Kuwait J. Sci. **41** (3) pp. 47-61, 2014

On Atanassov's intuitionistic fuzzy grade of the direct product of two hypergroupoids

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ABSTRACT

In this paper we continue the study of the sequences of join spaces and Atanassov's intuitionistic fuzzy sets associated with the direct product of two hypergroupoids with special properties and we determine the Atanassov's intuitionistic fuzzy grade of such a sequence.

Keywords: Atanassov's intuitionistic fuzzy set; direct product; fuzzy grade; fuzzy set; hypergroup; join space.

2000 Mathematics Subject Classification: 20N20, 20N25.

INTRODUCTION

Algebraic hyperstructures theory has been introduced by Marty (1934) at the 8th Congress of Scandinavia. In the last decades, many connections between hyperstructures and other branches of mathematics were considered leading to applications in geometry, hypergraphs, binary relations, combinatorics, artificial intelligence, automata and rough & fuzzy sets.

After the introduction of the concept of fuzzy set by Zadeh (1965), the study of hyperstructures connected with fuzzy sets represents a new line of research since the last twenty years. Rosenfeld defined the fuzzy subgroup of a group (Rosenfeld, 1971). Till then one can distinguish three principal approaches of this theme: the study of new crisp hyperoperation obtained by means of fuzzy sets, the study of fuzzy subhyperstructures and the study of structures endowed with fuzzy hyperoperation and called fuzzy hyperstructures. The notion of fuzzy group has been generalized by Davvaz (1999), introducing the concept of fuzzy subhypergroup of a hypergroup. Later on, this subject has been studied in depth, also in connection with other structures, like rings, modules (Davvaz, 2001), n-ary hypergroups (Davvaz *et al.*, 2009), hypersystems (Zhan & Davvaz, 2010) and complete hypergroups (Cristea & Darafsheh, 2010). The books (Corsini, 1993; Corsini & Leoreanu, 2003; Davvaz, 2013; Davvaz and Leoreanu, 2007; Vougiouklis, 1994) are surveys of the theory of algebraic hyperstructures and their applications, also see (Corsini & Leoreanu, 1995).

Two fundamental relations between hyperstructures and fuzzy sets were considered by Corsini; he associated a join space with a fuzzy set (Corsini, 1994), and then a fuzzy set with a hypergroupoid H (Corsini, 2003). These connections lead to a sequence of fuzzy sets and join spaces, which ends if two consecutive join spaces are isomorphic. The length of this sequence is called the fuzzy grade of the hypergroupoid H. Till now, one determined the fuzzy grade of the i.p.s. hypergroups of order less than or equal to 7 (Corsini & Cristea, 2003; Corsini & Cristea, 2004), of the complete hypergroups or 1-hypergroups which are not compete (Corsini & Cristea, 2005; Cristea, 2002). Moreover, several properties of the above sequence have been determined in the general case (Stefanescu & Cristea, 2008), and also for the direct product of two hypergroupoids (Cristea, 2010). Corsini *et al.* studied the same sequence associated with a hypergraph (Corsini & Leoreanu-Fotea, 2010; Corsini, *et al.,* 2008), and with multivalued functions (Corsini & Mahjoob, 2010). In Cristea \& Davvaz (2010), the authors extended the notion of fuzzy grade of a hypergroupoid to that of Atanassov's intuitionistic fuzzy grade. The study of Atanassov's intuitionistic fuzzy grade has been analyzed for i.p.s. hypergroups of order less than or equal to 7 (Davvaz *et al.,* 2012a) hypergraph (Davvaz *et al.,* 2012b).

The paper is structured as follows. Introductory concepts on hypergroups and on their intuitionistic fuzzy grade are briefly reviewed. Then, we study the study the sequence of join spaces associated with hypergroupoid $(H \times H, \otimes)$ and $(H_1 \times H, \otimes)$ in a particular case.

PRELIMINARIES

In this section we briefly recall some basic notions about hypergroups. For a comprehensive overview of this area, the reader is refereed to Corsini (1993; Corsini & Leoreanu (2003); Davvaz & Dudek (2006); Davvaz *et al.* (2008); Davvaz & Leoreanu-Fotea (2007) and Vougiouklis (1994).

Let H be a nonempty set and $P^*(H)$ be the set of all nonempty subsets of H. A set H endowed with a hyperoperation $\circ : H^2 \to P^*(H)$ is called a *hypergroupoid*. The image of the pair $(x, y) \in H \times H$ is denoted by $x \circ y$. If A and B are nonempty subsets of H,

then $A \circ B = \bigcup\limits_{\substack{a \in A \\ b \in B}} a \circ b$

If the hyperoperation satisfies the conditions:

(i) For any $(a, b, c) \in H^3, (a \circ b) \circ c = a \circ (b \circ c)$ (the *associativity*),

(ii) For any $a \in H, H \circ a = a \circ H = H$ (the *reproducibility*),

then the hypergroupoid $\langle H, \circ \rangle$ is a *hypergroup*. A hypergroup $\langle H, \circ \rangle$ is called *total hypergroup* if, for any $(x, y) \in H^2, x \circ y = H$.

For each pair $(a, b) \in H^2$, we denote:

$$a/b = \{x \in H \mid a \in xob\} \quad and\ b \backslash a = \{y \in H \mid a \in boy\}.$$

A commutative hypergroup $\langle H, \circ \rangle$ is called a *join space,* if for any four elements $a, b, c, d \in H$, such that $a/b \cap c/d \neq \phi$, it follows $a \circ d \cap b \circ c \neq \phi$. The notion of join space has been introduced by Prenowitz. Later, together with Jantosciak (Prenowits & Jantosciak, 1972). He reconstructed, from the algebraic point of view, several branches of geometry: the projective, the descriptive and the spherical geometry.

Let $\langle H, \circ \rangle$ and $\langle H', \circ' \rangle$ be two hypergroupoids and $f: H \to H'$ a map from H to H'. We say:

(i) f is a *homomorphism* if, for all $(x, y) \in H^2, f(x \circ y) \subseteq f(x) \circ' f(y)$;

(ii) f is a *good homomorphism* if, for all $(x, y) \in H^2, f(x \circ y) \subseteq f(x) \circ' f(y)$.

We say that the two hypergroups are *isomorphic*, and we write $H \cong H'$, if there is a good homomorphism between them, which is also a bijection.

ATANASSOV'S INTUITIONISTIC FUZZY GRADE OF HYPERGROUPS

In this paper we adopt the terminology and notation used in Corsini (2003); Cristea (2010) and Cristea & Davvaz (2010). For the beginning, we remember the construction of the sequence of join spaces and Atanassov's intuitionistic fuzzy sets associated with a hypergroupoid H.

As a generalization of the notion of fuzzy set in a nonempty set X, Atanassov has introduced the concept of *intuitionistic fuzzy set* in X (Atanassov, 1986; Atanassov, 1994) as an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$, where, for any $x \in X$, the *degree of membership* of x (namely $\mu_A(x)$) and the *degree of non-membership* of x (namely $\lambda_A(x)$) verify the relation $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For simplicity, we denote an Atanassov's intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$, by $A = (\mu, \lambda)$.

For any hypergroupoid $\langle H, \circ \rangle$, in Cristea & Davvaz (2010), the authors defined an Atanassov's intuitionistic fuzzy set $A = (\overline{\mu}, \overline{\lambda})$ in the following way: for any $u \in H$, we consider:

$$\overline{\mu}(u) = \frac{\sum_{(x,y)\in Q(u)} \frac{1}{|x\circ y|}}{n^2} = \frac{A(u)}{n^2}, \overline{\lambda}(u) = \frac{\sum_{(x,y)\in Q(u)} \frac{1}{|x\circ y|}}{n^2} = \frac{A'(u)}{n^2}.(\theta)$$

where $Q(u) = \{(a,b) \in H^2 | u \in a \circ b\}, \overline{Q}(u) = \{(a,b) \in H^2 | u \notin a \circ b\}$. If $Q(u) = \phi$, we set $\overline{\mu}(u) = 0$ and similarly, if $\overline{Q}(u) = \phi$ we set $\overline{\lambda}(u) = 0$. Moreover, it is clear that, for any $u \in H, 0 \leq \overline{\mu}(u) + \overline{\lambda}(u) \leq 1$.

Now, let $A = (\overline{\mu}, \overline{\lambda})$ be an Atanassov's intuitionistic fuzzy set on H. We may associate with H two join spaces $\langle {}_0H, \circ_{\overline{\mu}\wedge\overline{\lambda}} \rangle$ and $\langle {}^0H, \circ_{\overline{\mu}\wedge\overline{\lambda}} \rangle$, where, for any fuzzy set α on H, the hyperproduct "$\circ_\alpha$", introduced in Corsini (1994) is defined by

$$x \circ_\alpha y = \{u \in H | \alpha(x) \wedge \alpha(y) \leq \alpha(u) \leq \alpha(x) \vee \alpha(y)\}$$

Corsini has proved that the associated hypergroup $\langle H, \circ_\alpha \rangle$ is a join space (Corsini, 1994).

By using the same procedure as in (θ), from the join space $\langle {}_0H, \circ_{\overline{\mu}\wedge\overline{\lambda}} \rangle$ we can construct the Atanassov's intuitionistic fuzzy set $\overline{A}_1 = (\overline{\mu}_1, \overline{\lambda}_1)$ as in (θ); we associate again the join space $\langle {}_1H, \circ_{\overline{\mu}_1\wedge\overline{\lambda}_1} \rangle$, we determine, like in (θ), its Atanassov's intuitionistic fuzzy set $\overline{A}_2 = (\overline{\mu}_2, \overline{\lambda}_2)$ and we construct the join space $\langle {}_2H, \circ_{\overline{\mu}_2\wedge\overline{\lambda}_2} \rangle$ and so on. We obtain the sequence $({}_iH = \langle {}_iH, \circ_{\overline{\mu}_i\wedge\overline{\lambda}_i}; \overline{A}_i = (\overline{\mu}_i, \overline{\lambda}_i))_{i\geq 0}$ of join spaces and Atanassov's intuitionistic fuzzy sets with H. Similarly we may construct the second sequence $({}^iH = \langle {}^iH, \circ_{\overline{\mu}_i\wedge\overline{\lambda}_i}; \overline{A}_i = (\overline{\mu}_i, \overline{\lambda}_i)_{i\geq 0}$.

The length of the two corresponding sequences associated with H, called in Cristea & Davvaz (2010) the lower, and respectively, the upper Atanassov's intuitionistic fuzzy grade of H, more exactly:

Definition 3.1 *(Cristea & Davvaz, 2010) A set H endowed with an Atanassov's intuitionistic fuzzy set $A = (\overline{\mu}, \overline{\lambda})$ has the lower Atanassov's intuitionistic fuzzy grade $m, m \in N$, and we write $l.i.f.g.(H) = m$ if, for any $i, 0 \leq i < m-1$, the join spaces $\langle {}_iH, \circ_{\overline{\mu}_i\wedge\overline{\lambda}_i} \rangle$ and $\langle {}_{i+1}H, \circ_{\overline{\mu}_{i+1}\wedge\overline{\lambda}_{i+1}} \rangle$ associated with H are not isomorphic (where ${}_0H] \langle {}_0H, \circ_{\overline{\mu}\wedge\overline{\lambda}} \rangle$ and for any $s, s \geq m, {}_sH$, is isomorphic with ${}_{m-1}H$.*

Definition 3.2 *(Cristea & Davvaz, 2010) A set H endowed with an Atanassov's intuitionistic fuzzy set $A = (\overline{\mu}, \overline{\lambda})$ has the upper Atanassov's intuitionistic fuzzy grade $m, m \in N$, and we write $u.i.f.g.(H) = m$ if, for any $i, 0 \leq i < m-1$, the join spaces $\langle {}^iH, \circ_{\overline{\mu}_i\wedge\overline{\lambda}_i} \rangle$ and $\langle {}^{i+1}H, \circ_{\overline{\mu}_{i+1}\vee\overline{\lambda}_{i+1}} \rangle$ associated with H are not isomorphic (where ${}^0H = \langle {}^0H, \circ_{\overline{\mu}\vee\overline{\lambda}} \rangle$ and for any $s, s \geq m, {}^sH$, is isomorphic with ${}^{m-1}H$.*

Note that, if we start the construction of the above sequences with a hypergroupoid $\langle H, \circ \rangle$, and not with a set H endowed with an Atanassov's intuitionistic fuzzy set, then we obtain only one sequence of join spaces because the join spaces $\langle {}_0H, \circ_{\overline{\mu} \wedge \overline{\lambda}} \rangle$ and $\langle {}^0H, \circ_{\overline{\mu} \vee \overline{\lambda}} \rangle$ are isomorphic (Cristea & Davvaz, 2010). In order to explain this situation, we recall the following definition.

Definition 3.3 (Cristea & Davvaz, 2010) *We say that a hypergroupoid H has the Atanassov's intuitionistic fuzzy grade m, $m \in N$, and we write $i.f.g.(H) = m$, if $l.i.f.g.(H) = m$*

It is important to know when these join spaces are non-isomorphic. It is obvious that it has to be answered for two consecutive join spaces in the built in sequence, since in the case of isomorphism, the sequence ends. In order to resolve this problem, one introduces some notations. Let $({}_iH = \langle {}_iH, \circ_{\overline{\mu}_o \wedge \overline{\lambda}_i} \rangle; \overline{A}_i = (\overline{\mu}_i, \overline{\lambda}_i))_{i \geq 0}$ be the sequence of join spaces and Atanassov's intuitionistic fuzzy sets with a hypergroupoid H. Then, for any i, there are r, namely $r = r_i$ and a partition $\prod = \{{}^iC_j\}_{j=1}^r$ of ${}_iH$ such that, for any $j \geq 1, x, y \in^i Cj \Leftrightarrow \overline{\mu}_i(x) \wedge \overline{\lambda}_i(x) = \overline{\mu}_i(y) \wedge \overline{\lambda}_i(y)$. For $x \in H$, we denote $\lambda(x) = i_j$, when $x \in^i C_j$. On the set of classes $\{{}^iC_j\}_{j=1}^r$ we define the following ordering relation:

$$\overline{\mu}_i(x) \wedge \overline{\lambda}_i(x) < \overline{\mu}_i(y0 \wedge \overline{\lambda}_i(y) \ (therefore\ \lambda(x) < \lambda(y)).$$

With any ordered chain $({}^iC_{j_i}, {}^iC_{j_2}, ..., {}^iC_{j_r})$ we may associate an ordered r-tuple $(k_{j_1}, k_{j_2}, ..., k_{j_r})$, where $k_{j_1} = |{}^iC_{j_1}|$, for all $l, 1 \leq l \leq r$.

We conclude this section with a result expressing a necessary and sufficient condition such that two consecutive join spaces in the associated sequence of a hypergroupoid are isomorphic.

Theorem 3.4 *(Cristea & Davvaz, 2010) Let ${}^iH$ and ${}^{i+1}H$ be the join spaces associated with H determined by the membership functions $\tilde{\mu}_i$ and $\tilde{\mu}_{i+1}$, where ${}^iH = \bigcup_{l=1}^{r_1} C_l$, ${}^{i+1}H \bigcup_{l=1}^{r_2} C'_1$ and $(k_1, k_2, ..., k_{r_1})$ is the r_1-tuple associated with ${}^iH$, $(k'_1, k'_2, ..., k'_{r_2})$ is the r_2-tuple associated with ${}^{i+1}H$. The join spaces ${}^iH$ and ${}^{i+1}H$ are isomorphic if and only if $r_1 = r_2$ and $(k_1, k_2, ..., k_{r_1}) = (k'_1, k'_2, ..., k'_{r_2})$ or $(k_1, k_2, ..., k_{r_1}) = (k'_{r_1}, k'_{r_1 - 1}, ..., k'_1)$.*

Definition 3.5 *The equivalence relationR is called:*

1. regular on the right (on the left) if for all x of H, from aRb, it follows that $(a \circ x)\overline{R}(b \circ x)$ $((x \circ a)\overline{R}(x \circ b)$ respectively);
2. strongly regular on the right (on the left) if for all x of H, from aRb, it follows that $(a \circ x)\overline{\overline{R}}(b \circ x)$ $((x \circ a)\overline{\overline{R}}(x \circ b)$ respectively);
3. regular (strongly regular) if it is regular (strongly regular) on the right and on the left.

ATANASSOV'S INTUITIONISTIC FUZZY GRADE OF THE DIRECT PRODUCT OF TWO HYPERGROUPOIDS

Cristea investigated the fuzzy grade of the direct product of two hypergroupoids, when $|H/R_{\overline{\mu}}| = 2$, where $\tilde{\mu}$ is the fuzzy subset associated with it (Cristea, 2010). In this section we deal with the Atanassov's intuitionistic fuzzy grade of them with property that $|H/R_{\overline{\mu}\wedge\overline{\lambda}}| = 2$, where $R_{\overline{\mu}\wedge\overline{\lambda}}$ is the regular equivalence defined on H by

$$xR_{\overline{\mu}\wedge\overline{\lambda}}x' \Leftrightarrow (\overline{\mu} \wedge \overline{\lambda})(x) = (\overline{\mu} \wedge \overline{\lambda})(x')$$

We verify the Atanassov's intuitionistic fuzzy grade of a hypergroupoid $(H \times H, \otimes)$, in a particular case, that is when r-tuple $(k_1, k_2, ...k_r)$ associated with H has particular form (k, k), and also $H_1 \times H_2$, when (k, k), (l, l) associated with H_1 and H_2, respectively.

For any two hypergroupoids $(H_1, \circ_1)$ and $(H_2, \circ_2)$ endowed with membership functions and non membership functions $\overline{\mu}_i, \overline{\lambda}_i$, $i \in \{1, 2\}$, the hyperproduct is defined as it follows:

$$(a_1, a_2) \otimes (b_1, b_2) = \{(x, y)|x \in a_1 \circ_1 b_1, y \in a_2 \circ_1 b_2\}.$$

With the new hypergroupoid $(H_1 \times H_2, \otimes)$, we characterize a membership function $\overline{\mu}_\otimes : H_1 \times H_2 \to [0, 1]$ and non membership function $\overline{\lambda}_\otimes : H_1 \times H_2 \to [0, 1]$.

Now we determine the general formula for calculating the values of the membership function and non membership $\overline{\mu}_\otimes, \overline{\lambda}_\otimes$.

Theorem 4.1 *Let $\overline{\mu}_1, \overline{\mu}_2, \overline{\mu}_\otimes$ and $\overline{\lambda}_1, \overline{\lambda}_2, \overline{\lambda}_\otimes$, are the membership functions and non membership functions associated with H_1, H_2 and $H_1 \times H_2$, respectively, then for any $(x, y) \in H_1 \times H_2$, we have the relations:*

1. $\overline{\mu}_\otimes(x, y) = \overline{\mu}_1(x).\overline{\mu}_2(y)$.
2. $\overline{\lambda}_\otimes(x, y) = \overline{\lambda}_1(x).\overline{\lambda}_2(y) + \overline{\lambda}_1(x).\overline{\mu}_2(y) + \overline{\mu}_1(x).\overline{\lambda}_2(y)$..

Proof: We prove only the second part of the theorem. For the first part of the theorem Cristea (2010). For any $x \in H_1, y \in H_2$ we find:

$$|(a_1, a_2) \otimes (b_1, b_2)| = |\{(x, y)|x \in a_1 \circ_1 b_1, y \in a_2 \circ_2 b_2\}|$$

$$= |\{x, y)|x \in a_1 \circ_1 b_1, y \in a_2 \circ_2 b_2\}|$$

$$= |\{x \in H_1|x \in a_1 \circ_1 b_1|.|\{y \in H_2|y \in a_2 \circ_2 b_2\}|$$

$$= |a_1 \circ_1 b_1|.|a_2 \circ_2 b_2|$$

It follows that:

$$A(x,y) = \sum_{(x,y)\notin(a_1,a_2)\otimes(b_1,b_2)} \frac{1}{|(a_1,a_2)\otimes(b_1,b_2)|}$$

$$= \sum_{x\notin a_1\circ_1 b_1} \sum_{x\notin a_2\circ_2 b_2} \frac{1}{|(a_1 \circ_1 a_2)|} \cdot \frac{1}{(b_1 \circ_2 b_2)|}$$

$$+ \sum_{x\notin a_1\circ_1 b_1} \sum_{x\in a_2\circ_2 b_2} \frac{1}{|(a_1 \circ_1 a_2)|} \cdot \frac{1}{(b_1 \circ_2 b_2)|}$$

$$+ \sum_{x\in a_1\circ_1 b_1} \sum_{x\notin a_2\circ_2 b_2} \frac{1}{|(a_1 \circ_1 a_2)|} \cdot \frac{1}{(b_1 \circ_2 b_2)|}$$

$$= \left(\sum_{x\notin a_1\circ_1 b_1} \frac{1}{|(a_1 \circ_1 a_2)|}\right)\left(\sum_{x\notin a_2\circ_2 b_2} \frac{1}{(b_1 \circ_2 b_2)|}\right)$$

$$+\left(\sum_{x\notin a_1\circ_1 b_1} \frac{1}{|(a_1 \circ_1 a_2)|}\right)\left(\sum_{x\in a_2\circ_2 b_2} \frac{1}{(b_1 \circ_2 b_2)|}\right)$$

$$+\left(\sum_{x\in a_1\circ_1 b_1} \frac{1}{|(a_1 \circ_1 a_2)|}\right)\left(\sum_{x\notin a_2\circ_2 b_2} \frac{1}{(b_1 \circ_2 b_2)|}\right)$$

$$= A_1'(x).A_2'(y) + A_1'(x).A_2'(y) + A_1(x).A_2'(y$$

Set $|H_1| = n_1, |H_2| = n_2$, so $|H_1 \times H_2| = n = n_1 n_2$. Therefore

$$\overline{\lambda}_{\otimes}(x,y) = \frac{1}{(n_1 n_2)^2}(a_1'(x).A_2'(y) + A_1'(x).A_2(y) + A_1(x).A_2'(y))$$

$$\overline{\lambda}_1(x).\overline{\lambda}_2(y) + \overline{\lambda}_1(x).\overline{\mu}_2(y) + \overline{\mu}_1(x).\overline{\lambda}_2(y)$$

Proposition 4.2 *Let (k,l) be the pair associated with the hypergroupoid $(H,\circ)$, then we obtain triples $(k^2,l^2,2kl),(l^2,k^2,2kl),(2kl,l^2,k^2)$, $(k^2,2kl,l^2)$, $(k^2,2kl,k^2)$, $(l^2 2kl,k^2)$, associated with the hypergroupoid $(H\times H,\otimes)$.*

Proof. Since the hypergroupoid H has two classes of equivalence, so $H/R_{\overline{\mu}\wedge\overline{\lambda}} = \{C_,, C_2\}$, where $C_1 = \{x_1, x_2, ...x_k\}$ and $C_2 = \{x_{k+1}, x_{k+2}, ...x_{k+l}\}$; since $\overline{\mu}(x_1) < \overline{\mu}(x_{k+1})$, for any $i,j \in \{1,2,...,k\}$ and $i',j' \in \{k+1,k+2,....k+l\}$, we find

$$\overline{\mu}_{\otimes}(x_i,x_j) = \overline{\mu}(x_i).\overline{\mu}(x_j) = \overline{\mu}(x_1)^2, \overline{\lambda}_{\otimes}(x_i,x_j) = \overline{\lambda}(x)^2 + 2\overline{\mu}(x).\overline{\lambda}(x),$$

$$\overline{\mu}_{\otimes}(x_{i'},x_{j'}) = \overline{\mu}(x_{i'}).\overline{\mu}(x_{j'}) = \overline{\mu}(x_{k+1})^2, \overline{\lambda}_{\otimes}(x_{i'},x_{j'}) = \overline{\lambda}(x_{k+1})^2 + 2\overline{\mu}(x_{k+1}).\overline{\lambda}(x_{k+1}),$$

and finally

$$\overline{\mu}_{\otimes}(x_i, x_{j'}) = \overline{\mu}(x_i).\overline{\mu}(x_{j'}) = \overline{\mu}(x_1)\overline{\mu}(x_{k+1}),$$

$$\overline{\lambda}_{\otimes}(x_i, x_{j'}) = \overline{\lambda}(x_1).\overline{\lambda}(x_{k+1}) + \overline{\lambda}(x_1).\overline{\mu}(x_{k+1}) + \overline{\mu}(x_1).\overline{\lambda}(x_{k+1}).$$

Since $\overline{\mu}(x_1) < \overline{\mu}(x_{k+1})$ then $\overline{\mu}(x_1)^2 < \overline{\mu}(x_1).\overline{\mu}(x_{k+1}) < \overline{\mu}(x_{k+1})^2$, therefore $\overline{\mu}_{\otimes}(x_i, x_j) < \overline{\mu}_{\otimes}(x_i, x_{j'}) < \overline{\mu}_{\otimes}(x_{i'}, x_{j'})$ and $\overline{\lambda}_{\otimes}(x_{i'}, x_{j'}) < \overline{\lambda}_{\otimes}(x_i, x_{j'}) < \overline{\lambda}_{\otimes}(x_{\prime}, x_j)$

After some computations we result that there is no precise order between $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_j), (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'}), (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_{i'}, x_{j'})$ as we can see from the following examples. If

$$\overline{\mu}(x_1) = 1/13, \quad \overline{\mu}(x_{k+1}) = 3/16,$$

$$\overline{\lambda}(x_1) = 3/16, \quad \overline{\lambda}(x_{k+1}) = 1/16$$

then $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_j) = 1/16, (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'}) = 3/16, (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_{i'}, x_{j'}) = 7/16$, and therefore $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_j) < (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'}) < (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_{i'}, x_{j'})$ so the triple associated with the hypergroupoid $(H \times H, \otimes)$ is $(k^2, 2kl, l^2)$. Let us suppose

$$\overline{\mu}(x_1) = 10/225, \quad \overline{\mu}(x_{k+1}) = 11/225$$

$$\overline{\lambda}(x_1) = 5/225, \quad \overline{\lambda}(x_{k+1}) = 4/225$$

we find $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_j) = 100/225, (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'}) = 110/225, (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_{i'}, x_{j'}) = 104/225$, and so $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_j) < (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'}) < (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'})$ such that $(k^2, l^2, 2kl)$ is the triple associated with the hypergroupoid $(H \times H, \otimes)$. Similarly If

$$\overline{\mu}(x_1) = 24/900 \quad \overline{\mu}(x_{k+1}) = 25/900$$

$$\overline{\lambda}(x_1) = 6/900, \quad \overline{\lambda}(x_{k+1}) = 5/900$$

we obtain $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_{i'}, x_{j'}) < (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_{j'}) < (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x_i, x_j)$ and triple $(l^2, 2kl, k^2)$ is associated with the hypergroupoid $(H \times H, \otimes)$. Similar examples can be found, and therefore, we find 6 triples associated with $(H \times H, \otimes)$.

Theorem 4.3 (Cristea & Davvaz, 2010) Let $(k_1, k_2, ..., k_r)$ be the r-tuple associated with an arbitrary hypergroupoid $(H, \circ)$. If for any j, $1 < j < [\frac{r}{2}]$, $k_j = k_{r+1_{-j}}$, then, with the join space ${}^1H$ one associates the 1-tuple $(2k_1, 2k_2, ..., 2k_l)$, if $r = 2l$, (l+ 1)-tuple $(2k_1, 2k_2, ..., 2k_l, k_{l+1})$ if $r = 2l + 1$.

Now we recall here the Atanassov's intuitionistic fuzzy grade the hypergroupoid $(H, \circ)$ where $|H/R_{\overline{\mu} \wedge \overline{\lambda}}| \in \{2, 3\}$.

Theorem 4.4 (Anchluta & Cristea, 2012) *Let* $(H, \circ)$ *be a hypergroupoid of cardinality n*

(1) If $|H/R_{\overline{\mu}\wedge\lambda}| = 2$, that is with H one associates the pair (k_1, k_2), and

  (a) if $k_1 = k_2$, then we have $i.f.g.(H) = 2$.

  (b) if $k_1 \neq k_2$, then we have $i.f.g.(H) = 1$.

(2) If $|H/R_{\overline{\mu}\wedge\lambda}| = 3$, that is with H one associates the ternary (k_1, k_2, k_3),

  (a) if $k_1 = k_2 = k_3$, then $i.f.g.(H) = 2$.

  (b) if $k_1 \neq k_2 \neq k_3$, then $i.f.g.(H) = 2$ whenever $2k_1 \neq k_2$, and $i.f.g.(H) = 3$, whenver $2k_1 = k_2$.

  (c) if $k_1 < k_2 = k_3$, then $i.f.g.(H) = 1$, whenever $2k_2^2 > 3k_1k_2 + 3k_1^2$, and $i.f.g.(H) = 3$, otherwise.

  (e) if $k_1 \neq k_2 \neq k_3$, then there is no precise order between

  $\mu_1(x) \wedge \overline{\lambda}_1(x), \mu_1(y) \wedge \overline{\lambda}_1(y), \mu_1(z) \wedge \overline{\lambda}_1(z)$, and therefore $i.f.g.(H) \in \{1, 2, 3\}$.

Now, by using the above theorems, first we calculate the fuzzy grade of the Atanassov's intuitionistic fuzzy grade of the hypergroupoid $(H \times H, \otimes)$ when $|H/R_{\overline{\mu}\wedge\overline{\lambda}}| = 2$, and finally we obtain the Atanassov's intuitionistic fuzzy grade of the direct product of two distinct hypergroupoids, that is $(H \times H, \otimes)$.

Proposition 4.5 *Let us suppose that* (k, l) *be the pair associated with the hypergroupoid H,*

(a) If $k = 1$, then $i.f.g.(H \times H) \in \{2, 3\}$.

(b) If $k \neq 1, 2k = l$ we obtain $i.f.g.(H \times H) \in \{1, 2, 3\}$.

(c) If $k \neq l$, and $2k \neq 1$ we find $i.f.g.(H \times H) \in \{1, 2, 3\}$.

Proof: By using Proposition 4.2, we obtain that

(a) If $k = l$, triples associated with $H \times H$ are $(l^2, l^2, 2l^2)$, $(2l^2, l^2, l^2)$, $(l^2, 2l^2, l^2)$

  (a_1) If $(l^2, l^2, 2l^2)$ be the triple associated with $H \times H$, that is $k_1 = k_2 < k_3$, since $k_3 = 2k_1$, by Proposition 4.4 (d), we result that $i.f.g.(H \times H) = 2$.

  (a_2) Now, we consider triple $(2l^2, l^2, l^2)$, and therefore by Proposition 4.4 (c), we find $i.f.g.(H \times H) = 3$.

  (a_3) Similarly, if $(l^2, 2l^2, l^2)$, then by Proposition 4.4 (2) (b), since $k_2 = 2k_1$, we find $i.f.g.(H \times H) = 3$.

If $k \neq l$, we consider two below cases. (we suppose $(k < l)$).

(b) If $2k = l$, then the triples associated with $H \times H$ are (l^2, l^2, k^2), (k^2, l^2, l^2), (l^2, k^2, l^2)

(b_1) If (l^2, l^2, k^2) be the triple associated with $H \times H$, by Proposition 4.4 (d) we obtain $i.f.g.(H \times H) \in \{1, 2\}$.

(b_2) Let us consider (k^2, l^2, l^2), and so by Proposition 4.4 (c) we find $i.f.g.(H \times H) \in \{1, 3\}$.

(b_3) Similarly, if (l^2, k^2, l^2), by Proposition 4.4 (2) (b) we find $i.f.g.(H \times H) \in \{2, 3\}$.

(c) If $2k \neq 1$, then $k^2 \neq 2kl \neq l^2$ then by Proposition 4.4 (e), then there is no precise order between $(\overline{\mu_\otimes})_1(x) \wedge (\overline{\lambda_\otimes})_1(x), (\overline{\mu_\otimes})_1(y) \wedge (\overline{\lambda_\otimes})_1(y), (\overline{\mu_\otimes})_1(z) \wedge (\overline{\lambda_\otimes})_1(z)$, and so $i.f.g.(H \times H) \in \{1, 2, 3\}$.

Proposition 4.6 *Let $(H_1, \circ_1)$ and $(H_2, \circ_2)$ be two distinct hypergroupoids such that $|H_1/R_{\overline{\mu}_1 \wedge \overline{\lambda}_1}| = |H_2/R_{\overline{\mu}_2 \wedge \overline{\lambda}_2}| = 2$, we write H_1 as the union $H_1 = C_1 \cup C_2$, $|C_1| = k$, $|C_2| = l$ and $H_2 = C'_1 \cup C'_2 = m$, $|C'_2| = n$. If for any $x \in C_1, y \in C_2$, $x' \in C'_1, y' \in C'_2$ we have*

(i) $\overline{\mu}_\otimes(x, y') = \overline{\mu}_\otimes(y, x')$, then $\overline{\lambda}_\otimes(x, y') = \overline{\lambda}_\otimes(y, x')$, and therefore $|(H_1 \times H_2)|R_{\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes}| = 3$.

(ii) $\overline{\mu}_\otimes(x, y') \neq \overline{\mu}_\otimes(y, x')$, then $\overline{\lambda}_\otimes(x, y') \neq \overline{\lambda}_\otimes(y, x')$, and thereby $|(H_1 \times H_2)|R_{\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes}| = 4$

Proof: We consider

$$H_1 \times H_2 = \{(x, x'), (x, y'), (y, x'), (y, y')\} | x \in C_1, y \in C_2, x' \in C'_1, y' \in C'_2\}0$$

(i) If $(\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(x, y') = (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(y, x')$, as Proposition 4.2, we can obtain 6 triples associated with $H_1 \times H_2$, for example if $(\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(x, x') < (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(x, y') < (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(y, y')$, then $|(H_1 \times H_2)|R_{\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes}| = 3$ and the triple associated with $H_1 \times H_2$ is $(km, kn + lm, ln)$.

(ii) If $(\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(x, y') \neq (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(y, x')$, we find different 4-tuples associated with (H_1, H_2), for example if

$$(\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(x, x') < (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(x, y') < (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(y, x') < (\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes)(y, y')$$

then $|(H_1 \times H_2)|R_{\overline{\mu}_\otimes \wedge \overline{\lambda}_\otimes}| = 4$ and the 4-tuple associated with H_1, H_2 is (km, kn, lm, ln).

We analyze now the Atanassov's intuitionistic fuzzy grade of the hypergroupoid $H_1 \times H_2$ when the pairs associated with H_1 and H_2 are (k, k) and (l, l), respectively.

Proposition 4.7 *Let (k, k) and (l, l) be the pairs associated with the hypergroupoid $(H_1, \circ_1)$ and $(H_2, \circ_2)$, respectively, then $i.f.g.(H_1 \times H_2) \in \{1, 2, 3\}$.*

(i) If $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x, y') = (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(y, x')$, by Proposition 4.4 (e), we have $i.f.g.(H_1 \times H_2) \in \{1, 2, 3\}$.

(ii) If $(\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(x, y') \neq (\overline{\mu}_{\otimes} \wedge \overline{\lambda}_{\otimes})(y, x')$, by Proposition 4.6, we associate the 4-tuple $(kl, kl, kl, kl,)$ to $H_1 \times H_2$, then by Theorem 4.3, we associate the pair $(2kl, 2kl)$ to the join space ${}^1(H_1 \times H_2)$, and thereby by Proposition 4.4, (1) (a), $i.f.g.(H_1 \times H_2) = 3$.

Now, we give an example of a hypergroupoid $H_1 \times H_2$ such that $(3, 3)$ and $(2, 2)$ be the pairs associated with H_1 and H_2, respectively.

Example 1: *Let us suppose the hypergroupoid* $H_1 = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ *that is represented by the following table:*

$\circ_1$	b_1	b_2	b_3	b_4	b_5	b_6
b_1	b_1	b_2	b_3	b_4	b_5	b_6
b_2		b_3	b_1	b_4	b_5	b_6
b_3			b_2	b_4	b_5	b_6
b_4				b_6	$\{b_1, b_2, b_3\}$	b_3
b_5					b_6	b_4
b_6						$\{b_1, b_2, b_3\}$

then

$$\overline{\mu}_1(b_1) = \overline{\mu}_1(b_2) = \overline{\mu}_1(b_3) = 2/18, \quad \overline{\mu}_1(b_4) = \overline{\mu}_1(b_5) = \overline{\mu}_1(b_6) = 4/18$$

$$\overline{\lambda}_1(b_1) = \overline{\lambda}_1(b_2) = \overline{\lambda}_1(b_3) = 5/18, \quad \overline{\lambda}_1(b_4) = \overline{\lambda}_1(b_5) = \overline{\lambda}_1(b_6) = 3/18$$

As we observe the pair associated with H_1 is $(3, 3)$, $H_1 = C_1 \cup C_2$, $C_1 = \{b_1, b_2, b_3\}$, $C_2 = \{b_4, b_5, b_6\}$.

The hypergroupoid $H_2 = \{a_1, a_2, a_3, a_4\}$ has the below table:

$\circ_2$	a_1	a_2	a_3	a_4
a_1	a_1	a_2	a_3	a_4
a_2		a_3	$\{a_1, a_4\}$	a_2
a_3			a_2	a_3
a_4				a_1

then we calculate

$$\overline{\mu}_2(a_1) = \overline{\mu}_2(a_4) = 3/16, \quad \overline{\mu}_2(a_2) = \overline{\mu}_2(a_3) = 5/16$$

$$\overline{\lambda}_2(a_1) = \overline{\lambda}_2(a_4) = 12/16, \quad \overline{\lambda}_2(a_2) = \overline{\lambda}_2(a_3) = 10/16$$

therefore the pair associated with H_1 is $(2,2)$, $H_1 = C'_1 \cup C'_2$, $C'_1 = \{a_1, a_4\}$, $C'_2 = \{a_2, a_3\}$, and so

$$\overline{\mu}_{\otimes}(a_1, b_4) = 12/288,\ \overline{\mu}_{\otimes}(a_2, b_1) = 10/288$$

$$\overline{\lambda}_{\otimes}(a_1, b_4) = 12/288,\ \overline{\lambda}_{\otimes}(a_2, b_1) = 10/288$$

so we find

$$(\overline{\mu}_{\otimes} \wedge \overline{\mu}_{\otimes})(a_1, b_4) = 12/288 \neq (\overline{\mu}_{\otimes} \wedge \overline{\mu}_{\otimes})(a_2, b_1) = 10/288$$

Thereby by Proposition 4.7 (ii), we obtain that $(6,6,6,6)$ is 4-tuple associated with $H_1 \times H_2$, then by Theorem 4.3, it follwos that $(12,12)$ is the pair associated with ${}^1(H_1 \times H_2)$ and therefore by Proposition 4.4 (1) (a), $i.f.g.(H_1 \times H_2) = 3$.

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Submitted : *14/04/2013*
Revised : *22/01/2014*
Accepted : *17/03/2014*

حول درجة أتنسوف الحدسية المشوشة للجداء المباشر لفوزمريتين

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خلاصة

نقوم في هذا البحث بمتابعة دراسة متتاليات فضاءات الضم ومجموعات أتنسوف المشوشة المرتبطة بالجداء المباشر لفوزمريتين لهما صفات خاصة. ونتوصل في بحثنا إلى تحديد درجة أتنسوف الحدسية المشوشة لهذا النوع من المتتاليات.

المجلة العربية للعلوم الإدارية

Arab Journal of Administrative Sciences

رئيس التحرير: أ. د. آدم غازي العتيبي

- صدر العدد الأول في نوفمبر ١٩٩٣ .
- علمية محكمة تعنى بنشر البحوث الأصيلة في مجال العلوم الإدارية.
- تصدر عن مجلس النشر العلمي في جامعة الكويت ثلاثة إصدارات سنوياً (يناير - مايو - سبتمبر).
- تسهم في تطوير الفكر الإداري واختبار الممارسات الإدارية وإثرائها.
- مسجلة في قواعد البيانات العالمية.
- تخضع للتقييم الأكاديمي الخارجي.

- First issue, November 1993.
- Refereed journal publishing original research in Administrative Sciences.
- Published by Academic Publication Council, Kuwait University, 3 issues a year (January, May, September).
- Contributes to developing and enriching administrative thinking and practices.
- Listed in several international databases.
- Reviewed periodically by international referees for high academic standards.

الاشتراكات

الكويت: 3 دنانير للأفراد - 15 ديناراً للمؤسسات - الدول العربية: 4 دنانير للأفراد - 15 ديناراً للمؤسسات
الدول الأجنبية: 15 دولاراً للأفراد - 60 دولاراً للمؤسسات

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