

## Optimal selection of wastewater treatment and location for subway stations using mathematical techniques: Five stations at the eastern end of Tehran subway line 2

Sepideh Ghaderi<sup>1</sup>, Amirhossein Javid<sup>2,\*</sup>, Hamidreza Ghaffarzadeh<sup>3</sup>, Farhad Hosseinzadeh Lotf<sup>4</sup>

<sup>1</sup>Dept. of Environmental Management, Faculty of Environment and Energy, Science and Research Branch, Islamic Azad University, Tehran, Iran

<sup>2</sup>Dept. of Environmental Engineering-Water/ Wastewater Pollution, Faculty of Environment and Energy, Science and Research Branch, Islamic Azad University, Tehran, Iran

<sup>3</sup>Dept. of Environmental Economics, Faculty of Environment and Energy, Science and Research Branch, Islamic Azad University, Tehran, Iran

<sup>4</sup>Dept. of Mathematics, Faculty of Science, Science and Research Branch, Islamic Azad University, Tehran, Iran

\*Corresponding author: a.javid@srbiau.ac.ir

### Abstract

---

Several methods are available when choosing an optimum method and location for wastewater treatment facilities. However, making these two decisions at the same time depends on several factors. Multi-criteria decision making (MCDM) is one of the best options when controversial criteria must be considered. The best answer should be determined using mathematical programming methods. This study was an attempt to find an optimal location for wastewater facilities and an optimal wastewater treatment system for subway stations using mathematical techniques in Lingo software. The study was carried out as an applied quantitative study of the five stations of the eastern end of Tehran subway line 2. Linear binary programming was used to follow the minimizing objectives. Financial, spatial and capacity constraints were notable in this approach. Five scenarios were defined to choose the most appropriate location and the optimum method of wastewater treatment. Given the objective function and the constraints, scenario 1 (collecting all wastewater at Sarsabz station and pumping it to the urban ecosystem) was adopted as the optimum scenario. The study showed that finding the best method and location of a wastewater treatment system is a challenge. The study's decision-making method can yield the best scenario given the objective functions and constraints.

**Keywords:** Lingo; mathematical model; MCDM; treatment; wastewater.

---

### 1. Introduction

Wastewater contamination will affect the health of an ecosystem (Yan & Isegahyan, 2017). To prevent the negative consequences of discharging wastewater into the environment, a new set of regulations and requirements have been introduced to determine the acceptable volume of wastewater discharge after treatment into the environment (Metcalf and Eddy, 2003). Considering this, the number of wastewater treatment facilities in cities has surged dramatically. Such facilities are usually very expensive to build and operate, a fact which focuses attention on optimization processes in the design and construction phases of such projects. It is essential to model wastewater design as an optimization problem to find the best solutions.

The main challenge to wastewater management is to find an optimal location for the facilities and best treatment methods. Several models, such as linear programming, dynamic programming, nonlinear programming and meta-responsive algorithms have been introduced as possible alternatives to choosing the best wastewater treatment options (Wang & Jamieson, 2002; Lynn et al., 1962). However, none of these models was capable of

choosing the system and location of wastewater treatment system simultaneously. Several studies have relied on MADM to choose the optimum method of wastewater treatment in different industries (Singhirunnusorn & Stenstrom, 2009; Kaya, 2011; Anagnostopoulos & Vavatsikos, 2012; Yeonjoo et al., 2013; Pophali et al., 2011; Kalbar et al., 2013; Jing et al., 2013; Zeng et al., 2007).

Kamami and Avramenko (2010) used fuzzy logic to find the proper wastewater treatment method while taking into account environmental and economic criteria (Avramenko & Kamami, 2010). Melo and Camara (1994) studied the optimum design of a wastewater system and published their work as a review paper (Melo & Câmara, 1994). Curiel-Esparza et al. (2014) relied on VIKOR, AHP and Delphi techniques to find the best disinfection method for recycling wastewater. They adopted the optimum option among five options based on nine criteria. Zefrino et al. (2010) utilized a multi-objective model to solve the problem of a comprehensive optimum plan for a wastewater treatment facility and wastewater grid. They solved the problem using weighted vectors and a simulated annealing algorithm. Sousa et al. (2009) and Cunha et al. (2002) employed a simulated annealing algorithm to propose an optimum comprehensive design for waste

water collection grid and wastewater treatment system.

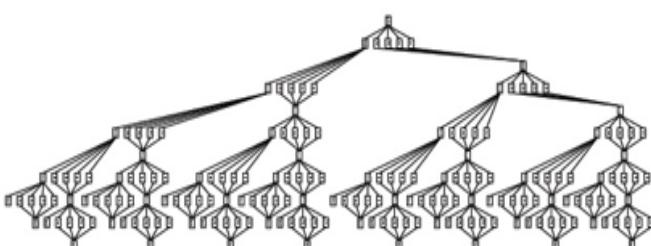
Ratnapriya and De Silva (2009) carried out a study to optimize a wastewater treatment system position using GIS in Sri Lanka that relied on MCDM. They also selected the criteria based on environmental, economic, topographical and technical factors and found the optimized position for the wastewater facility using GIS. Guo *et al.* (2008) carried out a review on the problem of optimum design of wastewater systems. Guangming *et al.* (2007) tried to find the best options for a wastewater facility using hierarchy analysis and gray-relationship analysis based on economic, technical and executive factors (e.g. funding, repair and maintenance cost, land value, and technology maturity). The anaerobic oxidation method was adopted as the best option among four alternatives.

Wastewater in subway stations are usually collected in catch basins and eventually transferred to seepage pits. Note that no treatment or pre-treatment process is carried out in the basins. The objective of the present study was to find an optimum location and system for wastewater systems of subway stations using mathematical modeling while considering environmental, economic and managerial factors. MCDM was adopted to find a suitable method for choosing an optimum alternative.

## 2. Methodology

### 2.1 Geographical region

Tehran subway line 2 is 26 km in length with 22 stations, five of which are located at the eastern end of the line (Farhangsara, Tehran Pars, Bagheri, Elm-o-Sanaat and Sarsabz). These were used as pilot stations. A total of 25 possible modes (two-way linear wastewater transfer grids) were examined in the process of designing the wastewater collection network, choosing the best method and positioning the facilities in the five stations. The structure of the wastewater transmission network is linear in these five stations. That is, from each station there is only one output and one entry to the next and previous stations (Fig. 1). The stations are equipped with a dewatering pool (DWP) for short-term storage of wastewater, which is then pumped to seepage pits.



**Fig. 1.** Modes of transferring wastewater to the five stations.

### 2.2 Method

After field visits and inspection of the DWPs in the five stations under study, the appropriate wastewater treatment solutions were evaluated based on the qualitative assessment of the wastewater (Table 1). Data was collected by questionnaire using the Delphi method on a group of 10 university professors and health and safety executives of the Tehran and Suburban Subway Operating Company. Four wastewater treatment methods were selected based on the specifications of the wastewater (Table 1) and the criteria for the optimum location and type of wastewater system. After determining the assumptions, parameters and variables of the model, the managerial pattern for choosing and prioritizing the alternatives (location and types of wastewater treatment system) were designed using binary linear programming in Lingo software.

### 2.3 Model development

Optimum design of a wastewater system for subway stations demands simultaneous decision-making about the location and type of treatment system. The comprehensive design of an optimum wastewater treatment is an optimization problem. The proposed model was developed using binary programming in Lingo software and several minimizing objectives. This optimization model defined an objective function and the constraints of the problem. Some of the constraints related to financing, space and capacity.

**Table 1.** Qualitative analysis of wastewater in subway stations

Test method	Value	Unit	Parameters
pH meter	<b>7.21</b>	-	pH
Conduct meter	<b>1007</b>	$\mu$ s/cm	Electrical conductivity (EC)
Gravimetric method	<b>453</b>	mg/Lit	Total dissolved substance (TDS)
Winkler method	<b>138</b>	mg/Lit	Biochemical oxygen demand (BOD) <sub>5</sub>
Reflex method	<b>262</b>	mg/Lit	Chemical oxygen demand (COD)
Volumetry by EDTA	<b>76.1</b>	mg/Lit	Calcium (Ca)
Computational method	<b>19.4</b>	mg/Lit	Magnesium (Mg)
Volumetry by EDTA	<b>270</b>	mg/Lit	Total hardness ( $\text{CaCO}_3$ )
Argentometry	<b>209</b>	mg/Lit	Chloride (Cl)
Spectro-photometry	<b>&gt;128</b>	mg/Lit	Nitrate ( $\text{NO}_3$ )
Spectro-photometry	<b>5.7</b>	mg/Lit	Nitrite ( $\text{NO}_2$ )
Spectro-photometry	<b>66.5</b>	mg/Lit	Phosphate ( $\text{PO}_4$ )
Spectro-photometry	<b>42.5</b>	mg/Lit	Sulfate ( $\text{SO}_4$ )

The variables of decision-making and the parameters used in the proposed mathematical model are introduced below.

#### 2.4 Objective function

Six elements were considered to create objective functions for choosing the optimum system and location for the wastewater facilities at the stations. The elements were the costs of:

1. Repair and maintenance services of the packages in each station;
2. The work force to operate the system;
3. Installing and developing the system at the stations
4. Pumping wastewater between stations
5. Repair and maintenance services of the pumping system; and,
6. Price of treated water for resale or reuse.

Based on the above elements, the objective function is defined as follows:

$$\text{Min} \quad (\sum_{j=1}^{25} \sum_{i=1}^4 t_{ij} y_{ij} + \sum_{j=1}^{25} \sum_{i=1}^4 d_{ij} y_{ij} + \sum_{j=1}^{25} \sum_{i=1}^4 p_{ij} y_{ij} + \sum_{i=1}^5 \sum_{j=1}^5 C_{ij} S_{ij} + \sum_{i=1}^5 \sum_{j=1}^5 a_{ij} S_{ij}) - (\sum_{j=1}^{25} \sum_{i=1}^4 r_{ij})$$

where

$n$  = Number of stations ( $n = 5$ ) and 25 modes

$n_1$  = Farhangsara station

$n_2$  = Tehranpars station

$n_3$  = Shahid Bagheri station

$n_4$  = Danshghah-e Elm-o sanat station

$n_5$  = Sarsabz station

$n_6$  = Pumping wastewater from Farhangsara station to Tehranpars station

$n_7$  = Pumping wastewater from Farhangsara and Tehranpars stations to Bagheri station

$n_8$  = Pumping wastewater from Farhangsara, Tehranpars and Bagheri stations to Danshghah-e Elm-o sanat station

$n_9$  = Pumping wastewater from Farhangsara, Tehranpars, Bagheri and Danshghah-e Elm-o sanat stations to Sarsabz station

$n_{10}$  = Pumping wastewater from Tehranpars station to Bagheri station

$n_{11}$  = Pumping wastewater from Tehranpars and Bagheri stations to Danshghah-e Elm-o sanat station

$n_{12}$  = Pumping wastewater from Tehranpars, Bagheri and Danshghah-e Elm-o sanat stations to Sarsabz station

$n_{13}$  = Pumping wastewater from Bagheri station to Danshghah-e Elm-o sanat station

$n_{14}$  = Pumping wastewater from Bagheri and Danshghah-e Elm-o sanat stations to Sarsabz station

$n_{15}$  = Pumping wastewater from Danshghah-e Elm-o sanat stations to Sarsabz station

$n_{16}$  = Pumping wastewater from Tehranpars station to Farhangsara station

$n_{17}$  = Pumping wastewater from Danshghah-e Elm-o sanat station to Bagheri station

$n_{18}$  = Pumping wastewater from Danshghah-e Elm-o sanat station to Tehranpars station

$n_{19}$  = Pumping wastewater from Danshghah-e Elm-o sanat station to Farhangsara station

$n_{20}$  = Pumping wastewater from Bagheri station to Tehranpars station

$n_{21}$  = Pumping wastewater from Tehranpars and Bagheri stations to Farhangsara station

$n_{22}$  = Pumping wastewater from Sarsabz and Danshghah-e Elm-o sanat stations to Bagheri station

$n_{23}$  = Pumping wastewater from Sarsabz, Danshghah-e Elm-o sanat and Bagheri stations to Tehranpars station

$n_{24}$  = Pumping wastewater from Sarsabz, Danshghah-e Elm-o sanat , Bagheri and Tehranpars stations to Farhangsara station

$X$  = Number of wastewater treatment plants ( $X = 4$ )

$X_1$  = Sequencing batch reactor (SBR) wastewater system

$X_2$  = Moving bed bioreactor (MBBR) wastewater system

$X_3$  = Extended aeration activated sludge wastewater system

$X_4$  = Connection to urban ego

$T_{ij}$  = Cost of repair and maintenance services of the packages in each station (ith type system and in jth station,  $t_{ij}$  Rials will be spent in 20 years)

$D_{ij}$  = Cost of human force needed to operate the system type ith in jth station in 20 years.

$Y_{ji}$  = Binary variable (wastewater system type ith installed jth station)  $\sum_{i=1}^4 Y_{ji} = 1$

$P_{ij}$  = Cost of installation and development of the ith wastewater system in jth station

$C_{ji}$  = Cost of pumping wastewater from ith point to jth for 20 years.

$a_{ji}$  = Cost of repair and maintenance services of pumps for 20 years.

$S_{ij}$  = 1 if the wastewater is pumped from ith station  $j^{\text{th}}$  station and 0 otherwise

$r$  = Price of treated water for reusing or selling purposes for 20 years.

In the objective function of the optimization problem,  $P_{ij}$  denotes the primary cost of the building and installation of the proposed treatment systems, which is generally a function of discharge, inlet wastewater volume and type of wastewater, and  $t_{ij}$  is the cost of repair and maintenance of each proposed system. The cost estimate for each year is equal to 10% of the total cost of building and installing the proposed system. For system type 4 (urban ecosystem), the cost of repair and maintenance would be determined by Tehran Water and Wastewater and paid in monthly statements.

The lifetime of the wastewater treatment system was assumed to be 20 years and all the cost of repair and maintenance services of the system, pumps and work force were computed based on a 14% annual inflation rate (based on historical data and similar studies conducted in previous years), and a 16% discount rate or difference in the cost of risk for a 20-year period. Factor  $d_{ij}$  is the cost of the work force needed to operate the system and is estimated at 8,100,000 Rials per employee, according to the Iran Labor Act.  $C_{ij}$  is the cost of pumping the wastewater from one station to another, which is a function of the price of the polyethylene pipes (the diameter of the pipes depends on the pumping discharge rate), cost of pumps needed to transfer the wastewater based on the head and discharge rate of the pumping system. In addition,  $a_{ij}$  is the cost of repair and maintenance services for the pumping system, which is a function of the electricity fee (based on the actual power of the pumps).

## 2.5 Constraints

The objective function in the optimization problem of choosing the wastewater system and positioning was minimized based on the following constraints:

$C$  = total budget dedicated by the managing director to the wastewater system of the subway system

$b_j$  = space available in the station  $j$  (location)

$y_{ji}$  = binary variable for wastewater system  $i$  in station  $j$

$\sum_{i=1}^4 y_{ij} = 1 \longrightarrow$  installation of wastewater system  $i$  in station  $j$

$\sum_{i=1}^4 y_{ij} = 0 \longrightarrow$  no installation if wastewater system  $i$  in station  $j$

$S_{ij} = 1$  if the wastewater is pumped from station  $i$  to station  $j$  and 0 otherwise

$V_j$  = volume of wastewater in station  $j$

$$1) \quad \sum_{j=1}^{25} \sum_{i=1}^4 p_{ij} y_{ij} \leq c \quad c=6000000000$$

$$2) \quad \sum_{j=1}^5 S_{ij} = 1 \quad i=1, \dots, 5$$

$$3) \quad S_{11} \geq \sum_{i=1}^4 y_{i1}$$

$$S_{22} \geq \sum_{i=1}^4 y_{i2}$$

$$S_{33} \geq \sum_{i=1}^4 y_{i3}$$

$$S_{44} \geq \sum_{i=1}^4 y_{i4}$$

$$S_{55} \geq \sum_{i=1}^4 y_{i5}$$

$$S_{12} + S_{22} \geq 2(\sum_{i=1}^4 y_{i6})$$

$$S_{13} + S_{23} + S_{33} \geq 3(\sum_{i=1}^4 y_{i7})$$

$$S_{14} + S_{24} + S_{34} + S_{44} \geq 4(\sum_{i=1}^4 y_{i8})$$

$$S_{15} + S_{25} + S_{35} + S_{45} + S_{55} \geq 5(\sum_{i=1}^4 y_{i9})$$

$$S_{23} + S_{33} \geq 2(\sum_{i=1}^4 y_{i10})$$

$$S_{24} + S_{34} + S_{44} \geq 3(\sum_{i=1}^4 y_{i11})$$

$$S_{24} + S_{34} + S_{44} \geq 3(\sum_{i=1}^4 y_{i11})$$

$$S_{25} + S_{35} + S_{45} + S_{55} \geq 4(\sum_{i=1}^4 y_{i12})$$

$$S_{34} + S_{44} \geq 2(\sum_{i=1}^4 y_{i13})$$

$$S_{35} + S_{45} + S_{55} \geq 3(\sum_{i=1}^4 y_{i14})$$

$$S_{45} + S_{55} \geq 2(\sum_{i=1}^4 y_{i15})$$

$$S_{21} + S_{11} \geq 2(\sum_{i=1}^4 y_{i16})$$

$$S_{43} + S_{33} \geq 2(\sum_{i=1}^4 y_{i17})$$

$$S_{42} + S_{32} + S_{22} \geq 3(\sum_{i=1}^4 y_{i18})$$

$$S_{41} + S_{31} + S_{21} + S_{11} \geq 4(\sum_{i=1}^4 y_{i19})$$

$$S_{32} + S_{22} \geq 2(\sum_{i=1}^4 y_{i20})$$

$$S_{31} + S_{21} + S_{11} \geq 3(\sum_{i=1}^4 y_{i21})$$

$$S_{54} + S_{44} \geq 2(\sum_{i=1}^4 y_{i22})$$

$$S_{53} + S_{43} + S_{33} \geq 3(\sum_{i=1}^4 y_{i23})$$

$$S_{52} + S_{42} + S_{32} + S_{22} \geq 4(\sum_{i=1}^4 y_{i24})$$

$$S_{51} + S_{41} + S_{31} + S_{21} + S_{11} \geq 5(\sum_{i=1}^4 y_{i25})$$

$$S_{ij} \in \{0, 1\}$$

$$y_{ij} \in \{0, 1\}$$

- 4) If  $\sum \sum y_{ij} = 1$ , all values except for  $\sum y_{i25}$  and  $\sum y_{i9}$  equal 0 and all wastewater will be pumped to one station.

$$\sum y_{i1} = 0$$

$$\sum y_{i14} = 0$$

$$\sum y_{i2} = 0$$

$$\sum y_{i15} = 0$$

$$\sum y_{i3} = 0$$

$$\sum y_{i16} = 0$$

$$\sum y_{i4} = 0$$

$$\sum y_{i17} = 0$$

$$\sum y_{i5} = 0$$

$$\sum y_{i18} = 0$$

$$\sum y_{i6} = 0$$

$$\sum y_{i19} = 0$$

$$\sum y_{i7} = 0$$

$$\sum y_{i20} = 0$$

$$\sum y_{i8} = 0$$

$$\sum y_{i21} = 0$$

$$\sum y_{i10} = 0$$

$$\sum y_{i22} = 0$$

$$\sum y_{i11} = 0$$

$$\sum y_{i23} = 0$$

$$\sum y_{i12} = 0$$

$$\sum y_{i24} = 0$$

$$\sum y_{i13} = 0$$

- 5) If  $\sum \sum y_{ij} = 2$ , quintuplet  $y_{ij}$  equals 0, and the condition in which all wastewater is pumped to one station is removed.

$$\sum y_{i9} = 0$$

$$\sum y_{i25} = 0$$

- 6) If  $\sum \sum y_{ij} = 3$ , quintuplet and quadruplet values of  $y_{ij}$  are equal to 0, and the option of aggregating the wastewater in 1 or 2 stations is removed.

- 6) If  $\sum \sum y_{ij} = 3$ , quintuplet and quadruplet values of  $y_{ij}$  are equal to 0, and the option of aggregating the wastewater in 1 or 2 stations is removed.

$$\begin{array}{ll} \sum y_{i9}=0 & \sum y_{i12}=0 \\ \sum y_{i25}=0 & \sum y_{i19}=0 \\ \sum y_{i8}=0 & \sum y_{i24}=0 \end{array}$$

- 7) If  $\sum \sum y_{ij} = 4$ , quintuplet, quadruplet and triple values of  $y_{ij}$  are equal to 0, and the wastewater must be handled in four stations and the option of aggregating wastewater in 1, 2, or 3 stations is removed.

$$\begin{array}{ll} \sum y_{i9}=0 & \sum y_{i7}=0 \\ \sum y_{i25}=0 & \sum y_{i11}=0 \\ \sum y_{i8}=0 & \sum y_{i14}=0 \\ \sum y_{i12}=0 & \sum y_{i18}=0 \\ \sum y_{i19}=0 & \sum y_{i21}=0 \\ \sum y_{i24}=0 & \sum y_{i23}=0 \end{array}$$

- 8) If  $\sum \sum y_{ij} = 5$ , quintuplet, quadruplet, triple, and double  $y_{ij}$  values equal 0, and the wastewater is handled in all five stations.

$$\begin{array}{ll} \sum y_{i9}=0 & \sum y_{i21}=0 \\ \sum y_{i25}=0 & \sum y_{i23}=0 \\ \sum y_{i8}=0 & \sum y_{i6}=0 \\ \sum y_{i12}=0 & \sum y_{i10}=0 \\ \sum y_{i19}=0 & \sum y_{i13}=0 \\ \sum y_{i24}=0 & \sum y_{i15}=0 \\ \sum y_{i7}=0 & \sum y_{i16}=0 \\ \sum y_{i11}=0 & \sum y_{i17}=0 \\ \sum y_{i14}=0 & \sum y_{i20}=0 \\ \sum y_{i18}=0 & \sum y_{i22}=0 \end{array}$$

### 9) Ninth constraint category

$$\begin{array}{ll} (\sum y_{i1}) \times (\sum y_{i16})=0 & (\sum y_{i4}) \times (\sum y_{i22})=0 \\ (\sum y_{i1}) \times (\sum y_{i19})=0 & (\sum y_{i5}) \times (\sum y_{i9})=0 \\ (\sum y_{i1}) \times (\sum y_{i21})=0 & (\sum y_{i5}) \times (\sum y_{i12})=0 \\ (\sum y_{i1}) \times (\sum y_{i25})=0 & (\sum y_{i5}) \times (\sum y_{i14})=0 \\ (\sum y_{i2}) \times (\sum y_{i6})=0 & (\sum y_{i5}) \times (\sum y_{i15})=0 \\ (\sum y_{i2}) \times (\sum y_{i18})=0 & (\sum y_{i6}) \times (\sum y_{i21})=0 \\ (\sum y_{i2}) \times (\sum y_{i20})=0 & (\sum y_{i6}) \times (\sum y_{i19})=0 \\ (\sum y_{i2}) \times (\sum y_{i24})=0 & (\sum y_{i6}) \times (\sum y_{i25})=0 \\ (\sum y_{i3}) \times (\sum y_{i7})=0 & (\sum y_{i21}) \times (\sum y_{i19})=0 \\ (\sum y_{i3}) \times (\sum y_{i10})=0 & (\sum y_{i21}) \times (\sum y_{i25})=0 \end{array}$$

$$\begin{array}{ll} (\sum y_{i3}) \times (\sum y_{i10})=0 & (\sum y_{i21}) \times (\sum y_{i25})=0 \\ (\sum y_{i3}) \times (\sum y_{i17})=0 & (\sum y_{i19}) \times (\sum y_{i25})=0 \\ (\sum y_{i3}) \times (\sum y_{i23})=0 & (\sum y_{i20}) \times (\sum y_{i18})=0 \\ (\sum y_{i4}) \times (\sum y_{i8})=0 & (\sum y_{i20}) \times (\sum y_{i24})=0 \end{array}$$

$$\begin{array}{ll} (\sum y_{i4}) \times (\sum y_{i11})=0 & (\sum y_{i18}) \times (\sum y_{i24})=0 \\ (\sum y_{i4}) \times (\sum y_{i13})=0 & (\sum y_{i17}) \times (\sum y_{i23})=0 \\ (\sum y_{i15}) \times (\sum y_{i14})=0 & (\sum y_{i12}) \times (\sum y_{i9})=0 \\ (\sum y_{i15}) \times (\sum y_{i12})=0 & (\sum y_{i13}) \times (\sum y_{i11})=0 \\ (\sum y_{i15}) \times (\sum y_{i9})=0 & (\sum y_{i13}) \times (\sum y_{i8})=0 \\ (\sum y_{i14}) \times (\sum y_{i12})=0 & (\sum y_{i11}) \times (\sum y_{i8})=0 \\ (\sum y_{i14}) \times (\sum y_{i9})=0 & (\sum y_{i10}) \times (\sum y_{i7})=0 \end{array}$$

$$10) \sum_{i=1}^5 s_{ij} v_i \leq b_j \quad j = 1, \dots, 5$$

The objective of this model was to decrease the fixed costs of construction and installation of a wastewater treatment system, decrease the cost of repair and maintenance of the system, and decrease the cost of the work force. Objective function (1) assumes that the cost of construction is at most equal to the target value determined in constraint (2) and supports pumping the wastewater based on constraint (3), when the wastewater grid is linear. In a linear grid, each station has only one inlet from the previous station and one outlet to the next station.

Five scenarios were proposed to solve the model in the constraint category (4, 5, 6, 7, 8). Given that several identical modes exist among the 25 modes, constraint category (9) was used to prevent such errors. Constraint (10) is the volume of the pumped wastewater, which must be less than or equal to the available space dedicated to the wastewater at each station. The model was solved in Lingo. After GAMS software, Lingo is the most powerful software of its kind. However, the model-type determination feature (without operator intervention) is a key advantage of Lingo over Lindo and GAMS (Zanjirani-Farahani and Askari, 2010).

## 3. Results

The model was executed using the data described above, and then a code was written in the Lingo environment. The results obtained for the five scenarios follow.

### 3.1. Scenario 1

It is assumed that only one of the four proposed methods to manage wastewater and one location will be selected based on the objective function. The results show that the optimum location for this scenario is mode

9, in which wastewater is pumped from Farhangsara, Tehran Pars, Bagheri and Elm-o-Sanaat stations to Sarsabz station at which point the wastewater is eventually fed into the urban ecosystem (Objective function: 2282659000).

<b>S(1, 5)</b>	<b>Y (4, 9)</b>
<b>S(2, 5)</b>	
<b>S(3, 5)</b>	
<b>S(4, 5)</b>	
<b>S(5, 5)</b>	

### 3.2. Scenario 2

It is assumed that the wastewater of all five stations is treated at only two stations, and the optimum choices for these two stations are Tehran Pars (wastewater of Farhangsara is pumped to Tehran Pars) and Bagheri (wastewater from Elm-o-Sanaat and Sarsabz stations is pumped to Bagheri). In addition, the optimum treatment system is designated as extended aeration active sludge (Objective function: 2320252414).

<b>S (1, 2)</b>	<b>Y (3, 6)</b>
<b>S(2, 2)</b>	
<b>S (3, 3)</b>	
<b>S (4, 3)</b>	
<b>S (5, 3)</b>	

### 3.3. Scenario 3

It is assumed that the wastewater of the five stations is treated in the three stations of Farhangsara, Sarsabz and Tehran Pars (wastewater from Elm-o-Sanaat is pumped to Tehran Pars). In addition, the optimum treatment system is the system type 1 (SBR) (Objective function: 3168519000).

<b>S (1, 1)</b>	<b>Y (1, 18)</b>
<b>S (5, 5)</b>	
<b>S (2, 2)</b>	
<b>S (3, 2)</b>	
<b>S (4, 2)</b>	

### 3.4. Scenario 4

It is assumed that the wastewater of the five stations is treated at four stations. The optimum choices for these four stations are Bagheri, Elm-o-Sanaat, Farhangsara and Sarsabz (the wastewater of Tehran Pars is pumped to Farhangsara; mode 16). In addition, the optimum treatment system is system type 3 (extended aeration active sludge). Objective function: 2973458100.

<b>S (3, 3)</b>	<b>Y (3, 16)</b>
<b>S (4, 4)</b>	
<b>S (5, 5)</b>	
<b>S (1, 1)</b>	
<b>S (2, 1)</b>	

### 3.5. Scenario 5

It is assumed that the wastewater of the five stations is treated at Bagheri, Elm-o-Sanaat, Farhangsara, Sarsabz and Tehran Pars stations. In addition, the optimum treatment system is system type 3 (extended aeration active sludge). (Objective function: 2957093000).

<b>S (1, 1)</b>	<b>Y (3, 5)</b>
<b>S (2, 2)</b>	
<b>S (3, 3)</b>	
<b>S (4, 4)</b>	
<b>S (5, 5)</b>	

## 4. Conclusions

The problem of positioning and choosing the best wastewater treatment system was solved by developing scenarios based on MCDM. Five scenarios were defined, and the problem was solved in Lingo based on the defined objective function and constraints. Given the results, and after comparison with the optimized value of the objective function in scenario 1, it is clear that:

$$\begin{aligned} \text{Min } \{z_1, z_2, z_3, z_4, z_5\} &= z_1 \\ &= 2282659000 \end{aligned}$$

The best alternative is to transfer wastewater from Farhangsara, Tehran Pars, Bagheri and Elm-o-Sanaat stations to Sarsabz station. The findings also indicate that the connection to the urban ecosystem, given the defined objectives, is the best alternative. Following scenario 1, scenarios 2, 5, 4, and 3 (in descending order) are the next top priorities.

## References

- Anagnostopoulos, K. & Vavatsikos, A. (2012).** Site suitability analysis for natural systems for wastewater treatment with spatial fuzzy analytic hierarchy process. *J. Water Resour. Plan. Mana*, 138:125–134.
- Avramenko, Y. & Kamami, M. (2010).** Fuzzy performance indicators for decision making in selection of wastewater treatment methods. *Computer Aided Chemical Engineering*, 28: 127-132.
- Curiel-Esparza, J.A., Cuenca-Ruiz, M., Martin-Utrillas, M. & Canto-Perello, J. (2014).** Selecting a sustainable disinfection technique for wastewater reuse projects. *Water Journal*, 6: 2732-2747.
- Cunha, M.C., Pinheiro, L., Zeferino J.A., Antunes, A. & Afonso, P. (2009).** Optimization model for integrated regional wastewater systems planning. *Journal of Water*

- Resources Planning and Management, 135(1): 23-33.
- Guo, Y., Walters, G. & Savic, D. (2008).** Optimal design of storm sewer networks: Past, present and future. In Proceeding of 11th International Conference on Urban Drainage, Edinburgh, Scotland, UK. pp 1-10.
- Guangming, Z., Ru, J., Guohe, H., Min, X. & Jianbing, L. (2007).** Optimization of wastewater treatment alternative selection by hierarchy grey relational analysis. Journal of Environmental Management, 82: 250-259.
- Jing, L., Chen, B., Zhang, B. & Li, P. (2013).** A hybrid stochastic-interval analytic hierarchy process approach for prioritizing the strategies of reusing treated wastewater. Mathematical Problems in Engineering, 2013: 1-10.
- Kaya, I. (2011).** Multicriteria location selection of wastewater treatment plant by fuzzy analytic hierarchy process. Multiple-Valued Log. Soft Comput., 17: 305–320.
- Kalbar, P.P., Karmakar, S. & Asolekar, S.R. (2013).** The influence of expert opinions on the selection of wastewater treatment alternatives: A group decision-making approach". Journal of Environmental Management, 128: 844–851.
- Lynn, W.R., Logan, J.A. & Charnes, A. (1962).** System analysis for planning wastewater treatment plants. J. of Water Pollution Control Federation, 34(6): 565-581.
- Melo, J.J., & Câmara, A.S. (1994).** Models for the optimization of regional wastewater treatment systems. Eur. J. Oper. Res., 73(1): 1-16.
- Pophali, G., Chelani, A. & Dhodapkar, R. (2011).** Optimal selection of full-scale tannery effluent treatment alternative using integrated AHP and GRA approach. Expert Syst. Appl., 38:10889–10895.
- Singhirunnusorn, W. & Stenstrom, M. (2009).** Appropriate wastewater treatment systems for developing countries: Criteria and indicator assessment in Thailand. Water Sci. Technol., 59: 1873–1884.
- Sousa, J., Ribeiro, A., Cunha, M.C. & Antunes, A. (2002).** An optimization approach to waste water systems planning at regional level. J. Hydroinform., 4(2): 115-123.
- Tchobanoglou, G. & Burton, F.L. (2003).** Waste-water engineering-treatment, disposal, reuse, 4th Ed. Metcalf and Eddy. McGraw-Hill, Inc.: New York.
- Ratnapriya, E.A. & De Silva, R.P. (2009).** Location optimization of wastewater treatment plants using GIS: A case study in upper Mahaweli catchment Sri Lan ka case. Applied Geoinformatics for Society and Environment. Stuttgart University of Applied Sciences.
- Wang, C.G. & Jamieson, D.G. (2002).** Objective approaches to regional wastewater treatment planning. Water Resour. Res., 38(3): 41-48.
- Yan, Z. & Iseghayan, N. (2017).** Effects of filter materials on the removal efficiency of phosphorus and nitrogen from wastewater in an enhanced AO-BAF system. Kuwait J. Sci. 44(3): 112-119.
- Yeonjoo, K., Eun-Sung, C., Sang-Mook, J. & Sang-Ug, K. (2013).** Prioritizing the best sites for treated wastewater instream use in an urban watershed using fuzzy TOPSIS. Resour. Conserv. Recycl. 73: 23–32.
- Zeng, G., Jiang, R., Huang, G., Xu, M. & Li, J. (2007).** Optimization of wastewater treatment alternative selection by hierarchy grey relational analysis. J. Environ. Manag. 82: 250–259.
- Zeferino, J.A., Antunes A.P. & Cunha M.C. (2010).** Multi-objective model for regional wastewater systems plane. Civil Engineering and Environmental Systems, 27(2): 95-106.

**Submitted:** 01-08-2017**Revised:** 01-10-2017**Accepted:** 29-10-2017

## الاختيار الأمثل لمعالجة مياه الصرف الصحي وموقع محطات مترو الأنفاق باستخدام الطرق الرياضية: خمس محطات لخط مترو الأنفاق 2 في الطرف الشرقي من طهران

<sup>1</sup> سبيده غاديري ، <sup>2,\*</sup> أمير حسين جاوید ، <sup>3</sup> حمیدرضا غفارزاده ، <sup>4</sup> فراتت حسين زاده لطفی

<sup>1</sup> قسم الإدارة البيئية، كلية البيئة والطاقة، فرع العلوم والأبحاث، جامعة آزاد الإسلامية، طهران، إیران

<sup>2</sup> قسم هندسة البيئة - تلوث المياه والصرف الصحي، كلية البيئة والطاقة، فرع العلوم والأبحاث، جامعة آزاد الإسلامية، طهران، إیران

<sup>3</sup> قسم الاقتصاد البيئي، كلية البيئة والطاقة، فرع العلوم والأبحاث، جامعة آزاد الإسلامية، طهران، إیران

<sup>4</sup> قسم الرياضيات، كلية العلوم، فرع العلوم والأبحاث، جامعة آزاد الإسلامية، طهران، إیران

\*[a.javid@srbiau.ac.ir](mailto:a.javid@srbiau.ac.ir)

### الملخص

توجد عدة طرق لاختيار الطريقة المثلثى والموقع المناسب لمراقبة مياه الصرف الصحي؛ ومع ذلك، فإن اتخاذ هذين القراراتين في نفس الوقت هو دالة لعدة معلمات. إن عملية صنع القرار متعدد المعايير (MCDM) هي واحدة من أفضل الخيارات عند النظر في المعايير المثلثة للجدل ويجب تحديد أفضل حل باستخدام طرق البرمجة الرياضية. وكانت الدراسة الحالية محاولة للعثور على موقع مثالى لمراقبة مياه الصرف الصحي ونظام مثالى لمعالجة مياه الصرف لمحطات مترو الأنفاق باستخدام طرق رياضية في برنامج لينجو (Lingo). وقد أجريت هذه الدراسة كدراسة كمية تطبيقية لمحطات الخمس خطوط مترو الأنفاق 2 في الطرف الشرقي من طهران. وقد استُخدمت البرمجة الخطية المزدوجة لمتابعة دوال الهدف المصغرة. وتم الوضع في الاعتبار كل من القيود المالية والمكانية والكافأة عند استخدام هذه الطريقة. تم تحديد خمس سيناريوهات في محاولة لاختيار أنساب موقع وطريقة المثلثى لمعالجة مياه الصرف الصحي. وبالنظر إلى دالة الهدف والقيود، تم اعتماد السيناريو 1 (وهو جمع كل مياه الصرف الصحي في محطة سارسابز (Sarsabz) وضخها إلى النظام البيئي الحضري) كسيناريو مثالى. أوضحت الدراسة أن إيجاد الطريقة المثلثى وموقع نظام معالجة مياه الصرف يشكل تحدياً وأن طريقة اتخاذ القرار المعتمدة هنا كانت قادرة على تقديم أفضل سيناريو بالنظر إلى دوال الهدف والقيود.