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The first integral method for the perturbed Wadati-Segur-Ablowitz equation with time dependent coefficient

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Abstract

The first integral method (FIM) is used to construct traveling wave solutions of perturbed Wadati-Segur-Ablowitz (pWSA) equation with time dependent coefficient in this manuscript. We obtained some different solutions by using Gauge transformation with time dependent coefficient of variable transformations. The method is an effective method to construct the different types of exact solutions of nonlinear partial differential equations (NPDE).

Keywords: Analytical solutions; FIM; pWSA; traveling wave solutions.

1. Introduction

Mathematical modeling of scientific events generally is expressed by nonlinear evolution equations. It is crucial to reach general solutions of NPDE in applied mathematics. The solutions of these equations provide much information about character and structure of the equations for researchers. Therefore, studies for solutions of NPDE, using Maple and Mathematica, are increasing day by day. Hence, these equations are mathematical modeling of scientific events that have been occurred in Mechanics and Physics... etc. Many efficient methods have been improved to provide much information for physicians and engineers. Most of these methods, which are tanh-function method studied by Parkes & Duffy (1996) and Sakthivel *et al*. (2010), G′/G expansion method studied by Wang *et al*. (2008) and Kim & Sakthivel (2012), Jacobi elliptic function method studied by Fu *et al*. (2001), Inverse scattering method studied by Drazin & Jhonson (1989), Hirota bilinear method studied by Hu & Ma (2002), exp-function method studied by Wu & He (2007) and Sakthivel & Chun (2010), Kudryashov method studied by Kim *et al.* (2014), sub-equation method studied by Ghany & Hydar (2014), q-differential transform method studied by Gang & Chanclani (2013) and first integral method studied by Feng (2002), use $\zeta = x \mp ct$ wave variable transformation to reduce the NPDE to ordinary differential equations (ODE) to acquire the solution. All of these methods are effective methods for acquiring traveling wave solutions NPDE.

FIM initially has been presented to the literature by solving Burgers-KdV equation by Feng (2002). This method has been successfully implemented to NPDE and some fractional differential equations. The first integral method has the advantageous over existing methods as mentioned above, that is, to obtain various solutions to obtain a uniform solution with Tanh, G′/G expansion, Jacobi elliptic function, Exp-function methods etc. So in this respect the FIM is an effective one than the others. In recent years, many studies on this method have been made. Raslan (2008) has used this method for the Fisher equation. Tascan & Bekir (2009) have used this method for Cahn-Allen equation. Abbasbandy & Shirzadi (2010) have investigated Benjamin Bona-Mohany equation by this method. Jafari *et al*. (2013) has researched for Biswas–Milovic equation. For this paper firstly, we present the method and secondly the FIM has been applied to the pWSA equation, which is introduced by Wadati *et al.* (1992) and studied by Liu & Yang (2004), then we give some conclusions in the last section.

2. The first integral method (fim)

The principal structures of the FIM are as:

Step 1. Take into account a usual NPDE in:

$$W(q, q_t, q_x, q_{xx}, q_{tt}, q_{xt}, \dots) = 0 \tag{1}$$

then the Equation (1) transforms the ODE as

$$H(Q, Q', Q'', Q''' \dots) = 0 \tag{2}$$

such that $\xi = x \mp ct$ and $Q' = \frac{\partial Q(\xi)}{\partial \xi}$.

Step 2. It could be taken of ODE (2) as:

$$q(x,t) = q(\xi) \tag{3}$$

Step 3. A new independent variable is presented by

$$Q(\xi) = q(\xi), \;\; G(\xi) = \frac{\partial q(\xi)}{\partial \xi} \tag{4}$$

that presents a new system of ODEs

$$\begin{cases} \dfrac{\partial Q(\xi)}{\partial \xi} = G(\xi) \\ \dfrac{\partial F(\xi)}{\partial \xi} = P\big(Q(\xi), G(\xi)\big) \end{cases} \tag{5}$$

Step 4. In accordance with the qualitative theory of ODE is given by Ding & Li (1996), if it is possible to reach the integrals for system (5), it could be obtained the solutions of system (5) immediately. On account of a certain independent plane system, there is no any approach that can guide how to reach its first integrals. The Division Theorem was given by Bourbaki (1972) for an idea that how to reach the first integrals.

3. Applications

In here, we illustrate the FIM for pWSA equation, which was introduced by Wadati and co-workers to study certain instabilities of the modulated wave trains, with time dependent coefficient as:

$$iq_x+q_{tt}+2\rho(t)|q^2|q-\lambda(t)q_{xt}=0. \tag{6}$$

Firstly the Equation (6) turns to the following ODE by using $q(x,t)=Q(\xi)e^{i(x-c(t))}$ and the wave variable $\xi=kx+wt$ then by separating the imaginary and real parts

$$k-w\lambda(t)-2wc'(t)+k\lambda(t)c'(t)=0 \tag{7}$$

$$w\big(w-k\lambda(t)\big)Q_{\xi\xi}-\begin{pmatrix}1+\lambda(t)c'(t)+c'(t)^2\\+ic''(t)\end{pmatrix}Q+2\rho(t)Q^3=0. \tag{8}$$

where $c'(t)=\frac{dc(t)}{dt}$. From the Equation (7) it is obtained that

$$c(t)=C+\int_1^t\frac{k-w\lambda(s)}{2w-k\lambda(s)}\mathrm{d}s \tag{9}$$

where C is arbitrary constant and then by replacing (9) into Equation (8) we have

$$Q_{\xi\xi}-\left(\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(t)+i(k^2-2w^2)\lambda'(\mathrm{t})}{(-2w+k\lambda(t))^2(w^2-kw\lambda(t))}\right)Q+\frac{2\rho(t)}{(w^2-kw\lambda(t))}Q^3=0 \tag{10}$$

Then, using (3) and (4) we have

$$\begin{cases}\dot{Q}(\xi)=G(\xi)\\ \dot{G}(\xi)=\left(\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(t)+i(k^2-2w^2)\lambda'(\mathrm{t})}{(-2w+k\lambda(t))^2(w^2-kw\lambda(t))}\right)Q(\xi)-\frac{2\rho(t)}{(w^2-kw\lambda(t))}Q^3(\xi)\end{cases} \tag{11}$$

In accordance with the FIM, it is supposed that $Q(\xi)$ and $G(\xi)$ are non-trivial solutions of system (11) and $F(Q,G)=\sum_{i=0}^{m}a_i(Q)G^i$ is an irreducible function in the complex domain $C[Q,G]$ such that

$$F\big(Q(\xi),G(\xi)\big)=\sum_{i=0}^{r}a_i(Q)G^i=0 \tag{12}$$

where $a_i(Q),(i=0,1,2,\dots,r)$ are polynomials of Q and $a_r(Q)\neq 0$. Equation (12) is the first integral for system (11), owing to the DT, there exists $g(Q)+h(Q)G$ in $C[Q,G]$ as:

$$\frac{dF}{d\xi}=\frac{dF}{dQ}\frac{dQ}{d\xi}+\frac{dF}{dG}\frac{dG}{d\xi}$$

$$=[g(Q)+h(Q)G]\sum_{i=0}^{r}a_i(Q)G^i \tag{13}$$

In this study, we consider $r=1$ and $r=2$ cases in Equation (13).

Case 1.1 If we equate the coefficients of $G^i(i=0,1,2,\ldots,r)$ on both sides of Equation (12) for $r=1$, we have

$$\dot{a}_1(X)=h(Q)a_1(Q) \tag{14}$$

$$\dot{a}_0(X)=a_1(Q)g(Q)+h(Q)a_0(Q) \tag{15}$$

$$a_0(Q)g(Q)=a_1(Q)(a\lambda Q(\xi)-(b+c)Q^3(\xi)+n) \tag{16}$$

Since $a_1(Q)(i=0,1)$ is polynomial of Q, $a_1(Q)$ is a constant and $h(Q)=0$ from (14). For convenience, it is obtained $a_1(Q)=1$, and equalization the degrees of $g(Q)$ and $a_0(Q)$ we conclude the degree of $g(Q)$ is equal to zero. Then, we assume that $g(Q)=A_0Q+A_1$, and we obtain from Equation (15) as follows

$$a_0(Q)=\frac{A_0}{2}Q^2+A_1Q+A_2 \tag{17}$$

Replacing $a_0(Q), a_1(Q)$ and $g(Q)$ in Equation (16) then equating the coefficients of Q^i to zero, we have:

Case 1:

$$A_0=\frac{\pm 2\sqrt{\rho(t)}}{\sqrt{-w^2+kw\lambda(t)}}, A_1=0, A_2=\frac{i(-ik^2-4iw^2+4ikw\lambda(t)+iw^2\lambda^2(t)-ikw\lambda^3(t)+k^2\lambda'(t)-2w^2\lambda'(t))}{Aw(w-k\lambda(t))(2w-k\lambda(t))^2} \tag{18}$$

setting (18) in (11), we have differential equations as follows

$$G(\xi)=\mp\frac{\sqrt{\rho(t)}}{\sqrt{-w^2+kw\lambda(t)}}Q^2(\xi)-\frac{i(-ik^2-4iw^2+4ikw\lambda(t)+iw^2\lambda^2(t)-ikw\lambda^3(t)+k^2\lambda'(t)-2w^2\lambda'(t))}{Aw(w-k\lambda(t))(2w-k\lambda(t))^2} \tag{19}$$

If we solve the Equation (19) by using (3) and (4) respectively, we have the solutions of Equation (6)

$$q(x,t)=-\frac{1}{\sqrt{2}}\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(t)+kw\lambda^3(t)+i(k^2-2w^2)\lambda'(t)}{\rho(t)\left(-2w+k\lambda(t)\right)^2}}$$

$$\tan\left[\frac{\mp\sqrt{2}}{2}(kx+wt)\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(t)+kw\lambda^3(t)+i(k^2-2w^2)\lambda'(t)}{w(w-k\lambda(t))(-2w+k\lambda(t))^2}}\right]e^{i(x-c(t))} \quad (20)$$

where C is arbitrary constant.

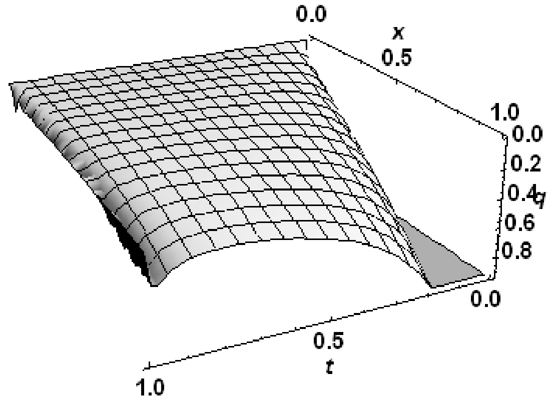

Fig. 1. The graphic of the solution (20) for $k = 1, w = 1, \lambda(t) = \rho(t) = t.$

Case 2:

$$\rho(t) = \frac{i(i-\lambda'(t)+2\lambda^2(t)\lambda'(t))}{2\lambda^2(t)}$$

$$k = \frac{\sqrt{i+(-1+2\lambda^2(t))\lambda'(t)}(-i\lambda^4(t)+\lambda^2(t)(4i-6\lambda'[t])+4(-i+\lambda'(t)))}{\lambda^3(t)\sqrt{\lambda'(t)[4+\lambda^4(t)+\lambda^2(t)(-4-4i\lambda'(t))+4i\lambda'(t)]}},$$

$$w = -\frac{2\sqrt{\lambda'(t)[i-\lambda'(t)+2\lambda^2(t)\lambda'(t)]}}{\sqrt{4-4\lambda^2(t)+\lambda^4(t)+4i\lambda'(t)-4i\lambda^2(t)\lambda'(t)]}}$$

setting the coefficients $\rho(t), k$ and w in (11), we have Riccati differential equations as follows

$$G(\xi) = \frac{1}{2}Q^2(\xi) - \frac{1}{2} \quad (21)$$

If we solve the Equation (21) by using (3) and (4) respectively, we have analytical solutions of Equation (6)

$$q(x,t) = (tanh[kx+wt] \pm i\text{sech}[kx+wt])e^{i(x-c(t))} \quad (22)$$

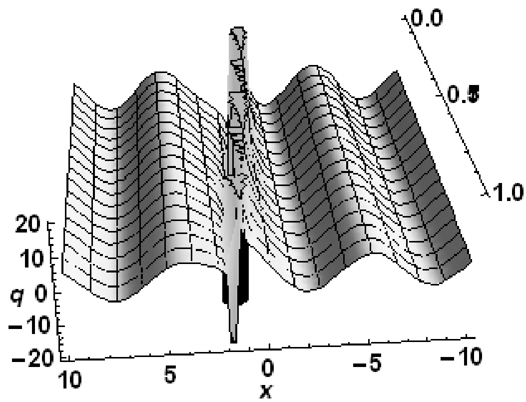

Fig. 2. The graphic of the solution (3.16) for $k=1, w=-\sqrt{3}, \lambda(t)=it$.

or

$$q(x,t)=(coth[kx+wt]\pm icsch[kx+wt])e^{i(x-c(t))} \tag{23}$$

where $i^2=-1$.

Case 1.2 If we equate the coefficients of $Y^i(i=0,1,2,\ldots,m)$ on both sides of Equation (12) for $m=2$, we have

$$\dot{a}_2(Q)=h(Q)a_2(Q) \tag{24}$$

$$\dot{a}_1(Q)=a_2(Q)g(Q)+h(Q)a_1(Q) \tag{25}$$

$$a_1(Q)g(Q)+h(Q)a_0(Q)=\dot{a}_0(Q)+2a_2(Q)(a\lambda Q(\xi)+(b+c)Q^3(\xi)+n) \tag{26}$$

$$a_1(Q)\dot{G}=a_0(Q)g(Q) \tag{27}$$

Since $a_2(Q)(i=0,1,2)$ is polynomial of Q, we conclude that $a_2(Q)$ is a constant and $h(Q)=0$ from (13). For convenience, it is obtained $a_2(Q)=1$, and equalization the degrees of $g(Q)$,$a_1(Q)$, $a_2(Q)$ we conclude the degree of $g(Q)$ is equal to one. Then, we suppose that $g(Q)=A_0Q+A_1$, and we obtain from Equation (25) as follows

$$a_1(Q)=\frac{A_0}{2}Q^2+A_1Q+A_2 \tag{28}$$

$$a_0(Q)=(\frac{A^2}{8}+\frac{\rho(t)}{w(w-k\lambda(t))})Q^4+\frac{1}{2}ABQ^3+$$

$$\left(\frac{1}{2}(B^2+AC)+\frac{-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\text{t})-kw\lambda^3(\text{t})-ik^2\lambda'(\text{t})+2iw^2\lambda'(\text{t})}{w(w-k\lambda(t))(2w-k\lambda(t))^2}\right)Q^2+BCQ-d \quad (29)$$

where d is integration constant from Equation (26).

Replacing $a_0(Q), a_1(Q)$ and $g(Q)$ in Equation (13) then equating the coefficients of Q^i to zero, we have:

$$A_0=0\,,\ \ A_1=0,\ \ A_2=0, \quad (30)$$

setting (28) and (29) in (11), we have differential equations as follows

$$\left(G(\xi)\right)^2=d-\left(\frac{-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\text{t})-kw\lambda^3(\text{t})-ik^2\lambda'(\text{t})+2iw^2\lambda'(\text{t})}{w(w-k\lambda(t))(2w-k\lambda(t))^2}\right)Q^2(\xi)-\frac{\rho(t)Q^4(\xi)}{w(w-k\lambda(t))} \quad (31)$$

If we solve the Equation (31) by using (3) and (4) respectively, we have analytical solutions of Equation (6). Equation (31) admits the following rational solution,

$$q(x,t)=\frac{1}{\sqrt{\frac{\rho(t)}{w(w-k\lambda(t))}}(kx+wt)}e^{i(x-c(t))} \quad (32)$$

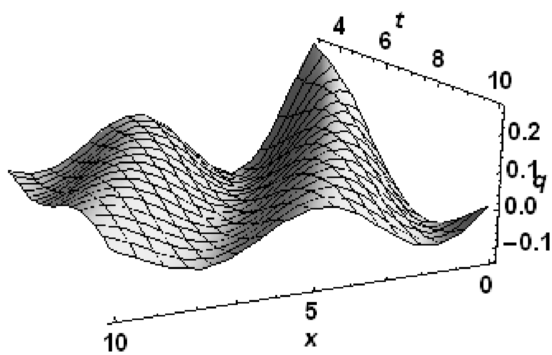

Fig. 3. The graphic of the solution (32) for $k=1, w=1, \lambda(t)=\rho(t)=t$.

where $d=0$ and $-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\text{t})-kw\lambda^3(\text{t})+\lambda'(\text{t})(2iw^2-ik^2)=0$.

Equation (31) admits the following triangular solutions

$$q(x,t)=\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\text{t})+kw\lambda^3(\text{t})+i(k^2-2w^2)\lambda'(t)}{\rho(t)\left(-2w+k\lambda(t)\right)^2}}$$

$$\sec\left[\sqrt{-\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda(t)^2+kw\lambda(t)^3+i(k^2-2w^2)\lambda'(t)}{(-2w+k\lambda(t))^2w(w-k\lambda(t))}}(kx+wt)\right]e^{i(x-c(t))} \quad (33)$$

where $d=0$.

$$q(x,t)=\sqrt{\frac{-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\mathrm{t})-kw\lambda^3(\mathrm{t})-ik^2\lambda'(t)+2iw^2\lambda'(t)}{\rho(t)(8w^2-8kw\lambda(t)+2k^2\lambda(t)^2)}}$$

$$\tan[\sqrt{-\frac{-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\mathrm{t})-kw\lambda^3(\mathrm{t})-i(k^2-2w^2)\lambda'(t)}{2w(w-k\lambda(t))(-2w+k\lambda(t))^2}}(kx+wt)]e^{i(x-c(t))} \quad (34)$$

where $d=0$.

Equation (31) admits the following hyperbolic solutions

$$q(x,t)=i\sqrt{\frac{-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\mathrm{t})-kw\lambda^3(\mathrm{t})-ik^2\lambda'(t)+2iw^2\lambda'(t)}{\rho(t)(8w^2-8kw\lambda(t)+2k^2\lambda(t)^2)}}$$

$$\tanh[-\sqrt{\frac{-k^2-4w^2+4kw\lambda(t)+w^2\lambda^2(\mathrm{t})-kw\lambda^3(\mathrm{t})-i(k^2-2w^2)\lambda'(t)}{2w(w-k\lambda(t))(-2w+k\lambda(t))^2}}(kx+wt)]e^{i(x-c(t))} \quad (35)$$

where $d=0$.

$$q(x,t)=\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(\mathrm{t})+i(k^2-2w^2)\lambda'(t)}{\rho(t)\left(-2w+k\lambda(t)\right)^2}}$$

$$\mathrm{sech}[\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(\mathrm{t})+i(k^2-2w^2)\lambda'(t)}{(-2w+k\lambda(t))^2w(w-k\lambda(t))}}(kx+wt)]e^{i(x-c(t))} \quad (36)$$

where $d=0$.

Equation (31) admits the following Jacobi elliptic function solutions as:

$$q(x,t)=\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(\mathrm{t})+i(k^2-2w^2)\lambda'(t)}{\rho(t)(-1+2m)(-2w+k\lambda(t))^2}}$$

$$cn[\sqrt{\frac{k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(\mathrm{t})+i(k^2-2w^2)\lambda'(t)}{w(w-k\lambda(t))(-1+2m)(-2w+k\lambda(t))^2}},m](kx+wt)]e^{i(x-c(t))} \quad (37)$$

$$q(x,t)=\sqrt{\frac{m\left(k^2+4w^2-4kw\lambda(t)-w^2\lambda^2(\mathrm{t})+kw\lambda^3(\mathrm{t})+ik^2\lambda'(t)-2iw^2\lambda'(t)\right)}{\rho(t)(4w^2+4mw^2-4kw\lambda(t)-4kmw\lambda(t)+k^2\lambda(t)^2+k^2m\lambda(t)^2)}}$$

$$sn[\sqrt{\frac{(k^2+4w^2-\lambda(t)(4kw-w^2\lambda(t)+kw\lambda^2(\mathrm{t}))+\lambda'(t)(ik^2-2iw^2))}{\rho(t)\left(-w^2+kw\lambda(t)\right)(1+\mathrm{m})(-2w+k\lambda(t))^2}},m](kx+wt)]e^{i(x-c(t))} \quad (38)$$

where m is the mod. of *cn* and *sn* functions and $d=0$.

4. Conclusions

We used the FIM for finding some new exact solutions for the pWSA equation with the time dependent coefficient. We have obtained different types exact solutions of this equation. The types of acquired solutions are denoted in terms of trigonometric, triangular, algebraic, cnoidal, snoidal wave solutions and exponential functions. Some of our reached solutions are new, as our research from literature. Consequently, the FIM is so crucial one to construct different types of the exact solutions of the NPDE and studied by many other authors He *et al.* (2013); Yuan-Quan & Jun (2005); Bekir & Ünsal (2012); Taghizadeh & Najand (2012); Jafari *et al.* (2012) etc. Some graphics of the solutions are illustrated in figures 1-3. For the next studies we are going to seek the dynamics of solitons with parabolic and dual-power law nonlinearities with the FIM.

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طريقة التكامل الأول لمعادلة واداتي – سيغور – أبولتز الراجفة ذات المعامل المعتمد على الزمن

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خلاصة

نقوم في هذا البحث باستخدام طريقة التكامل الاول لإيجاد حلول لمعادلة واداتي – سيغور – أبولتز الراجفة ذات المعامل المعتمد على الزمن و تكون هذه الحلول على شكل حلول موجات جوالة. كما نحصل على حلول مختلفة باستخدام تحويل غاوج ذو المعامل المعتمد على الزمن من تحويلات المتغيرات. و تعتبر طريقتنا طريقة فعالة لإنشاء أنواع مختلفة من الحلول المضبوطة للمعادلات التفاضلية الجزئية غير الخطية.