Kuwait J. Sci. **42 (2)** pp. 160-188, 2015

The political districting of Kuwait: Heuristic approaches

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ABSTRACT

This paper models the political districting of Kuwait as a multiple objective combinatorial optimization problem, where each political system is assessed in terms of population and voting equity, geographical contiguity, and social, religious, ethnic, family size, and educational homogeneity. First, it proposes four constructive heuristics and a specialized simulated annealing to generate alternative non-dominated districting plans that may "guarantee" a national consensus. Second, it searches for a districting plan that optimizes a set of criteria (classified as hard and soft constraints) using a tree-search based heuristic. The heuristic takes advantage and combines the orthogonal but complementary strengths of constraint and integer programming. Finally, it compares the proposed solution to both the existing and the previously applied patterns. Thus, this paper offer politicians a multiple criteria evaluation method that they may apply to choose the most "appropriate" political districting pattern.

Keywords: Combinatorial optimization; constraint propagation; heuristics; integer programming; multiple objective.

Mathematics Subject Classification (MSC): 90C10.

INTRODUCTION

The districting problem consists of partitioning a region, country or province into a set of different zones or districts subject to some constraints (Tavares *et al*., 2007). The districts, which are mutually exclusive and whose union is the territory being partitioned, are “homogeneous”, where homogeneity is assessed using multiple consistent criteria (Tavares *et al*., 2007). Generally, any feasible solution satisfies a set of technical, ethical, ecological, social, and political constraints (Tavares *et al*., 2007; Bozkaya *et al*., 2003). Such constraints include contiguity, population equality, adjacency, and socio-economic homogeneity. Contiguity or closeness implies that the voting population of a district shares a common continuous space. Population equality or equity insures a one-man one vote principle. Adjacency ensures that districts have more or less the same area and that access to voting centers is equally likely in all districts (Johnston, 2002; Ricca & Simone, 1997). Finally socio-economic homogeneity ensures a better representation of residents, who share common concerns or views, be them socially or economically driven (Bozkaya *et al*., 2003).

The reinforcement of these criteria usually prevents any gerrymandering and ensures "democratic" elections (Lewyn, 1993). In addition to these principles, and depending on the election context and historical background of the country, where the election is being conducted, other common sense criteria might apply. These include the respect of natural boundaries, the respect of existing administrative or political subdivisions to facilitate the registration and census of voters; similarity to the existing plan so that an incumbent runs again in a similar district, especially if incumbents are to approve the political districting for the upcoming election; the respect of integrity of communities to avoid splitting some communities between several districts; and equal probability of representation of minority groups.

Evidently, these criteria are context-dependent (Bozkaya *et al*., 2003; Ricca & Simone, 1997; Ricca *et al*., 2008). For instance, socio-economic heterogeneity may sometimes be desirable, whereas in other instances homogeneity is rather more important. Similarity to the existing districting plan can be viewed as protecting incumbents or bipartisan gerrymandering (Bourjolly *et al*., 1981; Kaiser, 1966; Nagel, 1965; Thoreson *et al*., 1961); while in other instances it may be perceived as voicing the population's opinion regarding the future political districting plan (Bozkaya *et al*., 2003; Chou & Li, 2006, 2007). Equal representation of ethnic groups may be viewed as racial or ethnic gerrymandering, if diluting the strength of political ethnic groups is sought, but may be considered desirable if voicing the opinion of minorities is fundamental (Bozkaya *et al*., 2003; Chou & Li, 2006, 2007). Formally, the political districting problem consists in partitioning a territory, which is a set **I** of mutually exclusive small units, into districts. It assigns each small unit $i \in I$ into one and only one district $j \in J$ as to optimize a number of multiple criteria, subject to a predefined set of constraints. It is equivalent to identifying the efficient (non-dominated) solution set X, where a solution $x \in X$ is efficient, if there is no $x\ 0 \in X$ such that $f\ (x0) \leq f(x)$ and $f\ (x0) \neq f\ (x)$, where f is the multiple criteria objective function to be minimized. This problem is NP-hard (Altman, 1997). Thus, the identification of an efficient solution x for the political districting problem using exact approaches is not possible for real life problems given their size, their large number of constraints, and their multiple objectives. It is therefore judicious to opt for approximate approaches.

Political Districting has attracted the attention of many researchers. The solution approaches can be classified as exact and approximate. For example Hojati (1996) and Johnston (2002) solve the single criterion districting problem exactly using a decomposition and a column generation algorithm, respectively. Examples of approximate approaches are numerous (Bourjolly *et al*., 1981; Bozkaya *et al*., 2003; Browdy, 1990; Chou & Li, 2007; Fleischmann & Paraschis, 1998; Nygreen, 1988; Ricca & Simone, 1997; Ricca *et al*., 2008, 2013). They are generally based on meta-heuristics such as tabu search, simulated annealing and evolutionary algorithms. King *et al*. (2014) developed geo-graph contiguity algorithms for geographic zoning and dynamic plane graph partitioning that examine these vertices more quickly than traditional search-based methods. Other approaches are based on mathematical modeling (Bozkaya *et al*., 2003; Garfinkel & Nemhauser, 1970; George *et al*., 1997; Hess *et al*., 1965; Hojati, 1996; Lewyn, 1993; Ricca & Simone, 2008, Thoreson *et al*., 1961), set-partitioning (Altman, 1997); graph-partitioning (Browdy, 1990; El-Farzi & Mitra, 1992; Nemoto & Hotta, 2003; Yamada, 2009), implicit enumeration (Garfinkel *et al*., 1970), network flow techniques (Pukelsheim *et al*., 2012), physical modeling (Chou & Li, 2006, 2007), computer-geometry approach (Kalcsics et. al., 2005; Ricca *et al*., 2013), and heuristics (Deckro, 1979; Forman & Yule, 2003; Ricca *et al*., 2008; Zoltners, 1979; Tavares *et al*., 2007). These methods deal with both the single (Garfinkel *et al*., 1970) and the multiple criteria problem (George *et al*., 1997; Hess *et al*., 1997; Hojati, 1996; Altman & McDonald, 2011).

This study follows the general trend of the literature and approximately solves the political districting problem of Kuwait using a simulated annealing and a tree search approach. The simulated annealing (SA) identifies approximate Pareto solutions. It initiates its search from good quality solutions obtained via four constructive heuristics. The tree search starts at the root node with a relaxed version of the problem; i.e., with the contiguity constraint dropped. If the relaxed problem has a feasible solution, the heuristic checks whether this solution is also feasible to the original problem. If that is the case, the heuristic stops; otherwise, the heuristic branches out of the node by adding more constraints to the relaxed problem. If, on the other hand, the relaxed problem has no feasible solution, the heuristic backtracks to the previous node and removes some of the added constraints. The heuristic pursues appending and removing constraints till a feasible solution for the relaxed problem satisfies the original problem.

BACKGROUND

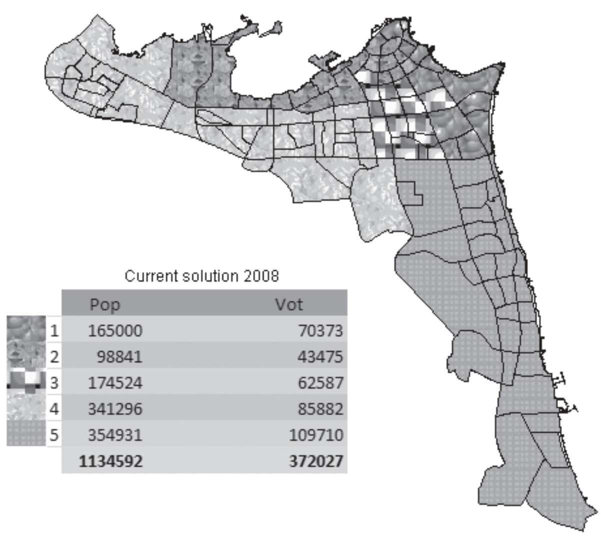
Kuwait has a very particular background and a unique social structure. It remains strongly influenced by its historical political system, which allowed for consultations among the representatives of the tribes, and strived to reach a consensus between the influential tribes, economic drivers, and social strata. Despite the absence of political parties, the population and its parliament representatives do share some socio-economic views or ethnic/religious characteristics that allow the classification of the voting population into social and economic groups.

Kuwait is a rich country with a relatively small population counting 3.4 million people. The number of Kuwaitis, more than a million, represents less than one third of the population. Kuwaiti males account for 49% of the Kuwaiti population. Until 2006, women along with other types of Kuwaiti citizens were not allowed to vote. This made the number of voters and their relative and absolute percentages very reduced.

Allowing women to vote increased the number of eligible voters from 139151 to 384790. The population growth rate in Kuwait (3.29%) is unfortunately not uniform across the country or among the social and economic strata.

The initial reform proposed by the Kuwaiti Parliament was a reduction in the number of electoral districts from twenty-five to ten. With twenty-five districts, each district had very few constituents, which increased the potential for vote buying and corruption. In particular, with few voters in each constituency, tribal leaders used family connections to position themselves to win. Then an important political movements developed in Kuwait and the districts were in fact voted down to five, where the areas where tribal tickets dominate the voting (4th and 5th districts).

Kuwait is composed of 85 units spread over six administrative governorates. According to the current political districting system, Kuwait is divided into 5 districts, as described in Figure 1. Each district contributes ten parliament deputies, with a total of 50 deputies. Each voter gets to choose one candidate among the runners in the district. The winners within a district are the candidates obtaining the largest number of votes. The distribution of eligible voters is not uniform across all districts as can be inferred from Figure 1 which displays the five districts, their population denoted as Pop and their corresponding number of voters denoted as Vot. For instance, a male voter in district 5 represents 52380 people (resp., a voter in district 5 represents 109710 people), whereas a voter in district 2 represents 20380 people (resp., a voter in district 2 represents 43475 people).



	Pop	Vot
1	165000	70373
2	98841	43475
3	174524	62587
4	341296	85882
5	354931	109710
	1134592	**372027**

Fig. 1. The 2008 five-district map of Kuwait.

MODEL

The political districting problem (PD) is a multiple criteria optimization problem. A common approach is to treat some criteria as hard constraints and others as soft requirements or as terms in an objective function (Bozkaya *et al*., 2003; Eiselt & Laporte, 1987; Mehrotra *et al*., 1998). The proposed model treats contiguity, assignment of a unit to a unique district, and socio-economic homogeneity as hard constraints, whereas it views voting and population equity as soft ones. It ignores adjacency as the country is very small and traveling distances are short. Let x_{ij}= 1 if unit $i \in I = \{1, . . . , 85\}$ is assigned to district $j \in J = \{1, . . . , 5\}$. A feasible political districting of Kuwait $x = (x_{ij})_{i \in I, j \in J}$ satisfies the following three sets of hard constraints.

The first set reflects the contiguity constraint, which requires that all the units of a district be adjacent to each other. Translating this requirement into a mathematical model is difficult, and necessitates the enumeration of a large number of special cases. Therefore, this constraint is herein relaxed: the model includes two special cases only. If a unit i is assigned to district j, then at least one unit i' from the set N_i of neighbors of i must be assigned to district j; i.e.,

$$x_{ij} - \sum x \quad \leq 0, i \in I, j \in J \tag{1}$$

In addition, if two units $i \in I$ and $i' \in N_i$ are assigned to the same district j, then at least one unit i^2 must be assigned to j where i^2 is different from i and i', but belongs to the union of the neighborhoods of i and i'. Differently stated, if $x_{ij} = x_{i'j} = 1$, and $i' \in N_i$, then at least a unit $i^2 \in N_i \cup Ni' \setminus \{i, i'\}$ must be assigned to j. This is equivalent

$$\sum_{i'' \in N_i \cup N_{i'} \setminus \{i,i'\}} x_{i''j} \geq x_{ij} + x_{i'j} - 1, \; i \in I, \; i' \in N_i, \; j \in J \tag{2}$$

Evidently, (1) and (2) do not guarantee contiguity, but eliminate a large number of non-contiguous solutions; thus, speed the convergence of the search toward a feasible solution. The second set requires that each unit i be assigned to exactly one district, i.e.

$$\sum_{j \in J} x_{ij} = 1, i \in 1, \tag{3}$$

The third set imposes that a political districting feasible solution provides socio-economic homogeneity (religious-shiite and sunite, ethnic-bedouin and town dweller, educational, economic class and family size)

- $s_i = 1$, i ∈ I , if the majority of the v_i voters of unit i is sunite (more than 50%), and 0 otherwise;

- $b_i = 1$, i ∈ I , if the majority of the v_i voters of unit i is bedouin, and 0 otherwise;
- $e_i^l = 1$, i ∈ I , if the majority of the v_i voters of unit i has education level l, and 0 otherwise, with = 1 for a high school education or less l= 2 for some college or technical school education, and l =3 for a university degree; and
- $c_i^l = 1$, i ∈ I , if the majority of the v_i voters of unit i are from the economic class l, and 0 otherwise, with l = 1 for a low economic class, l = 2 for middle class, l = 3 for upper class, and l = 4 for wealthy; and
- $z_i^l = 1$, i ∈ I , if the majority of the v_i voters of unit i are from the average family size l, and 0 otherwise, with l = 1 for average family size 4 and below, l = 2 for average family size 5, l = 3 for average family size 6, and l = 4 for average family size 7 and above.

Then, for any feasible solution x, the number of sunites to access the parliament is bounded by s and $\bar{s}$, where 0 ≤ s ≤ $\bar{s}$ ≤ 50.That is,

$$s \leq \sum_{j \in J} \left[10 \frac{\sum_{i \in I} s_i v_i x_{ij}}{\sum_{i \in I} v_i x_{ij}} \right] \leq \bar{s}. \tag{4}$$

Similarly, the number of Bedouins to access parliament is bounded:

$$\underline{b} \leq \sum_{j \in J} \left[10 \frac{\sum_{i \in I} b_i v_i x_{ij}}{\sum_{i \in I} v_i x_{ij}} \right] \leq \bar{b}; \tag{5}$$

where 0 ≤ b ≤ $\bar{b}$ ≤ 50. Similarly, the number of to be deputies representing the education level l, are bounded respectively as follows:

$$\underline{e}^l \leq \sum_{j \in J} \left[10 \frac{\sum_{i \in I} e_i^l v_i x_{ij}}{\sum_{i \in I} v_i x_{ij}} \right] \leq \bar{e}^l, \quad l = 1, 2, 3; \tag{6}$$

where $0 \leq e^l \leq \bar{e}^l \leq 50$. The number of to be deputies representing the economic class l, are bounded respectively as follows:

$$\underline{c}^l \leq \sum_{j \in J} \left[10 \frac{\sum_{i \in I} c_i^l v_i x_{ij}}{\sum_{i \in I} v_i x_{ij}} \right] \leq \bar{c}^l, \quad l = 1, 2, 3, 4; \tag{7}$$

where $0 \leq c^l \leq \bar{c}^l \leq 50$. Finally, the number of to be deputies representing the average family size l, are bounded respectively as follows:

$$\underline{z}^l \leq \sum_{j\in J}\left[10\frac{\sum_{i\in I} z_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right] \leq \bar{z}^l, \quad l = 1, 2, 3, 4 \tag{8}$$

where, $0 \leq z^l \leq \bar{z}^l \leq 50$. The quality of a feasible solution x is measured in terms of voting equity *fυ (x)* and population equity *fp (x)*. These two objectives can be aggregated into a single function *f(x)* consisting in the weighted sum of *f υ (x)* and *fp* (x):

$$f_\upsilon(\mathrm{x}) = \alpha_\upsilon f_\upsilon(\mathrm{x}) + (1-\alpha_\upsilon) f_p(\mathrm{x}) \ , \tag{9}$$

where $\alpha\upsilon$ and $1 - \alpha\upsilon$ reflect the relative importance of voting and population equity, respectively. Voting equity can be modeled as

$$f_\upsilon(\mathrm{x}) = \frac{\sum_{j\in J}\max\left\{V_j(x) - (1+\beta)\bar{V},\ (1-\beta)\bar{V} - V_j(x),\ 0\right\}}{\bar{V}} \tag{10}$$

where $0 \leq \beta \leq 1$ is a tolerance index defining the tolerance range $[(1-\beta)\bar{V}, (1+\beta)\bar{V}]$ of V_j $j \in J$, the eligible voting population within a district *j*, with $V_j(x) = \sum_{i\in I} v_i x_{ij}$. In this context, $\bar{V}$, the average voting population per district is the ratio of the total number of eligible voters to the number of political districts: $\bar{V} = \frac{\sum_{j\in J} v_j(x)}{|J|}$. Population equity can be measured using different approaches; however, given the population pyramid, and the distribution of the population of Kuwait, it is better to model it using the same measure as voting equity; that is,

$$f_p(\mathrm{x}) = \frac{\sum_{j\in J}\max\left\{P_j(x) - (1+\beta)\bar{P},\ (1-\beta)\bar{P} - P_j(x),\ 0\right\}}{\bar{P}} \tag{11}$$

where $\bar{P} = \frac{\sum_{j\in J} P_j(x)}{|J|}$ is the average size of the Kuwaiti population per district, and

$P_j(x) = \sum_{i\in I} p_i x_j\ , \quad j \in J$ is the size of the Kuwaiti population of district *j*, $j \in J$.

Solving this problem exactly is difficult. Indeed, the first set of constraints or contiguity constraints are too complex to model analytically. Including them as disjunctive constraints in an integer program generally yields infeasible solutions. In addition, the socio-economic homogeneity constraints are not linear. It is therefore approximately solved using constructive approaches, simulated annealing, and a constraint propagation tree search heuristic.

CONSTRUCTIVE HEURISTICS

Different approaches can be used to generate initial feasible solutions. Herein, four heuristics, H1-H4, are considered. H1 is a quasi-random constructive approach that adopts a myopic selection criterion to assign units to districts. H2 is a random approach that packs units into districts as long as the (knapsack) capacity of the district is not exceeded. H3 solves an integer programming problem, which is a relaxation of the districting problem at hand, and restores the feasibility of its optimal solution using a greedy approach. H4 modifies existing political districting systems to obtain feasible five-district systems.

H_1: Quasi random solutions

H1 starts by setting all $x_{ij}= 0$, $i \in I$, $j \in J$. It initializes the set of non-assigned units $I'=I$. It creates sets Sj , $j \in J$, and initializes them to the empty set. It then chooses five units from I that are unlikely to belong to a common set. For instance, it chooses five units that are geographically most distant or densest (in terms of population and/or voters) or that belong to five different governorates, etc. It includes each unit in a distinct S_j, and removes it from I'. Next, it iterates through all districts adding one unit to each of them. Any appended unit satisfies the contiguity constraint (since it is the neighbor of one of the units already included in the district) and the assignment to a single district (since it is removed from I' once it is assigned to a district). Formally, H_1 proceeds as detailed in Figure 2.

Objective	Generate an initial districting plan.
Input	I: The set of units to be assigned to districts.
	$\|J\|$: The number of political districts.
Output	Sj: The set of units assigned to political district $j \in J$
Initialization	
	Set $Sj = 0$.
	Set $I' = I$, where I' is the set of non-assigned units to districts.
	Choose, from I' , $\|J\|$ units ‘ any pair of these units is unlikely to belong to a same district.
	Insert each of these units in a distinct S_j , remove it from I' , and set the corresponding $x_{ij} = 1$.
Iterative step	
	Repeat for $j \in J$
	Choose $k \in N_i$, $i \in S_j$, $\exists$ $f(x)$ is minimum. Insert it in S_j, and remove it from I'.
	Until $I' = 0$ // all units are packed.
Final step	
	Compute f(x)

Fig. 2. Detailed algorithm of H1

The initial solution generated by H1 does not necessarily satisfy Equations (4) – (8). When that is the case, we restore feasibility using a greedy procedure which moves a unit *i* from its current district to a neighboring district. This move is only considered, if it does not violate the contiguity constraint for both districts. It is adopted, if it decreases the degree of violation of the homogeneity constraints; that is, if it decreases $\sum_{k=1}^{n_k} g_k(x)$ where nk=13;

$$g_1(x) = \max\left\{0, \underline{s} - \sum_{j\in J}\left[10\frac{\sum_{i\in I} s_i v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right], \sum_{j\in J}\left[10\frac{\sum_{i\in I} s_i v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right] - \overline{s}\right\}$$

$$g_2(x) = \max\left\{0, \underline{b} - \sum_{j\in J}\left[10\frac{\sum_{i\in I} b_i v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right], \sum_{j\in J}\left[10\frac{\sum_{i\in I} b_i v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right] - \overline{b}\right\}$$

$$g_{2+l}(x) = \max\left\{0, \underline{e}^l - \sum_{j\in J}\left[10\frac{\sum_{i\in I} e_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right], \sum_{j\in J}\left[10\frac{\sum_{i\in I} e_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right] - \overline{e}^l\right\}, l = 1, 2, 3,$$

$$g_{5+l}(x) = \max\left\{0, \underline{c}^l - \sum_{j\in J}\left[10\frac{\sum_{i\in I} c_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right], \sum_{j\in J}\left[10\frac{\sum_{i\in I} c_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right] - \overline{c}^l\right\}, l = 1, 2, 3, 4$$

$$g_{9+l}(x) = \max\left\{0, \underline{z}^l - \sum_{j\in J}\left[10\frac{\sum_{i\in I} z_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right], \sum_{j\in J}\left[10\frac{\sum_{i\in I} z_i^l v_i x_{ij}}{\sum_{i\in I} v_i x_{ij}}\right] - \overline{z}^l\right\}, l = 1, 2, 3, 4$$

A formal statement of the greedy procedure is detailed in Figure 3.

Initialization Set x*=x.

Iterative Step

Repeat for $j \in J$

Repeat for $j' \in J$ and $j' \neq j$

For each unit $i \in S_j$, and $i \in N_i$, ‘ $i \in S_j'$,

set $x_{ij} = 0$, $xij' = 1$; and compute $g_k(x)$, $k = 1, \ldots, n_k$.

If $\sum_{k=1}^{n_k} g_k(x) \leq \sum_{k=1}^{n_k} g_k(x^*)$ then set $x^* = x$; else set $x = x^*$

If $\sum_{k=1}^{n_k} g_k(x^*) = 0$, then exit.

Until x* is feasible.

Final step

Compute $f(x^*)$.

Fig. 3. A greedy heuristic that restores the feasibility of socio-economic homogeneity

H2: Random initial solutions

H2, detailed in Figure 4, initializes the set of non-assigned units $I' = I$, and orders it randomly. It initializes S_j, $j \in J$, to the empty set, and $P_j = V_j = 0$. Then chooses, from I', five units whose centers are as far (in terms of their Euclidian distance) as possible from each other and whose densities (in terms of population and voters) are largest. Next, it iterates through the units of I' trying to pack each unit in a district, while maintaining the contiguity constraint and ensuring that P_j , $j \in J$, does not exceed a threshold level $\tilde{P}$ and V_j , $j \in J$, does not exceed a threshold level $\tilde{V}$, where $\tilde{P} = (1 + \beta)\,\overline{P}$, and $\tilde{V} = (1 + \beta)\overline{V}$, with $\beta > 0$. A unit assigned to a district j is included in S_j and removed from I' whereas non-assigned unit (because it is not a neighbor of any of the currently assigned units) is kept into I' for further consideration during the next iterations. If it does not satisfy the socio-economic homogeneity constraints given by Equations (4)-(8), the obtained solution is subjected to the greedy approach of Figure 3.

Objective	Generate an initial districting plan.
Input	I: The set of units to be assigned to districts.
	$\|J\|$: The number of political districts.
Output	S_j : The set of units assigned to political district $j \in J$

Initialization

1. Set $S_j = 0$.
2. Set $I' = I$, where I' is the set of non-assigned units to districts.
3. Order the units of I' randomly.
4. Choose, from I', $|J|$ units i_j, $j = 1, \ldots, |J|$, whose centers are as far as possible from each other and whose densities in terms of population and voters are largest.
5. Include unit i_j, $j = 1, \ldots, |J|$, in district S_j, set $P_j = p_{ij}$, $V_j = v_{ij}$, and remove i_j from I'.

Iterative Step

Repeat for $i \in I'$

1. Set $j = 1$.
2. If $i \in N_i'$ where $i' \in S_j$, and $P_j + p_{ij} \leq \tilde{P}$, and $V_j + v_{ij} \leq \tilde{V}$, assign unit i to district j, update the population and the number of voters of district j : $P_j = P_j + p_{ij}$, and $V_j = V_j + v_{ij}$.

Else set j = j + 1.

3. If $j \in J$, goto step 2 of the Iterative step.

Until $I' = 0$ // all units are packed.

Final step

Compute $f(x)$.

Fig. 4. Detailed algorithm of H2

Two modifications could yield sizeable improvements of H2's solutions. The first concerns the criterion of choice of the initial five units. This criterion could be used to force some neighboring units with high densities to be part of different districts. The second concerns the annexation of units to districts. Despite its diversification via the random order of *I',* this assignment yields, in many instances, solutions where two or more highly dense units belong to the same district. To overcome this glitch, H2 either "shrinks" or extends the neighborhood matrix. It shrinks the matrix by making some of the highly dense units non-neighboring (despite their adjacent physical locations), and restoring it after some iterations. On the other hand, it extends the neighborhood matrix by adding to a unit's neighbor those corner adjacent ones. This matrix shrinkage/extension diversifies the population of initial solutions produced by H2 but at the cost of obtaining non-compact districts, as illustrated by the solution of Figure 5.

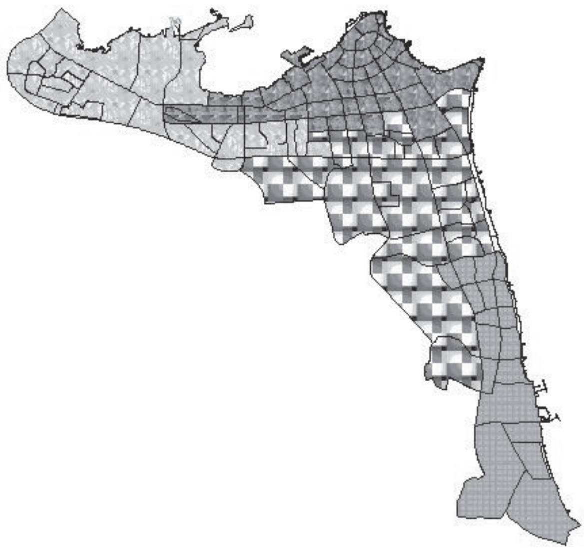


Fig. 5. Example of a solution obtained by allowing for corner districts

H3: An integer programming solution

H3 considers a relaxation of the problem, where the objective is to minimize the weighted sum of absolute deviations of the population size per district from the average population size and of the absolute deviations of the number of voters per district from the average number of voters; i.e. it assumes that $\beta = 0$. This objective is subject to the constraints given by Equations (1)-(3) and the integrality constraints of the binary variables x_{ij}. The relaxation computes the deviation of V_j, $j \in J$, from $\overline{V}$ as:

$$\sum_{i\in I} \nu_i x_{ij} - \overline{V} = \overline{\in}_j^{\upsilon} - \underline{\in}_j^{\upsilon} \tag{12}$$

and the deviation of P_j, $j \in J$ from $\overline{P}$ as:

$$\sum_{i\in I} p_i x_{ij} - \overline{P} = \overline{\in}_j^{p} - \underline{\in}_j^{p} \quad , \tag{13}$$

with $\overline{\in}_j, \overline{\in}_j^{\upsilon}, \underline{\in}_j^{\upsilon}$, $\overline{\in}_j^{p}$ and $\overline{\in}_j^{p}$, $j \in J$, being four positive decision variables.

The objective function consists therefore in minimizing

$$\alpha_{\upsilon} \sum_{i\in I} (\overline{\in}_j^{\upsilon} + \underline{\in}_j^{\upsilon}) + (1 - \alpha_{\upsilon}) \sum_{i\in I} (\overline{\in}_j^{p} + \underline{\in}_j^{p}) \tag{14}$$

If the optimal solution obtained by solving the resulting mixed integer programming model via Cplex does not satisfy the contiguity constraint, then the procedure of Figure 6 is applied.

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1. Let I'' denote the set of units causing the violation of the contiguity constraint.
2. For $i \in I''$, remove unit i from its assigned district j.
3. For j $\in$ J, recalculate the number of voters V_j and population size P_j .
4. For $i \in I''$,

- Determine the set J_i of districts to which unit i could be assigned without violating the contiguity constraint.
- Assign i to the district $j \in J_i$ such that the resulting solution x minimizes the objective function defined in Equation (8).
- Update V_j* and P_j* and remove i from I''.

Fig. 6. Restoring contiguity feasibility of a solution.

Let I^2 denote the set of units causing the violation of the contiguity constraint. Re- move each unit $i \in I^2$ from its assigned district j. Recalculate for each district $j\in J$ its number of voters V_j and population size P_j . Subsequently, determine for every $i \in I^2$ the set J_i of districts to which unit i could be assigned without violating the contiguity constraint. For each unit $i \in I^2$, assign i to the district $j^*\in J_i$ such that the resulting solution x minimizes the objective function value defined in Equation (9). If x does not satisfy the socio-economic homogeneity constraints, then the procedure of Figure 3 is applied.

H4: Modified existing political systems

H4 modifies one of the many proposed political districting systems. It restores contiguity feasibility, and combines/alters districts as to optimize the objective function. Specifically, H4 proceeds as described in Figure 7. If its solution does not satisfy the homogeneity constraints, the greedy procedure of Figure 2 is applied.

The districting plans herein considered are: a 25-district system which was applied in Kuwait from 1980 to 2006, a ten-district system that was proposed by the government, a six-district system, where each district corresponds to a governorate, and three five-district systems that were proposed by different deputies.

Initialization

- Set Sj = q, where Sj is the set of units to be included in political district $j \in J$.
- Set $S_j^c = \{i \in I \ni x_{ij'}^c = 1)\ j' = 1, \ldots, n^c$, where S_j^c is the set of units currently assigned to district j', and nc is the number of political district in the current solution.
- Set $J' = \{1, \ldots, nc\}$.
- Choose five of the current district such that they are unlikely to belong to the same district and are as distant as possible.
- Include each of these current districts in a distinct S_j, $j \in J$, and remove them from J'.

Iterative Step

Repeat for $j' \in J'$

For $j \in J$

If district j' is neighbor of district j, include S_j^c into S_j, remove j' from J' and go to next j'.

Until $J' = 0$ // all current districts are into the new districts.

For $j \in J$, compute P_j and V_j,

For $i \in I$,

Let $j \in J$ be the district to which i is assigned.

If $P_j > \widetilde{P}$ and $V_j > \widetilde{V}$, then

let $J_i = \{j' \in J \setminus \{j\} \ni \exists\, i' \in S_j$, and $i' \in N_i\}$;

for $j' \in J_i$,

If $P_{j'} > \widetilde{P}$ and $V_{j'} > \widetilde{V}$, then

move i from j to j';

update P_j, V_j, $P_{j'}$, and $V_{j'}$; and

exit for loop ; i.e. consider next $i \in I$;

Until $P_j \le \widetilde{P}$ and $V_j \le \widetilde{V}$

Final Step

Compute $f(x)$

Fig. 7. Detailed algorithm of H4

A simulated annealing heuristic

A successful heuristic addresses the two competing goals of powerful search techniques: diversification and intensification. The former is an extensive search of the solution space; it determines the part that has a higher chance of containing the global optimum. The latter refines the search and focuses on the part that has a high potential of containing the global optimum. Herein, the proposed SA heuristic achieves diversification by initiating the search from various initial solutions and intensification by undertaking a steepest descent in the neighborhood of each initial solution and of the incumbent of each plateau.

The temperature of SA is initially set to $T_0 = -1.1\tau_0$, where τ_0 is the positive difference between the cost of the first neighbor having a cost greater than that of the current solution and the cost of the current solution. It guarantees a 0.4 initial probability of acceptance. The temperature is adjusted every $L = 20$ iterations. It is decreased geometrically; that is, at plateau k, $T_k = 0.9Tk-1$. For a fixed T_k, small uphill moves have higher probabilities of acceptance than larger ones. However, as the temperature declines, the probability that an uphill move of size Δ is accepted diminishes since the acceptance of an uphill move is controlled by the probability $\exp(-\Delta / T_k)$. The algorithm is stopped, if the incumbent solution has not improved for 3 consecutive plateaus. A neighboring solution is obtained using one of two types of swaps. The first swap considers two units i_1 and i_2 belonging to the limits of two adjacent districts j_1 and j_2, $j_1 \in J, j_2 \in J$, and appoints i_1 to j_2 and i_2 to j_1 if this assignment does not cause the violation of the contiguity constraints. The second swap considers a pair (j_1, j_2), $j_1 \in J, j_2 \in J$, of neighboring political districts. If the population (resp. number of voters) of j_1 exceeds that of j_2 by a threshold level $\delta_p = \beta \ \bar{P}$ (resp. $\delta v = \beta \bar{V}$), the swap chooses a unit $i_1 \in j_1$ but neighboring j_2, and moves it to j_2. SA applies the first swap to generate neighbors of the current solution, and the second swap to improve the initial solution and the best current solution at each plateau. A detailed description of the proposed SA heuristic is provided in Figure 8.

1. Choose an initial solution x_0 and evaluate its fitness $f(x_0)$.
2. Apply the second swap to x_0 until no improvement is registered for three consecutive swaps.
   Let the resulting solution be x and its corresponding fitness $f(x)$.
3. Set $x^* = x$, and $f(x^*) = f(x)$.
4. Fix the initial temperature T_0 and the plateau size L.
5. Set $k = 0$.
6. Repeat L times.
   - Obtain, using the first swap, a neighboring solution x' and compute its fitness $f(x')$.
   - If $\Delta = f(x^*) - f(x) \leq 0$ then

     set $x = x'$ and $f(x) = f(x')$;

     if $f(x') < f(x^*)$, then $x^* = x'$ and $f(x^*) = f(x')$;
- else if $\exp(-\Delta / T_k)$ uniform[0, 1], then x = x' and $f(x) = f(x')$.
7. Apply the second swap to x^* until no improvement is registered for three consecutive swaps.

Let the resulting solution be x and its corresponding fitness f (x).

8. Increment k by 1, and compute T_k, the temperature at plateau k.
9. If the stopping criterion is not satisfied, go to Step 6.

Fig. 8. Detailed algorithm of the proposed SA heuristic

A constraint propagation based tree search

An approximate approach based on constraint propagation (CP) seems a more viable approach for this problem. Indeed, CP is known for its efficiency in generating feasible solutions (despite its weakness in identifying an optimal solution). CP offers a flexible modeling framework. It exploits the model structure to direct and accelerate the search. The proposed heuristic relies on the strength of CP techniques to identify a feasible solution and on the strength of integer programming to identify a near optimal one.

The tree search solves a relaxed districting problem, where some of the hard constraints are dropped. To restore feasibility, it adds iteratively constraints to RPD. It may also resort to removing some of the added constraints, if they yield infeasible solutions. It stops, when it generates a feasible solution to the original problem PD. When adding/removing constraints, the proposed approach mimics a best-first search tree, where the root node corresponds to the relaxed version, branching to adding a constraint, and removing a node to backtracking.

The heuristic proceeds as follows: it constructs a tree, whose root node corresponds to the problem that minimizes *f(x)* subject to (1) and (3). If *x* satisfies the contiguity constraints, then x offers the best districting that respects the contiguity constraints, where "best" infers the assignment that guarantees voting and population equity. It initiates branching from the current node, and creating new nodes at the next level of the tree. Each branch adds to the model a set of constraints that impose the inclusion (resp. exclusion) of a subset of areas within (resp. from) the same district. These additional constraints target those areas that are isolated from their districts. It forces them either to belong to one of their neighboring districts or to become connected to their assigned district; though, none of the constraints specifically forces the assignment of an area to a specific neighborhood.

At each node, the heuristic solves the resulting model. If it obtains an infeasible solution, the heuristic fathoms the node, backtracks (as if it were removing the last set of added constraints), and branches out with a different set of constraints. It proceeds as in a best first branch and bound. Any node that yields an objective function value worse than that obtained by the current districting plan is fathomed.

The heuristic keeps adding constraints and solving the resulting model. If obtains an infeasible solution, it removes the last added constraint, and replaces it with a different constraint.

Computational results

The objective of the computational investigation is to assess the performance of the constructive heuristics, simulated annealing and constraint propagation, where the solution quality is measured in terms of the satisfaction of voting equity, population equity and homogeneity of the parliament.

Data

The data relative to this study is available in the Annual Statistical Census of Kuwait (2009) and in the website http://gis1.baladia.gov.kw/. Data pertinent to this study, available from the PDF file https://www.dropbox.com/s/o4elwduq731ga7u/district09.pdf?dl=0, includes each unit's name, population size, number of male voters, number of female voters, economic level, educational level, religious orientation, ethnic background and family size. Figure 9 illustrates the socio-economic distributions of the areas of Kuwait.

Fig. 9. Distribution of the religious, ethnic-bedouin, education, economic and family size

All heuristics are coded in Java and run on an Intel Pentium IV, 3.20 GHz and 3.24 Gb of RAM. The mixed integer program is solved using Cplex which is evoked via GAMS 20.5, which is in turn called from the Java code.

Comparison of constructive heuristics H1-H4

Table 1 and Figure 10 present the best solutions obtained by H1-H4. They indicate, for each constructive heuristic, the number of voters Vj and population size P_j per district j, $j \in J$. In addition, Table 1 reports $\frac{V_j}{P_J}$, the weight of a vote in district j. Finally, it assesses in its last two rows the quality of each solution. The row before last displays, the standard deviation for each solution, the values of the objective functions relative to the population and voters' equity when $\beta = 0.1$. The last row indicates $f(x)$, the value of the aggregate objective function when α_v=0.5.

Table 1. Best solutions of H1-H4 with β= 0.1 and αv = 0.5

	H1			***H2***			***H3***			***H4***		
j	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$
4	63712	143914	0.44	61209	171911	0.36	65210	162603	0.40	68608	182713	0.38
2	71202	175152	0.41	79403	160173	0.50	75104	187374	0.40	75305	171276	0.44
1	74844	242119	0.31	72644	250109	0.29	74835	233202	0.32	74742	240005	0.31
3	79434	315193	0.25	79629	321193	0.25	78023	324199	0.24	72437	302291	0.24
5	82835	258214	0.32	79142	231206	0.34	78855	227214	0.35	80935	238307	0.34
Stdev	**7436**	**68140**		**7936**	**65047**		**5434**	**61651**		**4501**	**52517**	
f(x)		**0.8735**			**0.8771**			**0.7992**			**0.8182**	

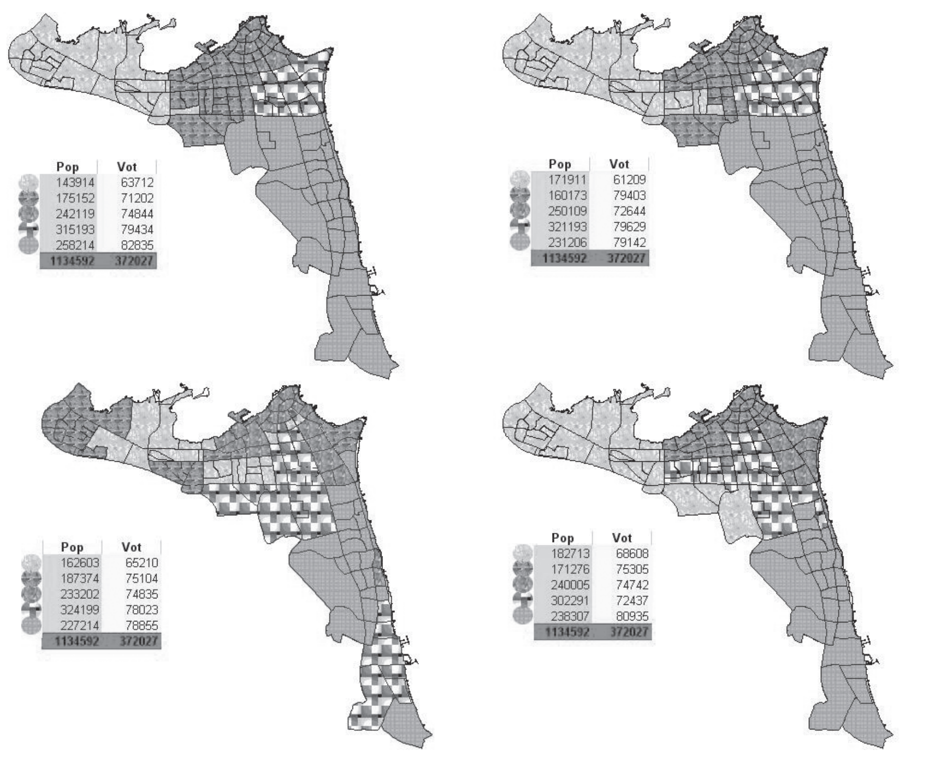


Fig. 10. Solutions generated by H1, H2 1st row and H3, H4 2nd row (β=0.1 & α_v=0.5)

The analysis of Table 1 shows that H3 outperforms the other three heuristics yielding *0.7992* value for *f(x)*. In addition, it yields the smallest deviation of the number of voters per district from the average number of voters and the smallest deviation of the population size per district from the average population size. Most importantly, it provides the smallest deviation of the worth of a voter's vote with all the weights being close to the 0.33 ideal weights where $0.3 \;\; = \sum V_j / \sum P_j = 303765/919033$. This is expected since H3 starts its search from a super-optimal solution and restores feasibility of the contiguity and socio-economic homogeneity constraints; thus, H3 is most likely to yield a better solution than either H1 or H2 which undertake a quasi random and random search, respectively. Similarly, H4 which reconstructs an existing political system yields a better solution than either H1 or H2. In fact, most existing systems are the fruit of careful studies and require relatively little modifications to make them optimize the aggregate objective function and satisfy the feasibility constraints.

To study the impact of diversification on H1 and H2, we run H1 and H2 using a reduced neighborhood. The resulting solutions, displayed in Table 2, are far better than the best solution obtained by H1 and H2. Using a reduced neighborhood eliminates many suboptimal solutions; thus, reduces the search space to the most promising regions. The solutions produced in this case are in many instances attractive to politicians who prefer that some "key" areas be kept separate.

Table 2. Impact of the reduction and extension of the neighborhood on H1 and H2 with $\beta = 0.1$ and $\alpha_v = 0.5$

Neighborhood

	Extend H1			***Reduce H1***			***Extend H2***			***Reduce H2***		
j	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$
4	70713	173915	0.41	73211	244713	0.30	71211	174913	0.41	74200	245714	0.30
2	75201	178171	0.42	75401	209272	0.36	75401	179172	0.42	74412	218271	0.34
1	74843	252108	0.30	74145	230209	0.32	74145	250109	0.30	74135	220108	0.34
3	73436	303194	0.24	74138	221192	0.34	73138	301192	0.24	74149	221273	0.34
5	77834	227204	0.34	75132	229206	0.33	78132	229206	0.34	75131	229226	0.33
Stdev	**2605**	**53940**		**878**	**13009**		**2586**	**52545**		**421**	**11310**	
f(x)		**0. 7025**			**0.3572**			**0.8472**			**0.4521**	

Assessment of the simulated annealing heuristic

Table 3 and Figure 11 present the best results obtained by SA after 20 runs that are started with the best solution obtained by each of the four heuristics H1 - H4. SA improves the solutions of H1, H2 and H4 with the improvements of standard deviation of voters and population and *f(x)* reaching 100% in many instances, and the values of the objective functions nearing or equaling zero in all instances. In addition, the solution of H3 is already optimal with respect to the objective function defined by Equation (9). However SA proposes a different solution with a zero value for $f_{\upsilon}(x)$, $f_p(x)$, and *f(x)*. In fact, SA prefers this solution to that of H3, given that it yields a smaller standard deviation for the distribution of the voters, of the population, and of a voter's weight among the five districts. Regardless of the quality of the initial solution, SA converges towards a good quality solution with $f_{\upsilon}(x)$, $f_p(x)$, and $f(x)$ equaling approximately zero.

Table 3. Best solutions by SA when started from H1-H4 solutions with $\beta=0.1$ & $\alpha_{\upsilon}=0$.

	H1 's solution			***SA solution***			***H2 's solution***			***SA solution***		
J	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$
4	63712	143914	0.44	70713	173915	0.41	61209	171911	0.36	71211	174913	0.41
2	71202	175152	0.41	75201	178171	0.42	79403	160173	0.50	75401	179172	0.42
1	74844	242119	0.31	74843	252108	0.30	72644	250109	0.29	74145	250109	0.30
3	79434	315193	0.25	73436	303194	0.24	79629	321193	0.25	73138	301192	0.24
5	82835	258214	0.32	77834	227204	0.34	79142	231206	0.34	78132	229206	0.34
Stdev	**7436**	**68140**		**2605**	**53940**		**7936**	**65047**		**2586**	**52545**	
f(x)		**0. 8735**			**0.703**			**0.8771**			**0. 703**	
	H3 's solution			**SA solution**			**H4 's solution**			**SA solution**		
J	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$
4	65210	162603	0.40	70312	172917	0.41	68608	182713	0.38	70610	172816	0.41
2	75104	187374	0.40	75002	177170	0.42	75305	171276	0.44	76304	181172	0.42
1	74835	233202	0.32	74843	253106	0.30	74742	240005	0.31	74741	252106	0.30
3	78023	324199	0.24	74035	304195	0.24	72437	302291	0.24	72439	300192	0.24
5	78855	227214	0.35	77835	227204	0.34	80935	238307	0.34	77933	228306	0.34
Stdev	**5434**	**61651**		**2702**	**54883**		**4501**	**52517**		**2934**	**52500**	
f(x)		**0.936**			**0.936**			**0.8182**			**0.833**	

To study the impact of the threshold level β, we run SA with four β levels: β=0.100, 0.125, 0.150, and 0.175. For each β level, we run SA eighty times: Each twenty runs starting from the solutions of H_{ℓ}, $\ell = 1, \ldots, 4$. Table 4 shows that a large b yields a solution with high values for f because it allows overfilling some of the districts and subsequently under filling some others whereas a small β makes SA more selective by gearing its choices towards a more balanced distribution of voters and population among the districts.

Table 4. Impact of the threshold level b on SA when $\alpha_\upsilon = 0.5$.

	b = 0.100			***b = 0.125***			***b = 0.150***			***b = 0.175***		
j	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$
4	70312	172917	0.41	59312	152915	0.39	54312	142906	0.38	44010	132906	0.33
2	75002	177170	0.42	85002	165173	0.51	75002	165173	0.45	75002	165173	0.45
1	74843	253106	0.30	74843	253106	0.30	74843	223109	0.34	54845	203109	0.27
3	74035	304195	0.24	68035	326192	0.21	78035	356197	0.22	98234	376197	0.26
5	77835	227204	0.34	84835	237206	0.36	89835	247207	0.36	99936	257207	0.39
Stdev	**54883**	**2702**		**11064**	**70597**		**12801**	**83680**		**25132**	**95419**	
f(x)		**0.9361**			**0. 9483**			**0. 9842**			**0. 9941**	

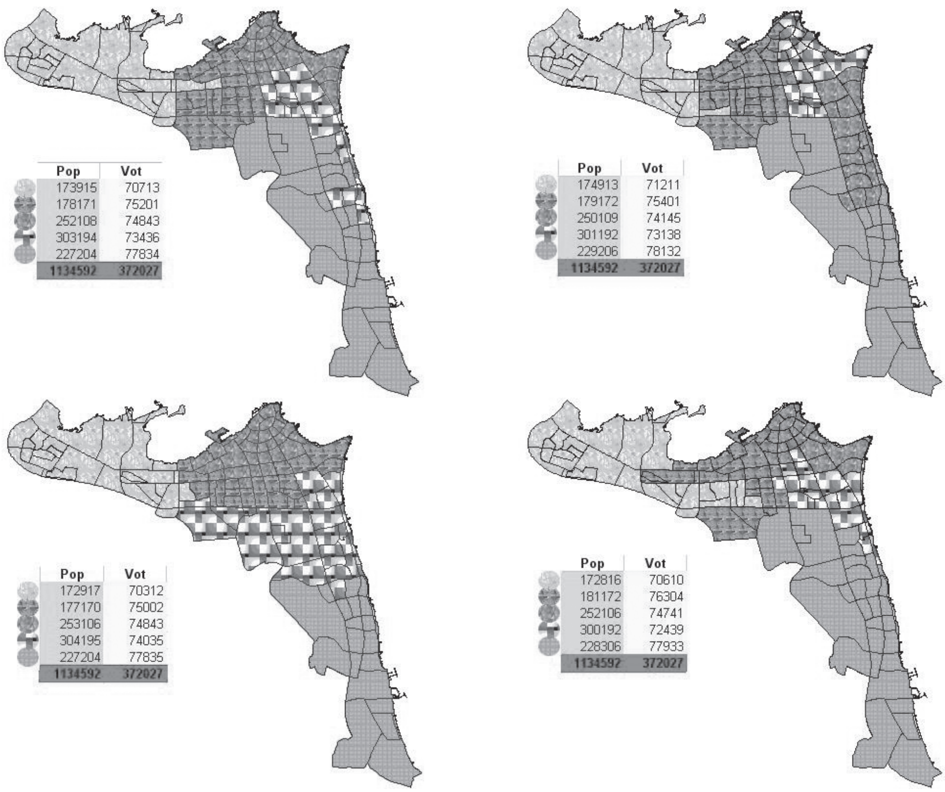


Fig. 11. Solutions generated by SA when initiated with the solutions of H1, H2 & H3, H4.

Figure 12 illustrates the impact of varying the weight factor αv from 0 to 1 on the problem's objective functions f_υ and *fp* when $\beta = 0$. The choice of this level of β is deliberate, as it reflects a worst case behavior. It forces SA to choose the solution having the minimum absolute deviation from the respective average levels. As expected f_v decreases as αv increases from 0 to 1 whereas f_p increases. This behavior may not be consistent as β increases.

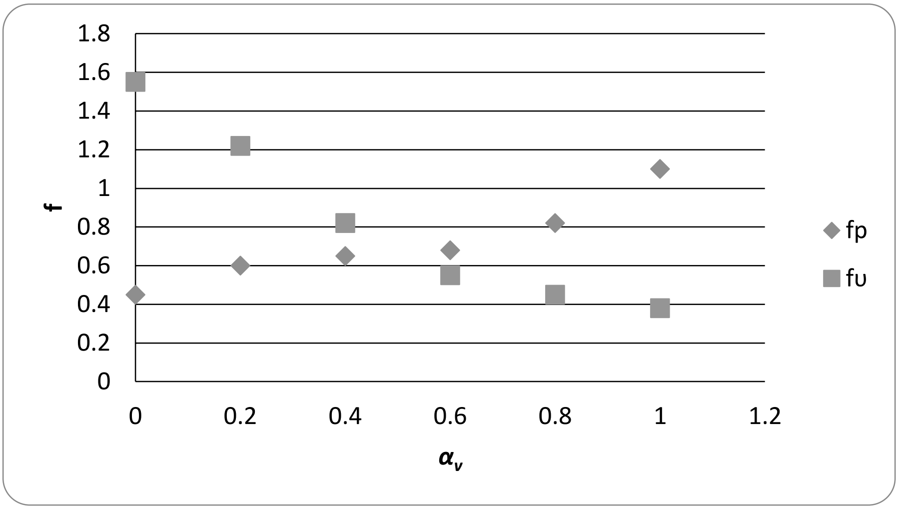


Fig. 12. Impact of the weight factor αv on the best solution obtained by SA when $\beta = 0$

Solution quality of the constraint propagation tree search

Table 5 (column 3) and Figure 13 present the best results obtained by the tree search, then the areas started moving to district 2, which has the much less population and voters size than other 4 districts. 7 areas moved from districts 4 to district 2 and 2 areas moved from 3 to district 2. Then 3 areas moved to district 3, thus 2 of them from district 5 and 1 area from district 1. Finally 2 areas (Al-Doha and Al-Solaibkhat) moved from district 2 to district 4. These 15 movements makes improvement the standard deviation of voters and population. See online animation tree search on the address https://www.dropbox.com/s/fexrz4jrxn076e1/animation1.wmv?dl=0.

Table 5. Best solutions obtained by Relaxed Model with population and voters equity in column 3 and the ratio 2:1 for districts 4 and 5 in column 5.

Dist.	**current solution**			**relaxed model**			**current solution ratio 2:1 district 4 &5**			**relaxed model ratio 2:1 district 4 &5**		
J	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$	V_j	P_j	$\frac{V_j}{P_j}$
1	70373	165000	0.43	76038	183777	0.41	70373	165000	0.43	57182	132780	0.43
2	43475	98841	0.44	74684	178001	0.42	43475	98841	0.44	54741	120783	0.45
3	62587	174524	0.36	72405	231440	0.31	62587	174524	0.36	57712	168169	0.34
4	85882	341296	0.25	68628	293502	0.23	42941	170648	0.25	49741	187281	0.27
5	109710	354931	0.31	80272	247872	0.32	54855	177466	0.31	54855	177465	0.31
Stdev	**24954**	**114512**		**4317**	**47820**		**11959**	**33008**		**3152**	**28976**	

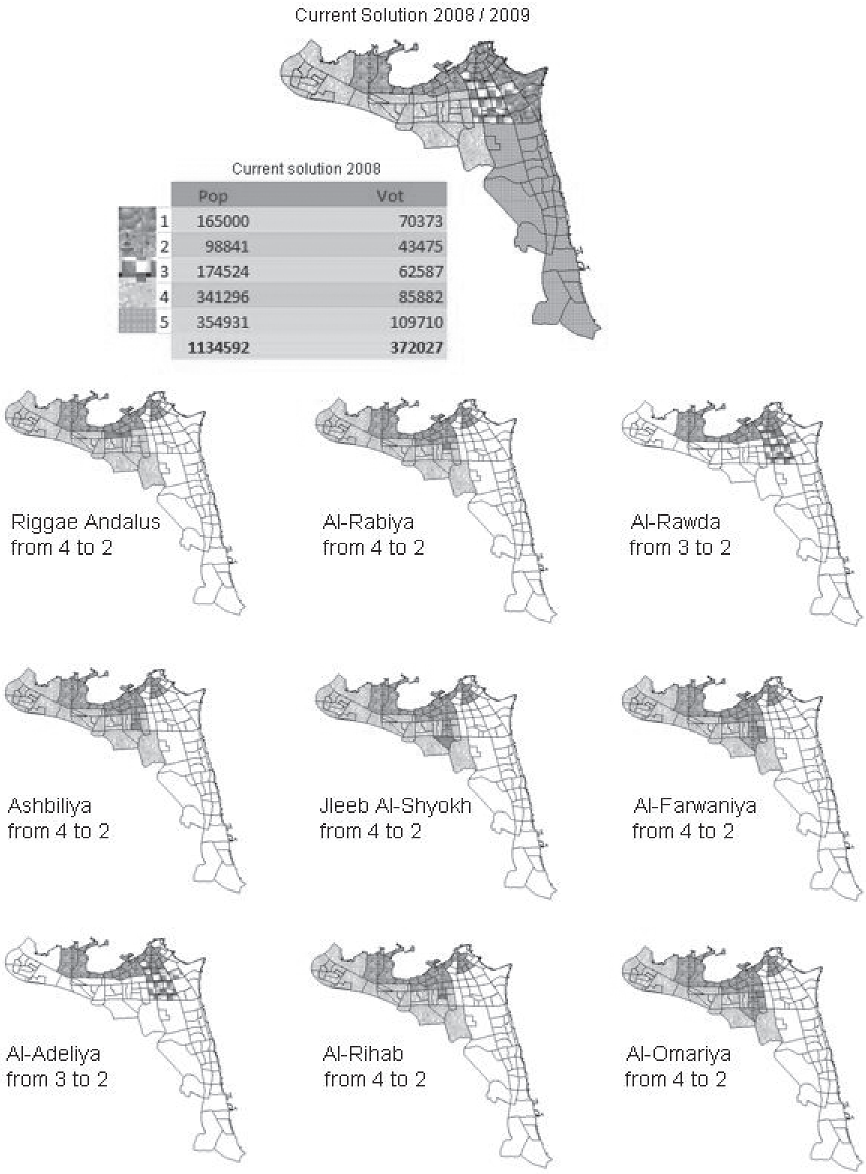
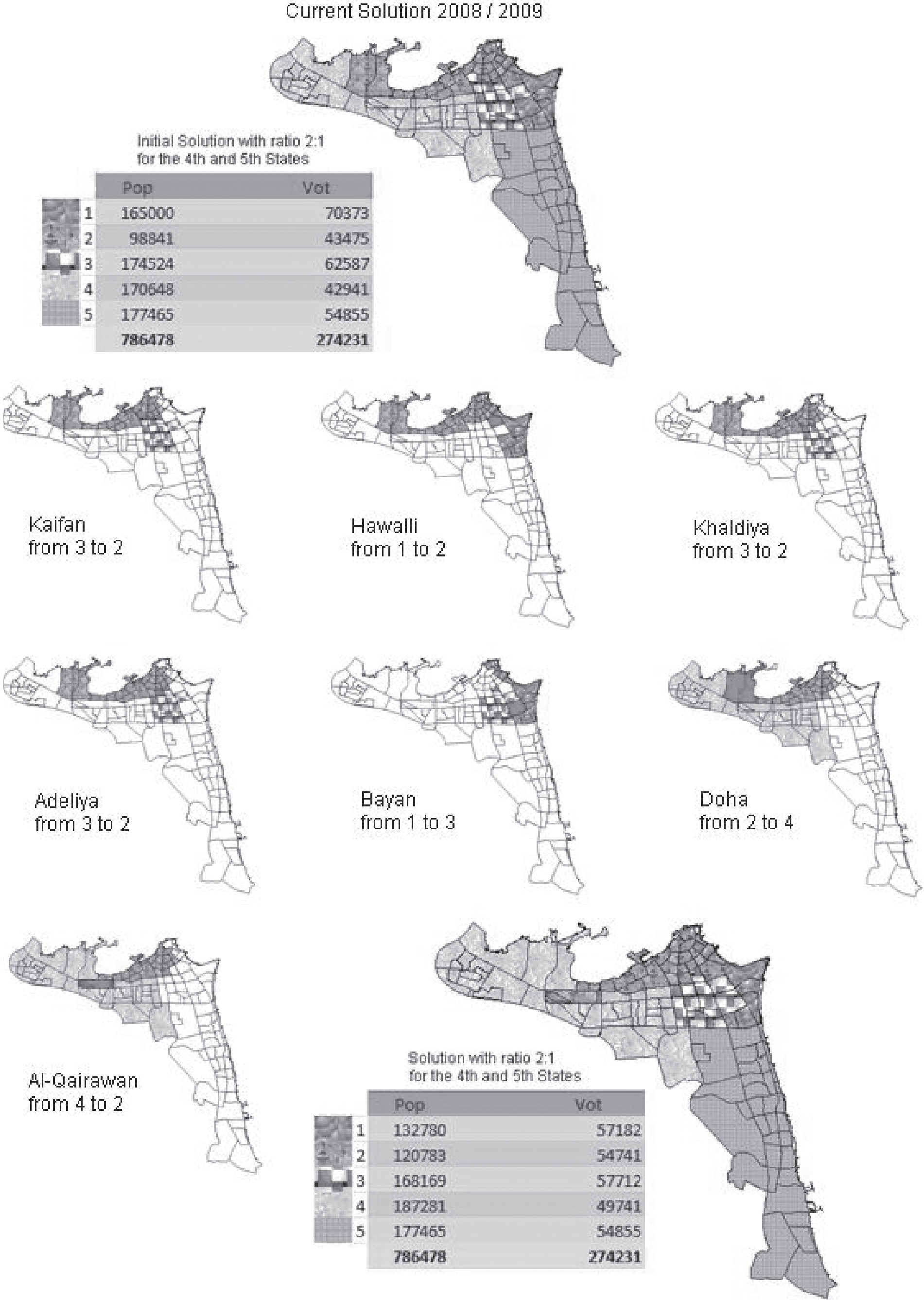


Fig. 13. Proposed relaxed solution using tree search when the objective is population and voters equity.

The weight of a vote is not consistent among the population and among the men and women voters with the current district system applied in Kuwait (See column 2 in Table 5). The weight of population and voters in districts 4 and 5 are much higher than other districts.

The main limitation of this model is non-uniform population growth rate across the country or among the social and economic strata. Districts 4 and 5 in the current political district have much higher growth rates (6.4 and 5.2 respectively) with respect to other districts (between 1.9 and 3.1). The majority of the districts 4 and 5 are Bedouin, who belong to certain tribes. Thus, women and men in these two districts do not vote independently, but tend to vote in blocs and support their candidates from their tribes by exchanging their support for other candidates with votes for their proper candidates. They resort to primary elections to decide how to channel and how to take advantage of their votes. Differently stated, in these two districts, gerrymandering takes many forms other than non-equity of population and voters among districts. Therefore, in these two districts, we proposed a model with objective that population and voters are duplicate, i.e. every 2 voters in districts 4 and 5 equivalent to one voter in districts 1, 2 and 3. Then we applied the same relaxed model applied above and the solution using tree search is given in the Figure 14 and Table 5 (column 5). Considering the ratio of population and voters in the western (4) and southern (5) districts is 2:1 in other districts, we get the best solution after only 7 movements starting the initial solution with current district system applied in Kuwait (dividing the population and voters in districts 4 and 5 by 2), while it took 15 movements with population and voters equity in all the districts. From these 7 movements 5 of them were to district 2 and one to district 3 and one to district 4 which improve the standard deviation of votes and population. From an utopist point of view, such a result is ideal; however, politicians may view it differently. The online animation tree search is available on the address https://www.dropbox.com/s/x5qapsipfcz0iw2/animation2.wmv?dl=0.



Initial Solution with ratio 2:1 for the 4th and 5th States

	Pop	Vot
1	165000	70373
2	98841	43475
3	174524	62587
4	170648	42941
5	177465	54855
	786478	**274231**

Solution with ratio 2:1 for the 4th and 5th States

	Pop	Vot
1	132780	57182
2	120783	54741
3	168169	57712
4	187281	49741
5	177465	54855
	786478	**274231**

Fig. 14. Proposed relaxed solution using tree search when the objective is population and voters of ratio 2:1 in districts 4 and 5.

CONCLUSION

This paper presents the political districting problem of Kuwait. It proposes alternative districting plans for the country. These plans are the result of four constructive heuristics, a simulated annealing based algorithm and a constraint propagation tree search. They seek voters and population equity among districts while respecting a subset of criteria related to social, religious, ethnic, and educational homogeneity, family size and geographical contiguity. The paper further shows that considering the ratio of population and voters in the western (4) and southern (5) districts is 2 to 1 in other districts allowing better presentation of political district and close to the current applied system in Kuwait. Politicians may apply a multiple-criterion evaluation method to these patterns and choose the "appropriate" one.

Even though specific to the political districting problem of Kuwait, the proposed approaches can be easily adapted to others districting problems encountered in many gulf countries and domains such as education, health, water distribution, and marketing.

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Submitted **:** 09/07/2013

Revised **:** 12/11/2014

Accepted **:** 20/01/2015

الدوائر الانتخابيه في الكويت : تحليل الاساليب الاستدلالية

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خلاصة

تتناول هذه الدراسة وضع نموذج الامثل لمشكلة توزيع الدوائر الانتخابيه في الكويت، عن طريق صياغة المشكلة كقضية متعددة الاهداف لايجاد الحل الامثل، حيث ان يتحدد توزيع كل دائرة انتخابيه بمتغيرات متعدده كالكثافة السكانية والمساواة في التصويت والتجاور الجغرافي والعوامل الاجتماعية والعرقية و الدينية وكذلك حجم الاسرة والتجانس التعليمي. نقترح اولا في هذه الورقة بتصميم نظام بديل غير مسيطر للانماط القطعية والذي يمكن ان يضمن اجماع وطني باربعة استدلالات بناءة بالاضافة الي اسلوب المحاكاة القوية التخصصية. ثانيا بحثنا عن نمط قطعي للتمثيل الافضل لمجموعة المعايير تصنف كقيود قوية و ضعيفة، ويتم حلها باسلوب الاستدلالي الشجري والتي تجمع بين برمجة العدد الصحيح وتقنيات النشر المقيدة. يتميز الاسلوب الاستدلالي بالقيد المتعامد والمكمل وبرمجة العدد الصحيح في شرح وحل مشكلة توزيع الدوائر. واخيرا اجرينا مقارنة بين النموذج المقترح والنمط الحالي المطبق بالاضافة الي الانماط المطبقة سابقا. فبامكان السياسيين ومتخذي القرار استخدام معايير التقييم المتعددة لهذه الانماط المقترحة واختيار الطريقة المناسبة منها.