Kuwait J. Sci. **47 (3)** pp. 2-13, 2020

Rarely convergent sequences in topological spaces

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Abstract

In this work, by using rare sets, we introduce a new type of convergence in topological spaces, called rare convergence, which is weaker than ordinary convergence. After presenting some examples, we investigate the relationship of rare convergence with continuity and rare continuity. Using rare convergence, we also construct a new topology, called rarely sequential topology, which is coarser than sequential topology.

Keywords: Rare sets; rare continuity; rare convergence; rarely sequential spaces

2010 Mathematics Subject Classification: 54D55, 40A05

1 Introduction and Preliminaries

A set R in a topological space is called rare if $int\,(R) = \emptyset$. Popa introduced the notion of rare continuity by using rare sets as follows:

Definition 1 (Popa (1979)) *Let $f : X \to Y$ be a function between topological spaces. Then f is called rarely continuous if for each x in X and each open set G containing $f\,(x)$, there is a rare set R_G with $G \cap cl\,(R_G) = \emptyset$ and an open set U containing x such that $f\,(U) \subseteq G \cup R_G$.*

Afterwards, many authors have investigated this notion (Jafari 1997, 1995; Long & Herrington 1982; Roy 2014). Weak and strong forms of rare continuity have also been studied (Caldas & Jafari 2005, 2006; Ekici & Jafari 2009; Jafari & Noiri 2000; Jafari & Sengül 2013; Jafari 2005). Multifunctions and their continuity in terms of rare sets have been studied by many authors (Caldas *et al.* 2005; Ekici & Jafari 2013; Popa 1989).

In this paper, in a similar manner, by using rare sets, we introduce a notion of rare convergence that is weaker than ordinary convergence. We give some examples showing that ordinary divergent sequences may be rarely convergent. Also, we investigate the relationship of rare convergence with continuity and rare continuity. Unfortunately, neither of these two preserves rare convergence. But if a function is continuous and one to one then it preserves rare convergence. However, we show that for a rarely continuous function, the image of a convergent sequence is rarely convergent. Moreover, by using rare convergence, we define the class of rarely sequentially open sets. Then it is proven that a subset is rarely sequentially open if and only if it is sequentially open and contains all rare sets. Hence the family of rarely sequentially open sets forms a topology which we call rarely sequential topology. We also note that the given topology and the induced rarely sequential topology on a given set cannot overlap (except when the given topology is the discrete or the indiscrete topology).

The following Definition and Theorem will be used in the sequel.

Definition 2 (Franklin (1965)) *A subset A of a topological space (X,τ) is said to be sequentially open if for every sequence $\{x_n\}$ in X converging to a point in A, there is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $x_n \in A$.*

Theorem 3 (Long & Herrington (1982)) *Let $f : X \to Y$ be a continuous and one-to-one function between two topological spaces. If R is rare in X then $f(R)$ is rare in Y.*

2 Results

Definition 4 *Let (X,τ) be a topological space, $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be rarely convergent to x in X if for any open set G containing x, there is a rare set R_G in X such that $G \cap cl(R_G) = \emptyset$ and there is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $x_n \in G \cup R_G$.*

Remark 5 *It is obvious that ordinary convergence implies rare convergence but the converse is not true , as shown by the next example.*

Example 6 *Consider the real numbers $\mathbb{R}$ with usual topology. The sequence $\{(-1)^n\}$ is ordinary divergent but it rarely converges to any point x in $\mathbb{R}$. In fact, any sequence in $\mathbb{R}$ rarely converges to any point in $\mathbb{R}$. Let $\{x_n\}$ be any sequence, $x \in \mathbb{R}$ and $D = \{x_n : n \in \mathbb{N}\}$. For any open set G containing x, we choose $R_G = D - G$. Since D has countably many elements and $R_G \subseteq D$, $int(R_G) = \emptyset$. On the other hand since G is open and $R_G \subseteq G^c$, $G \cap cl(R_G) = \emptyset$. As a result for all $n \in \mathbb{N}$, we have $x_n \in G \cup R_G$, which means that $\{x_n\}$ rarely converges to any point.*

We know that if X is a Hausdorff space and $\{x_n\}$ is a convergent sequence, then the limit is unique. But for a rarely convergent sequence, the limit is not unique as shown by the above example.

Example 7 *Similarly it can be shown that every sequence in $\mathbb{R}$ with lower (upper) limit topology rarely converges to any point in $\mathbb{R}$.*

Theorem 8 *Let (X,τ) be a topological space. If $\{x_n\}$ is a sequence rarely converging to x in X, then any subsequence of $\{x_n\}$ rarely converges to x.*

Proof. Obvious. ■

Example 9 *Let $X = \{x,y,z\}$ and $\tau = \{\emptyset, X, \{x\}, \{y\}, \{x,y\}\}$ be a topology on X. In this space $\{z\}$ is the only rare set. Let's determine the rarely convergence of following sequences.*
i) If $\{x_n\}$ is a sequence given by $x,x,x,\ldots$, then $\{x_n\}$ (rarely) converges to x and z but $\{x_n\}$ does not rarely converge to y.
ii) If $\{x_n\}$ is a sequence given by $z,z,z,\ldots$, then $\{x_n\}$ rarely converges to x,y and z.
iii) If $\{x_n\}$ is a sequence given by $x,y,x,y,x,y,\ldots$, then $\{x_n\}$ converges to z but $\{x_n\}$ does not rarely converge to x and y.
iv) If $\{x_n\}$ is a sequence given by $x,z,x,z,x,z,\ldots$, then $\{x_n\}$ converges to z, $\{x_n\}$ rarely converges to x but $\{x_n\}$ does not rarely converge to y.
v) If $\{x_n\}$ is a sequence given by $y,z,y,z,y,z,\ldots$, then $\{x_n\}$ converges to z, $\{x_n\}$ rarely converges to y but $\{x_n\}$ does not rarely converge to x.
vi) If $\{x_n\}$ is a sequence given by $x,y,z,x,y,z,x,y,z,\ldots$, then $\{x_n\}$ does not rarely converge to x and y but $\{x_n\}$ converges to z.

Now we generalize the above example.

Proposition 10 *Let R be a rare set in a topological space (X,τ) and $r \in R$. Then;*
i) If $\{x_n\}$ is a sequence given by $x,r,x,r,x,r,\ldots$, then $\{x_n\}$ rarely converges to x for any $x \in X$.
ii) If $\{x_n\}$ is a sequence given by $r,r,r,\ldots$, then $\{x_n\}$ rarely converges to x for any $x \in X$.

Proof. i) Consider any open set G containing x. If $r \in G$ then it is obvious. If $r \notin G$ then choose $R_G = \{r\} \subseteq R$ which is also rare in X and

$G \cap cl(R_G) = \emptyset$ since $R_G \subseteq G^c$. For all $n \in \mathbb{N}$, we have $x_n \in G \cup R_G$. Therefore $\{x_n\}$ rarely converges to x.
ii) The result follows by Theorem 8 and Part i). ■

Let X be a topological space. If there is at least one rare element say r, then by Proposition 10, the sequence $\{x_n\}$ given by $r, r, r, \ldots$ rarely converges to any x in X. If there are no rare elements in X, then it means that X is the discrete space and any rarely convergent sequence has a unique limit (see Example 12). Hence if X is a topological space having the property that every rarely convergent sequence $\{x_n\}$ has a unique limit, then X is the discrete space.

We know that if X is a first countable space and has the property that every convergent sequence $\{x_n\}$ has a unique limit, then X is Hausdorff. This statement is also true when ordinary convergence is replaced by rare convergence. But the only topological space that satisfies this condition is the discrete space.

Remark 11 *Let X be a topological space and $\{x_n\}$ be a divergent sequence rarely converging to x. Let R be the set of rare elements of $\{x_n\}$ which is not necessarily rare set in X. By the definition, there are some rare elements in the sequence causing $\{x_n\}$ not to converge to x. There are two cases: When we omit the rare elements in $\{x_n\}$, if there exist infinitely many elements then they form a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converging to x. If there exist finitely many elements, then it implies that there exists $n_0 \in \mathbb{N}$ such that $x_n \in R$ for $n \geq n_0$.*

Example 12 *In a discrete space, since there are no rare sets, a sequence is rarely convergent iff it is convergent (constant).*

It is well known that for a continuous function $f : X \to Y$, if $\{x_n\}$ converges to x in X then $\{f(x_n)\}$ converges to $f(x)$ in Y. But this fact is not valid for rarely continuous functions and rarely convergent sequences.

Example 13 *Let $X = \mathbb{R}$ and $\tau = \{\emptyset, \mathbb{R}, (-\infty, 0], (0, \infty)\}$. The sequence $\{(-1)^n\}$ rarely converges to 1. Indeed, there are two open sets containing 1. For $G = \mathbb{R}$, there is no need to look for any rare set since $\mathbb{R}$ contains all elements of the sequence. For $G = (0, \infty)$, we choose $R_G = \{-1\}$. It is clear that $int(R_G) = \emptyset$ and $cl(R_G) = (-\infty, 0]$. Hence R_G is a rare set in X and $G \cap cl(R_G) = (0, \infty) \cap (-\infty, 0] = \emptyset$. Therefore for all $n \in \mathbb{N}$, $x_n \in G \cup R_G = \mathbb{R}$. Now consider $f : (X, \tau) \to (\{-1, 1\}, \mathscr{D})$*

$$f(x) = \begin{cases} -1 & , x \in (-\infty, 0] \\ 1 & , x \in (0, \infty) \end{cases}$$

where $\mathscr{D}$ is discrete topology. Then f is continuous, hence rarely continuous and $\{x_n\}$ rarely converges to 1 but $\{f(x_n)\} = \{(-1)^n\}$ does not rarely converge to $f(1) = 1$ since it is not constant (see Example 12).

The above example also shows that continuous functions do not preserve rarely convergence of sequences. However, we have the following theorem.

Theorem 14 *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a continuous and one to one function. If $\{x_n\}$ rarely converges to x in X, then $\{f(x_n)\}$ rarely converges to $f(x)$ in Y.*

Proof. Let G be an open set containing $f(x)$ in Y. Since f is continuous, there is an open set U containing x in X such that $f(U) \subseteq G$. Since $\{x_n\}$ rarely converges to x and U is open set containing x, there is a rare set R_U in X such that $U \cap cl(R_U) = \emptyset$ and there is $n_0 \in \mathbb{N}$ such that for $n \geq n_0$, $x_n \in U \cup R_U$. Hence for $n \geq n_0$, we have

$$f(x_n) \in f(U \cup R_U) = f(U) \cup f(R_U) \subseteq G \cup f(R_U).$$

Since f is continuous and one to one, $f(R_U)$ is rare in Y by Theorem 3. Here we have two cases: $f(R_U)$ is a subset of G or not. It is obvious that if $f(R_U) \subseteq G$ then $\{f(x_n)\}$ rarely converges to $f(x)$. On the other hand if $f(R_U) \nsubseteq G$ then we choose $R_G = f(R_U) - G$ and since $f(R_U)$ is rare in Y and $R_G \subseteq f(R_U)$, R_G is rare in Y. Moreover, since $R_G \subseteq G^c$ we have $G \cap cl(R_G) = \emptyset$. As a result $\{f(x_n)\}$ rarely converges to $f(x)$. ■

The following example shows that rarely continuous functions do not necessarily preserve the convergence of sequences.

Example 15 *Let $\mathscr{U}$ be the usual topology on $\mathbb{R}$. Consider $f : (\mathbb{R}, \mathscr{U}) \to (\mathbb{R}, \mathscr{U})$ defined by*

$$f(x) = \begin{cases} 0 & , x \in \mathbb{Q} \\ 1 & , x \in \mathbb{Q}^c \end{cases} .$$

Then f is rarely continuous (Long & Herrington 1982). As $\mathbb{Q}$ is dense in $\mathbb{R}$, then there is a sequence $\{x_n\}$ in $\mathbb{Q}$ such that $\{x_n\}$ converges to x for any $x \in \mathbb{Q}^c$. But $\{f(x_n)\} = \{0\}$ does not converge to $f(x) = 1$.

In above example, however $\{f(x_n)\}$ rarely converges to $f(x)$. In the following theorem, we prove that this is not a coincidence.

Theorem 16 *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a rarely continuous function. If $\{x_n\}$ converges to x in X, then $\{f(x_n)\}$ rarely converges to $f(x)$ in Y.*

Proof. Let G be an open set containing $f(x)$ in Y. Since f is rarely continuous, there is a rare set R_G in Y such that $G \cap cl(R_G) = \emptyset$ and there is an open set U containing x in X such that $f(U) \subseteq G \cup R_G$. Since $\{x_n\}$ converges to x and U is open set containing x, there is $n_0 \in \mathbb{N}$ such that $x_n \in U$ for $n \geq n_0$. Hence $f(x_n) \in f(U) \subseteq G \cup R_G$ for $n \geq n_0$ which implies $\{f(x_n)\}$ rarely converges to $f(x)$. ■

In the following, we define rarely sequentially open sets in a similar manner to sequentially open sets.

Definition 17 *A subset A of a topological space (X, τ) is said to be rarely sequentially open set if for every sequence $\{x_n\}$ in X rarely converging to a point in A, there is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $x_n \in A$.*

An open set or a sequentially open set may not be rarely sequentially open.

Example 18 *Consider the real numbers $\mathbb{R}$ with the usual topology, then $A = (0, 2)$ is open and sequentially open but it is not rarely sequentially open. In particular, the sequence $\{x_n\}$ with $x_n = 3$ rarely converges to $1 \in A$ but $x_n \notin A$ for all $n \in \mathbb{N}$. As a result of Example 6, any sequence in $\mathbb{R}$ rarely converges to any point in $\mathbb{R}$. This implies that the only rarely sequentially open sets are $\mathbb{R}$ and $\emptyset$.*

Example 19 *Consider the real numbers $\mathbb{R}$ with co-countable topology. It is well known that in this space, any subset is sequentially open since convergent sequences are constant. On the other hand, it can be easily shown that (as in Example 6) any sequence rarely converges to any point which means that $\mathbb{R}$ and $\emptyset$ are the only rarely sequentially open sets. Therefore any open set different from $\mathbb{R}$ and $\emptyset$ is not rarely sequentially open.*

Example 20 *Consider $X = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ with the subspace topology of usual topology on $\mathbb{R}$. In this space, the only rare set is $\{0\}$. Let $\{x_n\}$ be a sequence in X. We have the following characterizations.*
i) $\{x_n\}$ rarely converges to $a \neq 0$ if and only if there is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $x_n = 0$ or a.
ii) $\{x_n\}$ rarely converges to 0 if and only if $x_n = 0$ or $\frac{1}{n}$ or subsequence of $\frac{1}{n}$ or combination of first and last (for example $x_n = 1/2, 0, 1/3, 0, 1/4, 0, \ldots, 1/n, 0, \ldots$).
In this space, the only rarely sequentially open sets are the sets containing $A_{n_0} = \left\{\frac{1}{n} : n \geq n_0, n_0 \in \mathbb{N}\right\} \cup \{0\}$ along with $\emptyset$.

In above examples, we observe that rarely sequentially open sets are sequentially open sets containing all rare sets.

Theorem 21 *Let (X, τ) be a topological space. A subset $A \subseteq X$ is rarely sequentially open if and* *only if it is sequentially open and contains all rare sets.*

Proof. Let A be a rarely sequentially open set in X. Now we show that A is sequentially open. Since a convergent sequence is also rarely convergent, then for all $x \in A$ and all sequences $\{x_n\}$ in X converging to x, there is $n_0 \in \mathbb{N}$ such that $x_n \in A$ for all $n \geq n_0$ which implies that A is sequentially open in X. Moreover, if R is a rare set in X, then by Proposition 10, the sequence $\{x, r, x, r, x, r, \ldots\}$ rarely converges to x for all $r \in R$. Hence, $r \in A$ and $R \subseteq A$ for any rare set R in X. On the other hand, we assume that A is a sequentially open set containing all rare sets. Let $a \in A$ and $\{x_n\}$ be a sequence rarely converging to x. Here we consider two cases.
Case 1) $\{x_n\}$ converges to x. Since A is sequentially open, there is $n_0 \in \mathbb{N}$ such that for $n \geq n_0$, $x_n \in A$ which implies that A is rarely sequentially open in X.
Case 2) $\{x_n\}$ rarely converges to x but $\{x_n\}$ does not converge to x. By Remark 11, here we have two cases.
Case 2-a) Suppose that there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converging to x. Since A is sequentially open, there is $n_0 \in \mathbb{N}$ such that for all $n_k \geq n_0$, $x_{n_k} \in A$. Since the other terms of $\{x_n\}$ are rare elements and by the hypothesis, they are also contained in A. Therefore A is rarely sequentially open in X.
Case 2-b) Suppose that there exists $n_0 \in \mathbb{N}$ such that x_n is a rare element for $n \geq n_0$. Again by the hypothesis, they are also contained in A. Therefore A is rarely sequentially open in X ■

Remark 22 *The above theorem says that the family of rarely sequentially open sets is a special class of the family of sequentially open sets containing all rare sets. Since the family of sequentially open sets denoted by τ_s is a topology on X then the family of rarely sequentially open sets denoted by τ_{rs} is also a topology on X.*

Definition 23 *Let (X, τ) be a topological space. Then τ_{rs} is called rarely sequential topology and (X, τ_{rs}) is called rarely sequential space.*

Remark 24 *If (X, τ) is sequential space, i.e. $\tau = \tau_s$, then τ_{rs} is coarser than τ, i.e. $\tau_{rs} \subseteq \tau$.*

Example 25 *If (X, τ) is indiscrete topological space then $\tau_{rs} = \tau$.*

Example 26 *If (X, τ) is discrete topological space then $\tau_{rs} = \tau$.*

Remark 27 *If (X, τ) is a topological space such that $\{x\}$ is rare in X for any $x \in X$ then $\tau_{rs} = \{\emptyset, X\}$.*

Remark 28 *Let (X, τ) be a topological space. If X contains no rare sets, then $\tau_{rs} = \tau_s = \tau$ is the discrete topology since $\{x\} \in \tau$ for any $x \in X$.*

Proposition 29 *Let (X, τ) be a topological space. If τ is different from indiscrete and discrete topology then $\tau \neq \tau_{rs}$.*

Proof. Assume that $\tau_{rs} = \tau$. Since τ is different from indiscrete and discrete topology, there is an open set U having at least two elements and $x \in U$ such that $\{x\}$ is rare in X. Consider an element $y \in U^c$. Here $\{y\}$ cannot be rare in X. Otherwise, since $U \in \tau = \tau_{rs}$ and $\{y\}$ is rare in X, $y \in U$, which yields a contradiction. Therefore $\{y\}$ is not rare in X. Then we have *int* $(\{y\}) \neq \emptyset$ implies *int* $(\{y\}) = \{y\}$ and $\{y\} \in \tau = \tau_{rs}$. Since $\{x\}$ is rare in X, we have $x \in \{y\}$ that is a contradiction. Therefore τ and τ_{rs} cannot be the same. ■

Theorem 30 *Let (X, τ) be a topological space. If $\{x_n\}$ is a sequence rarely converging to x in (X, τ) then $\{x_n\}$ converges to x in (X, τ_{rs}).*

Proof. Assume that $\{x_n\}$ rarely converges to x in (X, τ). Let G be an open set containing x in (X, τ_{rs}). Then G is rarely sequentially open set containing x in (X, τ). Hence there is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $x_n \in G$. Therefore $\{x_n\}$ converges to x in (X, τ_{rs}). ■

In the following, we give an example to show that the converse of the above theorem is not true.

Example 31 *Consider the information in Example 20. Let* $\left\{1, \frac{1}{2}, 1, \frac{1}{3}, \ldots, 1, \frac{1}{n}, \ldots\right\}$ *be a sequence in* X. *Then* $\{x_n\}$ *converges to* 1 *in* (X, τ_{rs}) *but* $\{x_n\}$ *does not rarely converge to* 1 *in* (X, τ).

Remark 32 *Let* x *be a rare element in* (X, τ). *Then the following questions arise: Whether or not* x *is a rare element in* (X, τ_{rs}). *The answer depends on the number of rare elements in* (X, τ). *If* x *is the only rare element in* (X, τ) *then* x *may not be a rare element in* (X, τ_{rs}). *Consider* $(\mathbb{R}, \tau)$ *where* $\tau = P(\mathbb{R} - \{0\}) \cup \{U \subseteq \mathbb{R} : 0 \in U,$ U^c *is countable*$\}$. *Then the only rare element in* (X, τ) *is* 0. *It is clear that* $\{0\}$ *is sequentially open since the only sequence converging to* 0 *is constant. Therefore* $\{0\} \in \tau_{rs}$ *which means* x *is not rare element in* (X, τ_{rs}). *For the other case, we give the following theorem.*

Theorem 33 *Let* (X, τ) *be a topological space. If the number of rare elements in* (X, τ) *is greater than 1 and* x *is a rare element in* (X, τ) *then* x *is also rare element in* (X, τ_{rs}).

Proof. Assume that x is not a rare element in (X, τ_{rs}). Then $\{x\} \in \tau_{rs}$ and by Theorem 21, it must contain all rare elements in (X, τ) which is a contradiction. ■

The following theorem discusses the case of non-rare elements in any given topological space except the discrete space.

Theorem 34 *Let* (X, τ) *be a topological space where* τ *is different from discrete topology. If* $\{x\}$ *is not rare in* (X, τ), *then* $\{x\}$ *is rare in* (X, τ_{rs}).

Proof. Assume that x is not a rare element in (X, τ_{rs}). Then $\{x\} \in \tau_{rs}$ and by Theorem 21, there are no rare elements in (X, τ). This implies that τ is discrete which is a contradiction. ■

Corollary 35 *Let* (X, τ) *be a topological space and the number of rare elements is greater than 1 then* $(\tau_{rs})_{rs}$ *is indiscrete topology.*

Proof. Since there are some rare elements, τ is different from discrete topology. By Theorem 33 and 34, any x is rare in (X, τ_{rs}) and by Remark 27, $(X, (\tau_{rs})_{rs})$ is indiscrete topological space. ■

ACKNOWLEDGEMENTS

The author would like to thank the referees for their valuable comments which helped to improve the manuscript.

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Submitted : 23/05/2019
Revised : 15/09/2019
Accepted : 15/10/2019

المتواليات نادرة التقارب في الفضاءات التوبولوجية

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ملخص

في هذا البحث نقدم نوعاً جديداً من التقارب في الفضاءات التوبولوجية، وذلك باستخدام مجموعات نادرة تُسمى التقارب النادر، وهي أضعف من التقارب العادي. وعقب تقديم عدد من الأمثلة، سنقوم بدراسة العلاقة بين التقارب النادر مع الاستمرارية والاستمرارية النادرة. وباستخدام التقارب النادر، يتم ايضاً انشاء توبولوجيا جديدة يُطلق عليها التوبولوجيا نادرة التوالي، والتي تُعتبر أضعف من التوبولوجيا التتابعية.