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حلول جديدة لمعادلة الأمواج المشتتة القابلة للتكامل مع مشتق الزمن الكسري الناشيء في نماذج هندسة المحيطات

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الملخص

تُستخدم معادلة Camassa-Holm الكسرية بوجه عام كأداة قوية في محاكاة الكمبيوتر للأمواج في نماذج المياه الضحلة والساحلية ومياه الموانيء. في هذا البحث، تم الحصول على حلول أمواج جديدة من هذه المعادلة باستخدام طريقة جبرية مباشرة وجديدة. تم الحصول على ستة وثلاثين حلاً جديداً وتم تمثيلها بيانياً بالكامل. هذه الحلول قد تحفز الأبحاث المستقبلية حول هذا الموضوع.

New wave solutions of an integrable dispersive wave equation with a fractional time derivative arising in ocean engineering models

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Abstract

The fractional Camassa-Holm equation is generally used as a powerful tool in computer simulations of water waves in shallow water, coastal and harbor models. In this paper, new wave solutions of this equation are obtained by using a new extended direct algebraic method. Thirty-six completely new solutions are obtained and are graphically represented. These solutions may motivate future research on the topic.

Keywords: Camassa-Holm equation; Conformable fractional derivative; new extended direct algebraic method; shallow water waves; wave solutions.

1. Introduction

On the evaluation of many events taking place in the universe, certain approaches called idealization are used. Idealization brings a deterministic perspective to our perception of physical events. From the physical point of view, in terms of predicting an event's pattern, idealization is quite successful in reflecting the main frame of the physical event, although it neglects some aspects of the event. This method works very well in systems where interactions are linear. However, it is impossible to use this method in systems where interactions are nonlinear, such as in oceans having large water waves. The results obtained by using linearization (ignoring nonlinear interactions) to idealize such systems, solitons, are unpredictable from the results obtained by linearization. On the other hand, solitons are of great importance not only for studying oceans, but also for modeling nonlinear wave interactions in nuclear physics (Birse, 1990), solid state physics (Ovid'ko, 1987) and fiber optics (Mitschke *et al.*, 2017).

In nature, most of the systems have nonlinear interactions, and this nonlinearity drives the system into a "chaotic" state, a phenomenon that is quite difficult to study. In physics, such systems are often referred to as dynamical systems. Water waves are especially the most common dynamical system in nature. In recent years, many researchers have focused on water waves (Qin *et al.*, 2018) due to their very rich mathematical structure. The use of unique nonlinear differential equations is inevitable for solving problems involving such systems (Marathe & Govindarajan, 2014). Equations such as Korteweg-de Vries (KdV) (Pelinovsky & Stepanyants, 2018), Kadomtsev-Petviashvili (Guo & Tian, 2018), Nonlinear Schrodinger (Huang *et al*, 2018) and Camassa-Holm (CH) (Crisan & Holm, 2018) are commonly used in particular nonlinear differential equations for the evaluation of water waves.

The fully nonlinear and generalized CH equation considered in Tian and Yin (2004) is as follows:

$$u_t + \kappa u_x + \beta_1 u_{xxt} + \beta_2 (u^l)_x + \beta_3 u_x (u^n)_{xx}, \\ + \beta_4 u (u^p)_{xxx} = 0$$

where $\beta_1, \beta_2, \beta_3, \beta_4$ are arbitrary real constants. Taking $\beta_1 = -1$, $\beta_2 = 3/2$, $\beta_3 = -2$, $\beta_4 = -1$, $l = 2$, $n = p = 1$, the above equation becomes a new shallow water wave equation called the CH equation.

The CH equation is a nonlinear dispersive wave equation (Crisan & Holm, 2018):

$$u_t + \kappa u_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \qquad (1)$$

which represents the gravitational propagation of unidirectional irrotational shallow water waves over a flatbed (Constantin & Holm, 2009) and water waves moving over an underlying shear flow (Constantin &

Ivanov, 2008), and where $u(x,t)$ is the velocity of the fluid, κ is the coefficient related to the critical shallow water wave speed. Due to its complete integrability and solvability via inverse scattering transform (Eckhardt, 2017) and Lax pair formulation (Chang & Szmigielski, 2018), the CH equation has attracted a lot of attention in recent years (Wang & Li, 2019; Matsuno, 2017; Luo, 2018). From a physical viewpoint, the integrability property makes it possible to accurately solve the initial value problem (Zhang & Li, 2003). The ordinary collision feature of the solitons on the real plane arises due to the integrability of the CH equation (Parker, 2008). One of the most important features of the CH equation is that it has a bi-Hamiltonian structure (Fokas & Fuchssteiner, 1980), which leads to solutions of breaking waves. This can be described as the transformation of wave energy into turbulent kinetic energy at a critical amplitude value of the wave (Kazolea & Ricchiuto, 2018). Because group velocity is lower at shallow waters, the occurrence of breaking waves is quite possible at the coastlines (Xiao *et al*., 2010). Because of these properties, the solution of the CH equation can help ocean engineers. Boyd (1997) found that if the solitary wave slowly varies with $\xi = x - ct$, then the extra terms on the right hand side of Eq. (1) will be small, and the soliton is given to the lowest order by the solutions of following:

$$u_t + 2\kappa u_x - u_{xxt} + buu_x = 0.$$

Wazwaz (2005) investigated a modified form of above-mentioned equation as

$$u_t + 2\kappa u_x - u_{xxt} + bu^n u_x = 0.$$

In this paper, we consider $n = 2$, so the equation reduces to a simplified and modified CH equation (Ali *et al*., 2016). Also considering $\beta_1 = -1, l = 3$ and $\beta_3 = \frac{\gamma}{3}$ yields modified a simplified CH equation (Ali *et al*., 2016).

Fractional calculus, which corresponds to arbitrary order differentiation and integration, has been a research focal point for the past decades. Many scientists believe that fractional models are the best way to explain the complexity and nonlinearity of nature (Garg & Manohar, 2013; Ghany & Hyder, 2014). In this regard, many definitions are used for fractional derivatives which are named after Caputo, Riemann-Liouville, Grünwald-Letnikov (Miller & Ross, 1993; Kilbas *et al*., 2006; Podlubny, 1998). However, it is seen that these definitions fall short in relation to satisfying basic properties of integer order differentiation and integration (Khalil *et al*., 2014). Some of these derivative definitions do not satisfy basic essential rules of pure mathematics such as the product, quotient, and chain rules, and the derivative of a constant. Recently a well-behaved, applicable, understandable, and effective fractional order derivative called the conformable fractional derivative was proposed by Khalil *et al.,* (2014). This new fractional derivative definition overcomes the deficiencies of the above-mentioned definitions. By using this new derivative definition, exact solutions of many models arising in ocean engineering, physics, social sciences can be elegantly obtained (Hosseini *et. al.*, 2017a; Hosseini *et. al.*, 2017b; Hosseini *et. al.*, 2018c; Hosseini *et. al.*, 2017d; Hosseini *et. al.*, 2017e).

In this study, new wave solutions of a time-fractional CH equation are obtained by using a new extended direct algebraic method. This method is applied to the time fractional CH equation for the first time and, to the best of our knowledge, the solutions derived in this paper are new and have never appeared in the literature.

2. Basics of conformable fractional calculus

In this section, the basic definitions and properties of conformable fractional calculus are expressed.

Definition 1. Let $f:(0,\infty)\to\mathbb{R}$ be a function. Then μ^{th} order conformable fractional derivative of *f* is defined as (Khalil *et al*., 2014):

$$D_t^{\mu} f(t) = \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\mu}) - f(t)}{\varepsilon},$$

for all $t > 0$, $\mu \in (0,1)$.

Definition 2. Suppose that $a \geq 0$ and $t \geq a$. Also let *f* be a function defined on $(a,t]$ and $\mu \in \mathbb{R}$. Then the $\mu-$fractional integral of *f* is defined by

$$_tI_a^{\mu} f(t) = \int_a^t \frac{f(x)}{x^{1-\mu}}dx$$

when the Riemann improper integral exists (Khalil *et al*., 2014).

Some basic properties of the conformable fractional derivative are given below (Khalil *et al*., 2014; Abdeljawad, 2015).

1. $\frac{d^{\mu}}{dt^{\mu}}(t^{\eta}) = \eta t^{\eta-\mu}$,

2. $\frac{d^{\mu}}{dt^{\mu}}\left(f(t)g(t)\right) = f(t)\frac{d^{\mu}}{dt^{\mu}}\left(g(t)\right) + g(t)\frac{d^{\mu}}{dt^{\mu}}\left(f(t)\right)$,

3. $\frac{d^{\mu}}{dt^{\mu}}\left((fog)(t)\right)=t^{1-\mu}g'(t)f'\left(g(t)\right)$,

where $\mu\in(0,1)$.

3. A brief description of the new extended direct algebraic method

Let us give the description of the new extended direct algebraic method (Rezazadeh *et al.*, 2017). Consider the following nonlinear time fractional partial differential equation of the form

$$S\left(u,D_t^{\mu}u,D_xu,D_t^{(2\mu)}u,D_x^2u,\ldots\right)=0. \qquad (2)$$

Here, u is an unknown function, S is a polynomial of u and $D_t^{(n\mu)}$ means the sequential conformable fractional derivative of order $n\,(n\in\mathbb{Z}^+)$ of the function u. Define the wave transformation as:

$$u(x,t)=U(\xi),\qquad \xi=kx+w\frac{t^{\mu}}{\mu}, \qquad (3)$$

where k and w are arbitrary constants to be determined later. Using the chain rule (Abdeljawad, 2015) and wave transform (3) in Equation (2) we get the following nonlinear ordinary differential equation

$$G\left(U,U',U'',\ldots\right)=0, \qquad (4)$$

where prime denotes the integer order derivative of function U with respect to ξ. Suppose that Eq. (4) has a formal solution of the form

$$U(\xi)=\sum_{j=0}^{N}b_jQ^j(\xi),\quad b_N\neq0, \qquad (5)$$

where $b_j\left(0\leq j\leq N\right)$ are constant coefficients to be determined later, N is a positive integer which is found by balancing procedure in Equation (4) and $Q(\xi)$ satisfies the ODE in the form

$$Q'\left(\xi\right)=Log(A)\left(\alpha+\beta Q(\xi)+\sigma Q^2(\xi)\right),\ A\neq0,1, \qquad (6)$$

where α,β and σ are constants. The solution set of Equation (6) is given as follows:

1) When $\beta^2-4\alpha\sigma<0$ and $\sigma\neq0$,

$$Q_1(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\times\tan_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\xi\right),$$

$$Q_2(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\times\cot_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\xi\right),$$

$$Q_3(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\times\left(\tan_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\pm\sqrt{pq}\sec_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\right)$$

$$Q_4(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\times\left(-\cot_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\pm\sqrt{pq}\csc_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\right),$$

$$Q_5(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4\sigma}\times\left(\tan_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4}\xi\right)-\cot_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4}\xi\right)\right).$$

2) When $\beta^2-4\alpha\sigma>0$ and $\sigma\neq0$,

$$Q_6(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\times\tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\xi\right),$$

$$Q_7(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\times\coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\xi\right),$$

$$Q_8(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\times\left(-\tanh_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\right),$$

$$Q_9(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\times\left(-\coth_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\right),$$

$$Q_{10}(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{4\sigma}\times\left(\tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\xi\right)+\coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\xi\right)\right).$$

3) When $\alpha\sigma>0$ and $\beta=0$,

$$Q_{11}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\tan_A\left(\sqrt{\alpha\sigma}\xi\right),$$

$$Q_{12}(\xi)=-\sqrt{\frac{\alpha}{\sigma}}\cot_A\left(\sqrt{\alpha\sigma}\xi\right),$$

$$Q_{13}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\sec_A\left(2\sqrt{\alpha\sigma}\xi\right)\right),$$

$$Q_{14}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\left(-\cot_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\csc_A\left(2\sqrt{\alpha\sigma}\xi\right)\right),$$

$$Q_{15}(\xi)=\frac{1}{2}\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)-\cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)\right).$$

4) When $\alpha\sigma<0$ and $\beta=0$,

$$Q_{16}(\xi)=-\sqrt{-\frac{\alpha}{\sigma}}\tanh_A\left(\sqrt{-\alpha\sigma}\xi\right),$$

$$Q_{17}(\xi)=-\sqrt{-\frac{\alpha}{\sigma}}\coth_A\left(\sqrt{-\alpha\sigma}\xi\right),$$

$$Q_{18}(\xi)=\sqrt{-\frac{\alpha}{\sigma}}\left(-\tanh_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$Q_{19}(\xi)=\sqrt{-\frac{\alpha}{\sigma}}\left(-\coth_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$Q_{20}(\xi)=-\frac{1}{2}\sqrt{-\frac{\alpha}{\sigma}}\left(\tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)+\coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)\right).$$

5) When $\beta=0$ and $\sigma=\alpha$,

$$Q_{21}(\xi)=\tan_A\left(\alpha\xi\right),$$

$$Q_{22}(\xi)=-\cot_A\left(\alpha\xi\right),$$

$$Q_{23}(\xi)=\tan_A\left(2\alpha\xi\right)\pm\sqrt{pq}\sec_A\left(2\alpha\xi\right),$$

$$Q_{24}(\xi)=-\cot_A\left(2\alpha\xi\right)\pm\sqrt{pq}\csc_A\left(2\alpha\xi\right),$$

$$Q_{25}(\xi)=\frac{1}{2}\left(\tan_A\left(\frac{\alpha}{2}\xi\right)-\cot_A\left(\frac{\alpha}{2}\xi\right)\right).$$

6) When $\beta=0$ and $\sigma=-\alpha$,

$$Q_{26}(\xi)=-\tanh_A\left(\alpha\xi\right),$$

$$Q_{27}(\xi)=-\coth_A\left(\alpha\xi\right),$$

$$Q_{28}(\xi)=-\tanh_A\left(2\alpha\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\alpha\xi\right),$$

$$Q_{29}(\xi)=-\coth_A\left(2\alpha\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\alpha\xi\right),$$

$$Q_{30}(\xi)=-\frac{1}{2}\left(\tanh_A\left(\frac{\alpha}{2}\xi\right)+\coth_A\left(\frac{\alpha}{2}\xi\right)\right).$$

7) When $\beta^2=4\alpha\sigma$,

$$Q_{31}(\xi)=\frac{-2\alpha(\beta\xi Log(A)+2)}{\beta^2\xi Log(A)}.$$

8) When $\beta=k$, $\alpha=mk$ $(m\neq 0)$ and $\sigma=0$,

$$Q_{32}(\xi)=A^{k\xi}-m.$$

9) When $\beta=\sigma=0$,

$$Q_{33}(\xi)=\alpha\xi Log(A).$$

10) When $\beta=\alpha=0$,

$$Q_{34}(\xi)=-\frac{1}{\sigma\xi Log(A)}.$$

11) When $\alpha=0$ and $\beta\neq 0$,

$$Q_{35}(\xi)=-\frac{p\beta}{\sigma\left(\cosh_A(\beta\xi)-\sinh_A(\beta\xi)+p\right)},$$

$$Q_{36}(\xi)=-\frac{q\beta}{\sigma\left(\cosh_A(\beta\xi)-\sinh_A(\beta\xi)+q\right)},$$

$$Q_{37}(\xi)=-\frac{\beta\left(\sinh_A(\beta\xi)+\cosh_A(\beta\xi)\right)}{\sigma\left(\sinh_A(\beta\xi)+\cosh_A(\beta\xi)+q\right)},$$

12) When $\beta=k$, $\sigma=mk$ $(m\neq 0)$ and $\alpha=0$,

$$Q_{38}(\xi)=\frac{pA^{k\xi}}{p-mqA^{k\xi}}.$$

Here, x is an independent variable, p and q are arbitrary constants greater than zero that are called deformation parameters. Substituting Equations (5) and (6) into Equation (4) and equating the coefficients of $Q^j(\xi)$ to zero, one gets the nonlinear algebraic system in b_j $(j=0,1,\ldots,N)$ and k and w. Then putting the obtained values of constants and solution set of Equation (6) into Equation (5) using the wave transform (3), we get the exact wave solutions for Equation (2).

Remark 1. The generalized hyperbolic and trigonometric functions are defined as

$$\sinh_A(\xi)=\frac{pA^{\xi}-qA^{-\xi}}{2},$$

$$\cosh_A(\xi)=\frac{pA^{\xi}+qA^{-\xi}}{2},$$

$$\tanh_A(\xi)=\frac{pA^{\xi}-qA^{-\xi}}{pA^{\xi}+qA^{-\xi}},$$

$$\coth_A(\xi)=\frac{pA^{\xi}+qA^{-\xi}}{pA^{\xi}-qA^{-\xi}},$$

$$\operatorname{sech}_A(\xi)=\frac{2}{pA^{\xi}+qA^{-\xi}},$$

$$\operatorname{csch}_A(\xi)=\frac{2}{pA^{\xi}-qA^{-\xi}},$$

$$\sin_A(\xi)=\frac{pA^{i\xi}-qA^{-i\xi}}{2i},$$

$$\cos_A(\xi)=\frac{pA^{i\xi}+qA^{-i\xi}}{2},$$

$$\tan_A(\xi)=-i\frac{pA^{i\xi}-qA^{-i\xi}}{pA^{i\xi}+qA^{-i\xi}},$$

$$\cot_A(\xi)=i\frac{pA^{i\xi}+qA^{-i\xi}}{pA^{i\xi}-qA^{-i\xi}},$$

$$\sec_A(\xi)=\frac{2}{pA^{i\xi}+qA^{-i\xi}},$$

$$\csc_A(\xi)=\frac{2i}{pA^{i\xi}-qA^{-i\xi}},$$

4. Wave solutions of time fractional CH equation with considered method

The time fractional simplified modified CH equation which is integrable shallow water wave equation (Kurt, 2019) is given by:

$$D_t^{\mu}u+2\kappa D_x u-D_t^{\mu}D_x^2u+\gamma u^2D_x u=0, \tag{7}$$

where $u(x,t)$ is the velocity of the fluid, κ is the coefficient related to the critical shallow water wave speed and γ is a nonzero constant. Applying the chain rule (Abdeljawad, 2015) with the help of wave transform (3) and integrating once we have

$$wU+2k\kappa U-k^2wU''+\frac{\gamma k}{3}U^3=0, \tag{8}$$

where prime indicates the derivative of function U with respect to ξ. Suppose that Equation (8) has the solution in the following form:

$$U(\xi)=\sum_{j=0}^{N}b_jQ^j(\xi),\quad b_N\neq 0.$$

Employing the balancing procedure in Equation (8) we have $N=1$, so the solution can be considered

$$V(\xi)=b_0+b_1Q(\xi). \tag{9}$$

Substituting Equations (9) and (6) into Equation (8), gathering the coefficients of $Q^j(\xi)$, and setting them equal to zero yields the following system of algebraic equations for b_0, b_1, k and w:

$$Q^0: b_0w+\frac{1}{3}b_0^3k\gamma+2b_0k\kappa-b_1k^2w\alpha\beta Log^2A=0$$

$$Q^1: b_1w+b_0^2b_1k\gamma+2b_1k\kappa-b_1k^2w\beta^2Log^2A$$
$$-2b_1k^2w\alpha\sigma Log^2A=0,$$

$$Q^2: b_0b_1^2k\gamma-3b_1k^2w\beta\sigma Log^2A=0,$$

$$Q^3: \frac{1}{3}b_1^3k\gamma-2b_1k^2w\sigma^2Log^2A=0.$$

Solving above the equation system with the aid of Mathematica, we have:

$$b_0 = \pm - \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2(\beta^2-4\alpha\sigma)Log^2A)}},$$

$$b_1 = \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2(\beta^2-4\alpha\sigma)Log^2A)}}, \quad (10)$$

$$w = - \frac{4k\kappa}{2+k^2(\beta^2-4\alpha\sigma)Log^2A}.$$

The solutions of (7) can be expressed for different cases as follows:

When $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$,

$$u_1(x,t) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} + \frac{\sqrt{-\Delta}}{2\sigma}\tan_A\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right),$$

$$u_2(x,t) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \times\left(-\frac{\beta}{2\sigma} - \frac{\sqrt{-\Delta}}{2\sigma}\cot_A\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right),$$

$$u_3(x,t) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} + \frac{\sqrt{-\Delta}}{2\sigma} \times\left(\tan_A\left(\sqrt{-\Delta}\xi\right) \pm \sqrt{pq}\sec_A\left(\sqrt{-\Delta}\xi\right)\right)\right)$$

$$u_4(x,t) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} + \frac{\sqrt{-\Delta}}{2\sigma} \times\left(-\cot_A\left(\sqrt{-\Delta}\xi\right) \pm \sqrt{pq}\csc_A\left(\sqrt{-\Delta}\xi\right)\right)\right),$$

$$u_5(x,t) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} + \frac{\sqrt{-\Delta}}{4\sigma} \times\left(\tan_A\left(\frac{\sqrt{-\Delta}}{4}\xi\right) - \cot_A\left(\frac{\sqrt{-\Delta}}{4}\xi\right)\right)\right)$$

where $\Delta = \beta^2 - 4\alpha\sigma$ and

$$\xi = kx + \left(-\frac{4k\kappa}{2+k^2(\beta^2-4\alpha\sigma)Log^2A}\right)\frac{t^\mu}{\mu}.$$

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$,

$$u_6(\xi) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} - \frac{\sqrt{\Delta}}{2\sigma}\tanh_A\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right),$$

$$u_7(\xi) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} - \frac{\sqrt{\Delta}}{2\sigma}\coth_A\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right),$$

$$u_8(\xi) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} + \frac{\sqrt{\Delta}}{2\sigma} \times\left(-\tanh_A\left(\sqrt{\Delta}\xi\right) \pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\Delta}\xi\right)\right)\right),$$

$$u_9(\xi) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} + \frac{\sqrt{\Delta}}{2\sigma} \times\left(-\coth_A\left(\sqrt{\Delta}\xi\right) \pm \sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\Delta}\xi\right)\right)\right),$$

$$u_{10}(\xi) = \pm \frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}} \pm \frac{2\sqrt{6}k\sqrt{\kappa\sigma}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{\beta}{2\sigma} - \frac{\sqrt{\Delta}}{4\sigma} \times\left(\tanh_A\left(\frac{\sqrt{\Delta}}{4}\xi\right) + \coth_A\left(\frac{\sqrt{\Delta}}{4}\xi\right)\right)\right)$$

where $\Delta = \beta^2 - 4\alpha\sigma$ and

$$\xi = kx + \left(-\frac{4k\kappa}{2+k^2(\beta^2-4\alpha\sigma)Log^2A}\right)\frac{t^\mu}{\mu}.$$

When $\alpha\sigma > 0$ and $\beta = 0$,

$$u_{11}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\times\left(\sqrt{\frac{\alpha}{\sigma}}\tan_A\left(\sqrt{\alpha\sigma}\xi\right)\right),$$

$$u_{12}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\times\left(-\sqrt{\frac{\alpha}{\sigma}}\cot_A\left(\sqrt{\alpha\sigma}\xi\right)\right),$$

$$u_{13}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\times\left(\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\sec_A\left(2\sqrt{\alpha\sigma}\xi\right)\right)\right),$$

$$u_{14}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\times\left(\sqrt{\frac{\alpha}{\sigma}}\left(-\cot_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\csc_A\left(2\sqrt{\alpha\sigma}\xi\right)\right)\right),$$

$$u_{15}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\frac{1}{2}\sqrt{\frac{\alpha}{\sigma}}\times\left(\tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)-\cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)\right)\right)$$

where $\Delta=-4\alpha\sigma$ and

$$\xi=kx+\left(-\frac{2k\kappa}{1-2k^2\alpha\sigma Log^2A}\right)\frac{t^\mu}{\mu}.$$

When $\alpha\sigma<0$ and $\beta=0$,

$$u_{16}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\sqrt{-\frac{\alpha}{\sigma}}\times\tanh_A\left(\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$u_{17}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\sqrt{-\frac{\alpha}{\sigma}}\times\coth_A\left(\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$u_{18}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\sqrt{-\frac{\alpha}{\sigma}}\times\left(-\tanh_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right)\right),$$

$$u_{19}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\sqrt{-\frac{\alpha}{\sigma}}\times\left(-\coth_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right)\right),$$

$$u_{20}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\frac{1}{2}\sqrt{-\frac{\alpha}{\sigma}}\times\left(\tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)+\coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)\right)\right)$$

where $\Delta=-4\alpha\sigma$ and

$$\xi=kx+\left(-\frac{2k\kappa}{1-2k^2\alpha\sigma Log^2A}\right)\frac{t^\mu}{\mu}.$$

When $\beta=0$ and $\sigma=\alpha$,

$$u_{21}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\tan_A\left(\alpha\xi\right)\right),$$

$$u_{22}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\cot_A\left(\alpha\xi\right)\right),$$

$$u_{23}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\tan_A\left(2\alpha\xi\right)\pm\sqrt{pq}\sec_A\left(2\alpha\xi\right)\right),$$

$$u_{24}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(-\cot_A\left(2\alpha\xi\right)\pm\sqrt{pq}\csc_A\left(2\alpha\xi\right)\right),$$

$$u_{25}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\frac{1}{2}\left(\tan_A\left(\frac{\alpha}{2}\xi\right)-\cot_A\left(\frac{\alpha}{2}\xi\right)\right)\right)$$

where $\Delta=-4\alpha^2$ and

$$\xi=kx+\left(-\frac{2k\kappa}{1-2k^2\alpha^2Log^2A}\right)\frac{t^\mu}{\mu}.$$

When $\beta=0$ and $\sigma=-\alpha$,

$$u_{26}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\tanh_A\left(\alpha\xi\right)\right),$$

$$u_{27}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\coth_A\left(\alpha\xi\right)\right),$$

$$u_{28}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\tanh_A\left(2\alpha\xi\right)\right. \mp i\sqrt{pq}\,\mathrm{sech}_A\left(2\alpha\xi\right)\Big),$$

$$u_{29}(\xi)=\pm\frac{2\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\coth_A\left(2\alpha\xi\right)\right. \mp\sqrt{pq}\,\mathrm{csch}_A\left(2\alpha\xi\right)\Big),$$

$$u_{30}(\xi)=\pm\frac{\sqrt{6}k\sqrt{\kappa}\alpha LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\left(\tanh_A\left(\frac{\alpha}{2}\xi\right)+\coth_A\left(\frac{\alpha}{2}\xi\right)\right)$$

where $\Delta=4\alpha^2$ and

$$\xi=kx+\left(-\frac{2k\kappa}{1+2k^2\alpha^2Log^2A}\right)\frac{t^\mu}{\mu}.$$

When $\beta^2=4\alpha\sigma$, we get

$$u_{31}(\xi)=\pm\frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-2\gamma}}\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-2\gamma}}\times\left(\frac{-2\alpha(\beta\xi LogA+2)}{\beta^2\xi LogA}\right)$$

where $\xi=kx+\left(-2k\kappa\right)\frac{t^\mu}{\mu}$.

For the case $\beta=\alpha=0$

$$u_{32}(\xi)=\frac{2\sqrt{3}k\sqrt{\kappa}}{\xi\sqrt{-\gamma}}$$

where $\xi=\left(kx-\frac{2kt^\mu\kappa}{\mu}\right)$.

When $\alpha=0$ and $\beta\neq0$,

$$u_{33}(\xi)=\pm\frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\beta^2Log^2A)}}\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\beta^2Log^2A)}}\times\left(\frac{p\beta}{\sigma\left(\cosh_A(\beta\xi)-\sinh_A(\beta\xi)+p\right)}\right),$$

$$u_{34}(\xi)=\pm\frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\beta^2Log^2A)}}\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\beta^2Log^2A)}}\times\left(\frac{q\beta}{\sigma\left(\cosh_A(\beta\xi)-\sinh_A(\beta\xi)+q\right)}\right),$$

$$u_{35}(\xi)=\pm\frac{\sqrt{6}k\beta\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\beta^2Log^2A)}}\pm\frac{2\sqrt{6}k\sqrt{\kappa}\sigma LogA}{\sqrt{-\gamma(2+k^2\beta^2Log^2A)}}\times\left(\frac{\beta\left(\sinh_A(\beta\xi)+\cosh_A(\beta\xi)\right)}{\sigma\left(\sinh_A(\beta\xi)+\cosh_A(\beta\xi)+q\right)}\right),$$

where $\xi=kx-2k\kappa\frac{t^\mu}{\mu}$.

When $\beta=l$, $\sigma=ml(m\neq0)$, $p=q$, $\alpha=0$,

$$u_{36}(\xi)=\pm\frac{\sqrt{6}kl\sqrt{\kappa}LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\pm\frac{2\sqrt{6}k\sqrt{\kappa}ml LogA}{\sqrt{-\gamma(2+k^2\Delta Log^2A)}}\times\left(\frac{A^{l\xi}}{1-mA^{l\xi}}\right)$$

where $\Delta=l^2$ and $\xi=kx+\left(-\frac{4k\kappa}{2+k^2l^2Log^2A}\right)\frac{t^\mu}{\mu}$.

5. Graphical Illustrations

In this section graphical representations of some remarkable solutions are given. Figures 1 and 3 show the traveling wave solutions of $u_6(x,t)$ and $u_9(x,t)$, respectively. Figures 2, 4 and 5 represent the soliton solutions of $u_7(x,t)$, $u_{10}(x,t)$ and $u_{17}(x,t)$, respectively.

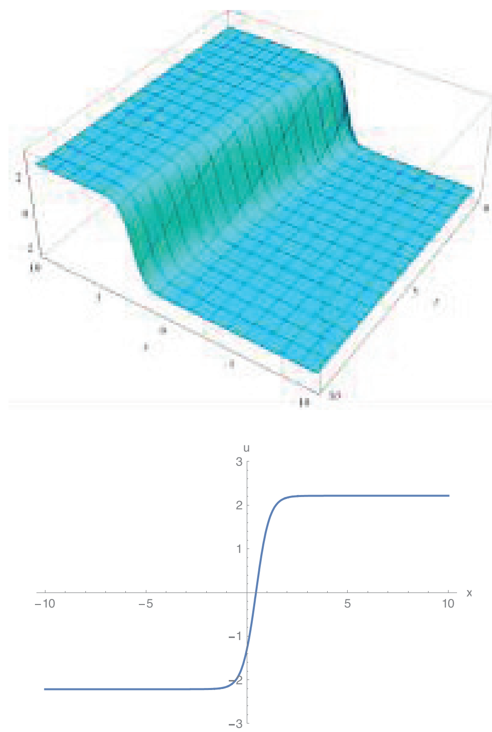

Fig. 1. The 3D (a) and 2D (b) graphs of the solution $u_6(x,t)$ for $A=\mathrm{e}$, $k=1$, $p=1$, $q=0$, $\mu=0.8$, $\kappa=1$, $\gamma=-1$, $\alpha=0$, $\beta=3$, $\sigma=1$.

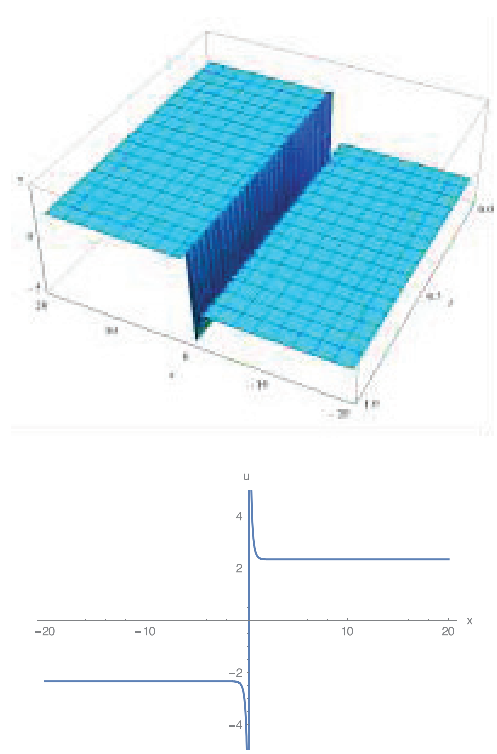

Fig. 2. The 3D (a) and 2D (b) graphs of the solution $u_7(x,t)$ for $A=\mathrm{e}$, $k=-2$, $p=1$, $q=1$, $\mu=0.8$, $\kappa=1$, $\gamma=-1$, $\alpha=1$, $\beta=3$, $\sigma=1$.

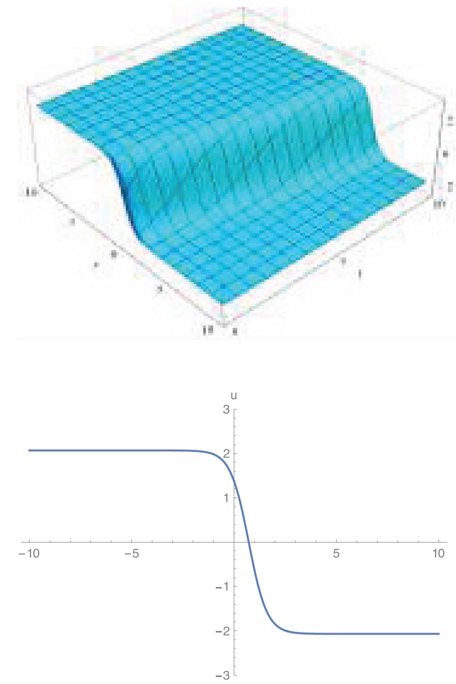

Fig. 3. The 3D (a) and 2D (b) graphs of the solution $u_9(x,t)$ for $A=\mathrm{e}$, $k=1$, $p=1$, $q=1$, $\mu=0.8$, $\kappa=1$, $\gamma=-1$, $\alpha=1$, $\beta=3$, $\sigma=1$.

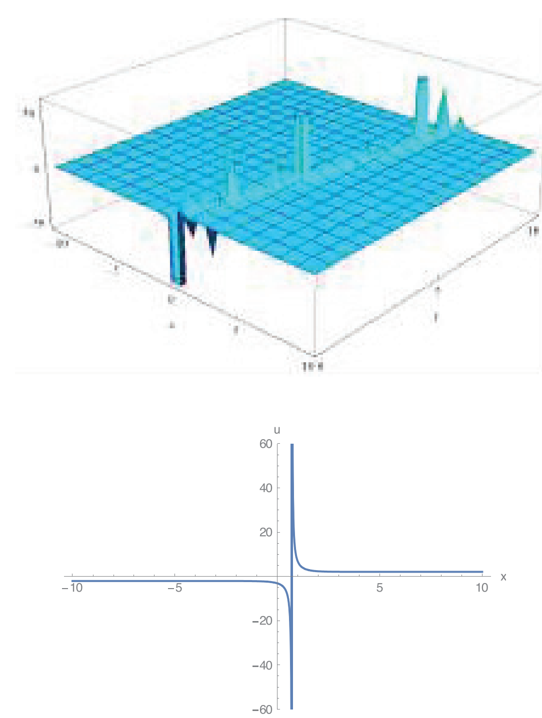

Fig. 4. The 3D (a) and 2D (b) graphs of the solution $u_{10}(x,t)$ for $A=\mathrm{e}$, $k=1$, $p=1$, $q=1$, $\mu=0.8$, $\kappa=1$, $\gamma=-1$, $\alpha=1$, $\beta=3$, $\sigma=1$.

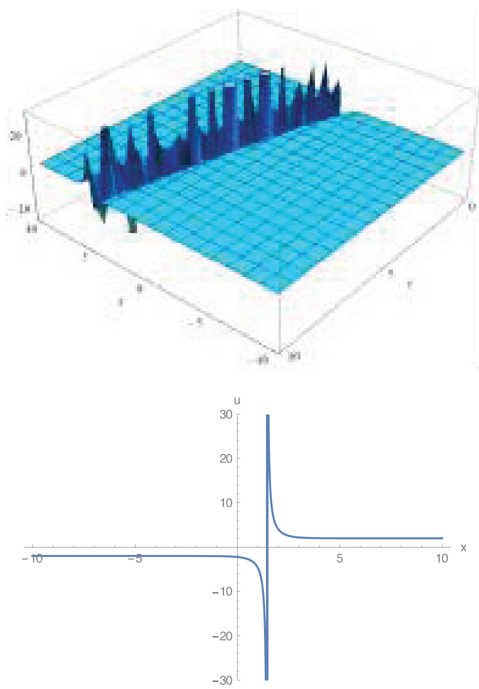

Fig. 5. The 3D (a) and 2D (b) graphs of the solution $u_{17}(x,t)$ for $A=e$, $k=1$, $p=1$, $q=1$, $\mu=0.8$, $\kappa=1$, $\gamma=-1$, $\alpha=1$, $\beta=0$, $\sigma=1$.

6. Conclusions

In this paper, a new direct algebraic method was used to obtain new wave solutions of the time fractional Camassa-Holm equation with a conformable derivative. This method depends on an auxiliary differential equation. The obtained solutions include the solutions that found by other analytical methods for the changing values of A. Also 3D and 2D graphical representations are given for different types of wave solutions. All the solutions were verified with the aid of *Mathematica*. The obtained results show that the considered method is a reliable, applicable, and efficient tool for evaluating waves which arise in shallow oceans water. The new wave solutions can be utilized in computer simulations of coastal and harbor modeling. These results can be useful for further research on ocean science and modelling.

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