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Balanced ternary and quaternary sequence pairs of odd period with three-level correlation

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Abstract

Sequence pairs with good correlation have wide applications in communication systems. In this paper, ternary and quaternary sequence pairs of odd period with three-level correlation are constructed based on cyclotomy. In our constructions, sequences are balanced and the maximum out-of-phase correlation magnitude is shown to be $\sqrt{7}$ for ternary sequence pairs and $\sqrt{5}$ for quaternary sequence pairs, both of which are better than the known sequence pairs of odd period with three-level correlation.

Keywords: Balance; cyclotomy; quaternary sequence pair; ternary sequence pair; three-level correlation

1. Introduction

Sequence $u=(u(i))_{i=0}^{N-1}$ is called the r -phase sequence of period N when $u(i)\in\mathbf{Z}_r$ for all $0\le i\le N-1$. Particularly, we say u is a binary sequence, ternary sequence, or quaternary sequence if $r=2$, 3, or 4, respectively. For $0\le t\le r-1$, define

$$N_t=\left|\{i\in\mathbf{Z}_N : u(i)=t\}\right|. \tag{1.1}$$

If $\max\{|N_s-N_t|: 0\le s\ne t\le r-1\}$ is equal to 0 for $N\equiv 0(\mathrm{mod}\, r)$ and 1 for other cases, then the sequence u is called balanced.

Let v be a r -phase sequence of period N. The periodic cross-correlation between u and v at shift $\tau(0\le\tau\le N-1)$ is

$$R_{u,v}(\tau)=\sum_{i=0}^{N-1}\omega^{u(i)-v(i+\tau)} \tag{1.2}$$

where $\omega=e^{2\pi\sqrt{-1}/r}$ is the primitive r th root of unity. In this paper, the exponent of ω is always performed modulo r. Furthermore, the values of $R_{u,v}(\tau)$ for $\tau\ne 0$ are called the out-of-phase correlation values. Specifically, the cross-correlation becomes the autocorrelation of u when $u=v$ and is denoted by $R_u(\tau)$.

In communication systems and cryptography, sequences usually need to have a small out-of-phase correlation magnitude (Fan & Darnell, 1996; Golomb & Gong, 2005). For example, in code division multiple access (CDMA), sequences with good correlation can be applied as address codes which acquire the correct timing information and distinguish multiple users. For $r=2$ or or 4, binary sequences and quaternary sequences with good autocorrelation have been widely studied (Arasu *et al*., 2001; Cai & Ding, 2009 ;Krone & Sarwate,1984; Tang & Lindner, 2009; Tang & Gong, 2010). With the growing need of high speed data communications, the polyphase sequences with good autocorrelation have a strong demand and attract much attention (Chung *et al*., 2011; Li *et al*., 2016; Lüke & Schotten, 2000). In order to find more discrete signals which can be used in practical communication, the discrete signal sequence pair (x, y) is proposed. If (x, y) has a small out-of-phase correlation magnitude, sequence x can be used as a sending sequence in a transmitter and we can use correlation computation to detect x by setting y as an address code in the receiver (Rohling & Plagge ,1989).

A sequence pair (x, y) is said to have a three-level correlation if

$$R_{x,y}(\tau)=\begin{cases}A, & \tau=0\\ B, & \tau\in T\\ C, & \text{otherwise}\end{cases} \tag{1.3}$$

where, A , B and C are pairwise distinct, and T is the subset of $\mathbf{Z}_N\setminus\{0\}$. We hope the absolute values of B and C are as small as possible. There are some conclusions about binary sequence pairs and quaternary sequence pairs with three-level correlation of even period.

(1) If (x, y) is a binary sequence pair with three-level

Table 1. Comparison of known sequence pairs with three-level correlation

Period	r-phase	Balanced	$R_{\max}$ *	Reference
$N \equiv 0 (\bmod 4)$	2	no	4	Peng *et al*. (2012a)
$N \equiv 3 (\bmod 4)$	2	no	3	Peng *et al*. (2011)
$N = 2p$ $p \equiv 1 (\bmod 4)$	4	yes	2	Peng *et al*. (2012b)
$N = 3p$ odd prime $p > 3$	2	no	3	Shen *et al*. (2017)
$N \equiv 1 (\bmod 4)$	3	yes	$\sqrt{7}$	this paper
$N \equiv 1 (\bmod 4)$	4	yes	$\sqrt{5}$	this paper

* $R_{\max}$ = the maximum out-of-phase correlation magnitude.

correlation, Peng *et al*. (2012a) proved that the smallest values for out - of - phase correlation are:

(a) $R_{x,y}(\tau) \in \{0, -4\}, \{0, 4\}, \text{or} \{2, -2\}$ for even N;

(b $R_{x,y}(\tau) \in \{1, -3\}, \text{or} \{-1, 3\}$ for odd N.

There are many binary sequence pairs with good three-level correlation which have been constructed by Jin and Xu (2010), Peng *et al*. (2011), and Shen *et al*. (2017).

(2) If (x, y) is a quaternary sequence pair of even period with three-level correlation, Peng *et al*. (2012b) showed that $\max\{|B|, |C|\} \geq 2$ and gave a class of quaternary sequence pairs of even period with out-of-phase correlation $\{0, -2\}$.

Cyclotomy is a powerful mathematical tool to construct sequences with good correlation. In this paper, we construct ternary sequence pairs and quaternary sequence pairs based on cyclotomy. In our constructions, ternary sequence pairs and quaternary sequence pairs both have odd period and three-level correlation. Also, the maximum out-of-phase correlation magnitudes are small, namely $\sqrt{7}$ for ternary sequence pairs and $\sqrt{5}$ for quaternary sequence pairs. Compared to known sequence pairs with three-level correlation, the maximum out-of-phase correlation magnitude in our constructions does not overlap with the known ones and is smaller for odd period (see Table 1). Thus, our constructions provide more discrete signals which can be used in practical communication.

The construction of the paper is as follows. Section 2 will presents some basic concepts of cyclotomy and the construction of balanced r-phase sequence of odd period. Section 3 gives the constructions of balanced ternary and quaternary sequence pairs with three-level correlation respectively. Section 4 offers some concluding remarks.

2. Cyclotomy and r -phase sequences

In this section, we first make a brief introduction of cyclotomy, then a class of balanced r-phase sequences of odd period are presented.

2.1 Cyclotomy

Cyclotomy is a powerful mathematical tool in sequence design. We give a simple introduction here. Readers can find more details in Storer (1967).

Let $N = rf + 1$ be an odd prime, where r and f are positive integers. Let α be a primitive element of finite field $\mathbf{Z}_N$, define a set

$D_k = \{\alpha^{rj+k} (\bmod N) : j = 0, 1, \cdots, f-1\}$ (2.1)

where $0 \leq k \leq r-1$. Then D_k is the subset of Z_n and is called the cyclotomic class of order r It is easy to check that

$$\mathbf{Z}_N \setminus \{0\} = \bigcup_{k=0}^{r-1} D_k \tag{2.2}$$

The cyclotomic number $(k,l)_r$ of order r is defined by

$$(k,l)_r = |(D_k + 1) \bigcap D_l|, \ 0 \le k,l \le r-1 \tag{2.3}$$

Storer (1967) and Sze *et al*. (2003) studied the properties of cyclotomic classes and cyclotomic numbers. We state some properties which will be used to prove our results.

Lemma 2.1. Let $N = rf + 1$ be an odd prime. Then

(1) $-1 \in D_0$ if f is even and $-1 \in D_{r/2}$ if f is odd;

(2) If $\beta \in D_l$, then $\beta D_k = D_{(k+l) \bmod r}$;

(3) $\sum_{k=0}^{r-1} (k,k+l)_r = \begin{cases} f-1, & \text{if } l = 0 \\ f, & \text{otherwise.} \end{cases}$

2.2 r -phase sequence

Let $N = rf + 1$ be an odd prime and the set $m = \{m_0, m_1, \ldots, m_{r-1}\}$ be a permutation of $\mathbf{Z}_r$. Let $0 \le j \le r-1$ the r -phase sequence of period N is constructed as

$$u_c(i) = \begin{cases} c, & i = 0 \\ j, & i \in D_{m_j} \end{cases} \tag{2.4}$$

where c is a constant from $\mathbf{Z}_r$ and D_{m_j} is the cyclotomic class of order r defined in (2.1). Then u_c is balanced and the set $\{m_0, m_1, \cdots, m_{r-1}\}$ is called the defining set for u_c.

Let u_{c_1} and u_{c_2} be the r -phase sequences of period N defined in Equation (2.4) where $c_1, c_2 \in \mathbf{Z}_r$, and two sequences have the same defining set. By (1.2), it is obvious that the value of $R_{u_{c_1}, u_{c_2}}(0)$ is equal to $N - 1 + \omega^{c_1 - c_2}$.

For $1 \le \tau \le N-1$, it is easy to see that $u_{c_1}(\tau) = u_{c_2}(\tau)$, the cross-correlation between u_{c_1} and u_{c_1} is given by

$$\begin{aligned} & R_{u_{c_1}, u_{c_2}}(\tau) \\ &= \omega^{c_1 - u_{c_1}(\tau)} + \omega^{u_{c_1}(-\tau) - c_2} \\ &+ \sum_{n=0}^{r-1} \omega^n \left(\sum_{j=0}^{r-1} \left| \{i : i \in D_{m_j}, i + \tau \in D_{m_{j-n}}\} \right| \right) \end{aligned} \tag{2.5}$$

where $j - n$ is performed modulo r, and the third formula comes by the fact that $u_{c_1}(i) - u_{c_2}(i+\tau) \equiv n (\bmod r)$ if $i \in D_{m_j}$ and $i + \tau \in D_{m_{j-n}}$ when $i \ne 0$ and $i \ne -\tau$.

We can determine the cross-correlation between u_{c_1} and u_{c_1} if the defining set satisfies some conditions.

Lemma 2.2. Let $m = \{m_0, m_1, \cdots, m_{r-1}\}$ be the defining set of u_{c_1} and u_{c_1}. If m satisfies the following conditions

(1) $m_{(j+1) \bmod r} - m_j \equiv d (\bmod r)$ is met for all $0 \le j \le r-1$;

(2) $\gcd(d, r) = 1$,

then we have

$$R_{u_{c_1}, u_{c_2}}(\tau) = \begin{cases} N - 1 + \omega^{c_1 - c_2}, & \tau = 0 \\ \omega^{c_1 - u_{c_1}(\tau)} + (-1)^f \cdot \omega^{u_{c_1}(\tau) - c_2} - 1, & \tau \ne 0. \end{cases}$$

Proof. For $1 \le \tau \le N-1$, by (1) and (2) of Lemma 2.1, we have

$$u_{c_1}(-\tau) = \begin{cases} u_{c_1}(\tau), & f \text{ even} \\ u_{c_1}(\tau) + \frac{r}{2}, & f \text{ odd}. \end{cases}$$

Then, Equation (2.5) becomes

$$R_{u_{c_1}, u_{c_2}}(\tau) = \omega^{c_1 - u_{c_1}(\tau)} + (-1)^f \cdot \omega^{u_{c_1}(\tau) - c_2} + s(\tau) \tag{2.6}$$

where $s(\tau)$ is given by

$$s(\tau) = \sum_{n=0}^{r-1} \omega^n \left(\sum_{j=0}^{r-1} \left| \{i : i \in D_{m_j}, i + \tau \in D_{m_{j-n}}\} \right| \right).$$

Next, we will calculate $s(\tau)$. For a fixed $n \in \mathbf{Z}_r$ note that

$$\begin{aligned} & |\{i : i \in D_{m_j}, i + \tau \in D_{m_{j-n}}\}| \\ &= |(D_{m_{j-n}} - \tau) \bigcap D_{m_j}| \\ &= |(D_{m_{j-n}} - \tau) \bigcap D_{m_{j-n}+nd}| \\ &= |((-\tau)^{-1} \cdot D_{m_{j-n}} + 1) \bigcap (-\tau)^{-1} \cdot D_{m_{j-n}+nd}| \end{aligned}$$

If we assume $-\tau \in D_{r-b}$, then it is easy to know that $(-\tau)^{-1} \in D_b$ By (2) and (3) of Lemma 2.1, we have

$$\begin{aligned} & \sum_{j=0}^{r-1} |\{i : i \in D_{m_j}, i + \tau \in D_{m_{j-n}}\}| \\ &= \sum_{j=0}^{r-1} (m_{j-n} + b, m_{j-n} + b + nd)_r \\ &= \sum_{t=0}^{r-1} (t, t + nd)_r \\ &= \begin{cases} f - 1, & \text{if } n = 0 \\ f, & \text{otherwise.} \end{cases} \end{aligned}$$

So, $s(\tau) = f - 1 + f \cdot \sum_{n=1}^{r-1} \omega^n = -1$.

Hence, the conclusion comes by Equation (2.6).

Corollary 2.3. Let u_{c_1} and u_{c_1} be the r -phase sequences defined in Lemma 2.2, then the biggest value of $|R_{u_{c_1},u_{c_2}}(\tau)|$ for $1\le\tau\le N-1$ is less than 3.

Tang & Lindner (2009) gave the autocorrelation of quaternary sequences of prime period $N=4f+1$, and Li *et al.* (2016) calculated the autocorrelation of polyphase sequences. If $c_1=c_2$ in Lemma 2.2, then the cross-correlation between u_{c_1} and u_{c_2} becomes the autocorrelation. Thus, Lemma 2.2 and Corollary 2.3 include the conclusions in (Tang & Lindner, 2009) and (Li *et al.*, 2016).

3. Sequence pairs with three-level correlation

By Corollary 2.3, we know that sequences u_{c_1} and u_{c_2} defined in Lemma 2.2 have a small out-of-phase correlation. In this section, we will study sequence pairs with three-level correlation in the cases $r=3$ and 4.

3.1 Ternary sequence pairs

Theorem 3.1. Let $N=3f+1$ be a prime. Let $m=\{m_0,m_1,m_2\}$ be the defining set of u_{c_1} and u_{c_2} where $c_1\ne c_2$. Then the balanced ternary sequence pair (u_{c_1},u_{c_2}) defined in Lemma 2.2 has three-level correlation, (given by)

$$R_{u_{c_1},u_{c_2}}(\tau)=\begin{cases} N-1+\omega^{c_1-c_2}, & \tau=0\\ \omega^{c_1-c_2}, & \tau\in D_{m_{c_1}}\cup D_{m_{c_2}}\\ 2\omega^{c_2-c_1}-1, & \text{otherwise}\end{cases} \quad (3.1)$$

where $\omega=e^{2\pi\sqrt{-1}/3}$ is the primitive 3rd root of unity.

Proof. Note that any permutation of $\mathbf{Z}_3$ satisfies the two conditions of Lemma 2.2. And f must be even if $N=3f+1$ is a prime, then for $1\le\tau\le N-1$,

$$R_{u_{c_1},u_{c_2}}(\tau)=\omega^{c_1-u_{c_1}(\tau)}+\omega^{u_{c_1}(\tau)-c_2}-1.$$

For $1\le\tau\le N-1$, we have

$$\omega^{c_1-u_{c_1}(\tau)}=\begin{cases}1, & u_{c_1}(\tau)=c_1\\ \omega^{c_1-c_2}, & u_{c_1}(\tau)=c_2\\ \omega^{c_2-c_1}, & \text{otherwise}\end{cases}$$

and

$$\omega^{u_{c_1}(\tau)-c_2}=\begin{cases}1, & u_{c_1}(\tau)=c_2\\ \omega^{c_1-c_2}, & u_{c_1}(\tau)=c_1\\ \omega^{c_2-c_1} & \text{otherwise}.\end{cases}$$

Hence, the proof is completed.

Corollary 3.2. Let u_{c_1} and u_{c_2} be the balanced ternary sequences defined in Theorem 3.1. Then, the maximum out-of-phase correlation magnitude is $\sqrt{7}$.

Example 3.3. Let $N=3\times6+1=19$ and $\alpha=2$, then we have $D_0=\{1,7,8,11,12,18\}$,
$D_1=\{2,3,5,14,16,17\}$, $D_2=\{4,6,9,10,13,15\}$.
Let $m=\{0,1,2\}$, $c_1=0$ and $c_2=1$. By (2.4), ternary sequences u_0 and u_1 of period 19 are as follows:
$u_0=(0,0,1,1,2,1,2,0,0,2,2,0,0,2,1,2,1,1,0)$ and
$u_1=(1,0,1,1,2,1,2,0,0,2,2,0,0,2,1,2,1,1,0)$.

Then the cross-correlation between u_0 and u_1 is as follows

τ	$R_{u_0,u_1}(\tau)$
0	ω^2+18
4,6,9,10,13,15	$2\omega-1$
1,2,3,5,7,8,11,12,14,16,17,18	ω^2

where $\omega=e^{2\pi\sqrt{-1}/3}$ and we can see that the maximum out-of-phase correlation magnitude is $|2\omega-1|=\sqrt{7}$.

3.2 Quaternary sequence pairs

Theorem 3.4. Let $N=4f+1$ be a prime. Let $m=\{m_0,m_1,m_2,m_3\}$ be the defining set of u_{c_1} and u_{c_2} and , where $c_1\ne c_2$. Assume that the elements of the set m satisfies $m_{j+1}-m_j\equiv1\text{ or }3(\text{mod }4)$ for all $0\le j\le3$, where $j+1$ is performed modulo 4. Let , the cross-correlation between u_{c_1} and u_{c_2} is as follows:

(1) If f is even and $c_1+c_2\equiv1(\text{mod }4)$, or f is odd and $c_1+c_2\equiv3(\text{mod }4)$, then.

$$R_{u_{c_1},u_{c_2}}(\tau)=\begin{cases} N-1+\omega^{c_1-c_2}, & \tau=0\\ \omega^{c_1}(1+\omega^3)-1, & \tau\in D_{m_0}\cup D_{m_1}\\ -\omega^{c_1}(1+\omega^3)-1, & \tau\in D_{m_2}\cup D_{m_3}.\end{cases}$$

(2) If f is even and $c_1+c_2\equiv3(\text{mod }4)$, or f is odd and $c_1+c_2\equiv1(\text{mod }4)$, then

$$R_{u_{c_1},u_{c_2}}(\tau)=\begin{cases} N-1+\omega^{c_1-c_2}, & \tau=0 \\ \omega^{c_1}(1+\omega)-1, & \tau\in D_{m_0}\bigcup D_{m_3} \\ -\omega^{c_1}(1+\omega)-1, & \tau\in D_{m_1}\bigcup D_{m_2}. \end{cases}$$

Proof. Note that f may be even or odd when $N=4f+1$ is a prime. We only give the proof in the case f is even, the method is similar for odd f.

By Lemma 2.2, for $1\le\tau\le N-1$, we have.

$$\begin{aligned} R_{u_{c_1},u_{c_2}}(\tau) &= \omega^{c_1-u_{c_1}(\tau)}+\omega^{u_{c_1}(\tau)-c_2}-1 \\ &= \omega^{c_1}(\omega^{-u_{c_1}(\tau)}+\omega^{u_{c_1}(\tau)-c_1-c_2})-1 \end{aligned}$$

If $c_1+c_2\equiv 1(\bmod 4)$, then

$$\begin{aligned} R_{u_{c_1},u_{c_2}}(\tau) &= \omega^{c_1}(\omega^{-u_{c_1}(\tau)}+\omega^{u_{c_1}(\tau)-1})-1 \\ &= \begin{cases} \omega^{c_1}(1+\omega^3)-1, & \text{if } u_{c_1}(\tau)=0,1 \\ -\omega^{c_1}(1+\omega^3)-1, & \text{if } u_{c_1}(\tau)=2,3 \end{cases} \end{aligned}$$

where the second result is given by the fact.

$\omega+\omega^2=-(1+\omega^3)$.

If $c_1+c_2\equiv 3(\bmod 4)$, then

$$\begin{aligned} R_{u_{c_1},u_{c_2}}(\tau) &= \omega^{c_1}(\omega^{-u_{c_1}(\tau)}+\omega^{u_{c_1}(\tau)-3})-1 \\ &= \begin{cases} \omega^{c_1}(1+\omega)-1, & \text{if } u_{c_1}(\tau)=0,3 \\ -\omega^{c_1}(1+\omega)-1, & \text{if } u_{c_1}(\tau)=1,2 \end{cases} \end{aligned}$$

where the second result is given by the fact.

$\omega^2+\omega^3=-(1+\omega)$.

With the above consideration, the proof is completed.

Corollary 3.5. Let u_{c_1} and u_{c_2} be the balanced quaternary sequences defined in Theorem 3.4. Then the quaternary sequence pair (u_{c_1},u_{c_2}) has a maximum out-of-phase correlation magnitude $\sqrt{5}$.

Example 3.6. Let $N=4\times 7+1=29$ and

$\alpha=2$, then

$$D_0=\{1,7,16,20,23,24,25\}$$

$$D_1=\{2,3,11,14,17,19,21\}$$

$$D_2=\{4,5,6,9,13,22,28\}$$

$$D_3=\{8,10,12,15,18,26,27\}$$

Let $m=\{0,3,2,1\}$, $c_1=2$ and $c_2=3$. By (2.4), the quaternary sequences u_2 and u_3 of period 29 are as follows:

$$u_2=(2,0,3,3,2,2,2,0,1,2,1,3,1,2,3,1,\\ 0,3,1,3,0,3,2,0,0,0,1,1,2)$$

and

$$u_3=(3,0,3,3,2,2,2,0,1,2,1,3,1,2,3,1,0,\\ 3,1,3,0,3,2,0,0,0,1,1,2)$$

The cross-correlation between u_2 and u_3 is as follows

τ	$R_{u_2,u_3}(\tau)$
0	ω^3+28
1,2,3,7,11,14,16, 17,19,20,21,23,24,25	ω^3-2
4,5,6,8,9,10,12,13, 15,18,22,26,27,28	ω

where $\omega=\sqrt{-1}$, and we can see that the maximum out-of-phase correlation magnitude is $|\omega^3-2|=\sqrt{5}$.

4. Concluding remarks

In this paper, we presented a class of ternary and quaternary sequence pairs of odd period. In our constructions, sequence pairs are balanced and have small out-of-phase correlation. Compared to known sequence pairs with three-level correlation, the maximum out-of-phase correlation magnitude in this paper does not overlap with the known ones (see Table 1). Though Table 1 shows that the maximum out-of-phase correlation magnitude can reach 2 when quaternary sequence pairs of even period have three-level correlation, the lower bound of the maximum correlation magnitude is still unknown for odd period. The maximum correlation magnitude in this paper is smaller than others for odd period.

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أزواج متواليات ثلاثية ورباعية متوازنة من الفترة الفردية ذات الارتباط ثلاثي مستويات

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الملخص

لأزواج المتوالية ذات الارتباط الجيد تطبيقات واسعة في أنظمة الاتصال. في هذا البحث، تم بناء أزواج متواليات من فترات فردية ثلاثية ورباعية مع ارتباط ثلاثي المستويات بناءً على السيكلوتومي. وفي التركيبات الخاصة بنا، كانت المتواليات متوازنة وظهر الحد الأقصى لحجم العلاقة خارج الطور بالنسبة لأزواج المتوالية الثلاثية $\sqrt{7}$ ولأزواج المتوالية الرباعية $\sqrt{5}$، وكلاهما أفضل من أزواج المتوالية المعروفة في الفترة الفردية ذات الارتباط ثلاثي المستويات.