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On several types of continuity and irresoluteness in L-topological Spaces

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Abstract

We use the concepts of ω-open L-sets, $\mathcal{N}$-open L-sets and $\mathcal{D}$-open L-sets to define several new types of continuity and irresoluteness in L-topological spaces. Several results have been proved. In particular, decomposition theorems of the new irresoluteness concepts are introduced.

Keywords: ω-open L-sets; $\mathcal{N}$-open L-sets; $\mathcal{D}$-open L-sets; fuzzy continuity; fuzzy irresoluteness.

Mathematics Subject Classification (2010): 54A05, 54A40.

1. Introduction and preliminaries

Fuzzy set theory still plays a vital role in research in almost all branches of mathematics and computer science (Rameez *et al.*, 2017; Ahmad *et al.*, 2016; Kia *et al.*, 2016; Pant *et al.*, 2015). Introducing new types of continuity or irresoluteness is still a hot area of research in L-topological spaces (See Chettri & Chettri, 2017; Vadivel & Swaminathan, 2017; Malathi & Uma, 2017; Swaminathan & Balasubramaniyan, 2016; Deb Ray & Chettri, 2016; Chettri *et al.*, 2014; Kharal & Ahmad, 2013; Shukla, 2012; Tripathy & Ray, 2014; Tripathy & Debnath, 2013; Tripathy & Ray, 2013; Tripathy & Ray, 2012).

Throughout this paper, we follow the notions and terminologies as they appeared in (Al Ghour, 2017).

As a weaker form of openness, the class of ω-open sets for a given topological space was introduced first by Hdeib in (Hdeib, 1982). This class of sets was used to introduce new topological notions in a number of previous works (See Al-Omari & Noorani, 2007; Al-Omari *et al.*, 2009; Hdeib, 1989; Al Ghour, 2006; Al Ghour & Zareer, 2016; Darwesh, 2013; Al-Zoubi, 2003). In particular, ω-continuity and ω-irresoluteness were introduced in (Hdeib, 1989) and (Al-Zoubi, 2003), respectively. For the purpose of characterizing compactness, in Al-Omari & Noorani (2009) introduced $\mathcal{N}$-open sets as a class of sets which lie between open sets and ω-open sets. Via $\mathcal{N}$-open sets, $\mathcal{N}$-continuity and $\mathcal{N}$-irresoluteness were introduced in (Al-Omari & Noorani, 2009) and (Al-Mshaqbeh, 2010), respectively.

For the purpose of introducing several types of compactness and Lindelöfness, and opening the door for future research in L-topological spaces, Al Ghour in Al Ghour (2017) fuzzified ω-openness and $\mathcal{N}$-openness by which he also introduced $\mathcal{D}$-open L-sets as a weaker form of $\mathcal{N}$-open L-sets as follows: Let (L^X,δ) be an L-ts and let $W, A \in L^X$. W is called an ω-open (resp. $\mathcal{N}$-open) L-set in (L^X,δ), if for every $x_a \in M(L^X)$ with $x_a \ll W$, there exist $U \in Q(x_a)$ and $G \in X_{coc}$ (resp. $G \in X_{cof}$) such that $x \in G$ and $U \wedge \mathcal{X}_G \leq W$. W is called a $\mathcal{D}$-open L-set in (L^X,δ) if for every $x_a \in M(L^X)$ with $x_a \ll W$, there exists $U \in Q(x_a)$ such that $U \wedge \mathcal{X}_{\{x\}} \leq W$. Denote the class of ω-open (resp. an $\mathcal{N}$-open, a $\mathcal{D}$-open) L-sets in (L^X,δ) by δ_ω (resp. $\delta_\mathcal{N}$, $\delta_\mathcal{D}$). Author in (Al Ghour, 2017) proved that each of δ_ω, $\delta_\mathcal{N}$ and $\delta_\mathcal{D}$ forms an L-topology on X and $\delta \subseteq \delta_\mathcal{N} \subseteq \delta_\omega \subseteq \delta_\mathcal{D}$ with each of the three inclusions being unreplaceable by equality in general. Also, via ω-open L-sets, $\mathcal{N}$-open L-sets and $\mathcal{D}$-open L-sets, he introduced and investigated several new types of compactness and Lindelöfness.

This paper is devoted to using the classes of ω-open L-sets, $\mathcal{N}$-open L-sets and $\mathcal{D}$-open L-sets to introduce and investigate $\mathcal{N}$-continuity, ω-continuity, $\mathcal{D}$-continuity, $\mathcal{N}$-irresoluteness, ω-irresoluteness, and $\mathcal{D}$-irresoluteness as new six L-topological properties. In Section 2, we characterize all of the new six concepts in terms of continuity. Also, we introduce a decomposition theorem for each of $\mathcal{N}$-irresoluteness and ω-irresoluteness. Moreover, we give several relationships between the new six concepts and between them and the continuity concept, in particular, we show that $\mathcal{D}$-continuity and $\mathcal{D}$-irresoluteness concepts are equivalent, and give sufficient conditions related to when two or more of the new concepts together with continuity are coincident to each other. Although we give some sufficient conditions for the validity of the implication "$\mathcal{N}$-irresoluteness $\Rightarrow$ ω-irresoluteness", we propose an open question about its validity in general. Moreover, we introduce composition and graph theorems related to the new concepts. In Section 3, we give examples to distinguish between our concepts as well as to highlight the significance of our results.

The following definition and proposition will be used in the sequel:

Definition 1.1. (Al Ghour, 2017) Let $(L^X,\delta), (L^Y,\mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping.

We say $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is a continuous mapping or call $f_L^{\rightarrow}$ continuous for short, if its L-reverse mapping $f_L^{\leftarrow}: L^Y \rightarrow L^X$ maps every open L-subset in (L^Y, μ) as an open L-subset in (L^X, δ), i.e. for every $V \in \mu$, $f_L^{\leftarrow}(V) \in \delta$.

Proposition 1.2. (Liu & Luo, 1997) Let $(L^X, \delta), (L^Y, \mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping. Then the following conditions are equivalent:
(i) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is continuous.
(ii) For any subbase μ_0 of μ, $f_L^{\leftarrow}(V) \in \delta$ $(\forall\, V \in \mu_0)$.

2. Continuity and irresoluteness

Based on Definition 1.1 and by means of ω-open, $\mathcal{N}$-open, $\mathcal{D}$-open L-sets in L-ts', we define six types of mappings:

Definition 2.1. Let $(L^X, \delta), (L^Y, \mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping.
(i) We say $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is an ω-continuous (resp. $\mathcal{N}$-continuous, $\mathcal{D}$-continuous) mapping or call $f_L^{\rightarrow}$ ω-continuous (resp. $\mathcal{N}$-continuous, $\mathcal{D}$-continuous) for short, if its L-reverse mapping $f_L^{\leftarrow}: L^Y \rightarrow L^X$ maps every open L-subset in (L^Y, μ) as an ω-open (resp. $\mathcal{N}$-open, $\mathcal{D}$-open) L-subset in (L^X, δ).
(ii) We say $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is an ω-irresolute (resp. $\mathcal{N}$-irresolute, $\mathcal{D}$-irresolute) mapping or call $f_L^{\rightarrow}$ ω-irresolute (resp. $\mathcal{N}$-irresolute, $\mathcal{D}$-irresolute) for short, if its L-reverse mapping $f_L^{\leftarrow}: L^Y \rightarrow L^X$ maps every ω-open (resp. $\mathcal{N}$-open, $\mathcal{D}$-open) L-subset in (L^Y, μ) as an ω-open (resp. $\mathcal{N}$-open, $\mathcal{D}$-open) L-subset in (L^X, δ).

We start by showing that $\mathcal{D}$-continuity and ω-irresoluteness are coincident with each other:

Theorem 2.2. Let $(L^X, \delta), (L^Y, \mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping. Then the following conditions are equivalent:
(i) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{D}$-continuous.
(ii) $f_L^{\rightarrow}: (L^X, \delta_{\mathcal{D}}) \rightarrow (L^Y, \mu)$ is continuous.
(iii) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{D}$-irresolute.

Proof. The proofs of (i) ⇔ (ii) and (iii) ⇒ (ii) are obvious, so we only show (ii) ⇒ (iii).
(ii) ⇒ (iii) : We apply Proposition 1.2. By Theorem 3.2 (vi) of (Al Ghour, 2017), $\mu \cup \tau^{sdisc}$ is a subbase of $\mu_{\mathcal{D}}$. By (ii), for every $V \in \mu$, $, f_L^{\leftarrow}(V) \in \delta_{\mathcal{D}}$, and for every $\mathcal{X}_B \in \tau^{sdisc}$, where $B \subset X, f_L^{\leftarrow}(\mathcal{X}_B) = \mathcal{X}_{f^{-1}(B)} \in \tau^{sdisc} \subset \delta_{\mathcal{D}}$.

Theorem 2.2 also characterizes $\mathcal{D}$-continuity and ω-irresoluteness in terms of continuity. As for continuity, Propositions 2.3, 2.4, and 2.5 will characterize ω-continuity, $\mathcal{N}$-continuity, ω-irresoluteness, $\mathcal{N}$-irresoluteness. In addition, Proposition 2.4 (resp. Proposition 2.5) will give a nice decomposition theorem for ω-irresoluteness (resp. $\mathcal{N}$-irresoluteness).

Proposition 2.3. Let $(L^X, \delta), (L^Y, \mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping. Then
(i) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is ω-continuous if and only if $f_L^{\rightarrow}: (L^X, \delta_\omega) \rightarrow (L^Y, \mu)$ is continuous.
(ii) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{N}$-continuous if and only if $f_L^{\rightarrow}: (L^X, \delta_{\mathcal{N}}) \rightarrow (L^Y, \mu)$ is continuous.

Proposition 2.4. Let $(L^X, \delta), (L^Y, \mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping. Then the following conditions are equivalent:
(i) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is ω-irresolute.
(ii) $f_L^{\rightarrow}: (L^X, \delta_\omega) \rightarrow (L^Y, \mu_\omega)$ is continuous.
(iii) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is ω-continuous, and $f_L^{\rightarrow}: (L^X, \delta_\omega) \rightarrow (L^Y, \tau^{scoc})$ is continuous.

Proof. (i) ⇔ (ii) is clear.
(ii) ⇔ (iii) By Theorem 3.2 (vi) of (Al Ghour, 2017), $\mu \cup \tau^{scoc}$ is a subbase of μ_ω. Applying Proposition 1.2, we get the result.

In view of the above theorem, we state the following result without proof.

Proposition 2.5. Let $(L^X, \delta), (L^Y, \mu)$ be L-ts', $f_L^{\rightarrow}: L^X \rightarrow L^Y$ be an L-mapping. Then the following conditions are equivalent:
(i) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{N}$-irresolute.
(ii) $f_L^{\rightarrow}: (L^X, \delta_{\mathcal{N}}) \rightarrow (L^Y, \mu_{\mathcal{N}})$ is continuous. (iii) $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{N}$-continuous, and $f_L^{\rightarrow}: (L^X, \delta_{\mathcal{N}}) \rightarrow (L^Y, \tau^{scof})$ is continuous.

Regarding the relationships between our new six concepts and the old continuity concept, the following diagram summarizes the implications that follow directly from the definitions and Theorem 2.2.

$$\begin{array}{ccccc}
\mathcal{N}\text{-irresolute} & & \omega\text{-irresolute} & \rightarrow & \mathcal{D}\text{-irresolute} \\
\downarrow & & \downarrow & & \updownarrow \\
\mathcal{N}\text{-continuous} & \rightarrow & \omega\text{-continuous} & \rightarrow & \mathcal{D}\text{-continuous} \\
\uparrow & & & & \\
\text{continuous} & & & &
\end{array}$$

Except the implication '$\mathcal{N}$-irresolute ⇒ ω-irresolute', in section 3, we will give counter-examples which distinguish between the concepts in the above diagram.

Recall that a function $f: X \rightarrow Y$ is called finite to one (resp. countable to one) if for each $y \in Y, f^{-1}(\{y\})$ is finite (resp. countable).

In each part of the next result, we give sufficient conditions for which two or more of the concepts in the above diagram are coincident to each other:

Theorem 2.6. Let $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ be an L-mapping. Then
(i) if (L^X, δ) is T_c, then the properties $f_L^{\rightarrow}$ is continuous and $f_L^{\rightarrow}$ is $\mathcal{N}$-continuous are equivalent.
(ii) if (L^X, δ) is T_c and P-L-ts, then the properties $f_L^{\rightarrow}$ is continuous, $f_L^{\rightarrow}$ is $\mathcal{N}$-continuous and, $f_L^{\rightarrow}$ is ω-continuous are equivalent.
(iii) if (L^X, δ) is CS, then the properties $f_L^{\rightarrow}$ is ω-continuous and $f_L^{\rightarrow}$ is $\mathcal{D}$-continuous are equivalent.
(iv) if (L^X, δ) is FS, then the properties $f_L^{\rightarrow}$ is $\mathcal{D}$-continuous, $f_L^{\rightarrow}$ is ω-continuous and $f_L^{\rightarrow}$ is $\mathcal{D}$-continuous are equivalent.
(v) if $f_L^{\rightarrow}$ is $\mathcal{N}$-irresolute such that (L^X, δ) is T_c, then $f_L^{\rightarrow}$ is continuous.
(vi) if $f_L^{\rightarrow}$ is ω-irresolute such that (L^X, δ) is T_c and P-L-ts, then $f_L^{\rightarrow}$ is continuous.

(vii) if (L^X,δ) and (L^Y,μ) are T_c, then the properties ${f_L}^{\rightarrow}$ is continuous and ${f_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute are equivalent.

(viii) if (L^X,δ) and (L^Y,μ) are T_c and P-L-ts, then the properties ${f_L}^{\rightarrow}$ is continuous, ${f_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute, and ω-irresolute are equivalent.

(ix) if (L^Y,μ) is T_c, then the properties ${f_L}^{\rightarrow}$ is $\mathcal{N}$-continuous and ${f_L}^{\rightarrow}$ is ω-irresolute are equivalent.

(x) if (L^Y,μ) is T_c and P-L-ts, then the properties ${f_L}^{\rightarrow}$ is ω-continuous and ${f_L}^{\rightarrow}$ is ω-irresolute are equivalent.

(xi) if $f:X\to Y$ is finite to one, and ${f_L}^{\rightarrow}$ is $\mathcal{N}$-continuous, then ${f_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute.

(xii) if $f:X\to Y$ is countable to one, and ${f_L}^{\rightarrow}$ is ω-continuous, then ${f_L}^{\rightarrow}$ is ω-irresolute.

Proof. The proofs of (i), (v), (vii), and (ix) follow directly by Theorem 3.4 of (Al Ghour, 2017).

The proofs of (ii), (vi), (viii), and (x) follow directly by Theorem 3.5 of (Al Ghour, 2017).

(iii) Follows by Theorem 3.7 (i) of (Al Ghour, 2017).

(iv) Follows by Theorem 3.7 (ii) of (Al Ghour, 2017).

(xi) By Theorem 4.4 of (Al Ghour, 2017), we only need to show ${f_L}^{\rightarrow}:(L^X,\delta_{\mathcal{N}})\to(L^Y,\tau^{scof})$ is continuous. For every $G\in Y_{cof}$ ${f_L}^{\leftarrow}(\mathcal{X}_G)=\mathcal{X}_{f^{-1}(G)}$ and $X-f^{-1}(G)=f^{-1}(Y-G)$ is finite, and thus ${f_L}^{\leftarrow}(\mathcal{X}_G)\in\tau^{scof}\subset\delta_{\mathcal{N}}$. This ends the proof.

(xii) Is similar to that used in (xi).

Question 2.7. Is it true that every $\mathcal{N}$-irresolute L-mapping is ω-irresolute?

The following result gives three sufficient conditions for an $\mathcal{N}$-irresolute L-mapping to be ω-irresolute.

Theorem 2.8. Let ${f_L}^{\rightarrow}:(L^X,\delta)\to(L^Y,\mu)$ be an $\mathcal{N}$-irresolute L-mapping. Then,

(i) if $f:X\to Y$ is countable to one, then ${f_L}^{\rightarrow}$ is ω-irresolute.

(ii) if (L^X,δ_ω) is P-L-ts, then ${f_L}^{\rightarrow}$ is ω-irresolute.

(iii) if (L^Y,μ) is FS, then ${f_L}^{\rightarrow}$ is ω-irresolute.

Proof. (i) Since ${f_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute, then by the above diagram ${f_L}^{\rightarrow}$ is ω-continuous. Therefore, by Theorem 2.7 (xii), it follows that ${f_L}^{\rightarrow}$ is ω-irresolute.

(ii) By Proposition 2.4, we only need to show that ${f_L}^{\rightarrow}:{f_L}^{\rightarrow}:(L^X,\delta_\omega)\to(L^Y,\tau^{scoc})$ is continuous. Let $G\in Y_{coc}$. It is not difficult to see that $\mathcal{X}_G=\wedge\{\mathcal{X}_{Y-\{y\}}:y\in Y-G\}$. Since $\{\mathcal{X}_{Y-\{y\}}:y\in Y-G\}\subset\mu_{\mathcal{N}}$ and ${f_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute, we have $\{{f_L}^{\leftarrow}(\mathcal{X}_{Y-\{y\}}):y\in Y-G\}\subset\delta_{\mathcal{N}}$. Since (L^X,δ_ω) is P-L-ts and ${f_L}^{\leftarrow}(\mathcal{X}_G)=\wedge\{{f_L}^{\leftarrow}(\mathcal{X}_{Y-\{y\}}):y\in Y-G\}$, then we have ${f_L}^{\leftarrow}(\mathcal{X}_G)\in\delta_\omega$.

(iii) Let $V\in\mu_\omega$. Then by Theorem 3.7 (ii) of (Al Ghour, 2017), $V\in\mu_{\mathcal{N}}$. Thus ${f_L}^{\leftarrow}(V)\in\delta_{\mathcal{N}}\subset\delta_\omega$.

The composition of two $\mathcal{N}$-continuous L-mappings do not need to be even ω-continuous.

Example 2.9. Take $X=\mathbb{R}, Y=\{0,1,2\}, Z=\{a,b\}$, L any F-lattice, $\delta=\{0_X,1_X\},\mu=\{0_Y1_Y,\mathcal{X}_{\{0\}},\mathcal{X}_{\{0,1\}}\}$, and $\gamma=\{0_Z1_Z,\mathcal{X}_{\{a\}}\}$. Define the function $f:X\to Y$ by $f(x)=2$ if $x=0$ and $f(x)=1$ otherwise, and define the function $g:Y\to Z$ by $g(0)=g(2)=a$ and $g(1)=b$. Then ${f_L}^{\rightarrow}:(L^X,\delta)\to(L^Y,\mu)$ and ${g_L}^{\rightarrow}:(L^Y,\mu)\to(L^Z,\gamma)$ are $\mathcal{N}$-continuous but ${(g\circ f)_L}^{\rightarrow}$ is not $\mathcal{N}$-continuous since ${(g\circ f)_L}^{\leftarrow}(\mathcal{X}_{\{a\}})=\mathcal{X}_{(g\circ f)^{-1}(\{a\})}=\mathcal{X}_{\{0\}}\notin\delta_{\mathcal{N}}$.

In what follows, we collect composition results related to our new continuity concepts. The results are easy to follow, and their proofs are left to the reader.

Proposition 2.10. Let ${f_L}^{\rightarrow}:(L^X,\delta)\to(L^Y,\mu)$ and ${g_L}^{\rightarrow}:(L^Y,\mu)\to(L^Z,\gamma)$ two L-mappings.

(i) If ${f_L}^{\rightarrow}$ is ω-continuous and ${g_L}^{\rightarrow}$ is continuous, then ${(g\circ f)_L}^{\rightarrow}$ is ω-continuous.

(ii) If ${f_L}^{\rightarrow}$ is $\mathcal{N}$-continuous and ${g_L}^{\rightarrow}$ is continuous, then ${(g\circ f)_L}^{\rightarrow}$ is $\mathcal{N}$-continuous.

(iii) If ${f_L}^{\rightarrow}$ is ω-irresolute and ${g_L}^{\rightarrow}$ is ω-continuous, then ${(g\circ f)_L}^{\rightarrow}$ is ω-continuous.

(iv) If ${f_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute and ${g_L}^{\rightarrow}$ is $\mathcal{N}$-continuous, then ${(g\circ f)_L}^{\rightarrow}$ is $\mathcal{N}$-continuousIf.

(v) ${f_L}^{\rightarrow}$ and ${g_L}^{\rightarrow}$ are ω-irresolute, then ${(g\circ f)_L}^{\rightarrow}$ is so.

(vi) If ${f_L}^{\rightarrow}$ and ${g_L}^{\rightarrow}$ are $\mathcal{N}$-irresolute, then ${(g\circ f)_L}^{\rightarrow}$ is so.

(vii) If ${f_L}^{\rightarrow}$ and ${g_L}^{\rightarrow}$ are $\mathcal{D}$-irresolute, then ${(g\circ f)_L}^{\rightarrow}$ is so.

The following lemma will be used in the proof of the next result. It generalizes a result in (Azad, 1981), where the result appears for the case $L=I$. Its proof can be established using standard techniques, so it is omitted here for brevity.

Lemma 2.11. Let $g: X\to X\times Y$ be the graph of the function $f:X\to Y$, and let L be an F-lattice. If $A\in L^X$ and $B\in L^Y$, then ${g_L}^{\leftarrow}(A\times B)=A\wedge {f_L}^{\leftarrow}(B)$.

In the following last result $\delta\times\mu$ will denote the product L-topology of δ and μ (for the definition of the product L-topology, one may refer to (Liu & Luo, 1997).

Theorem 2.12. Let ${f_L}^{\rightarrow}:(L^X,\delta)\to(L^Y,\mu)$ be an L-mapping and let ${g_L}^{\rightarrow}:(X,\delta)\to(X\times Y,\delta\times\mu)$ be its graph L-mapping.

(i) If ${g_L}^{\rightarrow}$ is ω-continuous, then so is ${f_L}^{\rightarrow}$.

(ii) If ${g_L}^{\rightarrow}$ is $\mathcal{N}$-continuous, then so is ${f_L}^{\rightarrow}$.

(iii) If ${g_L}^{\rightarrow}$ is $\mathcal{D}$-continuous, then so is ${f_L}^{\rightarrow}$.

(iv) If ${g_L}^{\rightarrow}$ is ω-irresolute, then so is ${f_L}^{\rightarrow}$.

(v) If ${g_L}^{\rightarrow}$ is $\mathcal{N}$-irresolute, then so is ${f_L}^{\rightarrow}$.

Proof. The proofs of all the parts are similar, so we prove only (i).

(i) Let $V\in\mu$. Then $1_X\times V\in\delta\times\mu$. Since ${g_L}^{\rightarrow}$ is ω-continuous, then ${g_L}^{\leftarrow}(1_X\times V)\in\delta_\omega$. By Lemma 2.12, we have ${g_L}^{\leftarrow}(1_X\times V)=1_X\wedge {f_L}^{\leftarrow}(V)={f_L}^{\leftarrow}(V)$. Therefore, ${f_L}^{\leftarrow}(V)\in\delta_\omega$. It follows that ${f_L}^{\rightarrow}$ is ω-continuous.

3. Examples

The following is an example of $\mathcal{N}$-continuous function that is not continuous. For the symbols that are not defined in Example 3.1, we refer to (Liu & Luo, 1997).

Example 3.1. Let L be any F-lattice which has a subset $\{a,b,c\}$ satisfying $0<a<b<c<1, a'=c$ and $b'=b$ (in particular $L=I$). Let $X=Y=I[L]$. Denote the L-

fuzzy unit interval on X by δ. Consider the mappings $x: \mathbb{R} \rightarrow L$

$$x(t) = \begin{cases} 1 & if\ t \in (-\infty, 0), \\ a & if\ t \in [0,1], \\ 0 & if\ t \in (1, \infty) \end{cases}.$$

It is proved in (Al Ghour, 2017) that $\mathcal{X}_{I[L]-\{[\mathrm{x}]\}} \notin \delta$. Let μ be the L-topology on Y generated by $\delta \cup \{\mathcal{X}_{I[L]-\{[\mathrm{x}]\}}\}$ as a subbase. Define $f: X \rightarrow Y$ by $f(x) = x$. Then $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{N}$-continuous but not continuous.

The following is an example of an ω-irresolute function that is not $\mathcal{N}$-continuous. It shows also that ω-irresoluteness does not imply $\mathcal{N}$-irresoluteness, in general.

Example 3.2. Let $X = Y = \mathbb{R}, L = I, \delta = \{0_X\} \cup \{A: A(x) > 0 \text{ for all } x \in X\}$ and μ be the L-topology on Y generated by $\{A: 0 < A(y) \leq 0.5 \text{ for all } y \in Y\} \cup \{0_Y, 1_Y, \mathcal{X}_{\{Y-\{0\}}\}$ as a subbase. Define $f: X \rightarrow Y$ by $f(x) = sin\, x$. Then $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is ω-irresolute but not $\mathcal{N}$-continuous.

$\mathcal{D}$-continuity does not imply ω-continuity in general as it can be seen from the following example:

Example 3.3. Let $X = Y = (0,1], L = I, \delta$ be the L-topology on X generated by $\{a\mathcal{X}_{(x,1]} : x \in X, a \in L\}$ as a base and μ be the L-topology on Y generated by $\{a\mathcal{X}_{(x,1]} : x \in X, a \in L\} \cup \{(0.5)\mathcal{X}_{\{0.3\}}\}$ as a subbase. Define $f: X \rightarrow Y$ by $f(x) = x$. Then $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is $\mathcal{D}$-continuous but not ω-continuous.

The following is an example of a continuous function that is neither ω-irresolute nor $\mathcal{N}$-irresolute. In particular, it shows that ω-continuity does not imply ω-irresoluteness, and also $\mathcal{N}$-continuity does not imply $\mathcal{N}$-irresoluteness, in general.

Example 3.4. Let $X = Y = \mathbb{R}, L = I, \delta = \{0_X\} \cup \{A: A(x) > 0 \text{ for all } x \in X\}$, and $\mu = \{A: 0 < A(y) \leq 0.5 \text{ for all } y \in Y\} \cup \{0_Y, 1_Y\}$. Define $f: X \rightarrow Y$ by $f(x) = 1$ for $x \in \mathbb{Q}$ and $f(x) = 2$ for $x \in \mathbb{R} - \mathbb{Q}$. Then $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is continuous but neither ω-irresolute nor $\mathcal{N}$-irresolute. To see that $f_L^{\rightarrow}$ is continuous, let $B \in \mu - \{0_Y\}$. Then for all $y \in Y, 0 < B(y) \leq 0.5$. So for all $x \in X$, we have $(f_L^{\leftarrow}(B))(x) = B(f(x)) > 0$. Hence $f_L^{\rightarrow}$ is continuous. Now, since $\mathcal{X}_{\mathbb{R}-\{2\}} \in \mu_{\mathcal{N}} = \mu_{\mathcal{N}} \cap \mu_\omega$ but $f_L^{\leftarrow}(\mathcal{X}_{\mathbb{R}-\{2\}}) = \mathcal{X}_{f^{-1}(\mathbb{R}-\{2\})} = \mathcal{X}_{\mathbb{Q}} \notin \delta_\omega = \delta_\omega \cup \delta_{\mathcal{N}}$, then $f_L^{\rightarrow}$ is neither ω-irresolute nor $\mathcal{N}$-irresolute.

ω-irresoluteness and $\mathcal{N}$-irresoluteness are not sufficient conditions to imply continuity as the following example clarifies:

Example 3.5. Let $X = Y = [0,1], L = I$ $\delta = \{0_X, 1_X\} \cup \{A: x^4 < A(x) \leq x \text{ for all } x \in X\}$ and μ be the L-topology on Y generated by $\{0_Y, 1_Y\} \cup \{A: y^2 < A(y) \leq \sqrt{y} \text{ for all } y \in Y\} \cup \{\mathcal{X}_{[0,1)}\}$ as a subbase. Define $f: X \rightarrow Y$ by $f(x) = x^2$. Then $f_L^{\rightarrow}: (L^X, \delta) \rightarrow (L^Y, \mu)$ is ω-irresolute and $\mathcal{N}$-irresolute but not continuous

Example 3.5 is a particular example of an $\mathcal{N}$-continuous function that is not continuous.

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عن أنواع متعددة للاتصال وانعدام العزم في فضاءات L الطوبولوجية

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الملخص

نستخدم المفاهيم ω -open L-sets و $\mathcal{N}$ -open L-sets و $\mathcal{D}$ -open L-sets لتعريف عدد من الأنواع الجديدة للاتصال وانعدام العزم في فضاءات L الطوبولوجية. وتم برهنة عدد من النتائج. على وجه الخصوص تم استحداث نظريات التفكيك لمفاهيم انعدام العزم الجديدة.