

On Z^* -open, Z^* -closed functions

MOHAMMAD AL-JUHANI* AND ALI MUBARKI**

Department of Mathematics, Faculty of Science, Taibah University, AL-Madinah AL-Munawarah, Saudi Arabia.

e-mail: mhm977@hotmail.com, alimobarki@hotmail.com***

ABSTRACT

The aim of this paper is this paper is to introduce Z^* -open, Z^* -closed, pre- Z^* -open and pre- Z^* -closed functions and investigate properties and characterizations of these new types of functions.

(2000) Mathematical Subject Classifications: 54C05; 54C08; 54C10.

Keywords: Z^* -open function; pre- Z^* -open function; super Z^* -open function.

INTRODUCTION

The topological spaces have been investigated from different aspects in the recent past Such as mixed fuzzy topological space by Tripathy & Ray (2012, 2013, 2014), from bitopological space by Tripathy & Sarma (2011, 2012, 2013) Tripathy & Debnath (2013) and others.

A subset A of a topological space (X, τ) is called regular open by Velicko (1968) (resp. regular closed) if $A = \text{int}(\text{cl}(A))$ (resp. $A = \text{cl}(\text{int}(A))$). The delta interior by Velicko (1968) of a subset A of X is the union of all regular open sets of X contained in A is denoted by $\text{int}_\delta(A)$. A subset A of a space X is called δ -open if $A = \text{int}_\delta(A)$. The complement of δ -open set is called δ -closed. Throughout this paper (X, τ) and (Y, σ) (Simply, X and Y) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure of subset A of X , the interior of A and the complement of A is denoted by $\text{cl}(A)$, $\text{int}(A)$ and A^c or $X \setminus A$, respectively. A subset A of a space (X, τ) is called preopen by Mashhour *et al.* (1982) or locally dense by Carson & Michael (1964) (resp. α -preopen by Raychaudhuri & Mukherjee (1993), δ -open by Njåstad (1965), b -open by Andrijević (1996) or γ -open by El-Atik (1997), semi-open by Levine (1963), e -open by Ekici (2008), e^* -open by Ekici (2009), Z^* -open by Mubarki (2012) if $A \subseteq \text{int}(\text{cl}(A))$ (resp. $A \subseteq \text{int}(\text{cl}_\delta(A))$, $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, $A \subseteq \text{cl}(\text{int}(A)) \text{int}(\text{cl}(A))$, $A \subseteq \text{cl}(\text{int}(A))$, $A \subseteq \text{cl}(\text{int}_\delta(A))$, $\text{int}(\text{cl}_\delta(A))$, $A \subseteq \text{cl}(\text{int}(\text{cl}_\delta(A))$, $A \subseteq \text{cl}(\text{int}(A)) \text{int}(\text{cl}_\delta(A))$). The complement of δ -preopen (resp. α -open, semi-open, γ -open, e -open, e^* -open, Z^* -open) set is

called δ -preclosed (resp. α -closed, semi-closed, γ -closed, e-closed, e^* -closed, Z^* -closed). The family of all δ -preopen (resp. semi-open, γ -open, e-open, e^* -open, Z^* -open) is denoted by δ -PO(X) (resp. SO(X), γ -O(X), e-O(X), e^* -O(X), Z^* O(X)). The intersection (resp. union) of Z^* -closed (resp. Z^* -open) sets containing (resp. contained in) A is called Z^* -closure (resp. Z^* -interior) and is denoted by Z^* -cl(A) (resp. Z^* -int(A)). The family of all Z^* -closed subsets of (X, τ) will be denoted by Z^* C(X).

PRELIMINARIES

In what follows, spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated and $f: (X, \tau) \rightarrow (Y, \sigma)$ or simply $f: X \rightarrow Y$ denotes a function f of a space (X, τ) into a space (Y, σ) .

We recall the following definitions by Mubarki (2012) which are useful in the sequel.

Definition 1. For a space (X, τ) , a point $p \in X$ is called a Z^* -limit point of A if for every Z^* -open set G containing p contains a point of A other than p. The set of all Z^* -limit points of A is called Z^* -derived set of A and is denoted by Z^* -d(A).

Definition 2. For a space (X, τ) :

- (i) A subset N of a space (X, τ) is called a Z^* -neighbourhood (briefly, Z^* -nbd) of a point p if there exists a Z^* -open set W such that $p \in W \subseteq N$,
- (ii) Z^* -b(A) = Z^* -cl(A) \setminus Z^* -int(A),
- (iii) Z^* -Bd(A) = $A \setminus Z^*$ -int(A).

The set of Z^* -boundary (resp. Z^* -border) of A is denoted by Z^* -b(A) (resp. Z^* -Bd(A)).

Proposition 1. Let (X, τ) be a topological space and $A \subseteq X$. Then, the following statements hold:

- (i) A is Z^* -closed if and only if Z^* -d(A) \subseteq A,
- (ii) A is Z^* -open if and only if it is Z^* -nbd for every point $p \in A$,
- (iii) Z^* -cl(A) = $A \cup Z^*$ -d(A).

Definition 3. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called semi-open by Biswas (1969) (resp. δ -preopen Raychaudhuri & Mukherjee (1993), γ -open by Mashhour *et al.* (1983), γ -open by El-Atik (1997), e-open by Ekici (2008), δ -open by Velicko (1968), e^* -open by Ekici (2009)) if, $f(V) \in$ SO(Y, σ) (resp. δ -PO(Y, σ), α -O(Y, σ), γ O(Y, σ), e-O(Y, σ), δ -O(Y, σ), e^* -O(Y, σ)) for each open set V of (X, τ) .

Z^* -OPEN AND Z^* -CLOSED FUNCTIONS

In this section, we define and study the concept Z^* -open function as the generalizations of open functions. Also, some of its properties are discussed.

Definition 4. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

- (i) Z^* -open if the image of each open set of (X, τ) is Z^* -open in (Y, σ) ,
- (ii) Z^* -closed if the image of each closed set of (X, τ) is Z^* -closed in (Y, σ) .

Remark 1. The implication between these types of functions and those as introduced above is given by the following diagram.

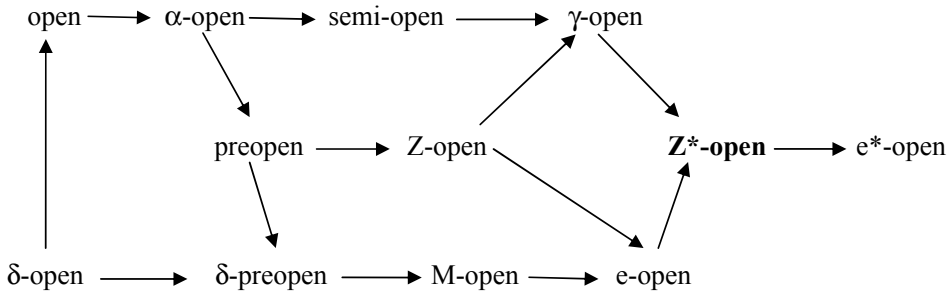


Fig.1. The relationship between the sets

The converse of these implications is not true in general as established by Biswas (1969); Raychaudhuri & Mukherjee (1993); Mashhour *et al.* (1982); Mashhour *et al.* (1983); El-Atik, (1997); Ekici (2008); Ekici (2009); Velicko (1968); EL-Magharabi & Mubarki (2012); EL-Magharabi & AL-Juhani (2013) and by the following examples.

Example 1. Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$ and $\sigma = \{Y, \phi, \{a, c\}, \{b, d\}, \{d\}, \{a, c, d\}\}$. Then the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is Z^* -open but not γ -open. Since, $f(\{b\}) \notin \gamma O(Y)$.

Example 2. Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{a, d\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Then the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is Z^* -open but not e -open. Since, $f(\{a, d\}) \notin eO(Y)$.

Example 3. Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Then the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is e^* -open but not Z^* -open. Since, $f(\{a, c\}) \notin Z^*O(Y)$.

Proposition 2. Let A be a subset of space (X, τ) . Then the following statements are held:

- (i) $Z^*O(A) \Leftrightarrow eO(A)$ if $\text{int}(A) = \phi$,
- (ii) $Z^*O(A) \Leftrightarrow ZO(A)$ if $\text{int}(A) = \text{int}_\delta(A)$.

Theorem 1. For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent :

- (i) f is Z^* -open,
- (ii) For each $x \in X$ and each neighbourhood U of x , there exists $V \in Z^*O(Y)$ containing $f(x)$ such that $V \subseteq f(U)$,
- (iii) $f(\text{int}(A)) \subseteq Z^*\text{-int}(f(A))$ for each $A \subseteq X$,
- (iv) $\text{int}(f^{-1}(B)) \subseteq f^{-1}(Z^*\text{-int}(B))$ for each $B \subseteq Y$,
- (v) $f^{-1}(Z^*\text{-Bd}(B)) \subseteq \text{Bd}(f^{-1}(B))$ for each $B \subseteq Y$,
- (vi) $f^{-1}(Z^*\text{-cl}(B)) \subseteq \text{cl}(f^{-1}(B))$ for each $B \subseteq Y$.

Proof.

- (i) \Rightarrow (ii). Let U be neighbourhood of x in X . Then there exists an open set G such that $x \in G \subseteq U$ and hence $f(x) \in f(G) \subseteq f(U)$. Since f is Z^* -open, then $f(G)$ is Z^* -open in Y . Put $f(G) = V$, then $f(x) \in V \subseteq f(U)$.
- (ii) \Rightarrow (i). Let U be an open set containing x in X for every $f(x) \in f(U)$. Then U is neighbourhood of each $x \in U$. By hypothesis, there exists $V \in Z^*O(Y)$ such that $f(x) \in V \subseteq f(U)$. Hence, $f(U)$ is Z^* -neighbourhood of each $f(x) \in f(U)$. By Proposition 1.1, $f(U)$ is Z^* -open in Y . Therefore, f is Z^* -open.
- (i) \Rightarrow (iii). Since $\text{int}(A) \subseteq A \subseteq X$ which is open and f is Z^* -open, then $f(\text{int}(A))$ is Z^* -open in Y . Hence, $f(\text{int}(A)) \subseteq Z^*\text{-int}(f(A))$ and since, $f(\text{int}(A)) \subseteq f(A)$, then $f(\text{int}(A)) \subseteq Z^*\text{-int}(f(A)) \subseteq f(A)$.
- (iii) \Rightarrow (iv). By replacing $f^{-1}(B)$ instead of A in (iii), we have $f(\text{int}(f^{-1}(B))) \subseteq Z^*\text{-int}(f(f^{-1}(B)))$ and then $\text{int}(f^{-1}(B)) \subseteq f^{-1}(Z^*\text{-int}(f(f^{-1}(B)))) \subseteq f^{-1}(Z^*\text{-int}(B))$
- (iv) \Rightarrow (i). Let $A \in \tau$. Then $f(A) \subseteq Y$ and by hypothesis, $\text{int}(f^{-1}(f(A))) \subseteq f^{-1}(Z^*\text{-int}(f(A)))$. This implies that, $\text{int}(A) \subseteq f^{-1}(Z^*\text{-int}(f(A)))$. Thus $f(\text{int}(A)) \subseteq Z^*\text{-int}(f(A))$. Therefore, f is Z^* -open.
- (iv) \Rightarrow (v). Let $B \subseteq Y$. Then by hypothesis, $f^{-1}(B) \setminus f^{-1}(Z^*\text{-int}(B)) \subseteq f^{-1}(B) \setminus \text{int}(f^{-1}(B))$ and hence, $f^{-1}(B \setminus Z^*\text{-int}(B)) \subseteq f^{-1}(B) \setminus \text{int}(f^{-1}(B))$. Therefore, $f^{-1}(Z^*\text{-Bd}(B)) \subseteq \text{Bd}(f^{-1}(B))$.

- (v) \Rightarrow (iv). Let $B \subseteq Y$. Then by Definition 2, we have $f^{-1}(B \setminus Z^*\text{-int}(B)) \subseteq f^{-1}(B) \setminus \text{int}(f^{-1}(B))$ and hence $f^{-1}(B) \setminus f^{-1}(Z^*\text{-int}(B)) \subseteq f^{-1}(B) \setminus \text{int}(f^{-1}(B))$. Therefore, $\text{int}(f^{-1}(B)) \subseteq f^{-1}(Z^*\text{-int}(B))$.
- (i) \Rightarrow (vi). Let $B \subseteq Y$ and $x \in f^{-1}(Z^*\text{-cl}(B))$. Then $f(x) \in Z^*\text{-cl}(B)$. Assume that U is an open set containing x . Since f is Z^* -open, then $f(U)$ is Z^* -open in Y . Hence, $B \cap f(U) \neq \emptyset$. Thus $U \cap f^{-1}(B) \neq \emptyset$. Therefore, $x \in \text{cl}(f^{-1}(B))$. So, $f^{-1}(Z^*\text{-cl}(B)) \subseteq \text{cl}(f^{-1}(B))$.
- (vi) \Rightarrow (i). Let $B \subseteq Y$. Then $Y \setminus B \subseteq Y$. By hypothesis, $f^{-1}(Z^*\text{-cl}(Y \setminus B)) \subseteq \text{cl}(f^{-1}(Y \setminus B))$ and hence $X \setminus f^{-1}(Z^*\text{-int}(B)) \subseteq X \setminus \text{int}(f^{-1}(B))$ that implies $\text{int}(f^{-1}(B)) \subseteq f^{-1}(Z^*\text{-int}(B))$. Then by (iv), f is Z^* -open.

Theorem 2. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Z^* -open function. If $W \subseteq Y$ and $F \subseteq X$ is a closed set containing $f^{-1}(W)$, then there exists a Z^* -closed set H of Y containing W such that $f^{-1}(H) \subseteq F$.

Proof. Let $H = Y \setminus (X \setminus F)$ and F be a closed set of X containing $f^{-1}(W)$. But f is Z^* -open function, then $f(X \setminus F)$ is Z^* -open set of Y . Therefore, H is Z^* -closed and $f^{-1}(H) = X \setminus [f^{-1}(X \setminus F)] \subseteq X \setminus (X \setminus F) = F$.

Remark 2. The converse of above theorem is not true in general. Suppose that $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Hence the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is satisfying the condition but it is not Z^* -open. Since $\{b, c\} \subseteq Y$ and $\{b, c, d\} \subseteq X$ is a closed set containing $f^{-1}(\{b, c\}) = \{b, c\}$, hence there exists $\{b, c\} \in Z^*C(Y)$ containing $\{b, c\}$ such that $f^{-1}(\{b, c\}) \subseteq \{b, c, d\}$ but, $\{c\} \in \tau$ and $f(\{c\}) = \{c\} \notin Z^*O(Y)$.

Theorem 3. For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:

- (i) f is Z^* -open,
- (ii) $f^{-1}(\text{cl}(\text{int}_\delta(B))) \cap f^{-1}(\text{int}(\text{cl}(B))) \subseteq \text{cl}(f^{-1}(B))$ for each $B \subseteq Y$,
- (iii) If f is bijective, then $\text{cl}(\text{int}_\delta(f(A))) \cap \text{int}(\text{cl}(f(A))) \subseteq f(\text{cl}(A))$ for each $A \subseteq X$.

Proof. (i) \Rightarrow (ii). Let $B \subseteq Y$ and f be a Z^* -open function. Then by Theorem 2, there exists a Z^* -closed set V of Y containing B such that $f^{-1}(\text{cl}(\text{int}_\delta(B))) \cap f^{-1}(\text{int}(\text{cl}(B))) \subseteq f^{-1}(\text{cl}(\text{int}_\delta(V))) \cap f^{-1}(\text{int}(\text{cl}(V))) \subseteq f^{-1}(B) \subseteq \text{cl}(f^{-1}(B))$ and hence, $f^{-1}(\text{cl}(\text{int}_\delta(B))) \cap f^{-1}(\text{int}(\text{cl}(B))) \subseteq \text{cl}(f^{-1}(B))$.

(ii) \Rightarrow (iii). Let f be a bijective function and $f(A) \subseteq Y$. Then by (ii), we have $f^{-1}(\text{cl}(\text{int}_\delta(f(A)))) \cap f^{-1}(\text{int}(\text{cl}(f(A)))) \subseteq \text{cl}(f^{-1}(f(A))) = \text{cl}(A)$. Hence, $\text{cl}(\text{int}_\delta(f(A))) \cap \text{int}(\text{cl}(f(A))) \subseteq f(\text{cl}(A))$.

(iii) \Rightarrow (i). Let $A \in \tau$. Then $X \setminus A$ is a closed set of X . Hence by hypothesis, $\text{cl}(\text{int}_\delta(f(X \setminus A))) \cap \text{int}(\text{cl}(f(X \setminus A))) \subseteq f(\text{cl}(X \setminus A)) = f(X \setminus A)$. By bijection of f , we have, $f(A) \subseteq \text{int}(\text{cl}_\delta(f(A))) \text{cl}(\text{int}(f(A)))$. Therefore, f is Z^* -open.

Theorem 4. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Z^* -closed function. Then the following statements hold:

(i) If f is a surjective and $f^{-1}(B), f^{-1}(C)$ have disjoint neighbourhoods of X , then B and C are disjoint of Y ,

(ii) $Z^*\text{-int}(Z^*\text{-cl}(f(A))) \subseteq f(\text{cl}(A))$ for each $A \subseteq X$.

Proof. (i) Let M, N be two disjoint neighbourhoods of $f^{-1}(B), f^{-1}(C)$. Then there exist two Z^* -open sets U, V such that $f^{-1}(B) \subseteq U \subseteq M, f^{-1}(C) \subseteq V \subseteq N$. But, f is a surjective, then $f f^{-1}(B) = B \subseteq f(U) \subseteq f(M), f f^{-1}(C) = C \subseteq f(V) \subseteq f(N)$. Since M, N are disjoint, then also $f(M \cap N) = \phi$ and hence $B \cap C \subseteq f(U \cap V) \subseteq f(M \cap N) = \phi$. Therefore, B and C are disjoint of Y .

(ii) Since $A \subseteq \text{cl}(A) \subseteq X$ and f is a Z^* -closed function, then $f(\text{cl}(A))$ is Z^* -closed in Y . Hence, $f(A) \subseteq Z^*\text{-cl}(f(A)) \subseteq f(\text{cl}(A))$. So $Z^*\text{-int}(Z^*\text{-cl}(f(A))) \subseteq f(\text{cl}(A))$.

Theorem 5. For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, then the following are equivalent :

(i) f is Z^* -closed,

(ii) $Z^*\text{-cl}(f(A)) \subseteq f(\text{cl}(A))$ for each $A \subseteq X$,

(iii) If f is surjective, then for each subset B of Y and each open set U in X containing $f^{-1}(B)$, there exists a Z^* -open set V of Y containing B such that $f^{-1}(V) \subseteq U$.

Proof. (i) \Rightarrow (ii). Let $\text{cl}(A) \subseteq X$ be a closed set. Since f is Z^* -closed, then $f(\text{cl}(A)) \in Z^*C(Y)$. Hence, $Z^*\text{-cl}(f(A)) \subseteq f(\text{cl}(A))$.

(ii) \Rightarrow (i). Let $A \subseteq X$ be a closed set. By hypothesis, $Z^*\text{-cl}(f(A)) \subseteq f(\text{cl}(A)) = f(A)$. Hence, $f(A) \in Z^*C(Y)$. Therefore, f is Z^* -closed.

(i) \Rightarrow (iii). Suppose that $V = Y \setminus f(X \setminus U)$ and U is an open set of X containing $f^{-1}(B)$. Then by hypothesis, V is Z^* -open in Y . But, $f^{-1}(B) \subseteq U$, then $B \subseteq f(U)$ and $f(X \setminus U) \subseteq Y \setminus B$, that is, $B \subseteq V$ and $f^{-1}(V) \subseteq U$.

(iii) \Rightarrow (i). Let $F \subseteq X$ be a closed set and y be any point of $Y \setminus f(F)$. Then $f^{-1}(y) \in X \setminus F$ which is open in X . Hence by hypothesis, there exists a Z^* -open set V containing y such that $f^{-1}(V) \subseteq X \setminus F$. But f is surjective, then $y \in V \subseteq Y \setminus f(F)$ and $Y \setminus f(F)$ is the union of Z^* -open sets and hence, $f(F)$ is Z^* -closed. Therefore, f is Z^* -closed.

Remark 3. The restriction of Z^* -open function is not Z^* -open.

Example 4. Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\{a, c, d\} = A \subseteq X$. Hence the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is Z^* -open. But, $f_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is not Z^* -open. Since $\{c\} \in \tau_A$ but $f(\{c\}) = \{c\} \notin Z^*O(Y)$.

The next theorem gives the restriction condition under which the intersection of two sets one of which is Z^* -open function will be Z^* -open.

Theorem 6. In a space (X, τ) , if $A \in \alpha\text{-}O(X, \tau)$ and $B \in Z^*O(X, \tau)$, then $A \cap B \in Z^*O(X, \tau_A)$.

Proof. Since $A \cap B \subseteq \text{int}(\text{cl}(\text{int}(A))) \cap (\text{cl}(\text{int}(B)) \text{int}(\delta\text{-cl}(B))) = (\text{int}(\text{cl}(\text{int}(A))) \cap \text{cl}(\text{int}(B))) \text{int}(\text{cl}(\text{int}(A))) \cap \text{int}(\delta\text{-cl}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(\text{int}(A)))) \cap \text{int}(B) \text{int}(\text{cl}(\text{int}(A)) \cap \delta\text{-cl}(B)) \subseteq \text{cl}(\text{cl}(\text{int}(A)) \cap \text{int}(B)) \text{int}(\text{cl}(\text{int}(A)) \cap \delta\text{-cl}(B)) \subseteq \text{cl}(\text{int}(A) \cap \text{int}(B)) \text{int}(\text{cl}(\text{int}(A)) \cap \delta\text{-cl}(B))$ and hence $A \cap B \subseteq (A \cap \text{cl}(\text{int}(A) \cap \text{int}(B))) (A \cap \text{int}(\text{cl}(\text{int}(A)) \cap \delta\text{-cl}(B))) \subseteq \text{cl}_A(\text{int}(A) \cap \text{int}(B)) \text{int}_A(\text{cl}(\text{int}(A)) \cap \delta\text{-cl}(B)) \subseteq \text{cl}_A(\text{int}(A) \cap \text{int}(B)) \text{int}_A(\delta\text{-cl}(\text{cl}(\text{int}(A)) \cap \delta\text{-cl}(B))) \subseteq \text{cl}_A(\text{int}(A) \cap \text{int}(B)) \text{int}_A(\delta\text{-cl}(\text{int}(A) \cap B)) \subseteq \text{cl}_A(\text{int}(A) \cap \text{int}(B)) \text{int}_A(\delta\text{-cl}(\text{int}(A) \cap B))$. Since $\text{int}(A) \cap \text{int}(B) \subseteq \text{int}(A) \subseteq A$ which is open in A , then $\text{cl}_A(\text{int}(A) \cap \text{int}(B)) \text{int}_A(A \cap \delta\text{-cl}(A \cap B)) \subseteq \text{cl}_A \text{int}_A(A \cap B) \text{int}_A \delta\text{-cl}_A(A \cap B)$. Therefore, $A \cap B \in Z^*O(X, \tau_A)$.

Proposition 3. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be Z^* -open and $A \subseteq X$. Then

$f_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is also Z^* -open, for every $f(A) \subseteq Y$ is α -open.

Proof. For every $U \in \tau_A$, then there exists $V \in \tau$ such that $U = A \cap V$ Hence by Theorem 6, $f_A(U) \in Z^*O(Y)$. Therefore, $f_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is Z^* -open.

Remark 4. The composition of two Z^* -open functions may not be Z^* -open.

Example 5. Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau_x = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $\tau_y = \{Y, \phi, \{a, b\}, \{c, d\}\}$ and $\tau_z = \{Z, \phi, \{a\}, \{c\}, \{a, c\}\}$. Hence the identity functions $f: (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g: (Y, \tau_y) \rightarrow (Z, \tau_z)$ are Z^* -open but $g \circ f$ is not Z^* -open. Since $\{b\} \in \tau_x$ and $(g \circ f)(\{b\}) = \{b\} \notin Z^*O(Z)$.

The following result gives some properties of the composition.

Theorem 7. Let $f: (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g: (Y, \tau_y) \rightarrow (Z, \tau_z)$ be two functions. Then the following statements hold:

- (i) If f is open and g is a Z^* -open functions, then $g \circ f$ is Z^* -open,
- (ii) If $g \circ f$ is Z^* -open and f is surjective continuous, then g is Z^* -open,
- (iii) If $g \circ f$ is open and g is injective Z^* -continuous, then f is Z^* -open.

Proof. (i) Let $U \in \tau_x$. Then by hypothesis, $f(U) \in \tau_y$. But g is Z^* -open function, then $g(f(U)) \in Z^*O(Z)$. Hence, $g \circ f$ is Z^* -open.

(ii) Let $U \in \tau_y$ and f be continuous function. Then $f^{-1}(U) \in \tau_x$. But $g \circ f$ is Z^* -open, then $(g \circ f)(f^{-1}(U)) \in Z^*O(Z)$. Hence by surjective of f , $g(U) \in Z^*O(Z)$. Hence, g is Z^* -open.

(iii) Let $U \in \tau_x$ and $g \circ f$ be open function. Then $(g \circ f)(U) = g(f(U)) \in \tau_z$. Since g is injective Z^* -continuous, hence $f(U) \in Z^*O(Y)$. Therefore, f is Z^* -open.

Definition 5. A space (X, τ) is called:

(i) Z^* - T_1 if for every two distinct points x, y of X , there exist two Z^* -open sets U, V such that $x \in U, y \notin U$ and $x \notin V, y \in V$,

(ii) Z^* - T_2 if for every two distinct points x, y of X , there exist two disjoint Z^* -open sets U, V such that $x \in U, y \in V$,

(iii) Z^* -compact if for every Z^* -open cover of X has a finite subcover,

(iv) Z^* -connected if it can not be expressed as the union of two disjoint non-empty Z^* -open sets of X ,

(v) Z^* -Lindelöff if every Z^* -open cover of X has a countable subcover.

Theorem 8. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective Z^* -open function. Then the following statements hold:

(i) If X is a T_1 -space, then Y is Z^* - T_1 where $i = 1, 2$.

(ii) If Y is a Z^* -compact space, then X is compact,

(iii) If Y is a Z^* -Lindelöff space, then X is Lindelöff.

Proof. (i) We prove that for the case of a T_1 -space. Let y_1, y_2 be two distinct points of Y . Then there exist $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is a T_1 -space, then there exist two open sets U, V of X such that $x_1 \in U, x_2 \notin U$ and $x_2 \in V, x_1 \notin V$. But, f is Z^* -open, then $f(U), f(V)$ are Z^* -open sets of Y with $y_1 \in f(U), y_2 \notin f(U)$ and $y_2 \in f(V), y_1 \notin f(V)$. Therefore, Y is Z^* - T_1 .

(ii) Let $\{U_i: i \in I\}$ be a family of open cover of X and f be a surjective Z^* -open function. Then $\{f(U_i): i \in I\}$ is a Z^* -open cover of Y . But, Y is Z^* -compact space, hence there exists a finite subset I_0 of I such that $Y = \{f(U_i): i \in I_0\}$. Then by injective of f , $\{U_i: i \in I_0\}$ is a finite subfamily of X . Therefore, X is compact.

(iii) Obvious.

Theorem 9. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a surjective Z^* -open function and Y is Z^* -connected space, then X is connected.

Proof. Suppose that X is a disconnected space. Then there exist two non-empty disjoint open sets U, V of X such that $X = U \cup V$. But f is a surjective Z^* -open, then $f(U)$ and $f(V)$ are non-empty disjoint Z^* -open sets of Y with $Y = f(U) \cup f(V)$ which is a contradiction with the fact that Y is Z^* -connected.

Theorem 10. For a bijective function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:

- (i) f^{-1} is Z^* -continuous,
- (ii) f is Z^* -open,
- (iii) f is Z^* -closed.

Proof. Obvious.

SUPER Z^* -OPEN AND SUPER Z^* -CLOSED FUNCTIONS

In the following, we introduce and study the concepts of super Z^* -open and super Z^* -closed functions and some of their properties have been investigated.

Definition 6. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called :

- (i) super Z^* -open if $f(U)$ is open in Y for each $U \in Z^*O(X, \tau)$,
- (ii) super Z^* -closed if $f(U)$ is closed in Y for each $U \in Z^*C(X, \tau)$.

Proposition 4. Every super Z^* -open function is Z^* -open.

Proof. Let $A \subseteq X$ be an open set and hence A is Z^* -open. But, f is super Z^* -open, then $f(A)$ is open in Y , hence $f(A)$ is Z^* -open in Y . Therefore, f is Z^* -open.

Remark 5. The converse of the above proposition is not necessarily true.

Example 6. In Example 1, f is Z^* -open but not super Z^* -open. Since $\{b\} \in Z^*O(X)$ and $f(\{b\}) = \{b\} \notin \sigma$.

Theorem 11. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a function, then the following statements are equivalent:

- (i) f is super Z^* -open,
- (ii) for each $x \in X$ and each Z^* -neighbourhood U of x , there exists a neighbourhood V of $f(x)$ such that $V \subseteq f(U)$,
- (iii) $f(Z^*\text{-int}(A)) \subseteq \text{int}(f(A))$ for each $A \subseteq X$,
- (iv) $Z^*\text{-int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B))$ for each $B \subseteq Y$,
- (v) $f^{-1}(\text{Bd}(B)) \subseteq Z^*\text{-Bd}(f^{-1}(B))$ for each $B \subseteq Y$,

- (vi) $f^{-1}(\text{cl}(B)) \subseteq Z^*\text{-cl}(f^{-1}(B))$ for each $B \subseteq Y$,
- (vii) If f is surjective, then for each subset B of Y and for any set $F \in Z^*C(X)$ containing $f^{-1}(B)$, there exists a closed subset H of Y containing B such that $f^{-1}(H) \subseteq F$.

Proof. (i) \Rightarrow (ii). Let U be a Z^* -neighbourhood of x in X . Then there exists $W \in Z^*O(X)$ such that $x \in W \subseteq U$ and hence $f(x) \in f(W) \subseteq f(U)$, for all $f(x) \in f(U)$. Hence by hypothesis, $f(W) \in \sigma$ and containing $f(x)$. Put $f(W) = V$, then $f(x) \in V \subseteq f(U)$.

(ii) \Rightarrow (i). Suppose that U is Z^* -open set of X and containing x for all $x \in X$. Then $f(x) \in f(U)$. Hence by hypothesis, there exists $V \in \sigma$ containing $f(x)$ such that $f(x) \in V \subseteq f(U)$. Hence, $f(U)$ is neighbourhood for all $f(x) \in f(U)$. Thus $f(U)$ is open in Y and hence f is super Z^* -open.

(i) \Rightarrow (iii). Since $Z^*\text{-int}(A) \subseteq A \subseteq X$ is Z^* -open set and f is super Z^* -open, then $f(Z^*\text{-int}(A)) \subseteq f(A)$ is open in Y . Hence, $f(Z^*\text{-int}(A)) \subseteq \text{int}(f(A))$.

(iii) \Rightarrow (iv). By replacing $f^{-1}(B)$ instead of A of (iii), we have $f(Z^*\text{-int}(f^{-1}(B))) \subseteq \text{int}(f(f^{-1}(B))) \subseteq \text{int}(B)$ and hence, $Z^*\text{-int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B))$.

(iv) \Rightarrow (v). Let $B \subseteq Y$. Then by hypothesis and Definition 2, we have $f^{-1}(B) \setminus f^{-1}(\text{int}(B)) \subseteq f^{-1}(B) \setminus Z^*\text{-int}(f^{-1}(B))$ and hence, $f^{-1}(\text{Bd}(B)) \subseteq Z^*\text{-Bd}(f^{-1}(B))$.

(v) \Rightarrow (iv). Let $B \subseteq Y$. Then by hypothesis and Definition 2, we have $f^{-1}(B) \setminus \text{int}(B) \subseteq f^{-1}(B) \setminus Z^*\text{-int}(f^{-1}(B))$ and hence $f^{-1}(B) \setminus f^{-1}(\text{int}(B)) \subseteq f^{-1}(B) \setminus Z^*\text{-int}(f^{-1}(B))$. Therefore, $Z^*\text{-int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B))$.

(iv) \Rightarrow (vi). Let $B \subseteq Y$. Then $Y \setminus B \subseteq Y$, hence by hypothesis, we have $Z^*\text{-int}(f^{-1}(Y \setminus B)) \subseteq f^{-1}(\text{int}(Y \setminus B))$ and hence $X \setminus Z^*\text{-cl}(f^{-1}(B)) \subseteq X \setminus f^{-1}(\text{cl}(B))$. Therefore, $f^{-1}(\text{cl}(B)) \subseteq Z^*\text{-cl}(f^{-1}(B))$.

(vi) \Rightarrow (iv). Let $B \subseteq Y$. Then $Y \setminus B \subseteq Y$. So by hypothesis, we have $f^{-1}(\text{cl}(Y \setminus B)) \subseteq Z^*\text{-cl}(f^{-1}(Y \setminus B))$ and hence $X \setminus f^{-1}(\text{int}(B)) \subseteq X \setminus Z^*\text{-int}(f^{-1}(B))$. Therefore, $Z^*\text{-int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B))$.

(iv) \Rightarrow (i). Let $A \in Z^*O(X)$. Then $f(A) \subseteq Y$ and by hypothesis, $Z^*\text{-int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{int}(f(A)))$. This implies that, $Z^*\text{-int}(A) \subseteq f^{-1}(\text{int}(f(A)))$. Thus $f(Z^*\text{-int}(A)) \subseteq \text{int}(f(A))$. Therefore by (iii), f is super Z^* -open.

(i) \Rightarrow (vii). Let $H = Y \setminus f(X \setminus F)$ and F be a Z^* -closed set of X containing $f^{-1}(B)$. Then $X \setminus F$ is a Z^* -open set. But f is a super Z^* -open function, then $f(X \setminus F)$ is open in Y . Therefore, H is a closed set of Y and $f^{-1}(H) = X \setminus f^{-1}f(X \setminus F) \subseteq X \setminus (X \setminus F) = F$.

- (vii) \Rightarrow (i). Let $U \in Z^*O(X)$ and put $B = Y \setminus f(U)$. Then $X \setminus U \in Z^*C(X)$ with $f^{-1}(B) \subseteq X \setminus U$. By hypothesis, there exists a closed set H of Y such that $B \subseteq H$ and $f^{-1}(H) \subseteq X \setminus U$. Hence, $f(U) \subseteq Y \setminus H$ and since $B \subseteq H$, then $Y \setminus H \subseteq Y \setminus B = f(U)$. This implies $f(U) = Y \setminus H$ which is open. Therefore, f is super Z^* -open.

Theorem 12. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective super Z^* -open function. Then the following statements hold:

- (i) If X is a Z^* - T_1 -space, then Y is T_i , where $i = 1, 2$.
- (ii) If Y is a Lindelöf space, then X is Z^* -Lindelöf.
- (iii) If Y is a compact space, then X is Z^* -compact,

Proof. (i) We prove that for the case of a Z^* - T_2 -space. Let y_1, y_2 be two distinct points of Y . Then there exist $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is a Z^* - T_2 -space, then there exist two disjoint Z^* -open sets U, V of X such that $x_1 \in U$ and $x_2 \in V$. But, f is super Z^* -open, then $f(U), f(V)$ are open sets of Y with $y_1 \in f(U), y_2 \in f(V)$ and $f(U) \cap f(V) = \emptyset$. Therefore, Y is T_2 .

- (ii) Let $\{U_i; i \in I\}$ be a family of Z^* -open cover of X and f be a surjective super Z^* -open function. Then $\{f(U_i); i \in I\}$ is an open cover of Y . But, Y is a Lindelöf space, hence there exists a countable subset I_0 of I such that $Y = \{f(U_i); i \in I_0\}$. Then by injective of f ,

$\{U_i; i \in I_0\}$ is a countable subfamily of X . Therefore, X is Z^* -Lindelöf.

- (iii) Obvious.

Theorem 13. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a surjective super Z^* -open function and Y is a connected space, then X is Z^* -connected.

Proof. Obvious.

PRE- Z^* -OPEN AND PRE- Z^* -CLOSED FUNCTIONS.

In this section, we introduce the concepts of pre- Z^* -open and pre- Z^* -closed functions and we study some of their basic properties and characterizations.

Definition 7. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

- (i) pre- Z^* -open if $f(V) \in Z^*O(Y)$ for each $V \in Z^*O(X)$,
- (ii) pre- Z^* -closed if $f(V) \in Z^*C(Y)$ for each $V \in Z^*C(X)$.

Proposition 5. (i) Every super Z^* -open function is pre- Z^* -open,

- (ii) Every pre- Z^* -open function is Z^* -open.

Proof. (i) Let $A \subseteq X$ be a Z^* -open set and f be super Z^* -open, then $f(A)$ is open in Y and hence $f(A)$ is Z^* -open. Therefore, f is pre- Z^* -open.

- (ii) Let $A \subseteq X$ be an open set and hence A is Z^* -open. But, f is pre- Z^* -open, then $f(A)$ is Z^* -open in Y . Therefore, f is Z^* -open.

Remark 6. According to the above proposition, we have the following diagram

$$\text{super } Z^*\text{-open} \rightarrow \text{pre-}Z^*\text{-open} \rightarrow Z^*\text{-open}$$

The converse of above implication is not true in general.

Example 7. Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{a, b, d\}\}$ and

$\sigma = \{Y, \phi, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Then the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is Z^* -open but not pre- Z^* -open. Since, $\{a\} \in Z^*O(X)$ and $f(\{a\}) = \{a\} \notin Z^*O(Y)$.

Example 8. In Example 1, f is pre- Z^* -open but not super Z^* -open. Since $\{b\} \in Z^*O(X)$ and $f(\{b\}) = \{b\} \notin \sigma$.

Theorem 14. For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:

- (i) f is pre- Z^* -open,
- (ii) For each $x \in X$ and each Z^* -neighbourhood U of x , there exists $V \in Z^*O(Y)$ containing $f(x)$ such that $V \subseteq f(U)$,
- (iii) $f(Z^*\text{-int}(A)) \subseteq Z^*\text{-int}(f(A))$ for each $A \subseteq X$,
- (iv) $Z^*\text{-int}(f^{-1}(B)) \subseteq f^{-1}(Z^*\text{-int}(B))$ for each $B \subseteq Y$,
- (v) $f^{-1}(Z^*\text{-Bd}(B)) \subseteq Z^*\text{-Bd}(f^{-1}(B))$ for each $B \subseteq Y$,
- (vi) $f^{-1}(Z^*\text{-cl}(B)) \subseteq Z^*\text{-cl}(f^{-1}(B))$ for each $B \subseteq Y$.

Proof. Obvious.

Theorem 15. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a surjective pre- Z^* -closed function and $f^{-1}(B), f^{-1}(C)$ have disjoint Z^* -neighbourhoods of X , then B, C are disjoint of Y .

Proof. Obvious.

Theorem 16. For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, then the following statements are equivalent:

- (i) f is pre- Z^* -closed,
- (ii) $Z^*\text{-cl}(f(A)) \subseteq f(Z^*\text{-cl}(A))$ for each $A \subseteq X$,
- (iii) If f is surjective, then for each subset B of Y and for each Z^* -open set U of X containing $f^{-1}(B)$, there exists a Z^* -open set V of Y containing B such that $f^{-1}(V) \subseteq U$.

Proof. Obvious.

Theorem 17. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective function. Then the following statements are equivalent:

- (i) f is pre- Z^* -closed,
- (ii) f is pre- Z^* -open,
- (iii) f^{-1} is Z^* -irresolute.

Proof. Obvious.

Theorem 18. Let $f: (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g: (Y, \tau_y) \rightarrow (Z, \tau_z)$ be two functions. Then the following statements hold:

- (i) $g \circ f$ is pre- Z^* -open function, if f, g are pre- Z^* -open,
- (ii) $g \circ f$ is Z^* -open function if f is Z^* -open and g is pre- Z^* -open,
- (iii) If f is a surjective Z^* -continuous function and $g \circ f$ is pre- Z^* -open, then g is Z^* -open.

Proof. (i) Let $U \in Z^*O(X)$ and f be pre- Z^* -open function. Then $f(U) \in Z^*O(Y)$. But, g is pre- Z^* -open, then $g(f(U)) \in Z^*O(Z)$. Hence, $g \circ f$ is pre- Z^* -open.

(ii) Let $U \in \tau_x$ and f be Z^* -open function. Then $f(U) \in Z^*O(Y)$. But, g is pre- Z^* -open, then $g(f(U)) \in Z^*O(Z)$. Hence, $g \circ f$ is Z^* -open.

(iii) Let $U \in \tau_y$ and f be Z^* -continuous function. Then $f^{-1}(U) \in Z^*O(X)$. But, $g \circ f$ is pre- Z^* -open, then $(g \circ f)(f^{-1}(U)) \in Z^*O(Z)$. Also, by surjective of $f, g(U) \in Z^*O(Z)$. Hence, g is Z^* -open.

Theorem 19. Let $f: (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g: (Y, \tau_y) \rightarrow (Z, \tau_z)$ be two functions such that $g \circ f: X \rightarrow Z$ is Z^* -irresolute. Then:

- (i) f is Z^* -irresolute, if g is an injective pre- Z^* -open function.
- (ii) g is Z^* -irresolute, if f is a surjective pre- Z^* -open function.

Proof. (i) Let $U \in Z^*O(Y)$. Then $g(U) \in Z^*O(Z)$. But, $g \circ f$ is Z^* -irresolute, then $(g \circ f)^{-1}(g(U)) \in Z^*O(X)$. Since g is injective, then $f^{-1}(U) \in Z^*O(X)$. Hence, f is Z^* -irresolute.

(ii) Let $V \in Z^*O(Z)$. Then $(g \circ f)^{-1}(V) \in Z^*O(X)$. But, f is pre- Z^* -open function, then $f[(g \circ f)^{-1}(V)] \in Z^*O(Y)$. Since f is surjective, then $g^{-1}(V) \in Z^*O(Y)$. Therefore, g is Z^* -irresolute.

Theorem 20. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be bijective pre- Z^* -open function. Then the following statements hold:

- (i) If X is a Z^*-T_1 -space, then Y is Z^*-T_i , where $i = 1, 2$.

- (ii) If Y is a Z^* -compact space, then X is Z^* -compact,
- (iii) If Y is a Z^* -Lindelspace, then X is Z^* -Lindel

Proof. Obvious.

Theorem 21. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a surjective pre- Z^* -open function and Y is a Z^* -connected space, then X is Z^* -connected.

Proof. Obvious.

ACKNOWLEDGMENT

The authors are highly and gratefully indebted to Taibah University, for providing necessary research facilities during the preparation of this paper.

REFERENCES

- Andrijević, D. 1996.** On b-open sets, *Matematički vesnik*. **48**: 59-64.
- Biswas, N. 1969.** On some mappings in topological spaces, *Bull. Calcutta Mathematical Society*. 61: 127-135.
- Carson, H. & Michael, E., 1964.** Metrizable of certain countable unions, *Illinois Journal of Mathematics*. **8**: 351-360.
- Ekici, E. 2008.** On e-open sets, DP*-sets and DPE*-sets and decompositions of continuity, *Arabian Journal for Science and Engineering*. **33** (2): 269 - 282.
- Ekici, E. 2009.** On e*-open sets and (D, S)*-sets, *Mathematica Moravica*. **13**(1): 29-36.
- El-Atik, A. A. 1997.** A study on some types of mappings on topological spaces, M.Sc. Thesis Tanta Univ. Egypt.
- EL-Magharabi, A. I. & Mubarki, A.M. 2012.** Characterizations of mappings via Z-open sets, *Studies in Mathematical Sciences*. **5**(2): 55-65.
- EL-Magharabi, A. I. & AL-Juhani, M. A. 2013.** New types of functions by M-open sets, *Journal of Taibah University for Science*. **7**: 137-145.
- Levine, N. 1963.** Semi-open sets and semicontinuity in topological spaces, *The American Mathematical Monthly*, **70**: 36-41.
- Mashhour, A. S., Abd EL-Monsef, M. E. & EL-Deeb, S. N. 1982.** On precontinuous and weak precontinuous mappings, *Society of Mathematics and Physics Egyptian*, **53**: 47-53.
- Mashhour, A. S., Hassanien, I. A. & EL-Deeb, S. N. 1983.** α -continuous and α -open mappings, *Acta Mathematica Hungarica*. **41**: 213-218.
- Mubarki, A. M. 2012.** Z^* -open sets and Z^* -continuity in topological spaces, *International Journal of Mathematical Archive*. **3**(1): 118-125.

- Njastad, O. 1965.** On some classes of nearly open sets, *Pacific Journal of Mathematics*. **15**: 961- 970.
- Raychaudhuri, S. & Mukherjee, N. 1993.** On δ -almost continuity and δ -preopen sets, *Bull. Institute of Mathematics Academia Sinica*. **21**: 357 - 366.
- Tripathy, B.C. & Debnath, S. 2013.** g-open sets and g-continuous mappings in fuzzy bitopological spaces, *Journal of Intelligent and Fuzzy Systems*. **24**(3): 631-635.
- Tripathy, B. C. & Ray, G. C. 2012.** On Mixed fuzzy topological spaces and count ability, *Soft Computing*. **16**(10): 1691-1695.
- Tripathy, B. C. & Ray, G. C. 2013.** Mixed fuzzy ideal topological spaces; *Applied Mathematics and Computations*. **220**: 602-607.
- Tripathy, B. C. & Ray, G. C. 2014.** On d-continuity in mixed fuzzy topological spaces, *Boletim da Sociedade Paranaense de Matematica*, **32**(2): 175-187.
- Tripathy, B. C. & Sarma, D. J. 2011.** On b-locally open sets in bitopological spaces; *Kyungpook Mathematical Journal*. **51**(4): 429-433.
- Tripathy, B. C. & Sarma, D. J. 2012.** On pairwise b-locally open and pairwise b-locally closed functions in bitopological spaces; *Tamkang Journal of Mathematics*. **43**(4): 533-539.
- Tripathy, B. C. & Sarma, D. J. 2013.** On weakly b-continuous functions in Bitopological spaces, *Acta Scientiarum Technology*. **35**(3): 521-525.
- Velicko, N. V. 1968.** H-closed topological spaces. *American Mathematical Society Translations*.**78**: 103-118.

Submitted : 10/07/2013

Revised : 16/02/2014

Accepted : 03/03/2014

حول الدوال المغلقة والدوال المفتوحة من النوع Z^*

* محمد الجهاني و ** علي مباركي

قسم الرياضيات - كلية العلوم - جامعة طيبة - المدينة المنورة - المملكة العربية السعودية

خلاصة

كان مباركي قد أدخل لأول مرة في العام 2012 مفهوم المجموعة المفتوحة من النوع Z^* وكذلك مفهوم علاقة Z^* . نقوم في هذا البحث بتقديم المفاهيم التالية: الدوال المغلقة من النوع Z^* ، الدوال المفتوحة من النوع Z^* ودوال أخرى قريبة منها. نقوم بعد ذلك بدراسة خصائص هذه الأنواع من الدوال.