

Study on the uncertainty of geological drilling parameters by the uncertainty theory

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Abstract

The geological geological drilling parameters reflect the integrated nature of basic oil and gas geological data. Drilling risks can be avoided by understanding these parameters. Due to the complexity of petroleum geology, the incompleteness of logging or seismic data, the precision of mathematical models, and other issues, the true value of geological geological drilling parameters is difficult to calculate as accurately as desired. The method of describing the uncertainty of the geological drilling parameters has been established based on the theory of sequence stratigraphy and uncertainty. First, the geological drilling parameters, which were interpreted by logging or seismic data in the same stratum, were regarded as the measured sample. Second, the probability distribution or interval range of geological drilling parameters at each measurement point was determined based on this sample. Finally, the uncertainty interval of parameters of geological drilling parameters along with the well depth was obtained. The study results show that the uncertain description of the geological drilling parameters was more relevant with practical engineering.

Keywords: Geological drilling parameters; interval analysis; sequence stratigraphy; uncertainty theory

1. Introduction

Geological drilling parameters are the basic data which reflect the integrated characteristics of a geological environment. The geological drilling parameters include rock mechanical parameters, in-situ stress and formation pressure, etc. Accurate description of geological drilling parameters is of great significance to drilling design and monitoring. It mitigates drilling risk during drilling engineering. Because of the complexity of a drilling geological environment, the incompleteness of explanatory data, the precision of mathematical models, and the exact values of geological parameters are difficult to obtain. At present, geological drilling parameters are mainly interpreted by using logging or seismic data. They are described only by a single certain value (Eaton, 1972; Fillippone, 1979; Bowers, 1995). Engineering uncertainty is now mainly accounted for by empirical design, regional statistical analysis or by adopting safety factors (Skogdalen *et al.*, 2012; Khakzad *et al.*, 2013; Sadiq *et al.*, 2014). However, these methods are subjective and less relevant in terms of practical engineering. A plentitude of drilling data shows that the true values of geological drilling parameters are varied in a certain interval and are characterized by their dispersion (Doyen *et al.*, 2003; Alberto *et al.*, 2004; Ke *et al.*, 2009; Sayers

et al., 2012; Sheng *et al.*, 2016). The degree of dispersion reflects the degree of reliability of the measurement results. The method to describe the uncertainty of the geological drilling parameters was established based on the uncertainty theory. In addition, a new calculation method of uncertainty transfer between direct and indirect measurement parameters was also developed.

2. Uncertainty model of the geological drilling parameters

Because uncertain factors rarely have the quantitative characteristics essential in the statistical probability evaluation, errors are common if statistical models are used to describe the parameters. When the statistical information is insufficient to describe the probability distribution of uncertain parameters, and only an interval range of engineering parameters can be obtained through a priori knowledge, it is of great advantage to use the uncertainty theory to describe the uncertainty of parameters.

The uncertainty is the latest understanding of the error. The error is classified into three categories: system error, random error and residual error. Because the system error and random error are sometimes difficult to distinguish, they can be transformed into each other under certain

conditions. Thus, the uncertainty theory is usually used to evaluate the results of measurement. The error and uncertainty have completely different meanings (Wang, 2004).

(1) Error

According to the definition of error, the error is equal to the measurement result minus its true value. That is:

$$e = m - t = (m - \bar{t}) + (\bar{t} - t) = r_e + s_e, \tag{1}$$

where: e represents the measurement error, m represents the measurement result, t represents the truth value, r_e is the random error, \bar{t} represents the general average, and s_e is the system error. The schematic diagram of the measurement error is as shown in Figure 1

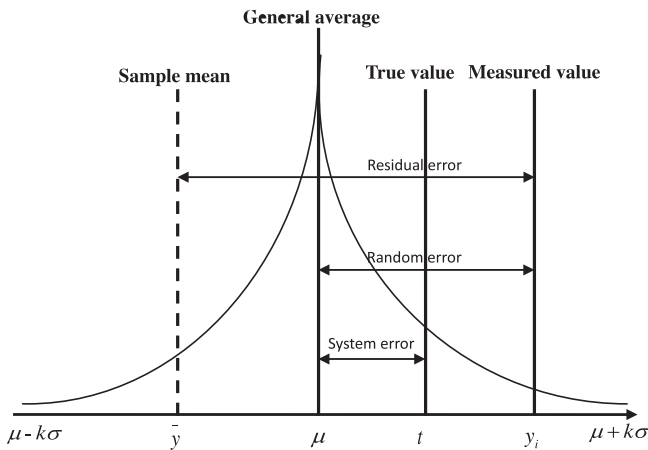


Fig. 1. Schematic diagram of the measurement error

In Figure 1, y represents the measured value, t represents the true value,.

Because of the existence of the measurement error, the measurement result (single measurement result y_i or measurement average \bar{y}) cannot equal the true value t . Assuming that the measurement values have a normal distribution, the location of the ensemble average of the distribution curves (μ), determines the size of the system error. The shape of the curve varies with the standard deviation σ which determines the distribution region of random error $[\mu - k\sigma, \mu + k\sigma]$ and the probability of the value in that range.

(2) Uncertainty

Uncertainty represents the incertitude degree to the true value due to the existence of error in the measurement. Uncertainty includes margin (interval) I and confidence probability P , representing that the possibility of measured

values falling into the interval I is P . The range of the true values falling into the interval is described by the degree of uncertainty. For example, the formation pressure is detected as 1.15 MPa \pm 0.15 MPa with a confidence probability of 95%. The results show that the possibility of the formation pore pressure lying between 1 MPa to 1.3 MPa is 95%.

The uncertainty represents the confidence probability of measured values falling into a certain range. Its size determines the reliability of the measurement results, and it is an important parameter for evaluating the reliability of the measurement. The smaller the degree of uncertainty, the closer the measured value is to the true value. The uncertainty can be divided into direct measurement uncertainty and indirect measurement uncertainty. These depend on whether the parameters can be directly measured.

2.1 Direct measurement uncertainty

1. Standard uncertainty

(1) Class A uncertainty

The standard uncertainty assessed by the method of statistical analysis of observation is called the Class A uncertainty component, which is represented by μ_A .

$$u_A = \sqrt{u_{A1}^2 + u_{A2}^2 + \dots + u_{AM}^2}, \tag{2}$$

where $u_{A1}, u_{A2} \dots u_{AM}$ in the formula are M uncertainty components.

The calculation methods of Class A uncertainty are the Bessel method, maximum residual method, maximum error method, and range method. If there is only one Class A uncertainty component, then the Class A uncertainty is represented by the mean standard deviation as:

$$u_A = s(\bar{x}) = \sqrt{\frac{\sum_{i=1}^n v_i^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n(n-1)}}, \tag{3}$$

where x_i is the i -th measurement result, \bar{x} is the arithmetic mean value of n times measurement results, and $v_i = x_i - \bar{x}$ is residual.

(2) Class B uncertainty

Without using the method of statistical analysis of observation, the assessed standard uncertainty is called the Class B uncertainty component which is represented by μ_B .

$$u_B = \sqrt{u_{B1}^2 + u_{B2}^2 + \dots + u_{BN}^2} \quad (4)$$

where $u_{B1}, u_{B2}, \dots, u_{BN}$ are N uncertainty components.

Class B uncertainty assesses the uncertainty on the basis of experience, other relevant information, or material. Assuming that the potential distributing range of measurement error is $[-\alpha, \alpha]$, the half-width of the error interval is α and the confidence factor is t . Thus, the uncertainty of the measurement value is:

$$u_B = \alpha / t \quad (5)$$

Prior information is the previous measurement data, experience or instruments' technical data, etc. To ensure a confidence factor, first, the measurement value's probability distribution can be assumed in $[-\alpha, \alpha]$. Then its accuracy is checked using Table 1.

We assume that probability distribution accords with the following principles:

- ① Random effect is assumed to be normal distribution.
 - ② There are both random and system effects, assumed to be uniform distribution.
 - ③ Based on prior information, assumed to be uniform distribution.
- (3) Combined standard uncertainty

If the measurement value is ensured by other variables' value, the uncertainty computed by other variables' variance and covariance is called the combined standard uncertainty:

$$u_C = \sqrt{\sum_{i=1}^M u_{Ai}^2 + \sum_{j=1}^N u_{Bj}^2} \quad (6)$$

Table 1 The confidence factor t of common probability distribution

Probability distribution	Confidence factor t
Triangular distribution	$\sqrt{6}$
Uniform distribution	$\sqrt{3}$
Arc-sine distribution	$\sqrt{2}$
Two-point distribution	1
Trapezoidal distribution	$\sqrt{6/(1+\beta^2)}$, β ratio of the upper and the bottom of the trapezoid
Normal distribution	According to the confidence probability p For example: $p=99\%$, $t=2.58$; $p=99.73\%$, $t=3$.

2. Expanded uncertainty

The coverage factor k is called the expanded uncertainty, which is also known as the total uncertainty. Expanded uncertainty gives the probable existing range of the measurement value:

$$u = k u_C \quad (7)$$

The problem of ensuring the expanded uncertainty is choosing k by using the probability confidence. There are two cases of choosing coverage factor k :

① When measurement distribution cannot be ensured, k is chosen in the range of 2~3 generally, and in most cases k is 2.

② Assuming that the measurement distribution is a normal distribution or an approximately normal distribution, we can choose coverage factor k according to the Table 2.

Table 2 The corresponding relationship between confidence probability and the coverage factor k

$p(100\%)$	0.5	0.68	0.95	0.99	0.997
k	0.67	1.0	2	2.6	3.0

2.2 Indirect measurement uncertainty

In practice, many engineering parameters cannot be measured directly. In other words, parameter values are obtained through indirect measurement. We should establish the modeling function relationship between the indirect measurement value and direct measurement value. Then through direct measurement value, the indirect measurement value can be estimated.

The assumption is that x_1, x_2, \dots, x_n are n direct measurement values which are independent of each

other. The function relationship is. $y = f(x_1, x_2, \dots, x_n)$
 If direct measurement variables' uncertainties are, $u(x_1), u(x_2), \dots, u(x_n)$ according to the International Organization for Standardization (ISO) measurement uncertainty evaluation guidelines (GUM), the criteria of using total differential formula for deducing indirect measurement value is:

$$u^2(y) = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad (8)$$

Table 3 is the uncertain propagation formula of commonly used functions. The relative uncertainty of indirect measurement y is:

$$u_r^2(y) = \left(\frac{u(y)}{y} \right)^2 = \sum_{i=1}^n \left(\frac{\partial \ln f}{\partial x_i} \right)^2 u^2(x_i) \quad (9)$$

Table 3. Uncertainty propagation formulas

Function expression	Uncertainty propagation formulas
$y = kx$	$u_y = ku_x$
$y = k\sqrt{x}$	$\frac{u_y}{y} = \frac{1}{2} \cdot \frac{u_x}{x}$
$y = \sin(x)$	$u_y = \cos(x) u_x$
$y = \ln(x)$	$u_y = \frac{u_x}{x}$
$y = x_1 \pm x_2$	$u_y = \sqrt{u_{x_1}^2 + u_{x_2}^2}$
$y = x_1 \cdot x_2$ or $y = x_1/x_2$	$\frac{u_y}{y} = \sqrt{\left(\frac{u_{x_1}}{x_1} \right)^2 + \left(\frac{u_{x_2}}{x_2} \right)^2}$
$y = \frac{x_1^k \cdot x_2^n}{x_3^m}$	$\frac{u_y}{y} = \sqrt{\left(k \frac{u_{x_1}}{x_1} \right)^2 + \left(n \frac{u_{x_2}}{x_2} \right)^2 + \left(m \frac{u_{x_3}}{x_3} \right)^2}$

Steps of analyzing indirect measurement uncertainty are:

Step 1. According to the function relationship between indirect measurement y and direct measurement variables $\{x_1, x_2, \dots, x_n\}$, the average value of computing indirect measurement variables is \bar{y} .

Step 2. By using the total differential equation, the uncertainty's transfer formula is deduced then the indirect measurement y uncertainty $u(y)$ is calculated. (Or first calculate relative uncertainty $u_r(y)$, then calculate $u(y)$).

Step 3. Write out the result: $y = \bar{y} \pm \mu(y)$.

2.3 Calculation method of uncertainty based on interval theory

Uncertainty defines a group of both-side ranges of measurement mean values, while parameter uncertainty is described by range interval. Interval mathematical theory is a branch of computational mathematics which is based on interval analysis. This paper proposes a kind of uncertainty calculation method based on interval theory (Moore *et al.*, 1979; Merlet *et al.*, 2006).

- (1) The fundamental principle of interval mathematics

If parameter p belongs to interval variables which cannot be ensured and its upper and lower bounds are p^u and p^l , then the interval variables belong to interval $[p^l, p^u]$, namely $p \in p^l = [p^l, p^u]$. According to interval mathematical theory, interval variables have two basic parameters: mean value (p^c) and deviation (p^r).

$$p^c = \frac{p^u + p^l}{2}, \quad p^r = \frac{p^u - p^l}{2}, \quad (10)$$

where interval p^l and variable p can be represented by $p^l = p^c + p^r \Delta^l$, $p^u = p^c + p^r \Delta^u$, among which $\Delta^l = [-1, 1]$ is the standard interval, and $\delta \in \Delta^l$ is the standard interval variable.

In regards to interval $x^l = [x^l, x^u]$ and $y^l = [y^l, y^u]$, the interval arithmetic is:

$$\begin{cases} x^l + y^l = [x^l + y^l, x^u + y^u] \\ x^l - y^l = [x^l - y^u, x^u - y^l] \\ x^l \times y^l = [\min(x^l y^l, x^l y^u, x^u y^l, x^u y^u)], \\ \max(x^l y^l, x^l y^u, x^u y^l, x^u y^u)] \\ x^l / y^l = [x^l, x^u] \times \left[\frac{1}{y^u}, \frac{1}{y^l} \right], \quad 0 \notin [y^l, y^u] \end{cases} \quad (11)$$

- (2) Calculation method of uncertainty based on interval theory

Assuming that x_1, x_2, \dots, x_n are n direct measurement values, which are independent of each other. By using the uncertainty analysis method of the direct measurement parameters, the ensured uncertainties are obtained as $u(x_1), u(x_2), \dots, u(x_n)$ and mean values are. x_1, x_2, \dots, x_n . Therefore, the description of the parameter uncertainty based on interval mathematics theory is:

$$\begin{cases} x_1 \in [x_1', x_1''] = [\bar{x}_1 - U(x_1), \bar{x}_1 + U(x_1)] \\ x_2 \in [x_2', x_2''] = [\bar{x}_2 - U(x_2), \bar{x}_2 + U(x_2)] \\ \dots \\ x_n \in [x_n', x_n''] = [\bar{x}_n - U(x_n), \bar{x}_n + U(x_n)] \end{cases} \quad (12)$$

We assumed that the function between the indirect measurement parameter y and direct measurement parameters $\{x_1, x_2, \dots, x_n\}$ is $y = f(x_1, x_2, \dots, x_n)$. According to interval arithmetic, the interval of the indirect measurement parameter can be calculated. The calculated interval is assumed to be $[y', y'']$, and the uncertainty of measurement parameters is:

$$u(y) = \frac{y'' - y'}{2} \quad (13)$$

3. Analysis of geological drilling parameters uncertainty

3.1 Fundamental principle

Due to the evolution of the structure and sediment, petroleum geology is divided into the erathem, system, series, group, and stratum by geological stratification. Geological stratification is based on information regarding the geological, seismic, logging, and laboratory analyses and other information, including structure, sequence stratigraphy, etc. The engineering practice proved that similar logging or seismic interpretation results are produced in the same stratum (Muto *et al.*, 2000; Catuneanu *et al.*, 2006; Herrera *et al.*, 2013). According to the theory, the measured sample is the geological drilling parameters interpreted by logging or seismic data in the same geological stratum. Then the probability distribution or interval ranges of drilling geological parameter at each measurement point are determined based on this sample. In a certain stratum, assuming that logging interpretation of a certain parameter of drilling geological characteristics is X , the depth interval is $\Delta H = [H_u, H_l]$, data points within the range are $(2n+1)$. Samples are described in Figure 2.

Finally, the uncertainty interval of the parameter of drilling geological characteristics in the data point i can be obtained based on the methods proposed above.



Fig. 2. Schematic diagram of sample

3.2 Uncertainty method

It is assumed that in a certain stratum group, the sampling interval of certain parameters X of drilling geological characteristics is $\Delta H = [H_u, H_l]$, the total data points are $(2n+1)$ (Figure 2) within range. The data point parameter in interval ΔH can be used as a measurement sample of geological drilling parameters at the point i : $\{x_{(i-n)}, x_{(i-n+1)}, \dots, x_{(i+n)}\}$. According to the uncertainty calculation method, the interval of uncertainty of drilling geological characteristics parameters can be determined (Figure 3).

① Class A Uncertainty

$$u_A = s(\bar{x}) = \sqrt{\frac{\sum_{j=i-n}^{i+n} v_j^2}{2n(2n+1)}} = \sqrt{\frac{\sum_{j=i-n}^{i+n} (x_j - \bar{x})^2}{2n(2n+1)}} \quad (14)$$

② Class B Uncertainty

Ignoring the influence of the logging tools error, only the impact of changes in lithology are considered within the same stratigraphic drilling geological characteristic parameters. If the sample consists of the interval of R_1, R_2, \dots, R_m the parameters of the drilling geological characteristic is $r_i \in [r_i^L, r_i^U]; (1 < i < m)$.

$$u_B = \sqrt{u_{B1}^2 + u_{B2}^2 + \dots + u_{Bm}^2} = \sqrt{\sum_{i=1}^m \left(\frac{r_i^U - r_i^L}{t} \right)^2} \quad (15)$$

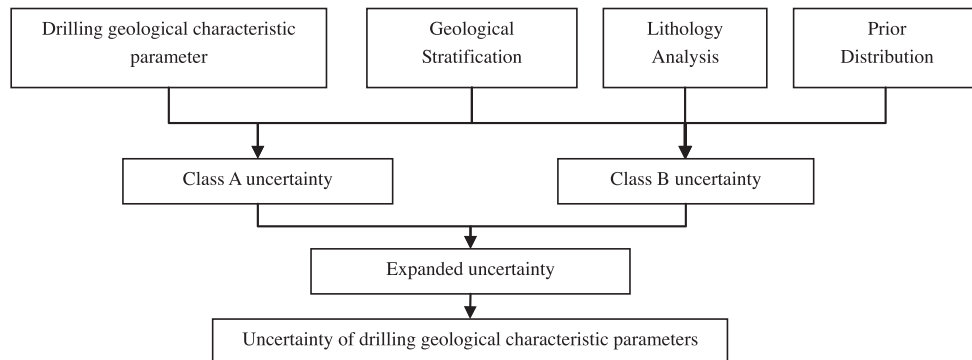


Fig. 3. Calculation process of uncertainty of drilling geological characteristic parameters

③ Expanded Uncertainty

Considering the practical engineering, determine the factor k , and then the uncertainty of parameter of drilling geological characteristics is:

$$u = k\sqrt{u_A^2 + u_B^2} \quad (16)$$

④ Interval range

The value of the parameter of drilling geological characteristics in the data point i is viewed as the interval range. Based on this assumption the uncertainty interval of the parameter of drilling geological characteristics in the data point i is:

$$\bar{X}(i) \in [X(i) - u, X(i) + u] \quad (17)$$

4. An example

The BD gas field is located offshore in the Madura Strait, which separates the Indonesian Islands of Java and Madura. Regional seismic exploration shows that there are many fault belts in this area, so it is hard to accurately predict pressure. The fuzzy understanding of the formation pressure causes drilling risks which mainly include loss, kicking, collapse, and stick. According to the methods proposed, the uncertainty interval of formation pressure of the BD-1 well can be obtained. Figure 4 shows the results. The tested formation pressure is in the uncertainty interval of formation pressure. The example results show that the uncertain description of the geological drilling parameters is more relevant with practical engineering.

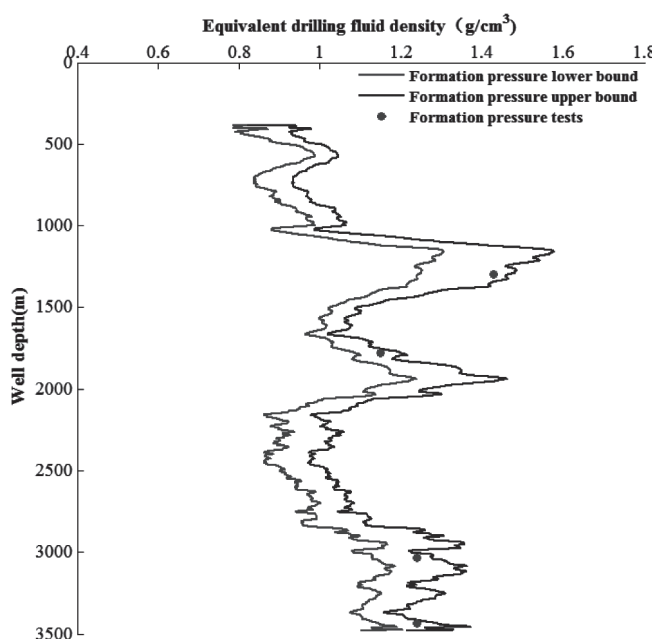


Fig. 4. Uncertainty interval of the formation pore pressure of BD-1 well

5. Conclusion

The uncertainty of geological drilling parameters creates a fundamental risk in drilling engineering. The uncertainty of geological drilling parameters can be divided into two categories, according to the causes of the uncertainty. One is the uncertainty caused by the time and space variability of the rock mass, characteristics of objective randomness. The other is caused by measurement error, or subjective uncertainty. This paper focused on the subjective uncertainty problem.

A method of using the uncertainty theory to describe the uncertainty of geological drilling parameters has been postulated. The geological drilling parameters interpreted by logging or seismic data in the same geological stratum were regarded as the measured sample on the theory of stratigraphic science and uncertainty. The size of the measured sample has a great influence on the analysis results, which can be determined by the spatial variability analysis of the geological drilling parameters in the actual project process.

In actual fieldwork, many parameters that cannot be obtained through direct measurement. However, they can usually be obtained by indirect measurement. In this paper, the method of determining the uncertainty of the indirect measurement parameters was established based on the theory of interval mathematics theory.

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دراسة عدم اليقين لمعلومات الحفر الجيولوجية من خلال نظرية عدم اليقين

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الملخص

إن المعلومات الجيولوجية للحفر تعكس الطبيعة المتكاملة للبيانات الجيولوجية الأساسية للنفط والغاز. فمعرفة مهمتهم لتجنب مخاطر الحفر. ونتيجة للتعقيدات الموجودة في جيولوجيا النفط، وعدم اكتمال بيانات التسجيل أو الزلازل، ودقة النموذج الرياضي وقضايا أخرى، فإنه لا يمكن الحصول على القيمة الحقيقية للمعلومات الجيولوجية للحفر. بناءً على نظرية طبقية التسلسل وعدم اليقين، تم تأسيس طريقة لوصف عدم اليقين من المعلومات الجيولوجية للحفر. أولاً، تم اعتبار المعلومات الجيولوجية للحفر التي تم تفسيرها بواسطة بيانات التسجيل أو الزلازل في نفس الطبقة كعينة مُقاسة. ثم تم تحديد توزيع الاحتمال أو المدى الفاصل للمعلومات الجيولوجية للحفر في كل نقطة قياس بناءً على هذه العينة. وأخيراً، يمكن الحصول على فترة عدم اليقين لمعلومات الحفر الجيولوجي إلى جانب عمق البئر. وأظهرت النتائج أن الوصف غير المؤكد للمعلومات الجيولوجية للحفر كان أكثر صلة بالهندسة العملية.