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Applications of soft intersection sets to Γ-hemirings

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ABSTRACT

In this paper, based on the results of soft sets and intersection operations of sets, we introduce a new kind of soft Γ-hemirings, which is called soft intersection Γ-hemirings. The main contribution of this paper is to give the concepts of *SI*-*h*-ideals (*SI*-*h*-interior ideals) and describe some characterizations of *h*-hemiregular and *h*-semisimple Γ-hemirings by means of *SI*-*h*-ideals and *SI*-*h*-interior ideals.

Keywords: Soft intersection Γ-hemiring; soft intersection *h*-ideal; soft intersection *h*- interior ideal; soft *h*-sum(product); Γ-hemirings.

INTRODUCTION

The fundamental concept of soft sets introduced by Molodtsov (1999) has been provided powerful and useful mathematical tool for deal with uncertain areas, such as decision making (Cagman & Enginoglu, 2010a; Cagman & Enginoglu, 2010b; Feng *et al.*, 2010a; Maji *et al.*, 2002; Maji *et al.,* 2003), data analysis (Zou & Xiao, 2008), forecasting and so on. Recently, the algebraic structures of soft sets have been studied increasingly, such as soft groups (Aktas & Cagman, 2007), soft rings (Acar *et al.,* 2010), soft-int groups (Cagman *et al.,* 2012), soft semirings (Feng *et al.,* 2008), soft BCK/BCI-algebras (Jun, 2008; Jun *et al.,* 2009), soft intersection near-rings (Sezgzin *et al.,* 2012), and soft rough sets (Feng *et al.*, 2010b).

On the other hand, the notion of a semiring, which provides a common generalization of a ring and a distributive lattice, was introduced by Vandiver, 1934. It has extensive applications in several fields, such as automata theory, formal languages, optimization theory, graph theory, theory of discrete event dynamical systems, generalized fuzzy computation, coding theory, analysis of computer programs and other branches of applied mathematics. Hemirings, as semirings with commutative addition and zero element, have also been proved to be an important algebraic tool in theoretical computer science, for examples, see (Torre, 1965; Wechler, 1978; Yin & Li, 2008; Zhan & Dudek, 2007).

Subsequently, the concept of Γ-semirings was then introduced by Rao, 1995 and some properties of such Γ-semirings have been studied, for example, see (Dutta & Sardar, 2002; Sardar & Mandal, 2009). In addition, Ma & Zhan (2010) and Zhan & Shum (2011) introduced the concept of *h*-hemiregular Γ-hemirings and gave a characterization of *h*-hemiregular Γ-hemirings in terms of fuzzy *h*-ideals. Nowadays, many researchers discussed this theory including their applications.

Recently, Ali *et al*. (2009) and Cagman *et al*. (2012) redefined the operations of soft sets to develop the soft set theory. By using their definitions, Segzin *et al*. (2012) defined soft int near-rings, which bring the soft set theory, set theory and near-ring properties, and then Ma & Zhan (2014) introduced the concepts of *SI*-hemirings and *SI*-*h*-ideals and obtain some related properties. As a continuation of this theory together, we introduce a new kind of soft Γ-hemirings called soft intersection Γ-hemirings and obtain some related properties. Some basic operations are also investigated. Finally, we describe some characterizations of *h*-hemiregular Γ-hemirings and *h*-semisimple Γ-hemirings by means of *SI*-*h*-ideals and *SI*-*h*-interior ideals.

PRELIMINARIES

Definition 2.1 (Rao, 1995) Let S and Γ be two addition semigroups. Then S is said to be a Γ-semiring if there exists a mapping $S\times\Gamma\times S \to S$ (images to be denoted by $a\ \alpha\ b$ for $ab \in S$ and $\alpha \in \Gamma$) satisfies the following conditions:

(i) $a\alpha(b+c) = a\alpha b + a\alpha c$;

(ii) $(a+b)\alpha c = a\alpha c + b\alpha c$;

(iii) $a(\alpha+\beta)c = a\alpha c + a\beta c$;

(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$.

By a zero of a Γ-semiring S, we mean an element $0 \in S$ such that $0\alpha x = x\alpha 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$ and $\alpha \in \Gamma$. A Γ-semiring with a zero is said to be a Γ-hemiring.

A left ideal of a Γ-hemiring S is a subset A of S which is closed under addition, such that $S\Gamma A \subseteq A$, where $S\Gamma A = \{x\alpha y | x, y \in S, \alpha \in \Gamma\}$.

A left ideal (right ideal, ideal) A of S is called a left *h*-ideal (right *h*-ideal, *h*-ideal) of S, respectively, if for any $x, z \in S$ and $a, b \in A$, $x + a + z = b + z \to x \in A$.

Right *h*-ideals are defined similarly.

The *h*-closure $\overline{A}$ of A in S is defined by

$$\overline{A} = \{x \in S | x + a_1 + z = a_2 + z \text{ for some } a_1, a_2 \in A, z \in S\}.$$

Clearly, if A is a left ideal of S, then $\overline{A}$ is the smallest left h-ideal of S containing A. We also have $\overline{\overline{A}} = \overline{A}$ for each $A \subseteq S$. Moreover, $A \subseteq B \subseteq S$ implies $\overline{A} \subseteq \overline{B}$.

An interior ideal A of S is called an h-interior ideal of S if A is closed under addition such that $\overline{A\Gamma A} \subseteq A, \overline{S\Gamma A\Gamma S} \subseteq A$ and $x + a + z = b + z$ implies that $x \in A$, for all $x, z \in S, a, b \in A$.

From now on, S is a hemiring, U is an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A, B, C \subseteq E$.

Definition 2.2 (Molodtsov, 1999) A soft set f_A of U is a set defined by $f_A : E \to P(U)$ such that $f_A(x) = \phi$ if $x \notin A$. Here f_A is also called an approximate function. A soft set over Ucan be represented by the set of ordered pairs $f_A = \{(x, f_A(x)) | x \in E, f_A(x) \in P(U)\}$. It is clear to see that a soft set is a parameterized family of subsets of the set U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definiton 2.3 (Cagman & Enginoglu, 2010b) Let f_A, f_B be soft sets over U. Then f_A is a soft subset of f_B, denoted by $f_A \tilde{\subseteq} f_B$, if $f_A \subseteq f_B, \forall x \in E$.

Definition 2.4 (Cagman & Enginoglu, 2010b) Let $f_A, f_B \in S(U)$, then

(i) The intersection of f_A and f_B, denoted by $f_A \tilde{\cap} f_B$, is defined as $f_A \tilde{\cap} f_B = f_{A \cap B}$, where $f_{A \cap B}(x) = f_A(x) \cap f_B(x)$, for all $x \in E$;

(ii) The union of f_A andf_B, denoted by $f_A \tilde{\cup} f_B$, is defined as $f_A \tilde{\cup} f_B = f_{A \cup B}$, where $f_{A \cup B}(x) = f_A(x) \cup f_B(x), \forall x \in E$.

Definition 2.5 (Cagman & Enginoglu, 2010b) Let $f_A, f_B \in S(U)$. Then $\wedge$-product and $\vee$-product of f_A and f_B, denoted by $f_A \wedge f_B$ and $f_A \vee f_B$, are defined by $f_{A \wedge B}(x, y) = f_A(x) \cap f_B(y), f_{A \vee B}(x, y) = f_A(x) \cup f_B(y) \forall x, y \in E$, respectively.

Definiton 2.6 (Cagman & Enginoglu, 2010b) Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, soft image of f_A under Ψ, denoted by $\Psi(f_A)$, is a soft set over U by

$$(\Psi(f_A))(b) = \begin{cases} \bigcup\{f_A(a) | a \in A, \Psi(a) = b\} & \text{if } \Psi^{-1}(b) \neq \phi, \\ \phi & \text{otherwise,} \end{cases}$$

for all $x \in B$. And soft pre-image of f_B under Ψ, denoted by $\Psi^{-1}(f_B)$, is a soft set over U by $(\Psi^{-1}(f_B))(a) = f_B(\Psi(a))$ for all $a \in A$.

SI-Γ-HEMIRINGS (*SI-H-IDEALS)*

In this paper, we introduce the concepts of soft intersection Γ-hemirings(soft intersection h-ideals) and obtain some related properties.

Definition 3.1 A soft set f_S over U is called a soft intersection Γ-hemiring of S (briefly, SI-Γ-hemiring) of S over U if it satisfies:

(SI_1) $f_S(x+y) \supseteq f_S(x) \cap f_S(y)$ for all $x, y \in S$,

(SI_2) $f_S(x\gamma y) \supseteq f_S(x) \cap f_S(y)$ for all $x, y \in S, \gamma \in \Gamma$,

(SI_3) $f_S(x) \supseteq f_S(a) \cap f_S(b)$ with $x+a+z = b+z$ for all $x, a, b, z \in S$.

Example 3.2 Let $U = S = \Gamma = \mathbf{Z}_4 = \{0, 1, 2, 3\}$ be the Γ-hemiring of non-negative integers modulo 4. Now define a soft set f_S over U by $f_S(0) = \{0, 1, 2, 3\}$ and $f_S = \{0, 2\}$. Then one can easily check that f_S is an SI-Γ-hemiring of S over U.

Proposition 3.3 If f_S is an SI-Γ-hemiring of S over U, then $f_S(0) \supseteq f_S(x)$ for all$x \in S$.

Proof. If f_S is an SI-Γ-hemiring of S over U. Let $x = 0$ and $a = b = x$ in (SI_3), we have $f_S(0) \supseteq f_S(x) \cap f_S(x) = f_S(x)$.

Definition 3.4 A soft set f_S over U is called a soft intersection left (right) h-ideal of S(briefly, SI-left(right) h-ideal) of S over U if it satisfies (SI_1), (SI_3) and:

(SI_4) $f_S(x\gamma y) \supseteq f_S(y)(f_S(x\gamma y) \supseteq f_S(x))$, for all $x, y \in S, \gamma \in \Gamma$.

A soft set over U is called a soft intersection h-ideal (briefly, SI-h-ideal) of S if it is both an SI-left h-ideal and an SI-right h-ideal of S over U.

Example 3.5 Assume that $U = \mathbf{Z}^+$ is the universal set and $S = \mathbf{Z}_3$ is the set of parameters. Define a soft set f_S as $f_S(0) = \{n | n \in \mathbf{Z}^+\}$,$f_S(1) = f_S(2) = \{2n | n \in \mathbf{Z}^+\}$. Then one can easily check that f_S is an SI-h-ideal of S over U.

Let $f_{S_1}$and f_{S_2} be two SI-Γ-hemirings. Now we can easily check that $f_{S_1} \times f_{S_2} = f_{S_1 \times S_2}$ is a SI-Γ-hemiring by the operations which we define as follows:

(i) $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$;

(ii) $(x_1, x_2)\gamma(y_1, y_2) = (x_1\gamma y_1, x_2\gamma y_2)$, for all $x_1, y_1 \in S_1, x_2, y_2 \in S_2, \gamma \in \Gamma$.

Propositon 3.6 Let f_{S_1} and f_{S_2} be two SI-Γ-hemirings of S_1 and S_2 over U, respectively. Then $f_{S_1} \wedge f_{S_2}$ is an SI-Γ-hemiring of $S_1 \times S_2$ over U.

Proof. Let f_{S_1} and f_{S_2} be two SI-Γ-hemirings of S_1 and S_2 over U, respectively. Then for all $(x_1, y_1), (x_2, y_2) \in S_1 \times S_2$, we have

$$(i)\ f_{S_1\wedge S_2}((x_1,y_1)+(x_2,y_2)) = f_{s_1\wedge s_2}(x_1+x_2+y_1+y_2)$$
$$= f_{s_1}(x_1+x_2)\cap f_{s_2}(y_1+y_2)$$
$$\supseteq (f_{s_1}(x_1)\cap f_{s_1}(x_1))\cap (f_{s_2}(y_1)\cap f_{s_2}(y_2))$$
$$= (f_{s_1}(x_1)\cap f_{s_2}(y_1))\cap (f_{s_1}(x_2)\cap f_{s_2}(y_2))$$
$$= f_{s_1\wedge s_2}(x_1,y_1)\cap f_{s_1\wedge s_2}(x_2,y_2).$$

$$(ii)\ f_{S_1\wedge S_2}((x_1,y_1)+(x_2,y_2)) = f_{s_1\wedge s_2}(x_1\gamma x_2, y_1\gamma y_2)$$
$$= f_{s_1}(x_1\gamma x_2)\cap f_{s_2}(y_1\gamma y_2)$$
$$\supseteq (f_{s_1}(x_1)\cap f_{s_1}(x_1))\cap (f_{s_2}(y_1)\cap f_{s_2}(y_2))$$
$$= (f_{s_1}(x_1)\cap f_{s_2}(y_1))\cap (f_{s_1}(x_2)\cap f_{s_2}(y_2))$$
$$= f_{s_1\wedge s_2}(x_1,y_1)\cap f_{s_1\wedge s_2}(x_2,y_2).$$

(*iii*) Let $(a_1,a_2),(b_1,b_2),(x_1,x_2),(z_1,z_2)\in S_1\times S_2$ be such that (x_1,x_2) $+(a_1,a_2)$ $+(z_1,z_2)=(b_1,b_2)+(z_1,z_2)$, and so $x_1+a_1+z_1=b_1+z_1$ and x_2+a_2 $+z_2=b_2+z_2$. Then

$$f_{S_1}\wedge S_2(x_1,x_1) = f_{S_1}(x_1)\cap f_{S_2}(x_2)$$
$$\supseteq (f_{S_1}(a_1)\cap f_{S_1}(b_1))\cap (f_{S_2}(a_2)\cap f_{S_2}(b_2))$$
$$= (f_{S_1}(a_1)\cap f_{S_2}(a_2))\cap (f_{S_1}(b_1)\cap f_{S_2}(b_2))$$
$$= f_{S_1\wedge S_2}(a_1,a_2)\cap f_{S_1\wedge S_2}(b_1,b_2).$$

Hence $f_{S_1\wedge S_2}$ is an*SI*-Γ-hemiring of $S_1\times S_2$ over U.

Similarly, we can obtain the following result:

Proposition 3.7 Let f_{S_1} and f_{S_2} be two *SI*-*h*-ideals of S_1 and S_2 over U, respectively. Then $f_{S_1}\wedge f_{S_2}$ is an *SI*-*h*-ideal of $S_1\times S_2$ over U.

Theorem 3.8 Let f_S and g_S be two *SI*-Γ-hemirings of S over U. Then $f_S\tilde{\cap} g_S$ is also an *SI*-Γ-hemiring of S over U.

Proof. Let f_S and g_S be two *SI*-Γ-hemirings of S over U. Then for all $x,y\in S$ and $\gamma\in\Gamma$, we have

$$\begin{aligned}(i)\quad (f_S\tilde{\cap}g_S)(x+y) &= f_S(x+y)\cap g_S(x+y)\\ &\supseteq (f_S(x)\cap f_S(y))\cap(g_S(x)\cap g_S(y))\\ &= (f_S(x)\cap g_S(x))\cap(f_S(y)\cap g_S(y))\\ &= (f_S\tilde{\cap}g_S)(x)\cap(f_S\tilde{\cap}g_S)(y).\end{aligned}$$

$$\begin{aligned}(ii)\quad (f_S\tilde{\cap}g_S)(x\gamma y) &= f_S(x\gamma y)\cap g_S(x\gamma y)\\ &\supseteq (f_S(x)\cap f_S(y))\cap(g_S(x)\cap g_S(y))\\ &= (f_S(x)\cap g_S(x))\cap(f_S(y)\cap g_S(y))\\ &= (f_S\tilde{\cap}g_S)(x)\cap(f_S\tilde{\cap}g_S)(y).\end{aligned}$$

(iii) Let $(a,b,x,z)\in S$ be such that $x+a+z=b+z$. Then

$$\begin{aligned}(f_S\tilde{\cap}g_S)(x) &= f_S(x)\cap g_S(x)\\ &\supseteq (f_S(a)\cap f_S(b))\cap(g_S(a)\cap g_S(b))\\ &= (f_S(a)\cap g_S(a))\cap(f_S(b)\cap g_S(b))\\ &= (f_S\tilde{\cap}g_S)(a)\cap(f_S\tilde{\cap}g_S)(b).\end{aligned}$$

Hence $f_S\tilde{\cap}g_S$ is an *SI*-Γ-hemiring of S over U.

Similarly, we can obtain the following theorem:

Theorem 3.9 Let f_S and g_S be two *SI*-*h*-ideals of S over U, then $f_S\tilde{\cap}g_S$ is also an*SI*-*h*-ideal of$S$ over U.

Theorem 3.10 Let f_{S_1} be an *SI*-Γ-hemiring (*SI*-*h*-ideal) of S_1 over $U$and Ψ an isomorphism from S_1 to S_2. Then $\Psi(f_{S_1})$is an *SI*-Γ-hemiring (*SI*-*h*-ideal) of S_2over U.

Proof. Since Ψ is surjective, we can let $x_1,x_2,x_3\in S_2$, then there exist $y_1,y_2,y_3\in S_1$, such that $\Psi(y_i)=x_i, i=1,2,3$. Then we have

$$
\begin{aligned}
(\Psi(f_{S_1}))(x_1+x_2) &= \bigcup\{f_{S_1}(y) | y \in S_1, \Psi(y) = x_1+x_2\} \\
&= \bigcup\{f_{S_1}(y) | y \in S_1, y = \Psi^{-1}(x_1+x_2)\} \\
&= \bigcup\{f_{S_1}(y) | y \in S_1, y = \Psi^{-1}(\Psi(y_1+y_2)) = y_1+y_2\} \\
&= \bigcup\{f_{S_1}(y_1+y_2) | y_i \in S_1, \Psi(y_i) = x_i, i = 1,2\} \\
&\supseteq \bigcup\{f_{S_1}(y_1) \cap f_{S_1}(y_2) | y_i \in S_1, \Psi(y_i) = x_i, i = 1,2\} \\
&= (\bigcup\{f_{S_1}(y_1) | y_1 \in S_1, \Psi(y_1) = x_1\}) \cap (\bigcup\{f_{S_1}(y_2) | y_2 \in S_1, \Psi(y_2) = x_2\}) \\
&= (\Psi(f_{S_1}))(x_1) \cap (\Psi(f_{S_1}))(x_2).
\end{aligned}
$$

Moreover, let $\gamma \in \Gamma$, we have

$$
\begin{aligned}
(\Psi(f_{S_1}))(x_1\gamma x_2) &= \bigcup\{f_{S_1}(y) | y \in S_1, \Psi(y) = x_1\gamma x_2\} \\
&= \bigcup\{f_{S_1}(y) | y \in S_1, y = \Psi^{-1}(x_1\gamma x_2)\} \\
&= \bigcup\{f_{S_1}(y) | y \in S_1, y = \Psi^{-1}(\Psi(y_1\gamma y_2)) = y_1\gamma y_2\} \\
&= \bigcup\{f_{S_1}(y_1\gamma y_2) | y_i \in S_1, \Psi(y_i) = x_i, i = 1,2\} \\
&\supseteq \bigcup\{f_{S_1}(y_1) \cap f_{S_1}(y_2) | y_i \in S_1, \Psi(y_i) = x_i, i = 1,2\} \\
&= (\bigcup\{f_{S_1}(y_1) | y_1 \in S_1, \Psi(y_1) = x_1\}) \cap (\bigcup\{f_{S_1}(y_2) | y_2 \in S_1, \Psi(y_2) = x_2\}) \\
&= (\Psi(f_{S_1}))(x_1) \cap (\Psi(f_{S_1}))(x_2).
\end{aligned}
$$

Furthermore, let $(a_1, b_1, x_1, z_1) \in S_2$be such that

$x_1 + a_1 + z_1 = b_1 + z_1$. Then there also exists $a_2, b_2, x_2, z_2 \in S_1$, such that $x_2 + a_2 + z_2 = b_2 + z_2$and $\Psi(a_2) = a_1, \Psi(b_2) = b_1$, then we have

$$
\begin{aligned}
(\Psi(f_{S_1}))(x_1) &= \bigcup\{f_{S_1}(y) | y \in S_1, \Psi(y) = x_1\} \\
&= \bigcup\{f_{S_1}(y) | y \in S_1, y = \Psi^{-1}(x_1)\} \\
&= \bigcup\{f_{S_1}(y) | y \in S_1, y = \Psi^{-1}(\Psi(y_1)) = y_1\} \\
&= \bigcup\{f_{S_1}(y_1) | y_1 \in S_1, \Psi(y_1) = x_1\} \\
&\supseteq \bigcup\{f_{S_1}(a_2) \cap f_{S_1}(b_2) | a_2, b_2 \in S_1, \Psi(a_2) = a_1, \Psi(b_2) = b_1\} \\
&= (\bigcup\{f_{S_1}(a_2) | a_2 \in S_1, \Psi(a_2) = a_1\}) \cap (\bigcup\{f_{S_1}(b_2) | b_2 \in S_1, \Psi(b_2) = b_1\}) \\
&= (\Psi(f_{S_1}))(a_1) \cap (\Psi(f_{S_1}))(b_1).
\end{aligned}
$$

Hence, $\Psi(f_{S_1})$ is an *SI*-Γ-hemiring of S_2over U. Similarly, we can prove the case for *SI-h*-ideal.

Similarly, we can obtain the following theorem:

Theorem 3.11 Let f_{S_2} be an *SI*-Γ-hemiring (*SI-h*-ideal) of S_2 over U and Ψ a homomorphism from S_1 to S_2. Then $\Psi^{-1}(f_{S_2})$ is an *SI*-Γ-hemiring (*SI*-*h*-ideal) of S_1 over U.

SI-H-INTERIORS IDEALS

In this section, we consider *SI*-Γ-interior ideals of Γ-hemirings and investigate some related properties.

Definition 4.1 A soft set f_S over U is called a soft intersection h-interior ideal (briefly, *SI*-*h*-interior ideal) of S over U if it satisfies (SI_1), (SI_2), (SI_3) and

$(SI_5) f_S(x\alpha a\beta y) \supseteq f_S(a)$ for all $x, y, a \in S$ and $\alpha, \beta \in \Gamma$

Example 4.2 Assume that $U = S_3$ is the universal set. Let $S = \Gamma = \mathbf{Z}_4 = \{0, 1, 2, 3\}$, non-negative positive integers modulo 4, be the Γ-hemiring of parameters. Define a soft set f_S over U by$f_S(0) = S_3$, $f_S(1)=\{(1), (2)\}$ and $f_S(2)=f_S(3)= \{(1), (12), (123), (132)\}$. The one can easily check that f_S is an *SI*-*h*-interior ideal of S over U.

Propositon 4.3 Every *SI*-*h*-ideal of Γ-hemiring S is an *SI*-*h*-interior ideal.

Proof. By Definitions 3.1 and 4.1, we only prove (SI_5) is satisfied. Assume f_S is an *SI*-*h*-ideal of S, let $x, y, z \in S, \alpha, \beta \in \Gamma$, we have $f_S(x\alpha y\beta z) \supseteq f_S(y\beta z) \supseteq f_S(y) \cap f_S(z)$ since f_S is an *SI*-left *h*-ideal of S, and also $f_S$is an *SI*-right *h*-ideal of S, so $f_S(x\alpha y\beta z) \supseteq f_S(x\alpha y) \supseteq f_S(y)$, hence, $f_S(x\alpha y\beta z) \supseteq f_S(y)$.

Theorem 4.4 A non-empty subset I of S is an h-interior ideal of S if and only if the soft subset f_S defined by

$$f_S(x) = \begin{cases} \alpha & \text{if } x \in I, \\ \beta & \text{if } x \in S/I, \end{cases}$$

is an *SI*-*h*-interior ideal of S over U, where $\alpha, \beta \subseteq U$ such that $\alpha \supseteq \beta$.

Proof. Let I be an h-interior ideal of S and $x, y \in S, \gamma \in \Gamma$.

(i) If $x, y, \gamma \in I$, then $x\gamma y, x + y \in I$. Hence $f_S(x + y) = f_S(x\gamma y) = f_S(x) = f_S(y) = \alpha$, and so $f_S(x + y) \supseteq f_S(x) \cap f_S(y)$ and $f_S(x\gamma y) \supseteq f_S(x) \cap f_S(y)$.

(ii) If either one of x and y does not belong to I, then $x + y \in I$ or $x + y \notin I$ and $x\gamma y \in I$ or $x\gamma y \notin I$. In any case, $f_S(x + y) \supseteq f_S(x) \cap f_S(y) = \beta$ and $f_S(x\gamma y) \supseteq f_S(x) \cap f_S(y) = \beta$. This proves that (SI_1) and (SI_2) hold.

Now, let $a, b, x, y, z \in I$ be such that $x + a + z = b + z$and $\gamma, \delta \in \Gamma$.

(i) If $a, b \in I$, then $x \in I$, and so $f_S(x) = f_S(a) \cap f_S(b) = \alpha$ and $f_S(x\gamma y\delta z) = f_S(y) = \alpha$.

(ii) If $a \notin I$ or $b \notin I$, then $f_S(x) \supseteq f_S(a) \cap f_S(b) = \beta$. This proves that (SI_3) and (SI_4) hold.

Thus, f_S is an *SI*-*h*-interior ideal of *S* over *U*.

Conversely, assume that f_S is an *SI*-*h*-interior ideal of *S* over *U*. Let $x, y, z \in I$, $\gamma, \delta \in \Gamma$, then $f_S(x+y) \supseteq f_S(x) \cap f_S(y) = \alpha$, $f_S(x\gamma y) \supseteq f_S(x) \cap f_S(y) = \alpha$ and $f_S(x\gamma y\delta z) = f_S(y) = \alpha$, which implies, $x + y, x\gamma y \in I, x\gamma y\delta z \in I$.

Now, let $x, z \in S$ and $a, b \in I$ be such that $x + a + z = b + z$. Then

$f_S(x) \supseteq f_S(a) \cap f_S(b) = \alpha$, and so $x \in I$. Hence, *I* is an *h*-interior ideal of *S*.

Corollary 4.5 Let *A* be a non-empty subset of Γ-hemiring *S*. Then *A* is an *h*-interior ideal if and only if characteristic function C_A is an *SI*-*h*-interior ideal of *S* over *U*.

CHARACTERIZATIONS OF Γ-HEMIRINGS

In this section, we describe the characterizations of *h*-hemiregular (*h*-semisimple) Γ-hemirings by means of *SI*-*h*-ideals and *SI*-*h*-interior ideals.

Definition 5.1 (Zhan & Shum, 2011) A Γ-hemiring *S* is called *h*-hemiregular if for each $x \in S$, there exist $a_1, a_2, z \in S$ and $\alpha, \alpha', \beta, \beta' \in \Gamma$ such that $x + x\alpha a_1 \beta x + z = x\alpha' a_2 \beta' x + z$.

Lemma 5.2 (Zhan & Shum, 2011)) If *A* and *B*, are, respectively, a right and a left *h*-ideal of *S*, then $\overline{A\Gamma B} \subseteq A \cap B$.

Lemma 5.3 (Zhan & Shum, 2011)) A Γ-hemiring *S* is *h*-hemiregular if and only if for any right *h*-ideal *A* and any left *h*-ideal *B*, we have $\overline{A\Gamma B} = A \cap B$.

Definition 5.4 Let $f_S, g_S \in S(U)$. Define soft *h*-sum and soft *h*-product of f_S and g_S as follows:

$$(1)(f_S \oplus_h g_S)(x) = \bigcup_{x+a_1+b_1+z=a_2+b_2+z} (f_S(a_1) \cap f_S(a_2) \cap g_S(b_1) \cap g_S(b_2))$$

and $(f_S \oplus_h g_S)(x) = \phi$ if *x* cannot be expressed as $x + a_1 + b_1 + z = a_2 + b_2 + z$.

$$(2)(f_S \otimes_h g_S)(x) = \bigcup_{x+a_1\gamma_1 b_1+z=a_2\gamma_2 b_2+z} (f_S(a_1) \cap f_S(a_2) \cap g_S(b_1) \cap g_S(b_2))$$

and $(f_S \otimes_h g_S)(x) = \phi$ if *x* cannot be expressed as$x + a_1\gamma_1 b_1 + z = a_2\gamma_2 b_2 + z$, where $\gamma_1, \gamma_2 \in \Gamma$.

Lemma 5.5 A Γ-hemiring S is h-hemiregular, let f_S and g_S be an SI-right h-ideal and an SI-left h-ideal of S over U, respectively, then $f_S \otimes_h g_S \subseteq f_S \tilde{\cap} g_s$.

Proof. If $(f_S \otimes_h g_S)(x) = \phi$, then it is clear that $f_S \otimes_h g_S \subseteq f_S \tilde{\cap} g_s$. Otherwise, we

$$\begin{aligned}(f_S \otimes_h g_S)(x) &= \bigcup_{x+a_1b_1+z=a_2b_2+z} (f_S(a_1) \cap f_S(a_2) \cap g_S(b_1) \cap g_S(b_2)) \\ &\subseteq \bigcup_{x+a_1b_1+z=a_2b_2+z} (f_S(a_1b_1) \cap f_S(a_2b_2) \cap g_S(a_1b_1) \cap g_S(a_2b_2)) \\ &\subseteq \bigcup_{x+a_1b_1+z=a_2b_2+z} (f_S(x) \cap g_S(x)) \\ &= f_S(x) \cap g_S(x) \\ &= (f_S \cap g_S)(x),\end{aligned}$$

which implies, $f_S \otimes_h g_S \subseteq f_S \tilde{\cap} g_S$.

Definition 5.6 Let $A \subseteq S$. We denote C_A the soft characteristic function of A and define as

$$C_A(x) = \begin{cases} U & \text{if } x \in A, \\ \phi & \text{if } x \notin A. \end{cases}$$

The following proposition is obvious and we omit the details.

Proposition 5.7 Let $A, B \subseteq S$. Then the following hold:

(1) $A \subseteq B \Rightarrow C_A \tilde{\subseteq} C_B$.

(2) $C_A \tilde{\cap} C_B = C_{A \cap B}$.

(3) $C_A \otimes_h C_B = C_{\overline{A\Gamma B}}$.

Theorem 5.8 For any Γ-hemiring S, then the following are equivalent:

(1) S is h-hemiregular;

(2) $f_S \otimes_h g_S = f_S \tilde{\cap} g_S$ for any SI-right h-ideal f_S and SI – left h – idealg_{S} of Sove U.

Proof. (1)⇒(2): Let S be an h-hemiregular Γ-hemiring, f_S and g_S an SI-right h-ideal and an SI-left h-ideal of S over U, respectively. By Lemma 5.5, we have $f_S \otimes_h g_S \subseteq f_S \tilde{\cap} g_S$. Let $x \in S$, then there exist $a, a', z \in S$ and $\alpha, \alpha', \beta, \beta' \in \Gamma$ such that $x + x\alpha a\beta x + z = x\alpha' a' \beta' x + z$ since S is h-hemiregular. Thus, we have

$$\begin{aligned}(f_S \otimes_h g_S)(x) &= \bigcup_{x+a_1\gamma_1 b_1+z=a_2\gamma_2 b_2+z} (f_S(a_1) \cap f_S(a_2) \cap g_S(b_1) \cap g_S(b_2)) \\ &\supseteq f_S(x\alpha a) \cap f_S(x\alpha' a') \cap g_S(x) \\ &\supseteq f_S(x) \cap g_S(x) \\ &= (f_S \cap g_S)(x),\end{aligned}$$

which implies $f_S \otimes_h g_S \supseteq f_S \tilde{\cap} g_S$. Thus, $f_S \otimes_h g_S = f_S \tilde{\cap} g_S$.

(2)⇒(1): Let R and L be any right h-ideal and left h-ideal of S, respectively. Then by Lemma 5.2, we have $\overline{R\Gamma L} \subseteq R \cap L$. Moreover, it is easy to check that C_R and C_L are an *SI*-right h-ideal and an *SI*-left h-ideal of S over U, respectively. Let $x \in R \cap L$, then, by Proposition 4.8, we have

$$C_{\overline{R\Gamma L}}(x) = (C_R \otimes_h C_L)(x) = (C_R \tilde{\cap} C_L)(x) = C_{R\cap L}(x) = U,$$

and so $x \in \overline{R\Gamma L}$. Then, $R \cap L \subseteq \overline{R\Gamma L}$. Thus, $R \cap L = \overline{R\Gamma L}$. It follows from Lemma 5.3 that S is h-hemiregular.

Definition 5.9 A subset A of S is called Γ-idempotent if $A = \overline{A\Gamma A}$. A Γ-hemiring S is called h-semisimple if every h-ideal of S is Γ-idempotent.

Lemma 5.10 A Γ-hemiring S is h-semisimple if and only if the following holds:

(1) S is h-semisimple;

(2) $a \in \overline{S\Gamma a\Gamma S\Gamma a\Gamma S}$, for all $a \in S$;

(3) $A \subseteq \overline{S\Gamma A\Gamma S\Gamma A\Gamma S}$, for all $A \subseteq S$.

Theorem 5.11 Let S be an h-semisimple Γ-hemiring and $f_S \in S(U)$. Then f_S is an *SI*-h-ideal of S over U if and only if it is an *SI*-h-interior ideal of S over U.

Proof. If f_S is an *SI*-h-ideal of S over U, then by Proposition 4.3, f_S is an *SI*-h-interior ideal of S over U.

Conversely, let f_S be an *SI*-h-interior ideal of S over U. Let $x, y \in S$ and $\alpha \in \Gamma$, then there exist $a_i, a'_i, z \in S (i = 1, 2, 3, 4)$ and $\gamma_i, \gamma'_i \in \Gamma (i = 1, 2, 3, 4, 5)$, such that

$$x + a_1\beta_1 x\beta_2 a_2\beta_3 a_3\beta_4 x\beta_5 a_4 + z = a'_1\beta'_1 x\beta'_2 a'_2\beta'_3 a'_3\beta'_4 x\beta'_5 a'_4 + z$$

since S is h-semisimple. Hence,

$$x\alpha y + a_1\beta_1 x\beta_2 a_2\beta_3 a_3\beta_4 x\beta_5 a_4\alpha y + z\alpha y = a'_1\beta'_1 x\beta'_2 a'_2\beta'_3 a'_3\beta'_4 x\beta'_5 a'_4\alpha y + z\alpha y.$$

Thus,

$$\begin{aligned} f_S(x\alpha y) &\supseteq f_S(a_1\beta_1x\beta_2a_2\beta_3a_3\beta_4x\beta_5a_4\alpha y)\cap f_S(a'_1\beta'_1x\beta'_2a'_2\beta'_3a'_3\beta'_4x\beta'_5a'_4\alpha y)\\ &\supseteq f_S(x)\cap f_S(x)\\ &= f_S(x), \end{aligned}$$

This proves that f_S is an *SI*-right *h*-ideal of *S* over *U*.

Similarly, we can prove that f_S is an *SI*-left *h*-ideal of *S* over *U*. Hence f_S is an *SI*-*h*-ideal of *S* over *U*.

Finally, we give a characterization of *h*-semisimple Γ-hemirings.

Theorem 5.12 A Γ-hemiring *S* is *h*-semisimple if and only if for any *SI*-*h*-interior ideals f_S and g_S, we have $f_S\otimes_h g_S = f_S\tilde{\cap} g_S$.

Proof. Assume that *S* is an *h*-semisimple hemiring and f_S and g_S are *SI*-*h*-interior ideals of *S* over *U*. Then, by Theorem 5.11, f_S and g_S are also *SI*-*h*-ideal of *S* over *U*. Thus, we have $f_S\otimes_h g_S\subseteq f_S\otimes_h \mathrm{C}\subseteq f_S$ and $f_S\otimes_h g_S\subseteq \mathrm{C}\otimes_h g_S\subseteq g_S$. This prove that $f_S\otimes_h g_S\subseteq f_S\tilde{\cap} g_S$.

For any $x\in S$, then there exist $a_i, a'_i, z\in S(i=1,2,3,4)$ and $\beta_i,\beta'_i\in\Gamma(i=1,2,3,4,5)$

such that

$$x+a_1\beta_1x\beta_2a_2\beta_3a_3\beta_4x\beta_5a_4+z = a'_1\beta'_1x\beta'_2a'_2\beta'_3a'_3\beta'_4x\beta'_5a'_4+z,$$

since *S* is *h*-semisimple. Thus,

$$\begin{aligned} (f_S\otimes_h g_S)(x) &= \cup_{x+a_1}\gamma_1b_1+z=a'_1\gamma'_1b'_1+z(f_S(a_1)\cap f_S(a'_1)\cap g_S(b_1)\cap g_S(b'_1))\\ &\supseteq f_S(a_1\beta_1x\beta_2a_2)\cap f_S(a'_1\beta'_1x\beta'_2a'_2)\cap g_S(a_3\beta_4x\beta_5a_4)\cap g_S(a'_3\beta'_4x\beta'_5a'_4)\\ &\supseteq f_S(x)\cap g_S(x)\\ &= (f_S\cap g_S)(x), \end{aligned}$$

which implies, $f_S\tilde{\cap} g_S\subseteq f_S\otimes_h g_S$. Thus, $f_S\tilde{\cap} g_S = f_S\otimes_h g_S$.

Conversely, let *A* be any *h*-ideal of *S*, then it is an *h*-interior ideal of *S*. Thus, by Corollary 4.5, C_A is an *SI*-*h*-interior ideal of *S* over *U*. By Proposition 5.7, we have $\mathrm{C}_A=\mathrm{C}_A\tilde{\cap}\mathrm{C}_A=\mathrm{C}_A\otimes_h\mathrm{C}_A=\mathrm{C}_{\overline{A\Gamma A}}$, and so $A=\overline{A\Gamma A}$. Hence, *S* is *h*-semisimple.

CONCLUSION

In this paper, we discuss soft intersection Γ-hemirings (soft intersection *h*-ideals) by soft theories and intersection operation of sets. In particular, we give some characterizations of *h*-hemiregular and *h*-semisimpleΓ-hemirings by means of *SI*-*h*-ideals and *SI*-*h*-interior ideals.

Gathering the former results, we except that the following topics will bring a new insight for further studies:

(1) To describe some new kinds of soft intersection *h*-ideals;

(2) To focus on studying of intuitionistic fuzzy set in Γ-hemirings and other algebraic structures of Γ-hemirings;

(3) To apply soft intersection Γ-hemirings to some applied fields, such as decision making, data analysis and computer science and so on.

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تطبيقات مجموعات التقاطع الضعيف على غاما نظر الحلقات

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خلاصة

نقدم في هذا البحث نوعاً من نظر الحلقات الضعيفة ونطلق عليها اسم غاما نظر حلقات التقاطع الضعيف، حيث أننا نستند إلى نتائج المجموعات الضعيفة وعمليات التقاطع على المجموعات. ويكمن الإسهام الأساسي لهذا البحث في تقديمنا مفهومي: "مثاليات إس آي – إتش" و"مثاليات إس آي – إتش الداخلية" وفي استخدامنا لهذين المفهومين لإيجاد بعض خصائص غاما نظر الحلقات التي تكون نظر المنتظمة وشبه البسيطة.