Kuwait J. Sci. **Vol. 40 (1) June 2013** pp. 81-92, 2013

Some topological and algebraic properties of paranorm *i*-convergent double sequence spaces

BIPAN HAZARIKA

Department of Mathematics, Rajiv Gandhi University, Doimukh, Itanagar-791 112, Arunachal Pradesh, India. E-mail: bh_rgu@yahoo.co.in

ABSTRACT

In this article some new double sequence spaces ${}_2c_0^I(p)$,${}_2c^I(p)$, ${}_2m_0^I(p)$ and ${}_2m^I(p)$ for $p=(p_{k,l})$ a double sequence of positive real numbers have been introduced. Some algebraic and topological properties of these spaces have been studied. The decomposition theorem and some inclusion relations are proved.

Keywords: Double sequence; ideal; *i*-convergence; *p*-convergence; paranorm.

INTRODUCTION

The notion of *I*-convergence was studied at the initial stage by Kostyrko *et al.* (2000). Later on, from sequence space point of view it was further investigated and linked with summability theory by Šalát *et al.* (2004, 2005), Tripathy & Hazarika (2009, 2011), Hazarika (2011), Savas (2010), Nabin *et al.* (2007), Dems (2004), Kostyrko *et al.* (2005) and many other authors. Also *I*-convergence has been discussed in more general abstract spaces such as 2-normed linear spaces by Gürdal (2006), *n*-normed linear spaces by Gürdal & Sahiner (2008). Tripathy & Tripathy (2005) introduced the concept of *I*-convergence and *I*-Cauchy sequence for double sequences and proved some properties related to the solidity, symmetricity, completeness and denseness. Kumar (2007) discussed the basic properties of *I*-convergence and I^*-convergence for double sequences. Also Pringsheim (1900), Robison (1926), Morciz (1991), Morciz & Rhoades (1988), Tripathy (2003), Gökhan & Colak (2004, 2005, 2006) studied on double sequences of real numbers.

DEFINITIONS AND NOTATIONS

Let X be a non empty set. Then a family of sets $I \subseteq 2^X$ (power sets of X) is said to be an *ideal* if I is additive (i.e. $A, B \in I \Rightarrow A \cup B \in I$) and hereditary (i.e. $A \in I, B \subseteq A \Rightarrow B \in I$). A non- empty family of sets $F \subset 2^X$ is said to be a *filter* on X if and only if $\phi \notin F$, for each A, $B \in F$ we have $A \cap B \in F$ and for each

$A \in F$ and $B \supset A$, implies $B \in F$. An ideal $I \subseteq 2^X$ is called *non-trivial* if $I \neq 2^X$. A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $I \supset \{\{x\} : x \in X\}$. A non-trivial ideal I is *maximal* if there cannot exists any non-trivial ideal $J \neq I$ containing I as a subset. For each ideal I, there is a filter $F(I)$ corresponding to I i.e. $F(I) = K \subseteq N$: $K^c \in I\}$,where $K^c = N - K$. We note I_2 to be an admissible ideal of $X \times X$.

Let $_2w$ be the set of all double sequences. A double sequence $x = (x_{k,l})$ is said to be convergent to a number L in the Pringsheim sense or P-convergent if for every $\varepsilon > 0$ there exists an $n \in N$ such that $|x_{k,l} - | < \varepsilon$ whenever $k, l > n$. We shall denote the space of all P-convergent sequences by $_2c$.

The double sequence $x = (x_{k,l})$ is bounded if there exists a positive number Msuch that $|x_{k,l}| < M$ for all k and l. We shall denote all bounded double sequences by $_2\ell_\infty$.

DEFINITION 1. A double sequence $(x_{k,l}) \in {_2w}$ is said to be double *I- convergent* to the number L if for every ε >0, $\{(k,l) \in N \times N: |x_{k,l} - L| \geq \varepsilon\} \in I_2$. We write I_2-*lim* $x_k = L$.

DEFINITION 2. A double sequence $(x_{k,l}) \in {_2w}$ is said to be double *I- null* if $L = 0$. We write I_2-*lim* $x_{k,l} = 0$.

DEFINITION 3. A double sequence $(x_{k,l}) \in {_2w}$ is said to be double *I-Cauchy* if for every $\varepsilon > 0$ there exist numbers $m = m(\varepsilon)$, $n = n(\varepsilon)$ such that $\{(k,l) \in N \times N : |x_{k,l} - x_{m,n}| \geq \varepsilon\} \in I_2$.

DEFINITION 4. A double sequence $(x_{k,l}) \in {_2w}$ is said to be double *I-bounded* if there exists $M > 0$ such that $\{(k,l) \in N \times N: |x_{k,l}| > M\} \in I_2$.

Let $(x_{k,l})$ and $(y_{k,l})$ be two sequences. We say that $x_{k,l} = y_{k,l}$ for *almost all k, l relative to* I_2 (here after we write *a.a.k.l.r.*I_2), if $\{(k,l) \in N \times N: x_{k,l} \neq y_{k,l}\} \in I_2$.

Throughout of the article $_2c^I$,$_2c_0^I$,$_2m^I$, $_2m_0^I$ represent the spaces of *I-convergent, I-null*, *bounded I-convergent* and *bounded I-null* double sequence spaces, respectively.

DEFINITION 5. A double sequence space $_2E$ is said to be *solid* (or *normal*) if $(\alpha_{k,l}x_{k,l}) \in {_2E}$, whenever $(x_{k,l}) \in {_2E}$ and for all sequence $(\alpha_{k,l})$ of scalars with $|\alpha_{k,l}| \leq 1$ for all $k, l \in N$.

DEFINITION 6. A double sequence space $_2E$ is said to be *monotone* if it contains the canonical preimages of its step spaces.

The notion of paranormed sequence space was studied at the initial stage by Nakano (1951) and Simons (1965). Later on it was further investigated by Maddox (1968), Lascardies (1971, 1983). In this article we introduce the following sequence space:

Let I_2 be an admissible ideal of $X \times X$. Let $p = (p_{k,l})$ a sequence of positive real numbers. Then for given $\varepsilon > 0$,

$$_2c^I(p) = \{(x_{k,l}) \in {_2w} : \{(k,l) \in N \times N :_{k,l} - L|^{p_{k,l}} \geq \varepsilon\} \in I_2 \quad, for\ some\ L \in C\};$$

$$_2c_0^I(p) = \{(x_{k,l}) \in {_2w} : \{(k,l) \in N \times N : |x_{k,l}|^{p_{k,l}} \geq \varepsilon\} \in I_2\};$$

$$_2\ell_\infty(p) = \{(x_{k,l}) \in {_2w} : \sup_{k,l} |x_{k,l}|^{p_{k,l}} < \infty\}.$$

We write

$$_2m^I(p) = {_2c^I(p)} \cap {_2\ell_\infty(p)}; {_2m_0^I(p)} = {_2c_0^I(p)} \cap {_2\ell_\infty(p)}.$$

From above definitions it is clear that $_2m^I(p) \subset {_2\ell_\infty(p)}$ and $_2m_0^I(p) \subset {_2\ell_\infty(p)}$.

The following results will be used for establishing some results of this article.

LEMMA 1. *A sequence space E is solid implies E is monotone* (Kamthan & Gupta, 1980).

LEMMA 2 [Šalăt, *et al.,* (2005), Lemma 2.5]. *Let $K \in F(I)$ and $M \subseteq N$. If $M \notin I$, then $M \cap K \notin I$.*

LEMMA 3 [Kostyrko *et al.* (2000), Lemma 5.1]. *If $I \subset 2^N$ is a maximal admissible ideal, then for each $A \subset N$ we have $A \in I$, or $N\text{-}A \in I$.*

MAIN RESULTS

THEOREM 1. *Let $(p_{k,l}) \in {_2\ell_\infty}$. Then $_2c^I(p)$, $_2c_0^I(p)$, $_2m^I(p)$ and $_2m_0^I(p)$ are linear spaces.*

Proof. Let $(x_{k,l})$, $(y_{k,l}) \in {_2c^I(p)}$ and α, β be two scalars Then for a given $\varepsilon > 0$, we have

$$\{(k,l) \in N \times N : |x_{k,l} - L_1|^{p_{k,l}} \geq \frac{\varepsilon}{2M_1} \quad, \text{ for some } L_1 \in C\} \in I_2;$$

$$\{(k,l) \in N \times N : |y_{k,l} - L_2|^{p_{k,l}} \geq \frac{\varepsilon}{2M_2}, \text{ for some } L_2 \in C\} \in I_2,$$

where $M_1 = D\ max\ \{1, \sup_{k,l} |\alpha|^{p_{k,l}}\}$; $M_2 = D\ max\ \{1, \sup_{k,l} |\beta|^{p_{k,l}}\}$, $D = \max(1, 2^{G-1})$ and $G = \sup_{k,l} p_{k,l} < \infty$.

Let $A_1 = \{(k, l) \in N \times N$: $|x_{k,l} - L_1|^{p_{k,l}} < \frac{\varepsilon}{2M_1}$, for some $L_1 \in C\}$;

$A_2 = \{(k, l) \in N \times N$: $|y_{k,l} - L_2|^{p_{k,l}} < \frac{\varepsilon}{2M_2}$, for some $L_2 \in C\}$ be such that $A_1^c, A_2^c \in I_2$.

Then

$$A_3 = \{(k,l) \in N \times N : \left|(\alpha x_{k,l} + \beta y_{k,l}) - (\alpha L_1 + \beta L_2)\right|^{p_{k,l}} < \varepsilon\}$$

$$\supseteq [\{(k,l) \in N \times N : |\alpha|^{p_{k,l}} |x_{k,l} - L_1|^{p_{k,l}} < \frac{D\varepsilon}{2M_1} |\alpha|^{p_{k,l}}\}$$

$$\cap \{(k,l) \in N \times N : |\beta|^{p_{k,l}} |y_{k,l} - L_2|^{p_{k,l}} < \frac{D\varepsilon}{2M_1} |\beta|^{p_{k,l}}\}]$$

Thus, $A_3^c = A_1^c \cup A_2^c \in I_2$.

Therefore $(\alpha(x_{k,l}) + \beta(y_{k,l})) \in {}_2c^I(p)$. Hence ${}_2c^I(p)$ is a linear space.

THEOREM 2. *Let* $(p_{k,l}) \in {}_2\ell_\infty$, *then the spaces* ${}_2m^I(p)$ *and* ${}_2m_0^I(p)$ *are paranormed* spaces, *paranormed by*

$$g((x_{k,l})) = \sup_{k,l} |x_{k,l}|^{\frac{p_{k,l}}{M}}, \ \textit{where } M = \max(1, \sup_{k,l} p_{k,l}).$$

The proof of this result is easy, so omitted.

THEOREM 3. ${}_2m^I(p)$ *is a closed subspace of* ${}_2\ell_\infty(p)$.

Proof. Let $(x_{k,l}^{(n)})$ be a Cauchy sequence in ${}_2m^I(p)$ such that

$x^{(n)} \to x$. To show that $x \in {}_2m^I(p)$.

Since $(x_{k,l}^{(n)}) \in {}_2m^I(p)$, then there exists a_n such that

$$\{(k,l) \in N \times N : |x_{k,l}^{(n)} - a_n|^{p_{k,l}} \geq \varepsilon\} \in I_2$$

We need to show that

(*i*) (a_n) converges to a (*ii*) if $U \{(k, l) \in N \times N$:$|x_{k,l} - a|^{p_{k,l}} < \varepsilon\}$, then $U^c \in I_2$.

(*i*) Since $(x_{k,l}^{(n)})$ is a Cauchy sequence in ${}_2m^I(p)$ then for given $\varepsilon > 0$, there exists $k_0 \in N$ such that

$$\sup_{k,l} |x_{k,l}^{(n)} - x_{k,l}^{(m)}| \overline{\frac{p_{k,l}}{M}} < \frac{\varepsilon}{3}, \text{ for all } n,m \geq k_0$$

Given $\varepsilon > 0$, *we have*

$$B_{nm} = \{(k,l) \in N \times N : |x_{k,l}^{(n)} - x_{k,l}^{(m)}|^{p_{k,l}} < \left(\frac{\varepsilon}{3}\right)^M\}; B_m = \{(k,l) \in N \times N : |x_{k,l}^{(m)} - a_m|^{p_{k,l}} < \left(\frac{\varepsilon}{3}\right)^M\};$$

and

$$B_n = \{(k,l) \in N \times N : |x_{k,l}^{(n)} - a_n|^{p_{k,l}} < \left(\frac{\varepsilon}{3}\right)^M\}$$

Then $B_{nm}^c, B_m^c, B_n^c \in I_2$.

Let $B^c = B_{nm}^c \cup B_m^c \cup B_n^c$, where $B = \{(k, l) \in N \times N: |a_m - a_n|^{p_{k,l}} < \varepsilon\}$. Then $B^c \in I_2$.

We choose $(k_0, l_0) \in B^c$. Then for each$n \geq k_0, m \geq_0$ we have

$$\{(k,l) \in N \times N : |a_m - a_n|^{p_{k,l}} < \varepsilon\} \supseteq [\{(k,l) \in N \times N : |a_m - x_k^{(m)}|^{p_k} < \left(\frac{\varepsilon}{3}\right)^M\} \cap$$

$$\{(k,l) \in N \times N : |x_{k,l}^{(m)} - x_{k,l}^{(n)}|^{p_{k,l}} < \left(\frac{\varepsilon}{3}\right)^M\} \cap \{(k,l) \in N \times N : |x_{k,l}^{(n)} - a_n|^{p_{k,l}} < \left(\frac{\varepsilon}{3}\right)^M\}]$$

Then (a_n) is a Cauchy sequence of scalars in C, so there exists a scalar 'a' in C such that $a_n \to a$, as $n \to \infty$.

(*ii*) Let $0 < \delta < 1$ be given. To show that if $U = \{k, l) \in N \times N: |x_{k,l} - a|^{p_{k,l}} < \delta\}$, then $U^c \in I_2$.

Since $x^{(n)} \to x$, then there exists $q_0 \in N$ such that

$$P = \{(k,l) \in N \times N : |x_{k,l}^{(q_0)} - x_{k,l}|^{p_{k,l}} < \left(\frac{\delta}{3D}\right)^M\} \text{ implies } P^c \in I_2 \qquad (1)$$

The number q_0 can be so chosen that together with (1), we have

$$Q = \{(k,l) \in N \times N : |a_{q_0} - a|^{p_{k,l}} < \left(\frac{\delta}{3D}\right)^M\} \text{ such that } Q^c \in I_2$$

Again since $\{(k, l) \in N \times N: |x_{k,l}^{(q_0)} - a_{q_0}|^{p_{k,l}} \geq \delta\} \in I$. Then we have a subset S of $N \times N$ such that

$$S^c \in I_2, where\ S = (k,l) \in N \times N : |x_{k,l}^{(q_0)} - a_{q_0}|^{p_{k,l}} < \left(\frac{\delta}{3D}\right)^M \}.$$

Let $U^c = P^c \cup Q^c \cup S^c$, where $U = \{(k, l) \in N \times N$: $|x_{k,l} - a|^{p_{k,l}} < \delta\}$.

Therefore for each $(k, l) \in U^c$ we have

$$\{(k,l) \in N \times N : |x_{k,l} - a|^{p_{k,l}} < \delta\} \supseteq [\{(k,l) \in N \times N : |x_{k,l} - x_{k,l}^{(q_0)}|^{p_{k,l}} < \left(\frac{\delta}{3D}\right)^M \}$$

$$\cap\{(k,l) \in N \times N : |x_k^{(q_0)} - a_{q_0}|^{p_k} < \left(\frac{\delta}{3D}\right)^M \} \cap \{(k,l) \in N \times N : |a_{q_0} - a|^{p_k} < \left(\frac{\delta}{3D}\right)^M \}]$$

Then the result follows.

COROLLARY 4. ${}_2m_0^I(p)$ *is a closed linear subspace of* ${}_2\ell_\infty(p)$.

THEOREM 5. *The spaces* ${}_2m^I(p)$ *and* ${}_2m_0^I(p)$ *are nowhere dense subsets of* ${}_2\ell_\infty(p)$.

Proof. The proof of this result follows from Theorem 1, Theorem 3 and Corollary 4.

THEOREM 6. *The spaces* ${}_2c_0^I(p)$ *and* ${}_2m_0^I(p)$ *are both solid and monotone.*

Proof. Let $(x_{k,l}) \in {}_2c_0^I(p)$ and $(\alpha_{k,l})$ *be a sequence of scalars with* $|\alpha_{k,l}| \leq 1$, for all $k,l \in N$.

Since $|\alpha_{k,l}x_{k,l}|^{p_{k,l}} \leq |x_{k,l}|^{p_{k,l}}$, for all $k, l \in N$.

The space ${}_2c_0^I(p)$ is solid follows from the following inclusion relation

$$\{(k,l) \in N \times N : |x_{k,l}|^{p_{k,l}} \geq \varepsilon\} \supseteq \{(k,l) \in N \times N : |\alpha_{k,l}x_{k,l}|^{p_{k,l}} \geq \varepsilon\}.$$

The space ${}_2c_0^I(p)$ is monotone by Lemma 1.

The other result follows similarly.

THEOREM 7. *Let* $(p_{k,l})$ *and* $(q_{k,l})$ *be two sequences of positive real numbers. Then* ${}_2m_0^I(p) \supseteq {}_2m_0^I(q)$ *if and only if* $\lim_{k,l \in K} \inf \frac{p_{k,l}}{q_{k,l}} > 0$, *where* $K \subseteq N \times N$ *such that* $K \in I_2$.

Proof. Let $\lim_{k,l \in K} \inf \frac{p_{k,l}}{q_{k,l}} > 0$ and $(x_{k,l}) \in {}_2m_0^I(q)$. Then there exists $\beta > 0$ such that $p_{k,l} > \beta q_{k,l}$, for all sufficiently large $(k, l) \in K$.

Since $(x_{k,l}) \in {}_2m_0^I(q)$. For given $\varepsilon > 0$, we have

$B_0 = \{(k, l) \in N \times N : |x_{k,l}|^{q_{k,l}} \geq \varepsilon\} \in I_2$.

Let $G_0 = K \cup B_0$. Then $G_0 \in I_2$.

Then for all sufficiently large $(k, l) \in G_0$,

$\{(k,l) \in N \times N : |x_{k,l}|^{p_{k,l}} \geq \varepsilon\} \subseteq \{(k,l) \in N \times N : |x_{k,l}|^{\beta q_{k,l}} \geq \varepsilon\} \in I_2$.

Therefore $(x_{k,l}) \in {}_2m_0^I(p)$.

The converse part of the result follows obviously.

COROLLARY 8. *Let* $(p_{k,l})$ *and* $(q_{k,l})$ *be two sequences of real numbers. Then* ${}_2m_0^I(p) = {}_2m_0^I(q)$ *if and only if* $\lim_{k,l \in K} \inf p_{k,l} q_{k,l} > 0$ and $\lim_{k,l \in K} \inf q_{k,l} p_{k,l} > 0$, *where* $K \subseteq N \times N$ *such that* $K \in I_2$.

THEOREM 9. *Let* $h = \inf_{k,l} p_{k,l}$ *and* $G = \sup_{k,l} p_{k,l}$, *then the following results are equivalent*:

(a) $G < \infty$ and $h > 0$;

(b) ${}_2c_0^I(p) = {}_2c_0^I$.

Proof. Suppose first that $h > 0$ and $G < \infty$, then the inequalities

$$\min(1, s^h) \leq s^{p_{k,l}} \leq \max(1, s^G)$$

hold for any $s > 0$ and for all $k, l \in N$.

Therefore the equivalent of (a) and (b) is obvious.

THEOREM 10. *Let* $G = \sup_{k,l} p_{k,l} < \infty$ *and* I_2 *is a maximal admissible ideal. Then the following are equivalent:*

(a) $(x_{k,l}) \in {}_2c^I(p)$;

(b) *there exists* $(y_{k,l}) \in {}_2c(p)$ *such that* $x_{k,l} = y_{k,l}$, *for a.a.k.l.r.*I_2;

(c) *there exists*$(y_{k,l}) \in {}_2c(p)$ *and* $(z_{k,l}) \in {}_2c_0^I(p)$ *such that* $x_{k,l} = y_{k,l} + z_{k,l}$, *for all k,l* $\in N$ *and* $\{(k, l) \in N \times N\text{: } |y_{k,l} - L|^{p_{k,l}} \geq \varepsilon\} \in I_2$

(d) *there exists a subset* $K = \{k_1 < k_2 < \ldots; l_1 < l_2 < \ldots\}$ *of* $N \times N$ *such that* $K \in F(I_2)$ *and* $\lim_{m,n \to \infty,\infty} |x_{k_m l_n} - L|^{p_{k_m l_n}} = 0$.

Proof. (a) $\Rightarrow$ (b). Let $(x_{k,l}) \in {}_2c^I(p)$. Then there exists $L \in C$ such that

$$\{(k,l) \in N \times N : |x_{k,l} - L|^{p_{k,l}} \geq \varepsilon\} \in I_2.$$

Let (m_t, n_s) be an increasing sequence with $(m_t, n_s) \in N \times N$ such that

$$\{k \leq m_t; l \leq n_s : |x_{k,l} - L|^{p_{k,l}} \geq \frac{1}{ts}\} \in I_2.$$

Define a sequence $(y_{k,l})$ as follows:

$$y_{k,l} = x_{k,l}, \text{for all } k \leq m_1; l \leq n_1$$

For $m_t < k \leq m_{t+1}, ; n_s < l \leq n_{s+1}, t, s \in N$,

$$y_{k,l} = \begin{cases} y_{k,l} = x_{k,l} \; if \, |x_{k,l} - L|^{p_{k,l}} < (ts)^{-1} \\ L, \; otherwise \end{cases}$$

Then $(y_{k,l}) \in {}_2c(p)$ and from the following inclusion

$$\{k \leq m; l \leq n : x_{k,l} \neq y_{k,l}\} \subseteq \{(k,l) \in N \times N : |x_{k,l} - L|^{p_{k,l}} \geq \varepsilon\} \in I_2,$$

we get $x_{k,l} = y_{k,l}$, *for a.a.k.l.r.*I_2.

$(b) \Rightarrow (c)$. For $(x_{k,l}) \in {}_2c^I(p)$, then there exists $(y_{k,l}) \in {}_2c(p)$ such that $x_{k,l} = y_{k,l}$, *for a.a.k.l.r.*I_2.

Let $K = \{(k, l) \in N \times N: x_{k,l} \neq y_{k,l}\}$, then $K \in I_2$.

Define $(z_{k,l})$ as follows:

$$z_{k,l} = \begin{cases} x_{k,l} - y_{k,l}, \; if \, (k,l) \in K; \\ 0, \; if \; (k,l) \notin K \end{cases}$$

Then $(z_{k,l}) \in {}_2c_0^I(p)$ and so $(y_{k,l}) \in {}_2c^I(p)$.

$(c) \Rightarrow (d)$. Suppose (c) holds. Let $\varepsilon > 0$ be given.

Let $P_1 = \{(k, l) \in N \times N: |z_{k,l}|^{p_{k,l}} \geq \varepsilon\}$ and $K = P_1^c = \{k_1 < k_2 <\ldots; l_1 <_2<\ldots\} \in F(I_2)$.

Then we have

$$\lim_{m,n \to \infty,\infty} |x_{k_m,l_n} - L|^{p_{k_m,l_n}} = 0.$$

$(d) \Rightarrow (a)$. Let $K = \{k_1 < k_2 <\ldots; l_1 < l_2 <\ldots\} \subset N \times N$ be such that $K \in F(I_2)$ and $\lim_{m,n \to \infty,\infty} |x_{k_m,l_n} - L|^{p_{k_m,l_n}} = 0$.

Then for any $\varepsilon > 0$ and Lemma 2, we have

$$\{(k,l) \in N \times N : |x_{k,l} - L|^{p_{k,l}} \geq \varepsilon\} \subseteq K^c \cup \{(k,l) \in K : |x_{k,l} - L|^{p_{k,l}} \geq \varepsilon\}.$$

Thus $(x_{k,l}) \in {}_2c^I(p)$.

PROPOSITION 11. *The spaces ${}_2c^I(p)$ and ${}_2m^I(p)$ are neither monotone nor solid, if I_2 is neither maximal nor $I_2 = I_2(f)$.*

Proof. We prove this result with the help of the following example.

EXAMPLE 1. Let $I_2 = I_2(\delta)$. Let $p_{k,l} = 1$, if k,l are even and $p_{k,l} = 2$, if k,l are odd.

Consider the K^{th} -step spaces ${}_2E_{k,l}$ of ${}_2E$ defined as follows:

Let $(x_{k,l}) \in {}_2E$ and $(y_{k,l}) \in {}_2E_{k,l}$ be such that

$$y_{k,l} = \begin{cases} x_{k,l}\,, \textit{if}\, k, l \textit{ are odd}; \\ 1\,, \textit{ otherwise} \end{cases}$$

Consider the sequence $(x_{k,l})$ as $x_{k,l} = (kl)^{-1}$, *for all* $k,l \in N$.

Then $(x_{k,l}) \in Z(p)$, but its K^{th} -step space preimage does not belong to $Z(p)$, where $Z = {}_2c^I$ and ${}_2m^I$.

Thus ${}_2c^I(p)$ and ${}_2m^I(p)$ are not monotone. By Lemma 1, it follows that the spaces ${}_2c^I(p)$ and ${}_2m^I(p)$ are not solid.

REFERENCES

Dems, K. 2004. On *I*-Cauchy sequences, Real Analysis Exchange **30**(1):123-128.

Gökhan, A. & Colak, R. 2004. The double sequence spaces $c^2(p)$ and $c_0^2(p)$. Applied Mathematics & Computations **157**(2): 491-501.

Gökhan, A. & Colak, R. 2005. Double sequence space $\ell^2(p)$. Applied Mathematics & Computations **160** (1): 147--153.

Gökhan, A. & Colak, R. 2006. On double sequence spaces $c^2(p)$ $c_0^2(p)$ and $\ell^2(p)$. International Journal of Pure & Applied Mathematics **30**(3): 309-321.

Gürdal, M. 2006. On ideal convergent sequences in 2-normed spaces. Thai Journal Mathematics **4**(1): 85-91.

Gürdal, M. & Sahiner, A. 2008. Ideal convergence in *n*-normal spaces and some new sequence spaces via *n*-norm. Journal of Fundamental Sciences **4**(1): 233-244.

Hazarika, B. 2011. On paranormed ideal convergent generalized difference strongly summable Ssequence spaces defined over *n*-normed spaces. ISRN Mathematical Analysis **2011** , no 17, doi:10.5402/2011/317423.

Kamthan, P. K. & Gupta, M. 1980. Sequence spaces and series. Marcel Dekker, New York.

Kostyrko, P., Macaj, M., Šalăt, T. & Sleziak, M. 2005. *I*-convergence and extermal *I*-limit points. Mathematica Slovaca **55**: 443-64.

Kostyrko, P., Šalăt, T. & Wilczynski, W. 2000. *I*-convergence. Real Analysis Exchange **26**(2): 669-686.

Kumar, V. 2007. On *I* and I^*- convergence of double sequences. Mathematical Communications **12**: 171-181.

Lascardies, C. G. 1971. A study of certain sequence spaces of Maddox and generalization of a theorem of Iyer. Pacific Journal Mathematics **38**(2): 487-500.

Lascardies, C. G. 1983. On the equivalence of certain sets of sequences. Indian Journal of Mathematics **25**(1): 41-52.

Maddox, I. J. 1968. Paranormed sequence spaces generated by infinite matrices. Proceedings of Cambridge Philosophical Society **64**: 335-340.

Morciz, F. 1991. Extension of the spaces c and c_0 from single to double sequences. Acta Mathematica Hungarica **57**(1-2): 129-136.

Morciz, F. & Rhoades, B. E. 1988. Almost convergence of double sequences and strong regularity summability matrices. Mathematical Proceedings of Cambridge Philosophical Society **104**: 283-294.

Nabin, A., Pehliven, S. & Gürdal, M. 2007. On *I*-Canchy sequences. Taiwanese Journal of Mathematics **11**(2): 569-76.

Nakano, H. 1951. Modular sequence spaces. Proceedings of Japan Academy **27:** 508-512.

Pringsheim, A. 1900. Zur theorie der zweifach unendlichen zahlenfolgen. Mathematical Annals Society **53:** 289-321.

Robison, G. M. 1926. Divergent double sequences and series. American Mathematical Society Translations **28**: 50-73.

Šalăt, T., Tripathy, B. C. & Ziman, M. 2004. On some properties of *I* - convergence. Tatra Mountain Mathematical Publications **28**: 279-286.

Šalăt, T., Tripathy, B. C. & Ziman, M. 2005. On *I* - convergence field. Italian Journal of Pure & Applied Mathematics **17**: 45-54.

Savas, E. 2010. Δ^m-strongly summable sequences in 2-normed spaces defined by ideal convergence and an Orlicz function. Applied Mathematics & Computations **217**: 271-276.

Simons, S. 1965. The spaces $\ell(p_\nu)$ and $m(p_\nu)$. Proceedings of London Mathematical Society **15**: 422-436.

Tripathy, B. C. 2003. Statistical convergence of double sequences. Tamkang Journal of Mathematics **34**(3): 231-237.

Tripathy, B. C. & Hazarika, B. 2009. Paranorm *I*-convergent sequence spaces. Mathematica Slovaca **59**(4): 485-494.

Tripathy, B. C. & Hazarika, B. 2011. *I*-monotonic and *I*-convergent sequences. Kyungpook Mathematics Journal **51:** 233-239, DOI 10.5666/KMJ.2011.51.2.233

Tripathy, B. K. & Tripathy, B. C. 2005. On *I*-convergent double sequences. Soochow Journal of Mathematics **31**(4):549-560.

Submitted : *28/2/2012*
Revised : *20/5/2012*
Accepted : *31/5/2012*

بعض الخواص الجبرية والطبولوجية لفضاءات متتالية مضاعفة ذات بارانورم متقارب- I

بيبان هازاريكا
قسم الرياضيات - جامعة راجيف غاندي - دويمخ - الهند
bh_rgu@yahoo.co.in

خلاصة

في هذا البحث جرى عرض فضاءات متتالية مضاعفة جديدة $_2c_0^1(p),_2 c^1(p),_2 m_0^1(p)$ and $_2m^1(p)$ لمتتالية مضاعفة لأعداد حقيقية موجبة $p = (p_{k,l})$. كما جرت دراسة بعض الخواص الجبرية والطبولوجية لهذه الفضاءات. وقد تم إثبات مبرهنة التفريق وبعض علاقات الاحتواء.