On paranormed type fuzzy I -convergent double multiplier sequences

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ABSTRACT

In this article, we introduce the classes of fuzzy real valued double sequences $_2c^{I(F)}(\Lambda, p)$ and $_2c_0^{I(F)}(\Lambda, p)$, where $\Lambda = (\lambda_{nk})$ is a multiplier sequence of non-zero real numbers and $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers. We study different topological properties of these classes of sequences. Also we characterize the multiplier problem and obtain some inclusion relation involving these classes of sequences.

Keywords: I-convergent; multiplier sequence spaces; sequence algebra; solid space; convergence free.

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh (1965). The notion of I-convergence of sequences of real numbers was studied at the initial stage by Kostyrko *et al.* (2000) which generalizes and unifies different notions of convergence of sequences. Tripathy & Sen (2008) introduced the concept of I-convergence of fuzzy real valued sequence. The initial works on double sequences of real or complex terms is found in Bromwich (1965). Tripathy & Dutta (2007, 2010) introduced and investigated different types of fuzzy real valued double sequence spaces. Different classes of paranormed sequence spaces have been introduced and studied by Tripathy (2003), Tripathy & Hazarika (2009), Tripathy & Sen (2003).

The scope for the studies on sequence spaces was extended by using the notion of associated multiplier sequences. The notion of multiplier sequences was first studied by Goes and Goes (1970) and later on it was followed by many workers. Goes & Goes (1970) defined the differentiated sequence space dE and integrated sequence space $\int E$ for a given sequence space E, by using multiplier sequences (k^{-1}) and (k) respectively. Tripathy & Sen (2003, 2006), Tripathy

(2004), Tripathy & Mahanta (2004), Tripathy & Hazarika (2008) used a general multiplier sequence (λ_k) of non-zero scalars for their studies on sequence spaces associated with multiplier sequences. In this paper we shall consider a general multiplier double sequence $\Lambda = (\lambda_{nk})$ of non-zero real numbers.

For a fuzzy real valued double sequence space E^F, the multiplier sequence space $E^F(\Lambda)$ associated with the multiplier double sequence Λ is defined as

$$E^F(\Lambda) = \{(X_{nk}) : (\lambda_{nk} X_{nk}) \in E^F\}.$$

Aim of this paper is to introduce some classes of fuzzy real valued double sequences with a multiplier sequence of non-zero real numbers and to obtain some inclusion relation involving these classes of sequences.

DEFINITIONS AND PRELIMINARIES

In this section we recall some notation and basic definitions which will be used in this paper. Let X be a non empty set. A non-void class $I \subseteq 2^X$ (power set of X) is called an ideal if I is additive (*i.e.* $A, B \in I \Rightarrow A \cup B \in I$) and hereditary (*i.e.* $A \in I$ and $B \subset A \Rightarrow B \in I$). A non-empty family of sets $F \subseteq 2^X$ is said to be a filter in X if $\phi \notin F$; $A, B \in F \Rightarrow A \cap B \in F$ and $A \in F$, $A \subseteq B \Rightarrow B \in F$. For each ideal I there is a filter $F(I)$ corresponding to I, given by $F(I) = \{K \subseteq N : N \backslash K \in I\}$.

A double sequence $x = (x_{nk})$ of numbers is said to be I_2-convergent to L, if for every $\varepsilon > 0$, the set $\{(n, k) \in N \times N : \bar{d}(X_{nk}, X_0) \geq \varepsilon\} \in I_2$.".

A double sequence $x = (x_{nk})$ of numbers is said to be I_2-bounded, if there exists a real number μ such that the set $\{(n, k) \in N \times N : |x_{nk}| > \mu\} \in I_2$.

Throughout $_2\ell_\infty^{I_2}$ denotes the space of all I_2-bounded double sequences. The usual convergence is a particular case of I-convergence. In this case $I = I_f$ (the ideal of all finite subsets of N). Throughout the article the ideals of 2^N will be denoted by I and the ideals of $2^{N \times N}$ will be denoted by I_2.

A fuzzy real number X is a fuzzy set on R, i.e. a mapping $X : R \rightarrow L (= [0, 1])$ associating each real number t with its grade of membership $X(t)$. Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows:

$$\bar{r}(t) = \begin{cases} 1 & if \qquad\qquad t = r \\ 0 & otherwise \end{cases}$$

The α-level set of a fuzzy real number X, $0 < \alpha \leq 1$, denoted by $[X]^\alpha$ is defined as

$$[X]^\alpha = \{t \in R : X(t) \geq \alpha\}.$$

The 0-level set is the closure of strong 0-cut i.e. cl $\{t \in R : X(t) > 0\}$. A fuzzy real number X is called convex if $X(t) \geq X(s) \wedge X(r) = \min[X(s), X(r)]$,. where $s < t < r$. If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called normal. A fuzzy real number X is said to be upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$,for all $a \in L$ is open in the usual topology of R. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $L(R)$. Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R. Then $X \leq Y$ if and only if $X^L \leq Y^L$ and $X^R \leq Y^R$.

Also let $d(X, Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$.Then (D, d) is a complete metric space. Let $\bar{d} : L(R) \times L(R) \rightarrow R$ be defined by $\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha)$, for $X, Y \in L(R)$. Then $\bar{d}$ defines a metric on $L(R)$.

A fuzzy real valued double sequence (X_{nk}) is said to be convergent in Pringsheim's sense to the fuzzy real number X, if for every $\varepsilon > 0$, there exists $n_0 = n_0(\varepsilon), k_0 = k_0(\varepsilon) \in N$ such that $\bar{d}(X_{nk}, X) < \varepsilon$ for all $n \geq n_0, k \geq k_0$.

A fuzzy real valued double sequence (X_{nk}) is said to be I_2- convergent to the fuzzy number X_0, if for all $\varepsilon > 0$, the set We write I_2-lim$X_{nk} = X_0$.

Throughout $_2w^F$, $_2\ell_\infty^F$, $_2c^F$, $_2c_0^F$, $_2c^{I(F)}$, $_2c_0^{I(F)}$ denote the spaces of all, bounded, convergent in Pringsheim's sense, null in Pringsheim's sense, I_2-convergent and I_2-null fuzzy real valued double sequences respectively.

A fuzzy real valued double sequence space E^F is said to be solid if $(Y_{nk}) \in E^F$,whenever $\bar{d}(Y_{nk}, \bar{0}) \leq \bar{d}(X_{nk}, \bar{0})$ for all $n, k \in N$ and $(X_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be monotone if E^F contains the canonical pre-image of all its step spaces.

A fuzzy real valued double sequence space E^F is said to be symmetric if $S(X) \subset E^F$, for all $X \in E^F$, where $S(X)$ denotes the set of all permutations of the elements of $X = (X_{nk})$.

A fuzzy real valued double sequence space E^F is said to be sequence algebra if $(X_{nk} \otimes Y_{nk}) \in E^F$, whenever $(X_{nk}), (Y_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be convergence free if $(Y_{nk}) \in E^F$, whenever $(X_{nk}) \in E^F$ and $X_{nk} = \bar{0}$ implies $Y_{nk} = \bar{0}$.

A multiplier from a fuzzy real valued double sequence space D^F into another fuzzy real valued double sequence space E^F is a real sequence $u = (u_{nk})$ such that $uX = (u_{nk}X_{nk}) \in E^F$ whenever $X = (X_{nk}) \in D^F$. The linear space of all such multipliers will be denoted by $m(D^F, E^F)$. Bounded multipliers will be denoted by $M(D^F, E^F)$.

Hence $M(D^F, E^F) = {}_2\ell_\infty^F \cap m(D^F, E^F)$.

Let $\Lambda = (\lambda_{nk})$ be a sequence of non-zero real numbers and $p = (p_{nk})$ be a double sequence of bounded strictly positive numbers. Then the following classes of sequences are introduced:

$$_2c^F(\Lambda, p) = \{X = (X_{nk}) : \lim[\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{p_{nk}} = 0, \text{ for some } X_0 \in L(R)\},$$

$$_2c^{I(F)}(\Lambda, p) = \{X = (X_{nk}) : I-\lim[\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{p_{nk}} = 0, for some X_0 \in L(R)\},$$

$$_2c_0^{I(F)}(\Lambda, p) = \{X = (X_{nk}) : I-\lim[\overline{d}(\lambda_{nk}X_{nk}, \overline{0})]^{p_{nk}} = 0\},$$

$$_2\ell_\infty^F(\Lambda, p) = \{X = (X_{nk}) : \sup_{n,k} [\overline{d}(\lambda_{nk}X_{nk}, \overline{0})]^{p_{nk}} < \infty\}.$$

Also we define

$$_2m^{I(F)}(\Lambda, p) = {}_2c^{I(F)}(\Lambda, p) \cap {}_2\ell_\infty^F(\Lambda, p)$$

$$\text{and} \quad _2m_0^{I(F)}(\Lambda, p) = {}_2c_0^{I(F)}(\Lambda, p) \cap {}_2\ell_\infty^F(\Lambda, p).$$

Let (X_{nk}) and (Y_{nk}) be two fuzzy real valued double sequences. Then we say that $X_{nk} = Y_{nk}$ for almost all n and k relative to I_2 (in short *a.a.n & kr I_2*) if the set $\{(n,k) \in N \times N : X_{nk} \neq Y_{nk}\} \in I_2.$.

Lemma 1. *If a sequence space E^F is solid, then it is monotone.*

For the crisp set case, one may refer to Kamthan & Gupta (1980).

Lemma 2. *If a fuzzy real valued double sequence space E^F is bounded and solid, then $(\lambda_{nk}) \in M(E^F, E^F)$ if and only iff $(\lambda_{nk}) \in {}_2\ell_\infty$.*

MAIN RESULTS

The proof of the following result is easy, so omitted.

Theorem 1. *For $\Lambda = (\lambda_{nk})$, a given multiplier sequence, the classes of sequences $_2m^{I(F)}(\Lambda, p)$ and $_2m_0^{I(F)}(\Lambda, p)$ are linear spaces.*

Theorem 2. *Let $\sup_{n,k} p_{nk} < \infty$. Then the following statements are equivalent:*

(i) $(X_{nk}) \in {}_2c^{I(F)}(\Lambda, p)$.

(ii) *There exists a sequence $(Y_{nk}) \in {}_2c^F(\Lambda, p)$ such that $X_{nk} = Y_{nk}$ for a.a.n & k r.I_2.*

(iii) *There exists a subset $M = \{(n_i, k_j) \in N \times N : i, j \in N\}$ of $N \times N$ such that $M \in F(I_2)$ and $(X_{n_i k_j}) \in {}_2c^F(\Lambda, p)$.*

Proof. (i) $\Rightarrow$ (ii) Let $(X_{nk}) \in {}_2c^{I(F)}(\Lambda, p)$. Then there exists $X_0 \in L(R)$ such that $I_2 - \lim(\overline{d}(\lambda_{nk}X_{nk}, X_0))^{p_{nk}} = 0$

So for any $\varepsilon > 0$, we have the set

$$\{(n, k) \in N \times N : [\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{P_{nk}} \geq \varepsilon\} \in I_2.$$

Let us consider the increasing sequences (T_j) and (M_j) of natural numbers such that if $p > T_j$ and $q > M_j$, then the set

$$\left\{(n, k) \in N \times N : n \leq p, k \leq q \text{ and } [\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{p_{nk}} \geq \frac{1}{j}\right\} \in I_2.$$

We define the sequence (Y_{nk}) as follows:

$Y_{nk} = X_{nk}$ if $n \leq T_1$ or $k \leq M_1$.

Also for all (n, k) with $T_j < n \leq T_{j+1}$ or $M_j < k \leq M_{j+1}$,

let $Y_{nk} = X_{nk}$ if $[\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{p_{nk}} < \frac{1}{j}$, otherwise $Y_{nk} = \lambda_{nk}^{-1}X_0$.

We show that $(Y_{nk}) \in {}_2c^F(\Lambda, p)$. Let $\varepsilon > 0$. We choose j such that $\varepsilon > \frac{1}{j}$. We see that for $n > T_j$ and $k > M_j$, $[\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{p_{nk}} < \varepsilon$.

Hence $(Y_{nk}) \in {}_2c^F(\Lambda, p)$.

We see that for $n > T_j$ and $k > M_j$, $(\overline{d}(\lambda_{nk}Y_{nk}, X_0))^{p_{nk}} < \varepsilon$. Hence $(Y_{nk}) \in {}_2c^F(\Lambda, p)$. Next we assume that $T_j < n \leq T_{j+1}$ and $M_j < k \leq M_{j+1}$, then the set

$$A = \{(n, k) \in N \times N : X_{nk} \neq Y_{nk}\} \subseteq \left\{(n, k) \in N \times N : [\overline{d}(\lambda_{nk}X_{nk}, X_0)]^{p_{nk}} \geq \frac{1}{j}\right\} \in I_2.$$

Hence $A \in I_2$ and so $X_{nk} = Y_{nk}$ for *a.a.n* & *kr.I_2*.

(ii) $\Rightarrow$ (iii) Suppose there exists a sequence $(Y_{nk}) \in {}_2c^F(\Lambda, p)$ such that $X_{nk} = Y_{nk}$ for *a.a.n* & *kr.I_2* . Let $M = \{(n, k) \in N \times N : X_{nk} = Y_{nk}\}$. Then $M \in F(I_2)$.

We can enumerate M as $M = \{(n_i, k_j) \in N \times N : i, j \in N\}$, on neglecting the rows and columns those contain finite number of elements. Then $(X_{n_i k_j}) \in {}_2c^{I(F)}(\Lambda, p)$.

(iii) $\Rightarrow$ (i) is obvious. This completes the proof of the theorem.

The following result can be proved easily using simple technique.

Theorem 3. *If $H = \sup\limits_{n,k} p_{nk} < \infty$, then the classes of sequences $_2m^{I(F)}(\Lambda, p)$ and $_2m_0^{I(F)}(\Lambda, p)$ are complete metric spaces with respect to the metric ρ defined by*

$$\rho(X, Y) = \sup_{n,k} [\bar{d}(\lambda_{nk} X_{nk}, \lambda_{nk} Y_{nk})]^{\frac{p_{nk}}{M}}, \text{where} = \max(1, H).$$

Theorem 4. *The class of sequences $_2m_0^{I(F)}(\Lambda, p)$ is solid as well as monotone.*

Proof. Let $(X_{nk}) \in {}_2m_0^{I(F)}(\Lambda, p)$ and (Y_{nk}) be such that $\bar{d}(Y_{nk}, \bar{0}) \leq \bar{d}(X_{nk}, \bar{0})$ for all $n, k \in N$. Let $\varepsilon > 0$ be given. Then the solidness of $_2m_0^{I(F)}(\Lambda, p)$ follows from the following relation:

$$\{(n, k) \in N \times N : [\bar{d}(\lambda_{nk} X_{nk}, \bar{0})]^{p_{nk}} \geq \varepsilon\} \supseteq \{(n, k) \in N \times N : [\bar{d}(\lambda_{nk} Y_{nk}, \bar{0})]^{p_{nk}} \geq \varepsilon\}.$$

Also by **Lemma 1**, it follows that the space is monotone.

Theorem 5. *The class of sequences $_2m^{I(F)}(\Lambda, p)$ is neither solid nor monotone in general.*

Proof. The result follows from the following example.

Example 1:   Let $A \in I_2, p_{nk} = \begin{cases} 2, & \text{if } (n, k) \in A \\ 1, & \text{otherwise} \end{cases}$

We consider the sequence (X_{nk}) defined by:

For all $(n, k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1 - (n+k) + \frac{t}{n+k}, for & (n+k-1)(n+k) \leq t \leq (n+k)^2 \\ 1 + (n+k) - \frac{t}{n+k}, for & (n+k)^2 \leq t \leq (n+k+1)(n+k) \\ 0, & otherwise \end{cases}$$

otherwise $X_{nk} = \overline{(n+k)^2}$.

Then taking $\lambda_{nk} = \frac{1}{(n+k)^2}$, for all $n, \ k \in N$, we have $(X_{nk}) \in {}_2m^{I(F)}(\Lambda, p)$.

Let $K = \{2i : i \in N\}$.

We consider the sequence (Y_{nk}) defined by:

$$Y_{nk} = \begin{cases} X_{nk}, & \text{if } (n, k) \in K \\ \bar{0}, & \text{otherwise} \end{cases}$$

Then (Y_{nk}) belongs to the canonical pre-image of K step space of $_2m^{I(F)}(\Lambda, p)$.

But $(Y_{nk}) \notin {}_2 m^{I(F)}(\Lambda, p)$. Hence the class of sequences ${}_2 m^{I(F)}(\Lambda, p)$ is not monotone. Therefore by **Lemma1**, the class of sequences is not solid.

Theorem 6. *The classes of sequences ${}_2 m^{I(F)}(\Lambda, p)$ and ${}_2 m_0^{I(F)}(\Lambda, p)$are not symmetric in general.*

Proof. The result follows from the following example.

Example 2 : Let $I_2 = I_2(\rho), p_{nk} = \begin{cases} 1, & \text{for } n \text{ even and all } k \in N \\ 2, & \text{otherwise} \end{cases}$

We consider the sequence (X_{nk})defined by:

For $n = i^2, \quad i \in N$and for all $k \in N$,

$$X_{nk}(t) = \begin{cases} 1 + \dfrac{t}{\sqrt{n}-1}, & \text{for} \quad 1 - \sqrt{n} \leq t \leq 0 \\[2mm] 1 - \dfrac{t}{\sqrt{n}-1}, & \text{for} \quad 0 < t \leq \sqrt{n}-1 \\[2mm] 0, & \text{otherwise} \end{cases}$$

otherwise $X_{nk} = \overline{0}$.

Then taking $\lambda_{nk} = \dfrac{1}{n}$, for all $n, k \in N$, we have $(X_{nk}) \in {}_2 m_0^{I(F)}(\Lambda, p), {}_2 m^{I(F)}(\Lambda, p)$.

We consider the rearrangement (Y_{nk}) of (X_{nk}) defined by:

For k odd and for all $n \in N$,

$$Y_{nk}(t) = \begin{cases} 1 + \dfrac{t}{n-1}, & \text{for} \quad 1 - n \leq t \leq 0 \\[2mm] 1 - \dfrac{t}{n-1}, & \text{for} \quad 0 < t \leq n-1 \\[2mm] 0, & \text{otherwise} \end{cases}$$

otherwise $Y_{nk} = \overline{0}$.

Then $(Y_{nk}) \notin {}_2 m_0^{I(F)}(\Lambda, p), {}_2 m^{I(F)}(\Lambda, p)$.

Hence the classes of sequences ${}_2 m_0^{I(F)}(\Lambda, p)$ and ${}_2 m^{I(F)}(\Lambda, p)$ are not symmetric in general.

Theorem 7. *The classes of sequences ${}_2 m_0^{I(F)}(\Lambda, p)$ and ${}_2 m^{I(F)}(\Lambda, p)$ are not sequence algebras in general.*

Proof. The result follows from the following example.

Example 3 :
$$\text{Let } A \in I_2, \quad p_{nk} = \begin{cases} \dfrac{1}{2}, & \text{if } (n,k) \in A \\[2mm] 1, & \text{otherwise} \end{cases}$$

We consider the sequences (X_{nk}) and (Y_{nk}) defined as follows:

For all $(n,k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1 + \dfrac{t}{(n+k)^2}, & \text{for} \quad -(n+k)^2 \le t \le 0 \\[3mm] 1 - \dfrac{t}{(n+k)^2}, & \text{for} \quad 0 < t \le (n+k)^2 \\[3mm] 0, & \text{otherwise} \end{cases}$$

otherwise $X_{nk} = \overline{0}$.

For all $(n,k) \notin A$,

$$Y_{nk}(t) = \begin{cases} 1 + \dfrac{t-1}{(n+k)^2}, & \text{for} \quad 1 - (n+k)^2 \le t \le 1 \\[3mm] 1 - \dfrac{t-1}{(n+k)^2}, & \text{for} \quad 1 \le t \le 1 + (n+k)^2 \\[3mm] 0, & \text{otherwise} \end{cases}$$

otherwise $Y_{nk} = \overline{0}$.

Then taking $\lambda_{nk} = \frac{1}{(n+k)^3}$, for all $n,k \in N$, we have (X_{nk}) , $(Y_{nk}) \in Z$, for $Z = {}_2m_0^{I(F)}(\Lambda, p)$ and ${}_2m^{I(F)}(\Lambda, p)$.

But $(X_{nk} \otimes Y_{nk}) \notin Z$, for $Z = {}_2m_0^{I(F)}(\Lambda, p)$ and ${}_2m^{I(F)}(\Lambda, p)$.

Hence the classes of sequences ${}_2m_0^{I(F)}(\Lambda, p)$ and ${}_2m^{I(F)}(\Lambda, p)$ are not sequence algebras.

Theorem 8. *The classes of sequences* $2m_0^{I(F)}(\Lambda, p)$ *and* ${}_2m^{I(F)}(\Lambda, p)$ *are not convergence free.*

Proof. The result follows from the following example.

Example 4:
$$\text{Let } A \in I_2, p_{nk} = \begin{cases} \dfrac{1}{2}, & \text{if} \quad (n,k) \in A \\[2mm] 2, & \text{otherwise} \end{cases}$$

We consider the sequence (X_{nk}) defined by:

For all $(n,k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1 + (n+k)(t-1) & for \quad 1 - \dfrac{1}{(n+k)} \le t \le 1 \\[2mm] 1 - (n+k)(t-1) & 1 < t \le 1 + \dfrac{1}{(n+k)} \\[2mm] 0, & otherwise \end{cases}$$

otherwise $X_{nk} = \overline{0}$.

Then taking $\lambda_{nk} = \dfrac{1}{n+k}$, for all $n,k \in N,$ we have $(X_{nk}) \in Z,$

$Z = {}_2 m^{I(F)}(\Lambda, p)$ *and* ${}_2 m_0^{I(F)}(\Lambda, p).$

We consider the sequence (Y_{nk}) defined by :

For all $(n,k) \notin A$,

$$Y_{nk}(t) = \begin{cases} 1 + \dfrac{t-1}{(n+k)^3}, & for \quad 1 - (n+k)^3 \le t \le 1 \\[2mm] 1 - \dfrac{t-1}{(n+k)^3}, & for \quad 1 \le t \le 1 + (n+k)^3 \\[2mm] 0, & otherwise \end{cases}$$

otherwise $Y_{nk} = \overline{0}$.

But $(Y_{nk}) \notin Z$, for $Z = {}_2 m^{I(F)}(\Lambda, p)$ and ${}_2 m_0^{I(F)}(\Lambda, p)$. Hence the classes of sequences ${}_2 m^{I(F)}(\Lambda, p)$ and ${}_2 m_0^{I(F)}(\Lambda, p)$ are not convergence free.

Theorem 9. $(\lambda_{nk}) \in M\left({}_2 c_0^{I(F)}(p), {}_2 c_0^{I(F)}(p)\right)$ *if and only if* $(\lambda_{nk}) \in {}_2 \ell_\infty^I(p).$

Proof. Let $(\lambda_{nk}) \in {}_2 \ell_\infty^I(p)$ and $(X_{nk}) \in {}_2 c_0^{I(F)}(\Lambda, p).$ Then there exists a $J > 0$ such that

$$P = \{(n,k) \in N \times N : |\lambda_{nk}|^{p_{nk}} \le J\} \in F(I_2) \text{ and}$$
$$Q = \left\{(n,k) \in N \times N : [\overline{d}(X_{nk}, \overline{0})]^{p_{nk}} < \frac{\varepsilon}{J}\right\} \in F(I_2).$$

Then $P \cap Q = \{(n,k) \in N \times N : [\overline{d}(\lambda_{nk} X_{nk}, \overline{0})]^{p_{nk}} < \varepsilon\} \in F(I_2).$

Thus $(\lambda_{nk} X_{nk}) \in {}_2 c_0^{I(F)}(p)$ and so $(\lambda_{nk}) \in M({}_2 c_0^{I(F)}(p), {}_2 c_0^{I(F)}(p)).$

The converse part is easy, so omitted.

Theorem 10. *If the class of sequences ${}_2 c^{I(F)}(p)$ is not solid, then*

$$(\lambda_{nk}) \notin M\left({}_2c^{I(F)}(p), {}_2c^{I(F)}(p) \right)$$

Proof. The result follows from **Lemma 2**.

Theorem 11. *If $\Omega\Lambda^{-1} = (w_{nk}\lambda_{nk}^{-1}) \in {}_2\ell_\infty^I(p)$ then $Z(\Lambda) \subset Z(\Omega)$ and the inclusion is proper, where $Z = {}_2c^{I(F)}(p), {}_2c_0^{I(F)}(p)$.*

Proof. The proof of the result is easy, so omitted. Also the inclusion is proper, which follows from the following example.

Example 5:
$$\text{Let} \quad A \in I_2, p_{nk} = \begin{cases} \dfrac{1}{2}, & if \quad (n,k) \in A \\ 2, & otherwise \end{cases}$$

We consider the sequence (X_{nk}) defined by:

For $((n,k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1 + \dfrac{t}{(n+k)}, & for \quad -(n+k) \le t \le 0 \\ 1 - \dfrac{t}{(n+k)}, & for \quad \le (n+k) \\ 0, & otherwise \end{cases}$$

otherwise $X_{nk} = \overline{0}$.

Taking $w_{nk} = \dfrac{1}{(n+k)}$, $\lambda_{nk} = 1$ for all $n, k \in N$, we have $(X_{nk}) \in Z(\Omega)$ and $(w_{nk}\lambda_{nk}^{-1})$ is bounded but $(X_{nk}) \notin Z(\Lambda)$ for $Z = {}_2c^{I(F)}(p), {}_2c_0^{I(F)}(p)$.

Hence the inclusion $Z(\Lambda) \subset Z(\Omega)$ is proper.

CONCLUSION

In this article, we introduced some classes of fuzzy real valued double sequences with a multiplier sequence of non-zero real numbers. We have studied the completeness of the introduced classes of sequences and also studied some other properties like solidness, symmetry etc. We have proved some inclusion results involving these classes of sequences.

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حول المتتاليات المتقاربة ضعيفة الضارب مثل المعيرة الشواشية النوع

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خلاصة

نقدم في هذا البحث، وللمرة الأولى، أصنافاً من المتتاليات الحقيقية الشواشية بالاعتماد على متتالية ضاربة ومتتالية ضعيفة من الأعداد الموجبة المحدودة. وندرس الخصائص الطبولوجية المختلفة لهذه الأصناف من المتتاليات. كما نقوم أيضاً بتمييز المسألة الضاربة ونحصل على علاقة إحتواء بين هذه الأصناف