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A note on the Friedmann universe with a higher order gravity

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Abstract

New images of black holes in recent times have created a strong validity of the Friedmann universe. Since the Friedmann universe possess singularity, synonymously black holes, its quantum aspect becomes more important for study. We have explored the Friedmann universe with reference to higher order gravity theory, a natural generalization of Einstein theory of gravity. Particle creation in vacuum space-time in the presence of strong gravitation field has been explained and thus emphasized upon gravitons creation as well as non-conformal invariance of the higher order theory in this context.

Keywords: Compton length; conformal invariance; Friedmann universe; quantum field theory; vacuum polarization.

1. Introduction

The quest for meaning to our existence is linked to the quest to understand nature. Gravitation affords an example. Albert Einstein (1915) proposed his theory of relativity as a more accurate and comprehensive description of gravitation than the then prevailing Newtonian theory. After introduction of general relativity in 1915, questions related to its limitations became more and more pertinent. Four years later, Weyl (1919) considered a modification to the theory, Eddington (1922) introduce higher-order invariants to the theory. Russian mathematician A.A. Friedmann (1922; 1923; 1924) revolutionized the then concepts of the universe. Friedmann postulated a big bang followed by expansion, at which point contraction and a possible big crunch occurred. This model supposes a closed universe. However, Friedmann also proposed comparable solutions including an open universe (which expands infinitely) or a flat universe (where expansion proceeds limitlessly, yet step by step, it approaches a pace of zero). The results were so striking that even Einstein at first considered it to be a mistake, but later he accepted it as well.

Now, with the knowledge of Hubble expansion, we realize the importance of Friedmann's ideas. As discussed, the main feature of Friedmann's solution is that the properties of space-time depend on time leading to the fact that the universe is non-stationary; it has a history. There were eras when there were no stars, no galaxies but something quite different from now. The most mysterious is the moment of beginning of the universe, as though that time itself, like temperature, grew from an absolute zero (singularity). It was only after the red shift discovery in spectra of galaxies that researchers start thinking about expansion of space. The discovery and understand of relic radiation and the necessity of singularity in an expanding universe developed into Friedmann Cosmology, such to the level of "physical reality that one must deal with. Friedmann's work was central to a clearer understand of the nature of the Universe. Now the problem regarding the beginning of the universe is of more interest both theoretically and experimentally.

2. Quantum effects in the Friedmann universe.

One of the main problems in Friedmann Cosmology is the problem of singularity. For example, one may ask what physical processes are most important near the point of the beginning of the universe? Can the density go to infinity near this point?

From quantum field theory, we know that if the classical electromagnetic or gravitational field is strong enough, then the vacuum state of quantum particles becomes unstable (Padmanabhan, 2016). Particles are created. This leads to an attractive idea to explain the origin of a visible Friedmann universe as a quantum creation from a vacuum because the density of matter never goes to infinity. Here it is worth mentioning how quantum processes of particle creation effectively grew for some value of a gravitational field corresponding to some definite time or, "from the beginning".

Besides particle creation,vacuum instability leads to vacuum polarization (Visser, 1996a; 1996b). Vacuum polarization goes to zero when the gravitational field becomes weak. On the other hand, particles created during this era having a strong field continue to exist (Grib, 1971).

Manifestation of vacuum polarization in the stress-energy tensor leads to additional terms in Einstein equations (Grib, 2005). So, we have a natural extension of Einstein's General Relativity in a strong gravitational field.

In the presence of sufficiently strong gravitational field, vacuum polarization creates particle-antiparticle pairs such that particle turn up from virtual to real and visible at macroscopic distances (Bordag, 1998). The effective distance between particles and a virtual pair is equal to the Compton length. Particle creation occurs if the tidal force caused by the gravitational field can produce work equal to $2m$. The geodesic deviation is (Kenneth, 2005):

$$\frac{d^2n^i}{da^2} = R^i_{klm}\, u^k n^l u^m, \tag{1}$$

n^i is the space-like vector connecting it with the second particle so that $n_i n^i \approx l_c^2$. Putting in equation (1), we have

$$|R^{\alpha}_{0\beta 0}|\alpha \gg l_c^2 = m^2$$

$$u^i = \delta^i_0\,, \qquad n^0 = 0, |n^\mu| \sim l_c\,, \mu = 1,2,3, \tag{2}$$

from the condition of equality of work of tidal forces equal to $2m$. Thus, from some time, the curvature of the Friedmann universe satisfies equation (2). This is the era of particle creation.

Furthermore, consider a charged massive scalar field with a conformal coupling satisfying the Klein-Gordon-Fock equation described by Grib (1980) and Hu (1977):

$$\left(\nabla^i\nabla_i + \frac{R}{6} + m^2\right)\varphi(x) = 0, \tag{3}$$

where R is the curvature scalar, and ∇_i is the covariant derivative in Friedmann metric

$$ds^2 = a^2(\eta)\,(d\eta^2 - dl^2)$$

$$dl^2 = dr^2 + f^2(r)(d\theta^2 + Sin^2\theta\, d\varphi^2) \tag{4}$$

where, $f(r) = \begin{cases} \sinh r & \varkappa = -1 \\ r & \varkappa = 0 \\ \sin r & \varkappa = 1 \end{cases}$,

and $\varkappa$ is the constant curvature of 3-space. The importance of $R/6$ and it's, conformal coupling is that only in this case, in the classical limit, do quantum particles move on geodesics.

The metrical stress energy tensor for φ in (3), $T^{(0)}_{ik}$ is obtained as

$$T^{(0)}_{ik} = T^{(0)can}_{ik} - \frac{1}{3}(R_{ik} + \nabla_i\nabla_k - g_{ik}\nabla^l\nabla_l)\varphi^*\varphi \tag{5}$$

Here, $T^{(0)can}_{ik} = 2\varphi^*_{,i}$

$$T^{(0)can}_{ik} = 2\varphi^*_{,i}\,\varphi_{,k} - g_{ik}[g^{lm}\varphi^*_{,l}\varphi_{,m} - \left(m^2 + \frac{R}{6}\right)\varphi^*\varphi\,] \tag{6}$$

With a scale factor $a(\eta)$ in equation (4), the solution of equation (3) is

$$\ddot{g}_J(\eta) + \omega^2(\eta)g_J(\eta) = 0;\; r^2(\eta) = \lambda^2 + m^2a^2(\eta) \tag{7}$$

where Φ_J are the eigenfunctions of the Laplace-Beltrami operator

$$\left(\nabla_2^{(3)} + K_J^2\right)\Phi_J(r,\theta,\varphi) = 0 \tag{8}$$

and

$$\ddot{g}_J(\eta) + \omega^2(\eta)g_J(\eta) = 0\; r^2(\eta)\;;\; = \lambda^2 + m^2a^2(\eta) \tag{9}$$

Equation (9) describes oscillations with time a dependent frequency. The parametric excitation of these oscillators is a classical analogue of the quantum effect of particle creation. The magnitude of this effect is defined by the characteristic quantity:

$$\delta = \frac{\dot{\omega}}{\omega^2} = m^2 a\dot{a}\omega^{-3}. \tag{10}$$

Its maximum value (for $\lambda = 0$) is $\delta_m = h/m$, $h = \dot{a}/a^2$ being the Hubble parameter of the metric. Since $h \sim 1/t$, it is obvious that for only small t the effect is substantial.

Needless to say, we do not want to focus on quantum field theory, where one can write the operator of thes calar field in terms of annihilation and creation, Fock operator for vacuum $|0>$ defined as the ground state of the instantaneous Hamiltonian at some initial moment. Instead, we consider the question of gravitons in Friedmann space-time. We are interested in gravitons as small perturbations on a given background (Pandey 2010). Therefore, we assume that

$$\gamma_{ik} = g_{ik} + h_{ik}\,, \tag{11}$$

where g_{ik} is the metric tensor of the background space-time and h_{ik} is the gravitons. The conditions

$$h \equiv h_{ik}g^{ik} = 0;\, h^i_{k;i} = 0\,, \tag{12}$$

are satisfied. Then, in linear approximation, one has for gravitons

$$h^{;l}_{ik;l} - 2R_{ilmk}h^{lm} = 0, \quad (13)$$

where R_{ilmk} is the background curvature tensor.

The main feature of equation (12) is that it is a non-conformal invariance (Pandey, 2014). Therefore gravitons can be created in conformally flat Friedmann space-time. Equation (13) with equation (4) is

$$h^{\beta''}_{\alpha} + \frac{2a'}{a}h^{\beta'}_{\alpha} + a^2 g^{\gamma\delta} h^{\beta}_{\alpha,\gamma,\delta} = 0\ , \quad (14)$$

where prime denotes the derivative with respect to time η, and a comma denotes the derivatives with respect to spatial coordinates. Following Lifshitz (1980), one can write:

$$h^{\beta}_{\alpha} = \left(\frac{\mu}{a}\right) M^{\beta}_{\alpha}\ , \quad (15)$$

where $\nabla^2 M^{\beta}_{\alpha} = -h^2 M^{\beta}_{\alpha}$; $M^{\alpha}_{\alpha} = M^{\beta}_{\alpha;\beta} = 0$,and n is a wave number, so that wave length is $\lambda = 2\pi a/n$. Then, μ satisfies (Pandey 2001):

$$\mu'' + \mu\left(n^2 - \frac{a''}{a}\right) = 0\ . \quad (16)$$

The effective potential a''/a in equation (16) distinguishes this equation from the ordinary wave equations in Minkowski space-time (Ozdemir, 2014;Demirbag, 2014). In fact, a''/a is not zero in general except for two cases:, (i) $a = \mathrm{const}$ and (ii) $a = a_0\eta$. This reflects a manifestation of the so-called conformal non-invariance of the gravitational wave equations, which are an inevitable consequence of the Einstein field equations derivable from the Hilbert Lagrangian (Mishra, 2015). The gravitational wave equations are meaningfully comparable to electromagnetic waves except for their non-conformal property. This motivates a higher order theory of gravity where it is expected that gravitational waves can behave as conformally invariant. Thus, in this framework, an attractive possibility is to consider Einstein's general relativity as a particular case of a more fundamental theory. This is an underlying philosophy of a higher order theory of gravity.

As shown, one can have equations of gravitons which are conformally invariant by adding some polynomial in scalar curvature to Hilbert Lagrangian of Einstein theory so that

$$\pounds_g = \sqrt{-g}\left[R - C_m \frac{(l^2 R)^m}{6l^2}\right], \quad (17)$$

where C_m are dimensionless coefficients which are to be determined from the condition of conformal invariance of graviton equations in aspatial background (Mishra, 2019). This Lagrangian strongly modifies the usual Einstein theory. This choice of Lagrangian is not disturbing because it is an observational fact that our universe is not asymptotically flat. There is enough matter on our past light cone to cause it to refocus. The total energy of the universe is exactly zero, the positive energy of gravitation and matter particles are exactly compensated by the negative gravitational potential energy. That is why the universe is expanding. Again, this Lagrangian only depends upon R, so the resulting field equations can be taken as a natural modification of Einstein's field equations. See discussions in Sotiriou (2006; 2010) and Pandey (2010). Here it is interesting to note that their trace in vacuum for $N = 2$ gives $\partial^i \partial_i R - \frac{R}{6a_2} = 0$, where $a_2 = C_2 l^2$. This is similar to the massless scalar field equation $\partial\emptyset + (R/6)\emptyset = 0$ and means that R has a wave nature in a higher order theory of gravity (Capozziello 2008; 2011).

Again, from equation (16), in the neighborhood of a singularity, the equation of state is

$$p = k\epsilon^{\gamma}, \quad (18)$$

where k is a constant, and γ is adiabatic index. Then, assuming the variation of $a(\eta)$ is sufficiently smooth,

$$\frac{a''}{a} = \alpha_0 + \alpha_1\eta + \alpha_2\eta^2 + \ldots, \quad (19)$$

and for $k\varepsilon^{\gamma-1} = 1$, we have conformally invariant gravitational wave equations so that gravitons are created.

3. Discussion

We have not gone further, but one can see that some primordial empty de Sitter universe (due to the instability in the quantum process of particle creation) can become the Friedmann universe with a dustlike equation of state. This means that a singularity in the Friedman universe exists only as long as we do not pay attention to the fact that the Friedmann universe originated from something else.

But what is it about the origin of an empty de Sitter Universe which is unstable to particle creation? Could it exists forever, or did it originate from something else? As in radioactive decay, we think that it originated from some quantum state of space-time itself because of non-stability in some arbitrary moment, which is the beginning of our "time". It is expected that proper under-standing can come if quantization of gravitation as quantization of space-time is achieved.

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ملاحظة حول الدرجة العليا من الجاذبية في نظرية الكون عند فريدمان

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ملخص

يوضح هذا البحث ان الصور الجديدة للثقوب السوداء في الآونة الأخيرة أعطت صلاحية قوية لنظرية الكون عند فريدمان. وحيث أن هذه النظرية تحظى بالتفرد وهى معنية بترادف الثقوب السوداء فقد أصبح العامل الكمي أكثر أهمية للدراسة. ولقد قمنا بدراسة نظرية الكون عند فريدمان بالإشارة إلى نظرية الدرجة العليا من الجاذبية وهى التعميم الطبيعي لنظرية الجاذبية عند اينشتين كما قمنا بتفسير تَكّون الجُزئ في الفراغ الزمكاني عند وجود مجال جاذبية قوى، ومن ثم تم التركيز على تَكّون الجرافيتون وكذلك الثبات غير المطابق لنظرية الدرجة العليا من الجاذبية الواردة في هذا السياق.