Kuwait J. Sci. **47 (4)** pp. 19-28, 2020

متوسط مخطط التحكم المرجح للتصاعد الهجيني باستخدام مخططات العينات المُصنفة المتباينة

محمد نور الأمين، نافارا شبير، نصرالله خان، أفشان رياز
قسم الإحصاء، جامعة اسلام اباد- لاهور-باكستان
جامعة العلوم البيطرية - لاهور- باكستان

الملخص

يوضح هذا البحث أنه تم استخدام مخططات متباينة لعينات مُصنَفة (RSS) لتصميم هجين جديد ذا مخطط تحكم متوسط مُرجح (HEWMA) لديه حساسية أكبر للتحولات الصغيرة والمتوسطة. تم استخدام متوسط طول المدى والانحراف المعياري لطول المدى على محاكاة لمونت كارلو كمقياس لتقييم حساسية مخطط التحكم المقترح. توضح تلك المعايير أن HEWMA الذي يستخدم مخططات RSS يُزيد بشكل كبير أداء مخطط التحكم. علاوة على ذلك، يشير البحث إلى أن مخطط التحكم المقترح يسيطر على متوسط مخطط التحكم المتحرك المُرجح بشكل تصاعدي (EWMA)، ويعتمد EWMA على مخطط تحكم RSS و HEWMA باستخدام عينة عشوائية بسيطة في الكشف عن التحولات البسيطة/ المتوسطة في العمليات المتوسطة. استعان البحث بمثال يشتمل على بيانات حقيقية لتقديم مزيد من الشرح التطبيقي.

Hybrid exponentially weighted moving average control chart for mean by using different ranked set sampling schemes

Muhammad Noor-ul-Amin[1,*], Navara Shabbir[2], Nasrullah Khan[3], Afshan Riaz[4]

[1,2,4]Dept. of Statistics, COMSATS University Islamabad, Lahore Campus, PAKISTAN.
[3]Jhang Campus, University of Veterinary and Animal Sciences, Lahore
[*]Corresponding Author: nooramin.stats@gmail.com

Abstract

In this paper, different schemes of ranked set sampling (RSS) are used to design a new hybrid exponentially weighted moving average (HEWMA) control chart having more sensitivity to small and moderate shifts. The average run length and standard deviation of run length based on Monte-Carlo simulations are used as an evaluation measure for the sensitivity of the proposed control chart. They indicate that the HEWMA using RSS schemes significantly increases the performance of the control chart. In addition, the suggested control chart dominates classical exponentially weighted moving average control charts (EWMA), EWMA based on RSS, and HEWMA control charts using simple random sampling in detecting the small/moderate shifts in the process mean. An example containing real data is also given to further explain the application.

Keywords: Average run length; EWMA; HEWMA; RSS; SRS.

1. Introduction

In manufacturing processes, products are subjected to variations which directly impact the quality of a product. To monitor and control this variation, manufacturers apply statistical process control (SPC), which is a combination of quality control and statistical techniques. SPC has a direct impact on increasing the quality of a product by reducing the amount of variation and enabling it to attain the desired satisfaction level. It differentiates between variation that is the result of an assignable cause and natural variation of a stable process. Natural variation is part of any stable system; it always exists when the process is in-control. It occurs due to an uncontrollable change in temperature, human inconsistency to perform a certain work due to fatigue, etc. It is beyond control and is difficult to avoid. In the presence of special or assignable causes, the process is statistically out-of-control. These causes are identified and removed by taking some quality enhancement measures. The control charts in SPC signal deviation from target or higher variability in a process. Each plotted statistic is equivalent to performing a hypothesis test about its corresponding process parameter. (see (Montgomery *et al.*, 2005)). Some popular control charts are Shewhart, exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts introduced by Shewhart (1924), Roberts (1959) and Page (1954) respectively.

Shewhart control charts are based on the current sample information, whereas the EWMA and CUSUM control charts are based on the current sample information along with information from previous samples. These latter charts are known as memory type control charts. They are used to monitor the small and moderate shifts. The EWMA control chart is easy to interpret and implement as compared to other memory type control charts. Let $W_1, W_2, .. W_t, ...$ be a sequence of independently and identically distributed (IID) random variables that represent the EWMA statistic for in-control processes with mean (μ_0) and variance (σ_0^2), then the statistic of EWMA control chart is given by

$$Z_t = (1-\omega) Z_{t-1} + \omega W_t, \qquad 0 < \omega \le 1, \tag{1}$$

where the smoothing constant is represented with ω, whose value ranges from 0 to 1.When ω is equal to 1, the EWMA chart becomes a Shewhart chart. The respective mean and variance of the EWMA statistic for n samples is defined as

$$E(Z_t) = Z_0 = \mu_0,$$
$$V(Z_t) = \left[\frac{\omega}{2-\omega} \left\{ 1-(1-\omega)^{2t} \right\} \right] \frac{\sigma^2}{n}.$$

The time-varying EWMA control limits are defined by

$$UCL = \mu_0 + P\left(\sqrt{V(Z_t)}\right)$$
$$CL = \mu_0$$
$$LCL = \mu_0 - P\left(\sqrt{V(Z_t)}\right),$$

where n is the sample size and P is a constant, used to specify the width of control limits. The value of P depends on the specified in-control average run length denoted by ARL_0. The chart is designed to detect a shift δ in the process mean, where $\delta = \frac{|\mu_1 - \mu_o|}{\sigma_o}$, and μ_1 is the mean when the process is out-of-control.The process is said to be out-of-control if $Z_t \geq UCL$ or $Z_t \leq LCL$. In process monitoring, ranked set sampling (RSS) and its modified schemes have successfully been used in-control charts when population units are time consuming or expensive. The reader may consult Haq *et al.* (2015) , Noor-ul-Amin *et al.* (2018a) and Tayyab *et al.* (2019). Haq (2013) developed the HEWMA control charts by using two EWMA statistics while Azam *et al.* (2015), Javaid *et al.* (2020) and Noor-ul-Amin *et al.* (2019) have used HEWMA control charts in different dimensions, but no one has enhanced the sensitivity of HEWMA control chart by changing the sampling design. In the current study, we presented the HEWMA control chart with RSS using sample sizes n=3 and 5, $ARL_0 = 370$ and 500. The comparisons are made on the basis of out-of-control run length (ARL_1) at various shifts for proposed control charts and existing charts. In section 2, the basic concept of the HEWMA control chart is presented. In section 3, the RSS schemes: extreme ranked set sampling (ERSS), median ranked set sampling (MRSS) and quartile ranked set sampling (QRSS) are formulated. The proposed control charts and the results of the simulation study are presented in section 4. An illustrative example is presented in section 5, for the implementation of the proposed chart on a real data set.

2. Hybrid Exponentially Weighted Moving Average Control Chart

Haq (2013) presented the HEWMA chart to detect the small to moderate shifts. Let $HE_1, HE_2, HE_3, \ldots HE_t, \ldots$ be the random variables of HEWMA statistic, ω and ω_1 be smoothing constants of the EWMA and HEWMA statistics, respectively. The HEWMA statistic is given by

$$HE_t = \omega_1 Z_t + (1-\omega_1) HE_{t-1}, \qquad HE_0 = 0 \tag{2}$$

where Z_t is the EWMA statistic defined in (1). The respective mean and variance of the HEWMA statistic is given by

$$E(HE_t) = \mu_0, \tag{3}$$

$$\sigma^2_{HE_t} = V(HE_t) = \left(\frac{\omega\omega_1}{\omega - \omega_1}\right)^2 \left[\sum_{i=1}^{2} \frac{(1-\omega_i)^2(1-(1-\omega_i)^{2t})}{1-(1-\omega_i)^2} - \frac{2(1-\omega)(1-\omega_1)\left\{1-(1-\omega)^t(1-\omega_1)^t\right\}}{1-(1-\omega)(1-\omega_1)}\right]\frac{\sigma^2}{n}. \tag{4}$$

The control limits of the HEWMA control chart are given as

$$UCL = \mu_0 + P_H \sigma_{HE_t}$$
$$CL = \mu_0 \tag{5}$$
$$LCL = \mu_0 - P_H \sigma_{HE_t},$$

where P_H is the control constant of HEWMA statistic. The chart is said to be out-of-control if $HE_t \geq UCL$ or $HE_t \leq LCL$. Crowder (1987) discussed the selection of smoothing constant and control limit coefficient. The sensitivity of the control chart increases as the value of the EWMA parameter ω decreases. In practice ω is set within the interval $[0.05 \leq \lambda \leq 0.25]$ with $\omega = 0.05$, 0.10 and 0.20 being popular choices. A rule of thumb is to use the small values of ω to detect smaller shifts. (Montgomery, 2009). Following this, we have used the value of ω as 0.05 and $\omega_1 < \omega$. Note that the HEWMA chart has a better performance than the EWMA chart when $\omega_1 > \omega$ but $\omega_1 \leq 0.75$ (Haq, 2013).

3. Ranked Set Sampling

RSS is a sampling scheme. It improves parametric estimation by ranking collected observations. It does not rank the entire population units but a subset of sampling units. This scheme is useful where the actual measurements of study variables are difficult, i.e., pricey, time-consuming and/or destructive. In this section, we discuss the formulation of the classical RSS and its modified schemes.

3.1 Classical RSS

From a target population, draw n independent random samples each of the size n units, and rank the units within the sample either by the judgment of experts, by an auxiliary variable, or by any qualitative method which doesn't include measurement (Chen *et al.*, 2004). After ranking all n sets, the lowest ranked unit is measured from the first set. Similarly, the second lowest ranked unit is measured from second set and so forth until the largest ranked unit is selected from the n^{th} set. Let $W_{i(j)k}$, $i, j = 1,2,3\ldots n$; $k = 1,2,..,r$, be the jth order statistic in the ith sample set with cycle r. It represents one cycle of RSS when r=1. Repeat the process r times to get a sample of size $n \times r$. The respective unbiased estimator for population mean and variance of the estimator based on RSS is given by

$$\overline{W}_{RSS} = \frac{1}{rn}\sum_{k=1}^{r}\sum_{i=1}^{n} W_{i(i)k}, \text{ where } E(\overline{W}_{RSS}) = \mu_w \quad (6)$$

and $$Var\left(\overline{W}_{RSS}\right) = \frac{\sigma^2}{rn} - \frac{1}{rn^2}\sum_{i=1}^{n}\left(\mu_{w(i,n)} - \mu_w\right)^2. \quad (7)$$

3.2 Extreme RSS

Samawi and Muttlak (1996) introduced the concept of extreme ranked set sampling (ERSS). This scheme is useful when collecting the i^{th} ranked unit is a tougher task than collecting extreme units. It selects *n* samples each of size *n* units from the population under study and ranks the units within samples according to the variable of interest. For even sample size *n,* select the first ranked unit from the first $(n/2)$ samples and the last ranked unit from the last $(n/2)$ samples. When the sample size is odd, select the first ranked unit from the first $(n-1)/2$ samples and the last ranked unit from the next $(n-1)/2$ samples and $\left((n+1)/2\right)th$ unit from the last sample. The whole process represents one cycle of ERSS with *r*=1, repeat the process *r* times to get a sample of size *rn.*

For even sample sizes, the population mean estimator using ERSS with one cycle is

$$\overline{W}_{(ERSS)e} = \frac{1}{n}\left[\sum_{i=1}^{n/2} W_{i(1)} + \sum_{i=1}^{n/2} W_{\frac{n}{2}+i(n)}\right], \quad (8)$$

with variance

$$Var\left(\overline{W}_{(ERSS)e}\right) = \frac{1}{2n}\left[\sigma_{(1)}^2 + \sigma_{(n)}^2\right]. \quad (9)$$

For odd sample sizes, the population mean estimator based on ERSS with one cycle is

$$\overline{W}_{(ERSS)o} = \frac{1}{n}\left[\sum_{i=1}^{(n-1)/2} W_{i(1)} + \sum_{i=1}^{(n-1)/2} W_{\frac{(n-1)}{2}+i(n)} + W_{n\left(\frac{n+1}{2}\right)}\right], \quad (10)$$

with variance

$$Var\left(\overline{W}_{(ERSS)o}\right) = \frac{n-1}{2n^2}\left[\sigma_{(1)}^2 + \sigma_{(n)}^2\right] + \frac{1}{n^2}\left[\sigma_{\left(\frac{n+1}{2}\right)}^2\right]. \quad (11)$$

3.3 Median RSS

The median RSS (MRSS) technique (Muttlak, 1997) yields a more efficient estimator than the RSS estimator by reducing errors in the ranking. The random selection of n^2 units from the target population follows the same pattern as used in RSS. The sample is partitioned into *n* sets of size *n* and units are ranked according to a variable of interest. For an even set size, select the lowest-ranked unit from the two middle sampling units of first $n/2$ sets and select the largest-ranked units from the two middle sampling units from other $n/2$ sets. To get a sample of size *rn,* repeat the whole process *r* times.

For even sample size, the population mean estimator with MRSS for one cycle is

$$\overline{W}_{(MRSS)e} = \frac{1}{n}\left[\sum_{i=1}^{n/2} W_{i\left(\frac{n}{2}\right)} + \sum_{i=1}^{n/2} W_{\frac{n}{2}+i\left(\frac{n+2}{2}\right)}\right], \quad (12)$$

with variance

$$Var\left(\overline{W}_{(MRSS)e}\right) = \frac{1}{2n}\left[\sigma_{\left(\frac{n}{2}\right)}^2 + \sigma_{\left(\frac{n+2}{2}\right)}^2\right]. \quad (13)$$

For odd sample size, the population mean estimator using MRSS with one cycle is

$$\overline{W}_{(MRSS)o} = \frac{1}{n}\left[\sum_{i=1}^{n} W_{i\left(\frac{n+1}{2}\right)}\right], \quad (14)$$

with variance

$$Var\left(\overline{W}_{(MRSS)o}\right) = \frac{1}{n}\left[\sigma_{\left(\frac{n+1}{2}\right)}^2\right]. \quad (15)$$

3.4 Quartile RSS

Quartile ranked set sampling (QRSS) (Muttlak, 2003) which reduces errors in ranking for some (for small sample sizes) cases. It yields an efficient mean estimator. For QRSS, select *n* samples of size *n* and rank them according to the variable of interest. For even sample size select $q_1(n+1)th$ ranked unit from first *n/2* samples and from the next *n/2* samples select $q_3(n+1)th$ ranked unit where $q_1=0.25$ and $q_1=0.25$. When the sample size is odd for actual measurement, select $q_1(n+1)th$ unit from first $(n-1)/2$ samples and from the further $(n-1)/2$ samples select $q_3(n+1)th$ unit and median unit from last sample data for QRSS. To get a sample of size *rn,* repeat the process *r* times.

For even sample size, the population mean estimator using QRSS with one cycle is

$$\overline{W}_{(QRSS)e} = \frac{1}{n}\left[\sum_{i=1}^{n/2} W_{i\left(\frac{n+1}{4}\right)} + \sum_{i=1}^{n/2} W_{\frac{n}{2}+i\left(\frac{3(n+1)}{4}\right)}\right], \quad (16)$$

with variance

$$Var(\bar{W}_{(QRSS)e}) = \frac{1}{2n}\left[\sigma^2_{\left(\frac{n+1}{4}\right)} + \sigma^2_{\left(\frac{3(n+1)}{4}\right)}\right]. \quad (17)$$

For odd sample size, the population mean estimator using QRSS with one cycle is

$$\bar{W}_{(QRSS)o} = \frac{1}{n}\left[\sum_{i=1}^{(n-1)/2} W_{i\left(\frac{n+1}{4}\right)} + \sum_{i=1}^{(n-1)/2} W_{\frac{(n-1)}{2}+i\left(\frac{3(n+1)}{4}\right)} + W_{n\left(\frac{n+1}{2}\right)}\right], \quad (18)$$

with variance

$$Var(\bar{W}_{(QRSS)o}) = \frac{n-1}{2n^2}\left[\sigma^2_{\left(\frac{n+1}{4}\right)} + \sigma^2_{\left(\frac{3(n+1)}{4}\right)}\right] + \frac{1}{n^2}\left[\sigma^2_{\left(\frac{n+1}{2}\right)}\right]. \quad (19)$$

4. Proposed HEWMA Control Chart

In this section, we present the HEWMA control chart under RSS and its modifications to monitor the process mean. Select a sample size *n* using sampling scheme *R*, at each time point *t,* where *R*=RSS, ERSS, MRSS, and QRSS. The HEWMA statistic under sampling scheme *R* is defined as

$$HE_{(R)t} = \omega_1 Z_{(R)t} + (1-\omega_1) HE_{(R)t-1} \quad (20)$$

$$Z_{(R)t} = (1-\omega) Z_{(R)t-1} + \omega \bar{W}_{(R)t} \quad (21)$$

$$E(HE_{(R)t}) = \mu_0,$$

$$\sigma^2_{HE_{(R)t}} = \left(\frac{\omega\omega_1}{\omega - \omega_1}\right)^2 \left[\sum_{i=1}^{2} \frac{(1-\omega_i)^2(1-(1-\omega_i)^{2t})}{1-(1-\omega_i)^2} - \frac{2(1-\omega)(1-\omega_1)\left\{1-(1-\omega)^t(1-\omega_1)^t\right\}}{1-(1-\omega)(1-\omega_1)}\right]\sigma^2_R.$$

The control limits of the proposed control chart under the various sampling schemes are given by

$$UCL_{HE_{(R)t}} = \mu_0 + P_{HE_{(R)}}\left(\sigma_{HE_{(R)t}}\right)$$
$$CL_{HE_{(R)t}} = \mu_0$$
$$LCL_{HE_{(R)t}} = \mu_0 - P_{HE_{(R)}}\left(\sigma_{HE_{(R)t}}\right), \quad (22)$$

where $P_{HE(R)}$ are the control constants of proposed statistics (specified according to scheme). The average run length (*ARL*) and standard deviation of run length (*SDRL*) are used as performance measures to check the sensitivity of the proposed control chart. To assure the in-control ARL_0 of the proposed control chart, the value of $P_{HE(R)}$ are obtained to achieve a specific level i.e. $ARL_0 = 370$ and 500. When the value of statistic falls beyond the control limits, the chart diagnoses an out-of-control signal. The shift (δ) is given different values i.e. 0.10, 0.25, 0.5, 0.75, 1.00, 1.25, 1.5, 1.75 and 2.00 values. For different values of *n* and δ, *ARLs* and *SDRLs* are calculated and the sensitivity of control chart is compared. There are different methods in literature to compute *ARL* and *SDRL* such as the Monte-Carlo simulation method, Markov chain and integral equation. In this study, Monte-Carlo simulations are conducted with sample sizes *n*=3 and 5 by using *10000* iterations to calculate the *ARLs* and *SDRLs* with R software. The large number of iterations in RSS is time consuming; to overcome this problem we used *10000* iterations. The value of ω for the EWMA control chart is 0.05 and for the HEWMA control charts the value of ω_1 is 0.03.

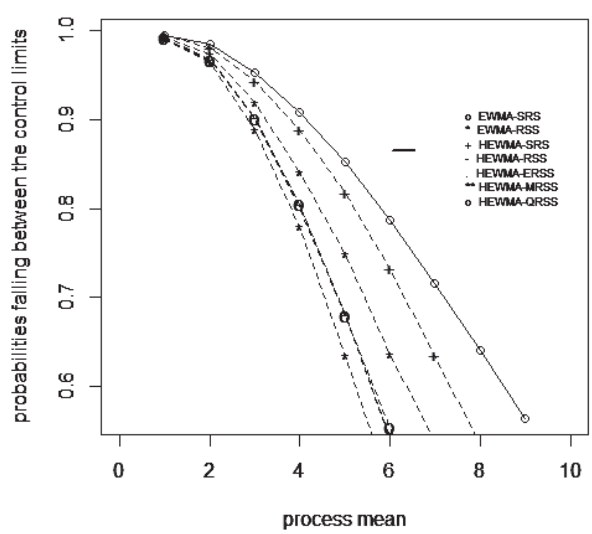

Fig. 1. OC Curve with *ARL*=370 for *n*=3

Figure 4.1 represents operating characteristic curve (OC curve) for the various control charts taking ARL_0=370 and *n*=3. The Figure 4.2 represents the OC curve for various control charts at ARL_0=500 and *n*=3. One can observe from Figures 4.1-4.2 that the probabilities falling between the controls limits quickly approaches to zero for the shifts in mean with the proposed control charts as compared to EWMA based control charts. The same pattern is observed in Figure 4.2. The computed values of the proposed control chart with the previous charts are presented in Tables 1-4. Table 1 presents the proposed control charts with sample size 3 at fixed $ARL_0 = 370$ (the same ARL_0 of 3 sigma limits of the Shewhart chart). The proposed chart under various RSS schemes is compared with existing EWMA based on simple random sampling (EWMA-SRS), hybrid EWMA based on simple random sampling (HEWMA-SRS), and EWMA-RSS charts.

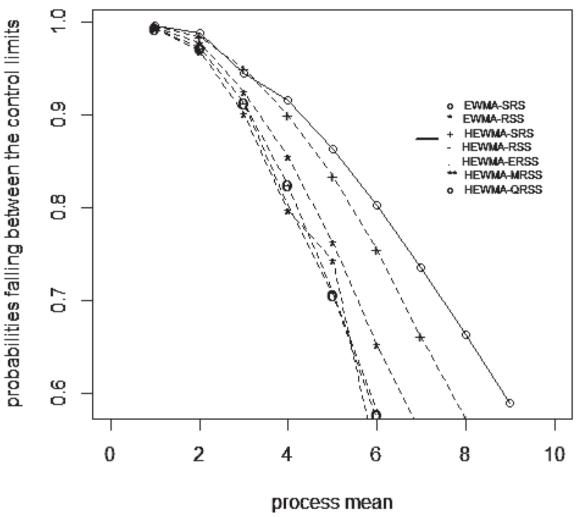

Fig. 2. *OC curve with ARL=500 with n=3*

From the results of Table 1, it is revealed that the proposed control chart under RSS and its modified schemes are more efficient than EWMA-SRS, EWMA-RSS, and HEWMA-SRS. For example, at δ = 0.1 the *ARLs* of EWMA-SRS, EWMA-RSS, and HEWMA-SRS are 210.4736, 143.6207, and 173.2291 respectively, while the respective *ARLs* of HEWMA-RSS, HEWMA-ERSS, HEWMA-MRSS, and HEWMA-QRSS are 120.8832, 117.4456, 108.0719 and 114.5822. Table 3 presented the results with sample size 5 at fixed $ARL_0 = 370$. As expected, the *ARL*s are decreasing with an increase in the sample size. For example in Table 3 with δ =0.1 the *ARLs* are decreased as compared to Table 1, such as 210.0399, 107.3397, 171.6541, 92.9715, 90.7006, 80.9843 and 85.8108 with EWMA-SRS, EWMA-RSS, HEWMA-SRS, and HEWMA-R schemes. Table 2 presented the results with sample size 3 for $ARL_0 = 500$ and Table 4 presented the results with *n*=5 at same ARL_0 A similar pattern has been observed in Tables 2 and 4 with the change of sample size as in Tables 1 and 3. It is also observed that from Tables 1-4 that HEWMA-MRSS has better performance among all the proposed control charts with minimum *ARLs*.

Table 1. The *ARLs* and *SDRLs* of control charts when $ARL_0 = 370$ and sample size *n*=3.

δ		EWMA-SRS	EWMA-RSS	HEWMA-SRS	HEWMA-RSS	HEWMA-ERSS	HEWMA-MRSS	HEWMA-QRSS
	P	2.523	2.517	1.860	1.852	1.851	1.850	1.851
0.00	*ARL*	371.5856	362.967	372.168	364.8851	372.1497	369.3698	370.1251
	SDRL	384.7957	369.0665	430.1302	429.9661	425.8971	420.3852	417.3808
0.10	*ARL*	210.4736	143.6207	173.2291	120.8832	117.4456	108.0719	114.5822
	SDRL	213.9865	142.932	192.127	128.566	126.1244	113.7807	125.1057
0.25	*ARL*	67.0834	38.2365	51.9980	31.2161	31.3841	28.1894	30.7485
	SDRL	60.7281	32.8118	50.0660	28.4433	28.6004	25.3046	28.3123
0.50	*ARL*	21.4001	12.3373	17.6457	10.2131	10.2032	9.0009	10.1892
	SDRL	16.5063	8.9086	15.4489	8.8244	8.7607	7.7099	8.7575
0.75	*ARL*	10.9209	6.3091	8.9766	5.1312	5.0288	4.5387	5.1242
	SDRL	7.7253	4.7665	7.6687	4.2223	4.1620	3.6778	4.2327
1.00	*ARL*	6.7465	3.9823	5.4700	3.1311	3.1285	2.7412	3.1178
	SDRL	4.4771	2.4337	4.5522	2.4130	2.4151	2.0607	2.4005
1.25	*ARL*	4.6827	2.7477	3.7306	2.1866	2.1816	1.9417	2.1943
	SDRL	2.9470	1.5703	2.9664	1.5244	1.5211	1.2895	1.5329
1.50	*ARL*	3.5215	2.1746	2.7422	1.6704	1.6783	1.5335	1.6717
	SDRL	2.1061	1.1476	2.0425	1.0231	1.0225	0.8635	1.0186
1.75	*ARL*	2.7811	1.7664	2.1463	1.3892	1.3936	1.2869	1.3913
	SDRL	1.5783	0.8629	1.4831	0.7092	0.7139	0.5887	0.7132
2.00	*ARL*	2.2901	1.4831	1.7684	1.2088	1.2159	1.1418	1.2090
	SDRL	1.2288	0.6639	1.1097	0.4915	0.5007	0.3953	0.4922

Table 2. The *ARLs* and *SDRLs* of control charts when $ARL_0 = 500$ and sample size *n*=3.

δ		**EWMA-SRS**	**EWMA-RSS**	**HEWMA-SRS**	**HEWMA-RSS**	**HEWMA-ERSS**	**HEWMA-MRSS**	**HEWMA-QRSS**
	P	2.640	2.620	1.988	1.985	1.983	1.981	1.984
0.00	*ARL*	499.1618	500.9285	500.505	499.0091	500.1144	500.8564	499.2481
	SDRL	512.6619	505.5442	533.4954	524.5461	522.8142	521.7661	520.7142
0.10	*ARL*	268.6158	183.0212	215.6258	148.3176	140.0241	136.016	138.8315
	SDRL	269.7042	179.7208	231.3073	152.2458	145.4476	140.0609	142.3081
0.25	*ARL*	78.1677	43.3267	59.5390	35.2444	35.2091	31.7277	35.4681
	SDRL	70.3916	35.8812	54.7743	30.6540	30.7310	27.192	30.9604
0.50	*ARL*	23.6538	13.3399	19.7851	11.4510	11.4204	10.0568	11.5518
	SDRL	17.8364	9.4447	16.3828	9.38498	9.3586	8.2156	9.3910
0.75	*ARL*	11.8860	6.8706	9.9784	5.7201	5.5843	4.9202	5.7079
	SDRL	8.2048	4.4082	8.1342	4.5233	4.4762	3.8881	4.5595
1.00	*ARL*	7.3124	4.2147	6.0438	3.4289	3.4173	2.9825	3.4060
	SDRL	4.7485	2.5148	4.8096	2.5849	2.5693	2.2137	2.5841
1.25	*ARL*	5.0723	2.9265	4.0811	2.3878	2.3361	2.1048	2.3651
	SDRL	3.1240	1.6613	3.1533	1.6519	1.6229	1.3959	1.6422
1.50	*ARL*	3.7836	2.268	2.9959	1.8106	1.8078	1.6177	1.7824
	SDRL	2.2211	1.1819	2.2033	1.1169	1.1109	0.9298	1.0990
1.75	*ARL*	2.9688	1.8078	2.3355	1.4496	1.4679	1.3390	1.4576
	SDRL	1.6550	0.8810	1.6050	0.7623	0.7802	0.6386	0.7711
2.00	*ARL*	2.4327	1.5762	1.9021	1.2551	1.2635	1.1805	1.2462
	SDRL	1.2987	0.7160	1.2105	0.5404	0.5512	0.4461	0.5346

Table 3. The *ARLs* and *SDRLs* of control charts when $ARL_0 = 370$ and sample size *n*=5.

δ		**EWMA-SRS**	**EWMA-RSS**	**HEWMA-SRS**	**HEWMA-RSS**	**HEWMA-ERSS**	**HEWMA-MRSS**	**HEWMA-QRSS**
	P	2.523	2.508	1.860	1.855	1.856	1.857	1.856
0.00	*ARL*	372.4275	370.8752	369.7005	369.0608	373.4179	368.5266	374.8436
	SDRL	389.3022	387.7831	437.7263	419.9176	424.1786	420.4862	421.732
0.10	*ARL*	210.0399	107.3397	171.6541	92.9715	90.7006	80.9843	85.8108
	SDRL	213.6267	108.2983	191.281	96.9568	96.7218	81.7770	87.4386
0.25	*ARL*	67.2383	27.7866	51.6684	23.8475	26.5032	19.436	20.6794
	SDRL	61.1125	22.8686	49.7242	21.1433	23.6482	17.1196	18.6366
0.50	*ARL*	21.4042	8.9203	17.5641	7.6201	8.5257	6.1245	6.5901
	SDRL	16.5871	6.2066	15.4007	6.4292	7.2289	5.1347	5.5540
0.75	*ARL*	10.8136	4.7213	8.9233	3.8866	4.2113	3.0817	3.2503
	SDRL	7.6273	2.9601	7.6319	3.0476	3.3808	2.3629	2.5331
1.00	*ARL*	6.7624	3.018	5.4432	2.3013	2.5793	1.9570	2.0602
	SDRL	4.5059	1.7368	4.5181	1.6454	1.9067	1.3006	1.4010
1.25	*ARL*	4.6978	2.1665	3.7207	1.6816	1.8248	1.4464	1.5163
	SDRL	2.9659	1.1468	2.9544	1.0246	1.1769	0.7734	0.8525
1.50	*ARL*	3.5209	1.6678	2.7324	1.3396	1.4468	1.1962	1.2437
	SDRL	2.1058	0.8064	2.0449	0.6567	0.7769	0.4751	0.5352
1.75	*ARL*	2.7737	1.3962	2.1458	1.1656	1.2380	1.0829	1.1070
	SDRL	1.5692	0.5972	1.4823	0.4321	0.5288	0.2979	0.3386
2.00	*ARL*	2.2892	1.2177	1.7696	1.0746	1.1191	1.0298	1.0403
	SDRL	1.2253	0.4466	1.1161	0.2793	0.3598	0.1738	0.2042

Table 4. The *ARLs* and *SDRLs* of control charts when $ARL_0 = 500$ and sample size *n*=5.

δ		EWMA-SRS	EWMA-RSS	HEWMA-SRS	HEWMA-RSS	HEWMA-ERSS	HEWMA-MRSS	HEWMA-QRSS
	P	2.652	2.610	1.977	1.975	1.972	1.970	1.974
0.00	*ARL*	503.6835	499.2638	496.0514	500.7819	500.312	500.5041	500.8484
	SDRL	519.1414	516.1574	566.5857	540.1909	533.9403	528.9114	530.418
0.10	*ARL*	269.4096	159.7485	210.6057	108.1894	110.8373	97.6546	103.2852
	SDRL	270.4187	143.1488	225.0733	109.0684	113.2717	90.1179	105.6024
0.25	*ARL*	78.1026	31.6040	58.9572	27.4820	29.7279	22.0286	25.6069
	SDRL	70.4539	25.4703	54.3211	22.8000	25.1995	18.4174	19.0421
0.50	*ARL*	23.8046	10.6354	19.6206	8.2126	9.1719	6.7854	7.1429
	SDRL	17.9686	8.5316	16.4129	6.6911	7.5158	5.4535	5.7986
0.75	*ARL*	11.8578	4.9679	9.9199	2.5335	4.6382	3.3545	3.5313
	SDRL	8.1659	3.0750	8.1169	1.7879	3.6218	2.5205	2.6817
1.00	*ARL*	7.3310	3.1402	6.0246	2.5335	2.8140	2.1059	2.1857
	SDRL	4.7553	1.7857	4.8047	1.9687	2.0449	1.4011	1.4820
1.25	*ARL*	5.0467	2.2554	4.0619	1.7842	1.9509	1.5226	1.5949
	SDRL	3.0954	1.1788	3.1503	1.0950	1.2660	0.8342	0.9099
1.50	*ARL*	3.7635	1.7638	2.9661	1.4095	1.5272	1.2463	1.2803
	SDRL	2.2062	0.8523	2.1790	0.7185	0.8417	0.5323	0.5743
1.75	*ARL*	2.9637	1.4437	2.3212	1.2084	1.2752	1.1033	1.1312
	SDRL	1.6515	0.6272	1.6056	0.4825	0.5681	0.3303	0.3751
2.00	*ARL*	2.4303	1.2516	1.8849	1.0985	1.1489	1.0414	1.0531
	SDRL	1.2947	0.47180	1.1977	0.3222	0.4012	0.2049	0.2335

Table 5. The HEWMA-RSS and EWMA-RSS control chart with $\omega = 0.25$ and $\omega_1 = 0.2$.

Sample number	Density (*Y*)	$HE_{(R)t}$	$UCL_{HE(R)t}$	$LCL_{HE(R)t}$	E_t	UCL_{Et}	LCL_{Et}
1	9.5	15.5	15.2	15.7	15.3	14.3	16.6
2	8.4	15.4	15.0	15.8	15.8	13.9	16.9
3	9.8	15.3	14.9	16.0	15.2	13.8	17.1
4	11.0	15.4	14.7	16.2	15.2	13.7	17.2
5	8.3	15.4	14.6	16.3	14.8	13.7	17.2
6	9.9	15.4	14.6	16.4	14.6	13.7	17.2
7	8.6	15.3	14.5	16.4	14.7	13.7	17.2
8	6.4	15.4	14.5	16.5	14.8	13.7	17.3
9	7.0	15.5	14.4	16.5	15.1	13.7	17.3
10	8.2	15.5	14.4	16.5	15.7	13.7	17.3
11	17.4	15.5	14.6	16.5	15.7	13.7	17.3
12	15.0	15.7	14.4	16.6	15.6	13.7	17.3
13	15.2	15.8	14.3	16.6	15.77	13.67	17.3
14	16.4	15.8	14.37	16.6	15.9	13.7	17.3
15	16.7	15.9	14.36	16.6	15.9	13.7	17.3
16	15.4	16.1	14.36	16.6	15.3	13.7	17.3
17	15.0	16.2	14.36	16.6	15.4	13.7	17.3
18	14.5	16.3	14.3	16.6	15.4	13.7	17.3
19	14.8	16.4	14.3	16.6	15.2	13.7	17.3

20	13.6	16.4	14.3	16.6	15.0	13.7	17.3
21	25.6	16.5	14.3	16.6	15.9	13.7	17.3
22	23.4	16.5	14.3	16.6	16.9	13.7	17.3
23	24.4	**16.6***	14.3	16.6	17.2	13.7	17.3
24	23.3	**16.8***	14.3	16.6	16.8	13.7	17.3
25	19.5	**17.1***	14.3	16.6	**17.8***	13.7	17.3
26	21.2	**17.3***	14.3	16.6	**18.0***	13.7	17.3
27	22.8	**17.4***	14.3	16.6	**17.8***	13.7	17.3
28	21.7	**17.5***	14.3	16.6	**17.8***	13.7	17.3
29	19.8	**17.6***	14.3	16.6	**18.4***	13.7	17.3
30	21.3	**17.7***	14.3	16.6	**18.5***	13.7	17.3

5. Illustrative Example

The application of the proposed chart is illustrated on the data set of (Yan and Su, 2009) where the quality characteristic under investigation is the density. The dataset consists of 30 samples drawn from a population whose in-control with μ_y =15.47 and σ_y^2 =34.023. The set is presented in Table 5. From the dataset, the first 20 samples are taken from the in-control process and the last 10 samples are taken from the shifted process, which is introduced as $0.5\times\sigma_y$. For a valid comparison between the EWMA-RSS and HEWMA-RSS, the ARL_0 is fixed at 200 with sample size 3 for both control charts. Figures 3 and 4 display the corresponding charts. The HEWMA-RSS control chart detects an out-of-control signal at sample 23, whereas the EWMA-RSS control chart detects an out-of-control signal at sample 25. In addition to signaling the out-of-control condition earlier; the HEWMA-RSS signal an upward trend that started around sample 12. From Figures 1 and 2 results, it is concluded that the proposed control chart has better performance than its competitor chart.

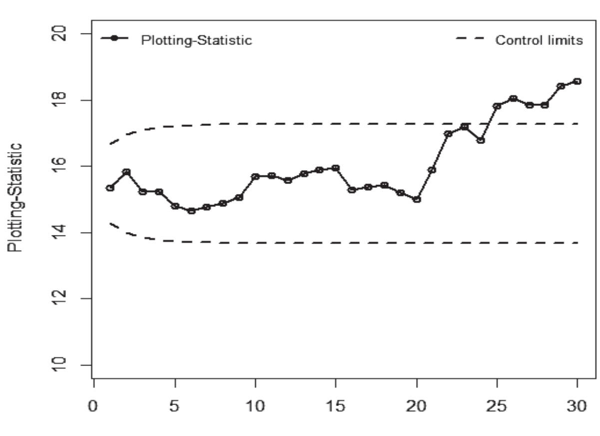

Fig. 3. The EWMA-RSS control chart

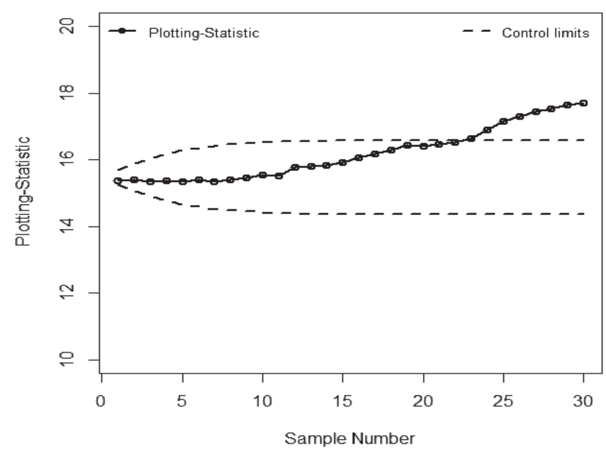

Fig. 4. The HEWMA-RSS control chart

6. Conclusion

The HEWMA control chart for monitoring the process mean with different sampling schemes called RSS, MRSS, QRSS, ERSS is introduced. The valid comparisons with their counterparts with different sampling schemes are presented, based on the values of *ARLs* and *SDRLs*. By fixing ARL_0 to 370 and 500 with sample sizes n=3 and 5, it is revealed that that the proposed control chart has more sensitivity and efficiency in detecting an out-of-control signal for various values of shifts. The application of the proposed control chart on the real dataset supports the results and confirms the superiority of the proposed control charts over the considered control charts. The HEWMA control chart with different schemes of RSS is recommended is a better control chart for the detection of small shifts in the mean of the production process. The proposed work can be extended by utilizing the auxiliary information under ranked set sampling following by authors such as Ismail *et al.* (2018) and Noor-ul-Amin *et al.* (2018b). Future, the effect of measurement error on proposed charts can also be studied.

References

Azam, M., Aslam, M. & Jun, C.H. (2015). Designing of a hybrid exponentially weighted moving average control chart using repetitive sampling. *The International Journal of Advanced Manufacturing Technology,* **77,** 1927-1933.

Chen, Z., Bai, Z. & Sinha, K. (2004). Ranked set sampling: theory and application lecture notes in statistics.

Crowder, S. (1987). A simple method for studying run–length distributions of exponentially weighted moving average charts. *Technometrics,* **29,** 401-407.

Haq, A. (2013). A new hybrid exponentially weighted moving average control chart for monitoring process mean. *Quality and Reliability Engineering International,* **29,** 1015-1025.

Haq, A., Brown, J., Moltchanova, E. & Al-Omari, A. I. (2015). Effect of measurement error on exponentially weighted moving average control charts under ranked set sampling schemes. *Journal of Statistical Computation and Simulation,* **85,** 1224-1246.

Ismail, M., Kawnal, N. & Shahbaz, M.Q. (2018). Generalized ratio-product-type estimator for variance using auxiliary information in simple random sampling. *Kuwait Journal of Science,* **45**.

Javaid, A., Noor-ul-Amin, M. & Hanif, M. (2020). A new Max-HEWMA control chart using auxiliary information. *Communications in Statistics-Simulation and Computation*, **49**, 1285-1305.

Montgomery, D.C. (2009). *Introduction to statistical quality control*, John Wiley & Sons (New York).

Montgomery, D.C., Myers, R.H., Carter Jr, W.H. & Vining, G.G. (2005). The hierarchy principle in designed industrial experiments. *Quality and reliability engineering international,* **21,** 197-201.

Muttlak, H. (1997). Median ranked set sampling. *J Appl Stat Sci,* **6,** 245-255.

Muttlak, H.A. (2003). Investigating the use of quartile ranked set samples for estimating the population mean. *Applied Mathematics and Computation,* **146,** 437-443.

Noor-ul-Amin, M., Arif, F. & Hanif, M. (2018a). A new scheme using the ranked sets. *Journal of Statistics and Management Systems,* **21,** 1565-1573.

Noor-ul-Amin, M., Khan, S. & Aslam, M. (2018b). An EWMA control chart using two parametric ratio estimator. *Journal of Industrial and Production Engineering,* **35,** 298-308.

Noor-ul-Amin, M., Khan, S. & Sanaullah, A. (2019). HEWMA control chart using auxiliary information. *Iranian Journal of Science and Technology, Transactions A: Science,* **43,** 891-903.

Page, E.S. (1954). Continuous inspection schemes. *Biometrika,* **41,** 100-115.

Roberts, S. (1959). Control chart tests based on geometric moving averages. *Technometrics,* **1,** 239-250.

Samawi, H.M. & Muttlak, H.A. (1996). Estimation of ratio using rank set sampling. *Biometrical Journal,* **38,** 753-764.

Shewhart, W. A. (1924). Some applications of statistical methods to the analysis of physical and engineering data. *Bell System Technical Journal,* **3,** 43-87.

Tayyab, M., Noor-ul-Amin, M. & Hanif, M. (2019). Exponential weighted moving average control charts for monitoring the process mean using pair ranked set sampling schemes. *Iranian Journal of Science and Technology, Transactions A: Science,* **43,** 1941-1950.

Yan, X. & Su, X. (2009). *Linear regression analysis: theory and computing*, World Scientific.

Submitted : 30/03/2019
Revised : 21/01/2020
Accepted : 06/02/2020