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Vehicle routing problem for minimizing consumption of energy in three dimensional space

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Abstract

The vehicle routing problem VRP is usually studied in two dimensional Euclidean spaces. In this paper a variant of VRP was proposed, when the points are lying in the three dimensional space, as it is often the case in the real problem. The cost matrix of the consumed energy was not symmetric. The minimum cost of total consumed energy was determined by identical vehicles. A new method was presented to compute the distance between every two points and the consumed energy.

Keywords: Consumed energy in three dimensional space; metaheuristics; transportation; vehicle routing problem.

1. Introduction

The vehicle routing problem appears naturally as a central problem in the fields of transportation, distribution, and logistics (Dantzig & Ramser, 1959). The vehicle routing problem (VRP) formulation was first introduced by Dantzig & Ramser (1959), as a generalization of the traveling salesman problem (TSP) presented by Flood (1956). The VRP is generally defined on a graph $G = \{V, E, C_{ij}\}$, where $V = \{v_0, \dots, v_n\}$ is the set of vertices, $E = \{(v_i, v_j) \mid (v_i, v_j) \in V^2, i \neq j\}$ is the arc set; and $C = (c_{ij})_{(v_i, v_j) \in E}$ is a cost matrix defined over E, representing distances, travel times, or travel costs. The vertex v_0 is called depot, while the remaining vertices in V represent customers (or requests) need to be served. The VRP is defined as the finding of routes set for K identical vehicles based at the depot, such that each of the vertices is visited exactly once, while minimizing the overall routing cost (Pillac *et al.,* 2013). VRP, like many other problems in the fields of transportation, is an NP - Hard problem. It is often desirable to obtain approximated solutions for NP Hard problems. Usually, this task is accomplished by employing various heuristic and metaheuristic methods. Many heuristic approaches such as sweep algorithm (Gillett & Miller, 1974), saving algorithm (Clarke & Wright, 1964), decomposition algorithm (Fisher & Jaikumar, 1978) and column generation (Desrosiers *et al.,* 1984) have been employed by researchers to solve vehicle routing problem. Other new kinds of popular heuristic methods are metaheuristic, which have appeared in the last 30 years. In general, metaheuristic methods are essential in solving complex optimization problems. Since metaheuristic approaches like tabu search (TS) (Taillard, 1993), simulated annealing (*SA*) (Osman, 1993), record -to- record travel (Li *et al.,* 2007), genetic algorithms (Nazif & Lee, 2012) and ant colony optimization (Zhang & Tang, 2009) are very efficient in escaping from local optimum values. They are among the best algorithms to solve combinatorial optimization problems. The VRP has been widely studied for over 50 years (Laporte, 2009 and Golden *et al.,* 2008). There are a lot of variants of VRP in view of new constraints in the basic model such as the vehicle routing problem with time windows VRPTW (Chen & Wang, 2012), open vehicle routing problem OVRP (Sedighpour *et al., 2*014), and so on with varying types of constraints (Geetha *et al.,* 2012 and Tas *et al.,* 2014). For example, in VRPTW the service at each customer must start within an associated time window and the vehicle must remain at the customer location during the service provision.

This paper aims to propose a variety of VRP and propose a method to solve this problem. In the real world, points (customers and depot) are on the non flat surface in the three dimensional space and the cost matrix is not symmetric. The offered problem is based on considering these facts. We define a variety of VRP as follows:

Consider a finite set of points in the three dimensional Euclidean space. We marked a subset of that as customers and depot. We intend to find minimal cost of total consumed energy of vehicles that are to serve a number of customers from a central depot. Here, the consumed energy is not symmetric from one customer to other and location of customers is not on the plane and the distance between every two points is not Euclidian. We wish to use our method to calculate the distance between every two points by finding a surface passing through two customers and computing the length of geodesic as a curve which connects every two points. Obtaining cost matrix in the three dimensional space is related to factors, which should be calculated. In Ghahremani-Gol *et al.* (2012), we gave the distance between every two points as desired in a special case. It is proved that there is only one two dimensional manifold for every two points (here two customers), which passes through them not for all exiting points. In this paper, we intend to consider the surface in the general case. A variety of VRP is three-dimensional loading capacitated vehicle routing problem, which considers the loads as a cube, but non of depot and customers are not in the three dimensional space (Tarantilis *et al.,* 2009 and Wisniewski *et al.,* 2011). As we know, the VRP is a generalization of the traveling salesman problem. Our problem can be considered as a generalization of asymmetric traveling salesman problem (ATSP) in the three dimensional space, which is an $NP - Hard$ problem. There are papers that study ATSP with some constraint (Ascheuer, 2001 and Roberti & Toth, 2012). Therefore, the variant of VRP presented here is a NP - Hard problem. The remaining parts of this paper are organized as follows: Notations and mathematical modeling of

VRP are explained in Section 2. The method applied to solve the problem is presented in Section 3. Computational results are shown in Section 4. The paper is concluded in Section 5.

2. Mathematical modeling of the problem

To clarify the problem addressed in this paper mathematical programming formulation is presented here. Considering $n+1$ points, $V=\{v_0,...,v_n\}$ such that every point has coordinate (x_i, y_i, z_i) in R^3 and v_0 is depot and other vertices are customers, $E=\{(v_i,v_j) \mid (v_i,v_j)\in V^2, i\neq j\}$ is the arc set; and $G=(G_{ij})_{(v_i,v_j)\in E}$ is a matrix of consumed energy defined over E and $D=(d_{ij})_{(v_i,v_j)\in E}$ is a distance matrix over E. In this problem $G_{ij}\neq G_{ji}$. $C=(c_{ij})$ is the cost matrix of consumed energy. Since the G_{ij} is not symmetric, it is evident that $c_{ij}\neq c_{ji}$. The major goal is to minimize the cost of total consumed energy. The objective function of the mathematical model is as follows:

$$min\sum_{i=0}^{n}\sum_{j=0}^{n} c_{ij} X_{ij}$$

subject to

$$\sum_{i=0}^{n} X_{ij} = 1 \quad \forall j = 1,\ldots,n \tag{2.1}$$

$$\sum_{i=0}^{n} X_{ij} - \sum_{i=0}^{n} X_{ji} = 0 \quad \forall j = 1,\ldots,n \tag{2.2}$$

$$\sum_{j=0}^{n} X_{0j} = k \quad \forall j = 1,\ldots,n \tag{2.3}$$

$$X_{ij} \in \{0,1\} \quad \forall i, j = 0,\ldots,n \tag{2.4}$$

(2.5) For every two points v_i and v_j, there is a two dimensional manifold $(S(v_i,v_j))$ such that it can be fitted to A and passed through v_i and v_j.

(2.6) Every two points v_i and v_j are related to each other by a geodesic on the $(S(v_i,v_j))$.

(2.7) The distance between v_i and v_j is the length of the geodesic connecting them. Where, Constraint (2.1) ensures that the vehicle arrives once at each customer. Constraint set (2.2) ensures that the vehicle leaves each customer once. Constraint set (2.3) determines the number of vehicles is equal to k. $X_{ij} = 1$ if the vehicle travels from customer v_i to customer v_j, X_{ij} is equal to zero otherwise.

3. Algorithm

The method applied to solve the problem is presented in this section. The algorithm adopted to minimize the cost of consumed energy consists of the following three parts:

- Developing an algorithm to obtain curves and distances among the costumers.
- Describing the mathematical formulas for G_{ij}.
- Applying one metaheuristic method to solve the problem.

The first part of the algorithm has been accomplished by a generalization of the method presented for distance among points in $\mathbb{R}^3$, as it is explained in Ghahremani-Gol *et al.* (2012). Let $\{v_0,...,v_n\}$ be a finite set of points in $\mathbb{R}^3$ (customers and depot). First, we find a surface (two dimensional manifold), which passes through two points v_i, v_j and then geodesic on the surface, whose joints v_i, v_j can be calculated. For the two given points, a geodesic (which is an analogue of a line in the Euclidean plane) is uniquely determined, that is a line passing through the two given points with the minimum length. In the second part, the cost of consumed energy from v_i to v_j is as follows: $c_{ij} = \omega G_{ij}$, where G_{ij} is the consumed energy moving from v_i to v_j and ω is the unit charge of energy. The consumed energy G_{ij} is related to consider some factors which should be evaluated moving from v_i to v_j. Finally, a generalization of ant colony algorithm introduced by Dorigo & Gambardella (1997) is applied to find the objective function.

3.1. Curve and distance among points

By generalizing the present method in Ghahremani-Gol *et al.* (2012), we give a surface on every two points of V and then exhibit a geodesic passing through them. Therefore, the distance among them is computable.

Remark 3.1. In application, we are not able to identify a specific surface passing through all of points. Actually, there are infinity many different ways to attribute a surface, but there is no way to prefer one to the other, even when we limit ourselves to polynomial surfaces. It is clear that since all of points are not on a known specific surface, we can not use arc length for distance between every two points. In Ghahremani-Gol *et al.* (2012) we gave a method to find a special surface passing through every specified two points and to solve problem. We have shown there, this definition is much plausible and in this section we employ this method but consider the surface in the general form. Indeed, the method to find distance between every two points in this paper is an extension of Ghahremani-Gol *et al.* (2012).

Step 1: Finding a surface which passes through two points v_i and v_j, using the least square method with conditions in this work. Consider $n+m+1$ points, $A=\{v_0,\ldots,v_n,p_{n+1},\ldots,p_{n+m}\}$ such that every point has its coordinate (x_i,y_i,z_i) in R^3. These points are extended to R^4 with $v_i'=(x_i,y_i,z_i,0)$ and $p_i'=(x_i,y_i,z_i,0)$. Through the least square method, the following surface is found that could be fitted to $\{v_1',\ldots,v_n',p_{n+1}',\ldots,p_m'\}$ and passes through v_i' and v_j'.

$$w=\sum_{i=0}^{n}\sum_{j=0}^{n-i}\sum_{k=0}^{n-i-j}a_{i,j,k}x^i y^j z^k \tag{3.1}$$

First, we should minimize the sum of square difference of the above equation with the given points, namely

$$D=\sum_{l=1}^{m+n+1}([\sum_{i=0}^{n}\sum_{j=0}^{n-i}\sum_{k=0}^{n-i-j}a_{i,j,k}x_l^i y_l^j z_l^k]-w_l)^2 \tag{3.2}$$

where in Eq. 3.2, parameters of w are obtained to solving the following system of equations:

$$\{\frac{\partial D}{\partial a_{i,j,k}}=0 \qquad ,for\quad i=0,\ldots,n\quad ,j=0,\ldots,n-i\quad ,k=0,\ldots,n-i-j\}$$

Now, it is evident that the following equation gives a favorite surface passing through v_i and v_j.

$$\sum_{i=0}^{n}\sum_{j=0}^{n-i}\sum_{k=0}^{n-i-j}a_{i,j,k}x^i y^j z^k=0 \tag{3.3}$$

Step 2: Solving the Euler Lagrange equations leads to obtaining the geodesic between the two points v_i and v_j.

Some theorems about computing geodesic between every two points and some of their properties are recalled in this section.

Theorem 3.2. (O'Neill, 1966) Let $r(u,v)=(u,v,f(u,v))$ be a parameterized surface *S*. A curve α on a surface *S* is a geodesic if and only if the following two equations are satisfied:

$$\frac{d^2u}{dt^2}+2\frac{du}{dt}\frac{dv}{dt}\Gamma_{12}^1+(\frac{du}{dt})^2\Gamma_{11}^1+(\frac{dv}{dt})^2\Gamma_{22}^1=0$$

$$\frac{d^2v}{dt^2}+2\frac{du}{dt}\frac{dv}{dt}\Gamma_{21}^{2}+(\frac{du}{dt})^2\Gamma_{11}^{2}+(\frac{dv}{dt})^2\Gamma_{22}^{2}=0$$

For a parameterized surface

$$\Gamma_{11}^{1}=r_{uu}.r_u,\quad \Gamma_{11}^{2}=r_{uu}.r_v$$

$$\Gamma_{12}^{1}=r_{uv}.r_u,\quad \Gamma_{12}^{2}=r_{uv}.r_v$$

$$\Gamma_{22}^{1}=r_{vu}.r_u,\quad \Gamma_{21}^{2}=r_{vu}.r_v$$

$$\Gamma_{22}^{1}=r_{vv}.r_u,\quad \Gamma_{22}^{2}=r_{vv}.r_v$$

According to the following theorem, when a surface is compact, the shortest path between two points p, q on the surface is the length of the geodesic passing through two points p, q.

Theorem 3.3. (Do Carmo, 1976) Hopf -Rinow

Let S be a complete surface. Given two points $p,q \in S$, there exists a minimal geodesic joining p, q.

Note that every compact surface is complete, i.e. for every point of $p \in S$, every geodesic $\gamma: I \to S$ with $\gamma(0)=p$ can be extended to all R. To solve the system of equations described in

Theorem (3.2), we can use the current numerical methods, which it has often error of order $o(h^m)$ (Griffiths & Higham, 2011).

The surface (3.3) can be parameterized in the form of $r(u,v)=(u,v,f(u,v))$, although we don't use it directly. We should calculate its derivatives which can be done implicitly.

Step 3: The distance between v_i and v_j is defined as the length of the above geodesic. The length of geodesic can be computed by the following simple formula.

$$d(v_i,v_j)=\int ds=\int_{t=t_0}^{t=t_1}\sqrt{x'(t)^2+y'(t)^2+z'(t)^2}\,dt$$

ds is the arc length parameter of $\gamma(t)$, the geodesic passing through v_i and v_j. $\gamma'(t)=(x'(t),y'(t),z'(t))$ is the tangent vector of the curve $\gamma(t)$.

3.2. Computing consumed energy.

We should compute G_{ij} which contains consumed energy due to friction, potential, and kinetic energy. Consider $G_{ij} = F_{ij} + P_{ij} + K_{ij}$ where F_{ij}, P_{ij}, and K_{ij} denote friction, potential, and kinetic energy respectively. Therefore;

$$F_{ij} = \int f_r ds \tag{3.4}$$

$$P_{ij} = Mg\Delta H_{ij} \tag{3.5}$$

$$K_{ij} = \frac{1}{2} m_{ij} v^2 \tag{3.6}$$

here M is the total mass of the vehicle and the vehicle load, g is the gravity of earth, and μ is the coefficient of kinetic friction. Let α_{ij} be the angle between curve connecting v_i to v_j and $z-$ axis, namely:

$$\cos\alpha_{ij} = \frac{z'(t)}{\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}} \tag{3.7}$$

Now f_r and ΔH_{ij} are represented as follow, respectively:

$$f_r = Mg\mu\cos\alpha_{ij},$$

$$\Delta H_{ij} = H_i - H_j$$

H_i and H_j are heights of v_i and v_j with respect to fixed point, respectively. We could assume depot $v_0 = (x_0, y_0, z_0)$ as a fixed point and let $H_i = z_0 - z_i, H_j = z_0 - z_j$. It is evident that, $F_{ij} = F_{ji}$ but $P_{ij} \neq P_{ji}$, however we have $P_{ij} = -P_{ji}$. Therefore, the cost energy matrix is not symmetric.

4. Computational results

In this section, the obtained computational results are presented for the proposed algorithm. The method defined in the previous sections of this paper has been applied to solve the ATSP in three dimensional space with the mentioned constraints and VRP with the variety which has been defined in this paper. For ATSP, 11 points have been considered in three dimensional space and $n = 3$ has been fixed (Table 1). The best solution has been obtained. when the matrix has been obtained based on the computations described in the previous section (Table 2). For the VRP variety defined in this paper, we consider 10 points as customers and one point as a depot in three dimensional space and $n = 3$ has been fixed. In addition, we consider five extra points to determine the surfaces more precisely. Let v_0 be a depot, $\{v_1,\ldots,v_{10}\}$ are costumers and $\{p_1,\ldots,p_5\}$ are extra points (Table 3 and Figures 1 and 2).

Table 1. The considered points in three dimensional space

	V0	**Vi**	**V2**	**V3**	**V4**	**V5**	**V6**	**V7**
Xi	1	-8	-12	21	-9	11	16	-3
Yi	1	-8	11	-7	-8	3	-9	-10
Zi	6	8	12	28	10	23	20	8
	V8	**V9**	**V1o**	**Pi**	**P2**	**P3**	**P4**	**P5**
Xi	3	12	-6	-10	5	12	2	16
Yi	-13	10	14	-8	-4	-8	-7	5
Zi	9	34	20	11	7	15	6	35

The following results for ATSP in three dimensional space are obtained:

Table 2. ATSP

	Best Cost	**Tour**
Based on computed energy	211.20863	10 4 6 7 3 11 5 2 1 8 9 10

The following results are obtained for VRP in three dimensional space:

Table 3. Cost of consumed energy for VRP

No. of customers	**No.of vehicle**	**Cost of computed energy**
10	3	262.91

Solving Euler-Lagrange equations to obtain geodesic between every two points, needs two parameters (V_1, V_2) whereas, $\overleftarrow{V} = (V_1, V_2)$ is the velocity at the initial point. In addition, computing the consumed energy requires two parameters g and μ which are the gravity of earth and the coefficient of kinetic friction, respectively. We consider the following values for the mentioned parameters (Table 4):

Table 4. Parameters

V1	**V2**	**g**	**μ**
1	1	9.8	0.02

4.1. Metaheuristic algorithm

The metaheuristic algorithm is based on Ant colony Algorithm for solving the given example, a well known approach to solving VRP. A similar algorithm has been presented by Yousefikhoshbakht *et al.* (2014) to give a solution for a variant of VRP.

Remark: As we said before in the real world, points (customers and depot) are on the non flat surfaces. This means the real distance between points is not Euclidian. Considering basic concepts of differential geometry, for example Theorem (3.3) we define distance between points as the length of geodesics which joints them. In Ghahremani-Gol *et al.*(2012) we discussed why it is plausible. To our knowledge, no study has so far considered the costumers and depot of VRP in the three dimensional space such that the distance between every tow points could be non-Euclidian. Moreover, computing consumed energy with factors considered in this paper is not done in the other work. Therefore, we cannot compare our methods with other papers. In other words, there were not a paper, which is exactly equivalent with the variant introduced here, to be compared to our method.

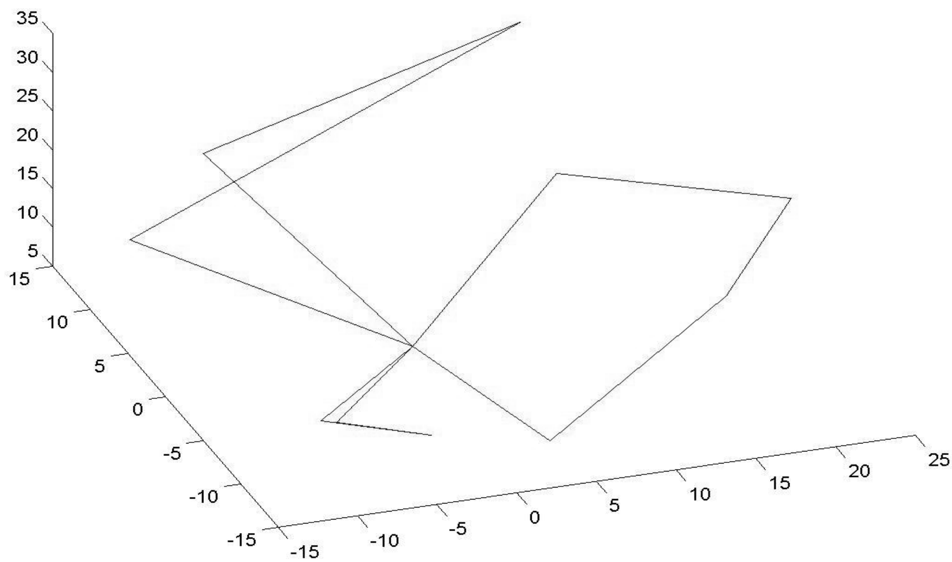

Fig. 1. Tour based geodesic and computed energy for VRP

5. Conclusions

The aim of this paper is to give a variant of the VRP and propose a suitable algorithm to solve this problem. Here, the points (customers and depot) are on the three dimensional space whereas, the distance between them is not Euclidian. Moreover, the matrix cost of consumed energy is not symmetric. A good suggestion to calculate the distance between every two points is computing the length of geodesic connecting them on a two dimensional manifold. The matrix cost of consumed energy can be obtained using formulas which compute consumed energy between every two points on a non-flat surface. The minimum cost of consumed energy has been found based on the above discussion by using a metaheuristic algorithm.

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مسألة توجيه المركبات لتقليل استهلاك الطاقة إلى أدنى حد في الفضاء ثلاثي الأبعاد

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خلاصة

عادة تتم دراسة مسألة توجيه المركبات VRP في الفضاءات الإقليدية ثنائية الأبعاد. في هذا البحث تم اقتراح نهج مختلف للمسألة VRP حينما تقع النقاط في الفضاء ثلاثي الأبعاد كما هو الحال غالبا في المسألة الفعلية الحقيقية. ومصفوفة تكاليف الطاقة المستهلكة لم تكن متماثلة. وقد تم تحديد أقل تكاليف للطاقة المستهلكة بمركبات متطابقة. وتم عرض طريقة جديدة لحساب المسافة بين كل نقطتين والطاقة المستهلكة.

الكلمات المفتاحية:

الطاقة المستهلكة في الفضاء ثلاثي الأبعاد، الأدلة العليا، التنقل، مسالة توجيه المركبات.