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Analytical solution of the reaction-diffusion equation with space-time fractional derivatives by means of the generalized differential transform method

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ABSTRACT

In the present paper we use generalized differential transform method to derive analytical solution of linear and non-linear space-time fractional reaction-diffusion equations on a finite domain. The space and time fractional derivatives are considered in Caputo sense. Some examples are given and it has been observed that the generalized differential transform method is very effective and convenient and overcomes the difficulty of Adomian decomposition method and homotopy perturbation method.

Keywords: Adomian decomposition method; Caputo fractional derivative; fractional reaction-diffusion equation; generalized differential transform method.

INTRODUCTION

Fractional calculus is a generalization of ordinary differentiation and integration to arbitrary non-integer order. The subject is as old as classical calculus. The idea of fractional calculus has been a subject of interest not only among mathematicians, but also among physicists and engineers. Fractional differential equations have gained considerable importance due to their application in various sciences, such as physics, mechanics, chemistry, engineering, etc. In recent years, there has been a significant development in ordinary and partial differential equations involving fractional derivatives (Samko *et al*., 1993; Kilbas *et al*., 2006; Miller & Ross, 1993; Oldham & Spanier, 1974; Podlubny, 1999). Numerous problems in these areas are modeled mathematically by systems of fractional differential equations. These new models are more adequate than the previously used integer order models, because fractional order derivatives and integrals describe the memory and hereditary properties of different substances (Podlubny, 1999). This is the most significant advantage of the fractional order models, in comparison with integer order models, in which such effects are neglected.

Finding accurate and efficient methods for solving fractional differential equations has been an active research undertaking. In the last decade, various analytical and numerical methods have been employed to solve linear and nonlinear problems. For example, Adomian decomposition method (ADM) (Bildik & Konuralp, 2006; Bildik *et al.*, 2006; Hassan, 2008; Ray, 2007; Ray 2009; Yu *et al.*, 2008), homotopy-perturbation method (HPM) (Yildirim & Sezer 2010), homotopy-analysis method (HAM) (Alomari *et al.*, 2008), variational iteration method (VIM) (Bildik & Konuralp, 2006), generalized differential transform method (GDTM) (Momani & Odibat 2008; Momani *et al.*, 2007; Odibat & Momani 2008; Odibat *et al.*, 2008), multi-step differential transform method (Erturk *et al.*, 2011; Odibat *et al.,* 2010), and matrix method (Garg & Manohar 2010; Podlubny *et al.*, 2009) have all been applied to solve fractional equations both ordinary and partial. In the present paper, we use generalized differential transform method to obtain analytical solution of linear and non-linear space-time fractional reaction diffusion equations on a finite domain.

The differential transform method was first introduced by Zhou (1986) who solved linear and nonlinear initial value problems in electric circuit analysis. This method constructs an analytical solution in the form of a series. It is different from the traditional higher order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method computationally takes longer time for large orders. The differential transform is an iterative procedure for obtaining analytic Taylor series solution of ordinary or partial differential equations.

Recently, Momani & Odibat (2008) ; Momani *et al.* (2007) ; Odibat & Momani (2008) and Odibat *et al.* (2008) have developed the generalized differential transform method for solving two-dimensional linear and non-linear partial differential equations of fractional order. The method is based on the differential transform method given in (Bildik & Konuralp, 2006; Hassan, 2008) generalized Taylor's formula (Odibat & Shawagfeh, 2007) and Caputo fractional derivative (Caputo 1969, 2008).

In the present paper we first consider linear and non-linear space-time fractional reaction-diffusion equations as

$$ {}_0D_t^{\alpha}u(t,x) = b(x)\,{}_0D_x^{\beta}u(t,x) - c(x)u(t,x) + f(t,x) \tag{1} $$

and

$$ {}_0D_t^{\alpha}u(t,x) = b(x)\,{}_0D_x^{\beta}u(t,x) + g(u) + f(t,x) \tag{2} $$

on a finite domain $0 < x < L, t > 0$ with $0 < \alpha \leq 1$ and $1 < \beta \leq 2$ where the coefficient of diffusion $b(x) > 0$, reaction term $c(x) > 0$, $g(u)$ is non-linear reaction term and the function $f(t,x)$ represents source or sink. The fractional derivatives ${}_0D_t^{\alpha}$ and ${}_0D_x^{\beta}$ are considered in Caputo sense, as defined by equation (3). It is assumed that the fractional reaction-diffusion equations under consideration have unique and sufficiently smooth solutions under suitable initial conditions.

The fractional reaction-diffusion equations (1) and (2) have been considered earlier by Yu *et al.* (2008) and Yildirim & Sezer (2010) who used Adomian decomposition method and homotopy perturbation method respectively to obtain numerical solutions of these equations. In both papers, solutions are obtained in terms of series and truncated series provides numerical solutions of these equations. In the present paper, we solve these equations by generalized differential transform method (Momani & Odibat, 2008; Momani *et al.*, 2007; Odibat & Momani, 2008; Odibat *et al.*, 2008) and show that this method is more effective and convenient and provides analytical solution without approximations.

PRELIMINARIES

Caputo fractional derivative of order α, is defined as in (Caputo 1969, 2008):

$$ {}_aD_x^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)} \int_a^x \frac{f^{(m)}(\xi)}{(x-\xi)^{\alpha-m+1}} d\xi = {}_aI_x^{m-\alpha} D^m f(x), (m-1 < \alpha \le m), m \in \mathrm{N} \quad (3) $$

where $D^m = \dfrac{d^m}{dx^m}$, ${}_aI_x^{\alpha}$ stands for the **Riemann-Liouville fractional integral operator** of order $\alpha > 0$ given by (Samko *et al.*, 1993; (Miller & Ross, 1993; (Kilbas *et al.*, 2006).

$$ {}_aI_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t) dt. \quad (4) $$

Clearly

$$ {}_aI_x^{\alpha} (x-a)^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu+\alpha+1)} (x-a)^{\mu+\alpha}, \mu > -1. \quad (5) $$

Generalized differential transform (Momani & Odibat, 2008; Momani *et al.*, 2007; Odibat & Momani, 2008; Odibat *et al.*, 2008) is as given below:

Consider a function of two variables $u(x,y)$ and suppose that it can be represented as product of two single-variable functions, i.e. $u(x,y) = f(x)g(y)$. If function $u(x,y)$ is analytic and differentiable continuously with respect to x and

y in the domain of interest, then the generalized differential transform of the function $u(x, y)$ is given by

$$U_{\alpha,\beta}(k, h) = \frac{1}{\Gamma(\alpha k + 1)\Gamma(\beta h + 1)} \left[\left({}_aD_x^{\alpha} \right)^k \left({}_bD_y^{\beta} \right)^h u(x, y) \right]_{(a,b)}, \tag{6}$$

where $0 < \alpha, \beta \leq 1$, $\left({}_aD_x^{\alpha} \right)^k \equiv {}_aD_x^{\alpha} {}_aD_x^{\alpha} \cdots {}_aD_x^{\alpha}$ (k times) and $U_{\alpha,\beta}(k, h)$ is the transformed function.

The generalized differential transform inverse of $U_{\alpha,\beta}(k, h)$ is given by:

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U_{\alpha,\beta}(k, h)(x - a)^{k\alpha}(y - b)^{h\beta}. \tag{7}$$

We now mention a theorem established by the authors (communicated for publication), which gives the conditions under which the exponent law holds for Caputo fractional derivatives.

Theorem 1. Suppose that $f(x) = (x - a)^{\lambda} h(x)$, where $\lambda > 0$ and $h(x)$ has the generalized power series expansion $h(x) = \sum_{n=0}^{\infty} a_n (x - a)^{n\alpha}$ with radius of convergence $R > 0, 0 < \alpha \leq 1$. Then

$${}_aD_x^{\gamma} {}_aD_x^{\beta} f(x) = {}_aD_x^{\gamma+\beta} f(x), \tag{8}$$

for all $(x - a) \in (0, R)$, the coefficients $a_n = 0$ for n given by $n\alpha + \lambda - \beta = 0$ and either

(a) $\lambda > \mu, \mu = \max(\beta + [\gamma], [\beta + \gamma])$

or

(b) $\lambda \leq \mu, a_k = 0$ for $k = 0, 1, ..., \left[\frac{\mu - \lambda}{\alpha} \right]$,

where $[x]$ denotes the greatest integer less than or equal to x.

Theorem 2. (Momani & Odibat 2008, Momani *et al.*, 2007, Odibat & Momani 2008, Odibat *et al.*, 2008) Some fundamental properties for generalized differential transform are given below.

Let $U_{\alpha,\beta}(k, h)$, $V_{\alpha,\beta}(k, h)$ and $W_{\alpha,\beta}(k, h)$ be generalized differential transforms of functions $u(x, y)$, $v(x, y)$ and $w(x, y)$ respectively, then

(i) If $u(x, y) = v(x, y) \pm w(x, y)$, then $U_{\alpha,\beta}(k, h) = V_{\alpha,\beta}(k, h) \pm W_{\alpha,\beta}(k, h)$,

(ii) If $u(x, y) = av(x, y)$, a is constant, then $U_{\alpha,\beta}(k, h) = aV_{\alpha,\beta}(k, h)$,

(iii) If $u(x,y) = v(x,y)w(x,y)$, then $U_{\alpha,\beta}(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} V_{\alpha,\beta}(r,h-s)W_{\alpha,\beta}(k-r,s)$,

(iv) If $u(x,y) = (x-a)^{n_1\alpha}(y-b)^{n_2\beta}, n_1, n_2 \in \mathrm{N}$, then $U_{\alpha,\beta}(k,h) = \delta(k-n_1)\delta(h-n_2)$, where δ is defined as

$$\delta(k) = \begin{cases} 1, & when \quad k=0 \\ 0, & otherwise \end{cases}.$$

(v) If $u(x,y) = \left({}_aD_x^{\alpha}\right)^m v(x,y)$ where $0 < \alpha \le 1$, $m \in \mathrm{N}$ then

$$U_{\alpha,\beta}(k,h) = \frac{\Gamma(\alpha(k+m)+1)}{\Gamma(\alpha k+1)} V_{\alpha,\beta}(k+m,h),$$

(vi) If $u(x,y) = \left({}_bD_y^{\beta}\right)^n v(x,y)$ where $0 < \beta \le 1$, $n \in \mathrm{N}$, then

$$U_{\alpha,\beta}(k,h) = \frac{\Gamma(\beta(h+n)+1)}{\Gamma(\beta h+1)} V_{\alpha,\beta}(k,h+n).$$

SOLUTION OF SOME FRACTIONAL REACTION-DIFFUSION EQUATIONS

Example 1. Consider the linear space-time fractional reaction-diffusion equation

$${}_0D_t^{1/2}u(t,x) = b(x)\left({}_0D_x^{1/5}\right)^9 u(t,x) - c(x)u(t,x) + f(t,x), 0 \le x \le 1, 0 \le t \le 1, \quad (9)$$

with initial condition

$$u(0,x) = x^2 - x^3, \quad (10)$$

and

$$f(t,x) = 3\left(4t^2+1\right)x^3 + 323\sqrt{\pi}t^{1.5}\left(x^2-x^3\right), \quad b(x) = \Gamma(1.2)x^{1.8} \text{ and } c(x) = 2.$$

Applying the generalized differential transform (6) with $a = 0 = b$, $\alpha = 1/2$, $\beta = 1/5$, to both sides of equation (9) and making use of Theorem 2, (9) transforms to

$$U_{\frac{1}{2},\frac{2}{5}}(k+1,h) = \frac{\Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(\frac{(k+1)}{2}+1\right)}\left[\Gamma(1.2)\sum_{r=0}^{k}\sum_{s=0}^{h}\delta(h-s-9)\delta(r)\frac{\Gamma\left(\frac{s+9}{5}+1\right)}{\Gamma\left(\frac{s}{5}+1\right)}U_{\frac{1}{2},\frac{1}{5}}(k-r,s+9)\right.$$
$$-2U_{\frac{1}{2},\frac{1}{5}}(k,h) + 12\delta(k-4)\delta(h-15) + 3\delta(k)\delta(h-15) \quad (11)$$
$$\left.+\frac{32}{3\sqrt{\pi}}\{\delta(k-3)\delta(h-10) - \delta(k-3)\delta(h-15)\}\right].$$

The generalized differential transform of initial condition (10) is given by

$$U_{\frac{1}{2},\frac{1}{5}}(0,10) = 1,\; U_{\frac{1}{2},\frac{1}{5}}(0,15) = -1. \tag{12}$$

Utilizing the recurrence relation (11) and the transformed initial condition(12), we obtain

$$\begin{aligned} &U_{\frac{1}{2},\frac{1}{5}}(0,10) = 1, \quad U_{\frac{1}{2},\frac{1}{5}}(0,15) = -1, \quad U_{\frac{1}{2},\frac{1}{5}}(4,10) = 4,\; U_{\frac{1}{2},\frac{1}{5}}(4,15) = -4, \\ &U_{\frac{1}{2},\frac{1}{5}}(k,h) = 0 \text{ for } \; k \neq 0,4 \text{ and } h \neq 10,15. \end{aligned} \tag{13}$$

From (7), we have

$$u(t,x) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_{\frac{1}{2},\frac{1}{5}}(k,h)t^{k/2}x^{h/5}. \tag{14}$$

Using the values of $U_{\frac{1}{2},\frac{1}{5}}(k,h)$ from (13) in (13), the exact solution of linear space-time fractional reaction-diffusion equation (9) is obtained as

$$u(t,x) = 4\left(1+t^2\right)\left(x^2 - x^3\right). \tag{15}$$

This may be verified by direct substitution in (9).

Further in view of Theorem 1, we find that $\left({}_0D_x^{1/5}\right)^9 u(t,x) = {}_0D_x^{9/5}u(t,x)$. Thus the linear space-time fractional reaction-diffusion equation (9) can be taken as in (Yildirim & Sezer, 2010; Yu *et al.*, 2008)

$${}_0D_t^{1/2}u(t,x) = b(x){}_0D_x^{9/5}u(t,x) - c(x)u(t,x) + f(t,x), 0 \le x \le 1, 0 \le t \le 1, \tag{16}$$

with solution given by (14).

Example 2. Consider the non-linear space-time fractional reaction-diffusion equation

$${}_0D_t^{9/10}u(t,x) = \left({}_0D_x^{1/10}\right)^{11} u(t,x) + g(u(t,x)) + f(t,x), 0 \le x \le 1, t > 0, \tag{17}$$

with initial condition

$$u(0,x) = 0, 0 \le x \le 1, \tag{18}$$

where the non-linear reaction term is $g(u) = 0.25u(1-u)$ and $f(t,x) = -0.0104649t^{0.9} + 0.00961766x^{1.1} - 0.0025t^{0.9}x^{1.1} + 0.000025t^{1.8}x^{2.2}$.

Applying the generalized differential transform (6) with $a = 0 = b$, $\alpha = 9/10, \beta = 1/10$, to both the sides of (17) and making use of Theorem 2, (17) transforms to

$$\begin{aligned} U_{\frac{9}{10},\frac{1}{10}}(k+1,h) = \frac{\Gamma\left(\frac{9}{10}k+1\right)}{\Gamma\left(\frac{9}{10}(k+1)+1\right)} & \left[\frac{\Gamma\left(\frac{h+11}{10}+1\right)}{\Gamma\left(\frac{h}{10}+1\right)} U_{\frac{9}{10},\frac{1}{10}}(k,h+1) + 0.25 U_{\frac{9}{10},\frac{1}{10}}(k,h)\right. \\ & -0.25\sum_{r=0}^{k}\sum_{s=0}^{h} U_{\frac{9}{10},\frac{1}{10}}(r,h-s)U_{\frac{9}{10},\frac{1}{10}}(k-r,s) \\ & -0.01046498(k-2)\delta(h)+0.0096176 6\delta(k)\delta(h-11) \\ & \left.-0.0025\delta(k-1)\delta(h-11) - 0.000025\delta(k-2)\delta(h-22)\right]. \end{aligned} \tag{19}$$

The generalized differential transform of initial condition (18) is given by

$$U_{\frac{9}{10},\frac{1}{10}}(0,h) = 0. \tag{20}$$

Utilizing the recurrence relation (19) and the transformed initial condition (20), we obtain

$$U_{\frac{9}{10},\frac{1}{10}}(1,11) = 0.01 \text{ and } U_{\frac{9}{10},\frac{1}{10}}(k,h) = 0 \text{ for } k \neq 1 \text{ and } h \neq 11. \tag{21}$$

From(7), we have

$$u(t,x) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_{\frac{9}{10},\frac{1}{10}}(k,h) t^{9k/10} x^{h/10}. \tag{22}$$

Using the values of $U_{\frac{9}{10},\frac{1}{10}}(k,h)$ from equation (21) in equation (22), the exact solution of non-linear space-time fractional reaction-diffusion equation (17) is obtained as

$$u(t,x) = 0.01 t^{0.9} x^{1.1}. \tag{23}$$

This may be verified by direct substitution in the equation (17).

Further in view of Theorem 1, we find that $\left({}_0D_x^{1/10}\right)^{11} u(t,x) = {}_0D_x^{11/10} u(t,x)$. Thus the non-linear space-time fractional reaction-diffusion equation (17) can be taken as in (Yildirim & Sezer, 2010; Yu *et al.*, 2008)

$$_0D_t^{0.9}u(t,x) = {}_0D_x^{11/10}u(t,x) + g(u(t,x)) + f(t,x), 0 \le x \le 1, t > 0 \quad (24)$$

with solution given by (23).

Example 3. Consider the non-linear space-time fractional reaction-diffusion equation

$$_0D_t^{4/5}u(t,x) = \left({}_0D_x^{1/6}\right)^5 u(t,x) + g(u(t,x)) + f(x,t), 0 \le x \le 1, t > 0, \quad (25)$$

with initial condition

$$u(0,x) = 0, 0 \le x \le 1, \quad (26)$$

where the non-linear reaction term $g(u) = u^3 - u^2$ and $f(t,x) = -0.0011018t^{0.8} +0.000931384x^{1.2} + 1 \times 10^{-6}t^{1.6}x^{2.4} - 1 \times 10^{-9}t^{2.4}x^{3.6}$.

Applying the generalized differential transform(6), with $a = 0 = b, \alpha = {}^4/_5, \beta = {}^1/_5$, to both sides of (25) and making use of Theorem 2, (25) transforms to

$$\begin{aligned} U_{\frac{4}{5},\frac{1}{5}}(k+1,h) &= \frac{\Gamma\left(\frac{4k}{5}+1\right)}{\Gamma\left(\frac{4}{5}(k+1)+1\right)}\left[\frac{\Gamma\left(\frac{h+6}{5}+1\right)}{\Gamma\left(\frac{h}{5}+1\right)}U_{\frac{4}{5},\frac{1}{5}}(k,h+6)\right. \\ &+ \sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{p=0}^{h-s} U_{\frac{4}{5},\frac{1}{5}}(r,h-s-p)U_{\frac{4}{5},\frac{1}{5}}(l,s)U_{\frac{4}{5},\frac{1}{5}}(k-r-l,p) \\ &+ \sum_{r=0}^{k}\sum_{s=0}^{h} U_{\frac{4}{5},\frac{1}{5}}(r,h-s)U_{\frac{4}{5},\frac{1}{5}}(k-r,s) \\ &-0.0011018\delta(k-1)\delta(h) + 0.0009313 84\delta(k)\delta(h-6) \\ &+(1\times 10^{-6})\delta(k-2)\delta(h-12) - (1\times 10^{-9})\delta(k-3)\delta(h-18) \end{aligned} \quad (27)$$

The generalized differential transform of initial condition (26) is given by

$$U_{\frac{4}{5},\frac{1}{5}}(0,h) = 0. \quad (28)$$

Utilizing the recurrence relation (27) and the transformed initial condition (28), we obtain

$$U_{\frac{4}{5},\frac{1}{5}}(1,6) = 0.001, U_{\frac{4}{5},\frac{1}{5}}(k,h) = 0 \ \ for \ \ k \ne 1 \ \ and \ \ne 6 \quad (29)$$

From (7), we have

$$u(t,x) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_{\frac{4}{5},\frac{1}{5}}(k,h)t^{4k/5}x^{h/5}. \tag{30}$$

Using the values of $U_{\frac{4}{5},\frac{1}{5}}(k,h)$ from equation (28) in equation (30), the exact solution of non-linear space-time fractional reaction-diffusion equation (25) is obtained as

$$u(t,x) = 0.001t^{0.8}x^{1.2}. \tag{31}$$

This may be verified by direct substitution in the (25).

Further in view of Theorem 1 we find that $\left({}_0D_x^{1/5}\right)^6 u(t,x) = {}_0D_x^{6/5}u(t,x)$. Thus the non-linear space-time fractional reaction-diffusion equation (25) can be taken as in (Yildirim & Sezer 2010, Yu *et al.* 2008)

$${}_0D_t^{0.8}u(t,x) = {}_0D_x^{1.2}u(t,x) + g(u(t,x)) + f(x,t), 0 \leq x \leq 1, t > 0. \tag{32}$$

CONCLUSION

In this paper we have obtained analytical solutions of some linear and non-linear reaction-diffusion equations by means of generalized differential transform method. The same equations have been considered earlier by Yu *et al.* (2008) and Yildirim & Sezer (2010). They have used Adomian decomposition method and homotopy perturbation method respectively, to obtain numerical solutions of these equations. In both the papers, the solutions were obtained in terms of series and the truncated series provided numerical solutions of these equations, whereas the solutions obtained by generalized differential transform method are in closed form. This shows that generalized differential transform method is more effective and convenient and overcomes the difficulty of Adomian decomposition method and homotopy perturbation method.

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خلاصة

نقوم في هذا البحث بإستخدام طريقة التحويل التفاضلي المعمم لإيجاد حلول تحليلية لمعادلات الأنتشار – رد الفعل الزمكانية الخطية وغير الخطية، وذلك على مجال منته. مشتقات الزمان والمكان الكسرية تؤخذ كما جاء به كابوتو. نعطي بعض الأمثلة ونلاحظ أن هذة الطريقة فائقة الفعالية ومناسبة وتتغلب على الصعاب الناتجة عن طريقة تفريق أدومين وطريقة الرجفان المتحاول.