\alpha^2\beta_2^{\wedge}(x) + \\ 4\alpha^2\beta_{xy}^2\beta_2^{\wedge}(x) - 6a' + 4\alpha a' + 4\alpha\beta_{xy}a' - \\ 12\alpha\beta_2^{\wedge}(x) - 12\alpha\beta_{xy}\beta_2^{\wedge}(x) + 8\alpha^2\beta_{xy}\beta_2^{\wedge}(x) \end{bmatrix}$$

Where the optimal value of α is attained by reducing the above equation and is given as

$$\alpha = \left[\frac{(-a' + 3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2 + 2\beta_{xy})}\right]$$

By utilizing the optimal value of α the minimum mean squared error of proposed estimator is

$$MSE(t_{sa})_{\min} = \lambda S_y^4 \begin{bmatrix} \beta_2^{\wedge}(y) + 9\beta_2^{\wedge}(x) + 4\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right]^2 \beta_2^{\wedge}(x) + \\ 4\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right]^2 \beta_{xy}^2\beta_2^{\wedge}(x) - \\ 6a' + 4\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right] a' + \\ 4\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right] \beta_{xy}a' - \\ 12\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right] \beta_2^{\wedge}(x) - \\ 12\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right] \beta_{xy}\beta_2^{\wedge}(x) + \\ 8\left[\frac{(-a'+3\beta_2^{\wedge}(x))(1+\beta_{xy})}{2\beta_2^{\wedge}(x)(1+\beta_{xy}^2+2\beta_{xy})}\right]^2 \beta_{xy}\beta_2^{\wedge}(x) \end{bmatrix}$$

3. Special case of new proposed estimator of population variance

If we put α = 1 and β_{xy} = 0, the proposed estimator reduces to Isaki, 1983 ratio estimator.

4. Results

This section provides twofold studies. Firstly, an empirical analysis is carried out by considering two real populations; later on, a simulation study is conducted to prove the suitability of the new estimator t_{sa} .

4.1 Empirical analysis

To carry out the empirical analysis, 2 data sets are occupied from Basic Econometrics (2004) by Gujarati and Sangeetha.

Data 01: Data is collected about 81cars where variable y is the average no of covered miles, and variable x is engine horsepower. The data summary measures are given in Table 1, whereas the computed values of mean squared error and root mean squared errors are given in Table 2.

Table 2 compares the proposed estimator t_{sa} over traditional ratio-type variance estimator and widely used modified ratio-type variance estimators. It can be seen that as the sample size (n) increases, the mean squared error (MSE) and root mean squared error (RMSE) decreases. The smaller the value of mean squared error (MSE) and root mean squared error (RMSE) indicates the more extensive the percentage relative efficiency (PRE). Table 2 shows that the proposed estimator t_{sa} has less MSE, RMSE, and larger percentage relative efficiency (PRE) overall

existing estimators

Table 1. Data description.

C_y	C_x	S_y^2	S_x^2	$\beta_2^{'}(y)$
0.2971934	0.4862001	101.1113	3261.952	3.591309
$\beta_2^{'}(x)$	a	ρ_{xy}	n	
5.707267	2.230698	-0.7944504	10,20	

Table 2. Mean squared error (MSE), and root mean squared error (RMSE), and percentage relative efficiency (PRE) of different estimators.

Estimators	MSE		RMSE		PRE	
	n = 10	n = 20	n = 10	n = 20	n = 10	n = 20
t_0	2322.159	997.547	48.18878	31.58397	100	100
t_{Is}	4334.759	1862.115	65.83888	43.15223	53.57065	53.57065
t_{upsi}	4323.889	1857.445	65.75628	43.09809	53.70533	53.70533
t_{KC1}	4345.693	1866.812	65.92187	43.20662	53.43587	53.43587
t_{KC2}	4357.317	1871.805	66.00998	43.26437	53.29332	53.29332
t_{KC3}	4335.688	1862.514	65.84594	43.15685	53.55918	53.55918
t_{KC4}	4334.921	1862.185	65.84012	43.15304	53.56865	53.56865
t_{suku1}	4153.154	1784.101	64.44497	42.23862	55.91314	55.91314
t_{suku2}	4186.259	1798.323	64.70131	42.40663	55.47098	55.47098
t_{suku3}	4085.482	1755.031	63.91777	41.89309	56.83929	56.83929
t_{suku4}	4225.391	1815.133	65.00301	42.60438	54.95725	54.95725
t_{suku5}	4279.252	1838.27	65.41599	42.87505	54.26553	54.26553
t_{suku6}	4135.128	1776.358	64.30496	42.14686	56.15688	56.15688
t_{suku7}	3980.021	1709.727	63.08741	41.34885	58.34539	58.34539
t_{sa}	2033.817	873.6821	45.09786	29.55811	114.1773	114.1773

Data 02: Data is collected about literacy rate of 64 countries where

y: no of children died under age five in a year per 1000 live births   x: total fertility rate

Table 3. Data description.

C_y	C_x	S_y^2	$\beta_2'(y)$	$\beta_2'(x)$
0.5369475	0.271906	5772.667	2.378336	2.816908
a	ρ_{xy}	S_x^2	n	
1.481965	0.6711349	2.27706	8,16	

Table 4. Mean squared error (MSE), and root mean squared error (RMSE), and percentage relative efficiency (PRE) of different estimators.

Estimators	MSE		RMSE		PRE	
	n = 8	n = 16	n = 8	n = 16	n = 8	n = 16
t_0	5023729	2153027	2241.368	1467.32	100	100
t_{Is}	8132649	3485421	2851.78	1866.928	61.77235	61.77235
t_{upsi}	4776488	2047066	2185.518	1430.757	105.17620	105.17620
t_{KC1}	137660302	58997272	11732.87	7680.968	3.64936	3.64936
t_{KC2}	6539055	2802452	2557.158	1674.053	76.82652	76.82652
t_{KC3}	9573998	4103142	3094.188	2025.621	52.47263	52.47263
t_{KC4}	8576397	3675599	2928.549	1917.185	58.57621	58.57621
t_{suku1}	4558229	1953527	2135.001	1397.686	110.21229	110.21229
t_{suku2}	4592052	1968022	2142.907	1402.862	109.40052	109.40052
t_{suku3}	4558361	1953583	2135.032	1397.706	110.20910	110.20910
t_{suku4}	4966698	2128585	2228.609	1458.967	110.11482	110.11482
t_{suku5}	5690191	2438653	2385.412	1561.619	88.28752	88.28752
t_{suku6}	4562076	1955175	2135.902	1398.276	110.11936	110.11936
t_{suku7}	4754320	2037566	2180.44	1427.433	105.66661	105.66661
t_{sa}	4557748	1953321	2134.888	1397.613	110.2239	110.2239

Table 4 represents the suitability of the proposed estimator t_{sa} over traditional ratio-type variance estimator and widely used modified ratio-type variance estimators. It is evident from table 4 that as sample size (n) increases, the mean squared error (MSE) and root mean squared error (RMSE) decreases. The smaller the value of mean squared error (MSE) and root mean squared error (RMSE) indicates the more extensive the percentage relative efficiency (PRE). In table 4, the proposed estimator t_{sa} shows less MSE, RMSE, and larger percentage relative efficiency (PRE) overall existing estimators.

4.2. Simulation study results

In this section, a simulation study is done to see the performance of the suggested estimator t_{sa}. A simulation study is carried out by using a multivariate normal distribution, and we generated a population of size 10,000. This process is repeated 1000 times, and then we computed MSE, RMSE, and PRE of the proposed and existing estimators.

Table 5 represents that the new suggested estimator t_{sa} is showing less (MSE) and (RMSE) as well as larger percentage relative efficiency (PRE) among all existing estimators.

Table 5. Mean squared error (MSE), and root mean squared error (RMSE), and percentage relative efficiency (PRE) at different values of correlation coefficients.

Estimators	MSE	RMSE	PRE
	r = 0.88	**r = 0.88**	**r = 0.88**
t_0	0.0001987291	0.01441634	100
t_{Is}	8.829262e-05	0.009461709	216.9821
t_{upsi}	0.0001345604	0.011834	147.9304
t_{KC1}	0.0003945465	0.02020014	49.55449
t_{KC2}	8.561994e-05	0.009442035	221.7021
t_{KC3}	0.0001954534	0.01440238	101.1279
t_{KC4}	0.0001889825	0.01437263	103.3942
t_{suku1}	9.075502e-05	0.009523128	216.2621
t_{suku2}	0.001098177	0.03672747	15.1962
t_{suku3}	8.437891e-05	0.009226042	231.697
t_{suku4}	0.0001024148	0.01024249	191.5811
t_{suku5}	8.440008e-05	0.009237856	230.711
t_{suku6}	8.836804e-05	0.00950407	213.4412
t_{suku7}	8.826834e-05	0.009461637	216.9873
t_{sa}	7.817046e-05	0.0088582	247.0853

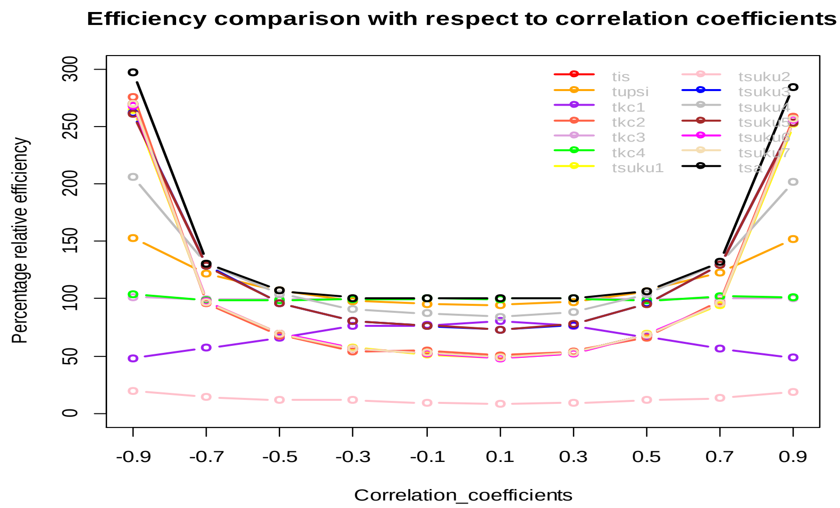

Fig. 1. This shows that the suggested estimator performs better for all the correlation coefficient values (r) over modified ratio-type existing estimators.

5. Conclusion

In this paper, we suggested an efficient transformed ratio-type estimator for estimation of population variance of study variable (y) by utilizing single auxiliary information (x) under simple random sampling without replacement. We perceived that our proposed estimator provided Isaki, 1983, a ratio-type estimator, as a particular case of the new estimator t_{sa}. The mean squared error and bias of the suggested estimator are derived up-to 1st order approximation. The suggested estimator's performance has been seen through empirical analysis and simulation study, and it can be evaluated that the mean squared error (MSE) and root mean squared error (RMSE) of the suggested estimator is less than the MSE and RMSE of the well-known existing estimators. We proved that our proposed estimator performs better than the competing estimators used in the study as our proposed estimator has a smaller mean squared error over competing estimators. This indicates that our proposed estimator is likely to estimate the variance with less error than other estimators. This also indicates that the confidence intervals for variance based upon our proposed estimator will be more reliable than the confidence intervals constructed by using some other estimator. In addition to this, our proposed estimator is a generalized type estimator and hence can be used in broader situations. Various choices of α will lead to a different estimator, and hence the estimator provides broader applicability compared to the existing estimators. The estimator can be effectively used to estimate variability in different areas of life, e.g., in the medical field, a physician can use it to know the variation in the degree of human blood pressure, body temperature, and heartbeat and pulse rate for an adequate prescription. An agriculturist can use this method to identify all those factors that can affect crop yield. The manufacturer can also use this method to recognize consumers' variation level and disapprove of newly launched products.

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