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Satisfiability in intuitionistic fuzzy logic with realistic tautology

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Abstract

Any variable in Intuitionistic Fuzzy Logic (*IFL*) is either a Realistic Fuzzy Tautology (*RFT*) with a Truth exceeding one half, or a non-Realistic Fuzzy Tautology (*nRFT*) with a Truth less than or equal to one half. This results in a dichotomy somewhat similar to that of the *Excluded Middle* in Ordinary Logic (*OL*) albeit allowing both *Falsity* and *Hesitancy* in addition to *Truth* in an *IFL* variable. Consequently, many problems (and solutions) in Boolean logic can be fuzzified without any significant change in their essence. We show herein that one such problem is that of Boolean satisfiability. We handle this problem by converting a *CNF* expression into a *disjoint DNF* one, and solving the resulting two-valued Boolean equation. This solution strategy is essentially retained in *IFL*, thanks to the *RFT* concept. All steps needed in the fuzzification process are proved, and a demonstrative example illustrates the method in both crisp and intuitionistic fuzzy cases.

Keywords: disjoint DNF; intuitionistic fuzzy logic; realistic fuzzy tautology; satisfiability.

1. Introduction

Boolean Satisfiability (SAT) is the problem of deciding whether a propositional logic formula can be satisfied, given suitable value assignments to the variables of the formula. The SAT problem is the first problem proved to be NP-complete. SAT has a potpourri of solution techniques, notable among which are efficient search strategies in terms of the Davis-Putnam algorithm and its successors (Biere *et al*., 2009).

There are many extensions of SAT that either use the same algorithmic techniques as used in SAT, or use SAT as a core engine (Marques-Silva, 2008). These include problems of hard combinatorics, test-pattern generation (Larrabee, 1992; Stephen *et al*., 1996) and decision making (Lin *et al*., 2007; Wei, 2010). Research concerning propositional (crisp) logic is now being extended to fuzzy logic (Sen & Ray, 2013; Ma & Zhan, 2014; Chauhan *et al.,* 2014; ET *et al.,* 2014; Davvaz & Sadrabadi, 2014; Pant *et al.,* 2015). SAT research is no exception, as fuzzy approaches to it are emerging (Pedrycz *et al.,* 2002).

This paper presents a simple method for handling SAT via Boolean-equation solving, and then adapting the solution to *IFL*. Fuzzification of the method utilizes the fact that any variable in *IFL* is either a Realistic fuzzy tautology (*RFT*) with a Truth exceeding one half, or a non-realistic fuzzy tautology (*nRFT*) with a Truth less than or equal to one half (Rushdi *et al.,* 2015). This fact results in a dichotomy somewhat similar to that of ordinary logic (OL) despite allowing both *Falsity* and *Hesitancy* in addition to *Truth* in an *IFL* variable. Consequently, SAT solutions are fuzzified without any significant change in their essence. Our solution strategy is to first convert a conjunctive normal form (*CNF*) into a *disjoint* disjunctive normal form (*disjoint DNF*), and then solve the resulting two-valued Boolean equation. This strategy is retained in *IFL*, thanks to the *RFT* concept.

The organization of the rest of this paper is as follows. Section 2 derives exhaustive solutions of SAT via Boolean-equation solving, and demonstrates this via a small example. Section 3 reviews *IFL* and *RFT*. Section 4 surveys and proves properties of *RFT* needed in adapting the method of Section 2 to *IFL*. Section 5 presents fuzzy satisfiability via Boolean-equation solving and applies it to a fuzzified version of the example in Section 2. Section 6 concludes the paper.

2. Handling SAT via Boolean-equation solving

This section reviews a method (Rushdi & Ahmad, 2016) for handling SAT by solving the Boolean equation

$$f(\mathbf{X}) = 1, \qquad (1)$$

where $f(\mathbf{X})\colon \boldsymbol{B}^n \rightarrow \boldsymbol{B}$ is a switching function of n variables

$\mathbf{X}$ = [X_1 X_2 …..X_n]^T, and $\boldsymbol{B}$ = {0, 1}. The method uses orthogonalization techniques to convert a CNF-form $f(\mathbf{X})$ into a disjoint DNF. Since $f(\mathbf{X})$ is typically in CNF form, we address (1) in three stages, namely:

a. Converting $f(\mathbf{X})$ from CNF into DNF.

b. Converting the DNF $f(\mathbf{X})$ into a disjoint or orthogonal DNF.

c. Interpreting the disjoint DNF $f(\mathbf{X})$ as a solution of [$f(\mathbf{X}) = 1$].

In the disjoint-multiply procedure (DMP) (Rushdi & Ahmad, 2016), stages (a) and (b) above are swapped as shown by the following example.

Example 1:

Consider a 5-variable 4-clause function, taken from Steinbach & Posthoff (2015), namely

$$f = S_1 \wedge S_2 \wedge S_3 \wedge S_4, \quad (2)$$

where

$$S_1 = X_1 \vee \overline{X_2} \vee \overline{X_3}, \quad (3a)$$

$$S_2 = X_2 \vee \overline{X_4} \vee \overline{X_5}, \quad (3b)$$

$$S_3 = \overline{X_1} \vee X_4 \vee X_5, \quad (3c)$$

$$S_4 = X_2 \vee X_3 \vee \overline{X_5}. \quad (3d)$$

We now multiply S_1 and S_2, and disjoint the result (Rushdi & Rushdi, 2017), namely,

$$S_1 \wedge S_2 = \overline{X_2}(\overline{X_4} \vee \overline{X_5}) \vee X_2(X_1 \vee \overline{X_3}), \quad (4a)$$

$$(S_1 \wedge S_2)_{dis} = \overline{X_2}(\overline{X_4} \vee X_4\overline{X_5}) \vee X_2(X_1 \vee \overline{X_1}\,\overline{X_3}), \quad (4b)$$

and similarly disjoint the product of S_3 and S_4:

$$S_3 \wedge S_4 = X_5(X_2 \vee X_3) \vee \overline{X_5}(\overline{X_1} \vee X_4), \quad (5a)$$

$$(S_3 \wedge S_4)_{dis} = X_5(X_2 \vee \overline{X_2}X_3) \vee \overline{X_5}(\overline{X_1} \vee X_1X_4), \quad (5b)$$

Since the product of two disjoint DNFs is also a disjoint DNF, we performed the disjointing operation before further multiplication. Figure 1 demonstrates a multiplication matrix used to multiply $(S_1 \wedge S_2)_{dis}$ in (4b) with $(S_3 \wedge S_4)_{dis}$ in (5b). The result is the original f of (2) cast as a disjoint DNF. Eight solutions result, and are reduced into 5 by combining certain terms to obtain the final set of solutions of Table 1.

	X_1X_2	$\overline{X_1}X_2\overline{X_3}$	$\overline{X_2}\,\overline{X_4}$	$\overline{X_2}X_4\overline{X_5}$
$\overline{X_1}\,\overline{X_5}$	-	$\overline{X_1}X_2\overline{X_3}\,\overline{X_5}$	$\overline{X_1}\,\overline{X_2}\,\overline{X_4}\,\overline{X_5}$	$\overline{X_1}\,\overline{X_2}X_4\overline{X_5}$
$X_1X_4\overline{X_5}$	$X_1X_2X_4\overline{X_5}$	-	-	$X_1\overline{X_2}X_4\overline{X_5}$
X_2X_5	$X_1X_2X_5$	$\overline{X_1}X_2\overline{X_3}X_5$	-	-
$\overline{X_2}X_3X_5$	-	-	$\overline{X_2}X_3\overline{X_4}X_5$	-

Fig. 1. Multiplication matrix for $(S_1 \wedge S_2)_{dis}$ and $(S_3 \wedge S_4)_{dis}$. The symbol (-) for 0 expresses *nRFT* upon fuzzification

Table 1. A set of solutions of Example 1, where the values 1 and 0 are replaced by *RFT* and *nRFT* upon fuzzification

Equational form	Variable form				
	X_1	X_2	X_3	X_4	X_5
$\overline{X_1}X_2\overline{X_3} = 1$	0	1	0	-	-
$\overline{X_1}\,\overline{X_2}\,\overline{X_5} = 1$	0	0	-	-	0
$\overline{X_2}X_3\overline{X_4}X_5 = 1$	-	0	1	0	1
$X_1X_2X_5 = 1$	1	1	-	-	1
$X_1X_4\overline{X_5} = 1$	1	-	-	1	0

3. Review of intuitionistic fuzzy logic and realistic tautology

There are two different notions of intuitionistic fuzzy sets. The first is due to Takeuti & Titani (1984) and is characterized as the first-order Gödel logic (Metcalfe *et al*., 2008). The second was developed by Atanassov (1986; 1999), and identifies with vague sets (Bustince & Burillo, 1996). Based on this second notion, variants of *IFL* were developed by Atanassov & Gargov (1998). In any variant of *IFL*, a variable X_i is represented by its validity which is the ordered couple

$$V(X_i) = < a_i, b_i >, \quad (6)$$

where a_i and b_i are degrees of Truth and Falsity of X_i, respectively, such that each of the real numbers a_i, b_i, $a_i + b_i \in [0, 1]$. Note that when $a_i + b_i = 1$, then *IFL* reduces to ordinary fuzzy logic (*OFL*), in which a_i alone suffices as a representation for X_i, since b_i is automatically determined by $b_i = 1 - a_i$. The condition $\{a_i + b_i \leq 1\}$ allows a degree of *Hesitancy*, *Ignorance* or *Uncertainty* when one can designate a variable neither as true nor as false. The complementation, conjunction (meet), and disjunction (join) operations are defined herein, respectively, by:

$$V(\bar{X}_i) = < b_i, a_i >, \quad (7)$$

$$V(X_1 \wedge X_2) = < min\ (a_1, a_2),\ max\ (b_1, b_2) >, \quad (8)$$

$$V(X_1 \vee X_2) = < max\ (a_1, a_2),\ min\ (b_1, b_2) >. \quad (9)$$

With these definitions, *IFL* enjoys the usual properties of idempotency, commutativity, associativity, absorption, and distributivity (Rushdi *et al*., 2015). The conjunction $(X_i \wedge \bar{X}_i)$ and the disjunction $(X_i \vee \bar{X}_i)$ are not necessarily of validities $< 0, 1 >$, and $< 1, 0 >$, respectively. Instead, one has

$$V(X_i \wedge \bar{X}_i) = < min\ (a_i, b_i),\ max\ (b_i, a_i) >, \quad (10)$$

$$V(X_i \vee \bar{X}_i) = < max\ (a_i, b_i),\ min\ (b_i, a_i) >. \quad (11)$$

Therefore, $(X_i \wedge \bar{X}_i)$ and $(X_i \vee \bar{X}_i)$ are complementary in the *IFL* sense (7). Now, we discuss the tautology concept. Atanassov (1999) defined intuitionistic fuzzy tautology (*IFT*) by:

$$\{ X_i = IFT\} \leftrightarrow \{a_i \geq b_i\}. \quad (12)$$

Rushdi *et al*. (2015) introduced realistic fuzzy tautology (*RFT*) as

$$\{ X_i = RFT\} \leftrightarrow \{a_i > 0.5\}. \quad (13)$$

Note that an RFT is necessarily an IFT, while an IFT might not be an RFT. If $b_i = 1 - a_i$, then the RFT reduces to the Fuzzy Tautology given by Lee (1972). A related definition in Rushdi *et al*. (2015) is that of non-realistic fuzzy tautology ($nRFT$), namely

$$\{ X_i = nRFT\} \leftrightarrow \{a_i \leq 0.5\}. \quad (14)$$

For convenience, we restate (13) and (14) as

$$V(RFT) = < G, l >, \quad G > 0.5, \ l < 0.5 \quad (15)$$

$$V(nRFT) = < L, u >, \ L \leq 0.5. \quad (16)$$

As a result, an *IFL* variable is either an RFT or an $nRFT$; a dichotomy somewhat similar to that of ordinary logic (OL) despite (a) allowing both Falsity and Hesitancy in addition to Truth in an *IFL* variable, and (b) allowing a variable and its complement to be of the opposing types or to be both $nRFT$, but disallowing them to be both RFT. The RFT value is characterized by Truth, Falsity, and Hesitancy which $\in (0.5, 1.0]$, $[0.0, 0.5)$, and $[0.0, 0.5)$, respectively, while the $nRFT$ value is characterized by Truth, Falsity and Hesitancy, which $\in [0.0, 0.5]$, $[0.5, 1.0]$, and $[0.5, 1.0]$, respectively.

4. Some properties of realistic fuzzy tautology

We list, explain and prove certain (mostly new) properties of general RFT and $nRFT$ concepts. Additional more specific results could be obtained for RFT and $nRFT$ variables with specific Truth and Falsity values.

Property 1: The conjunction of any IFL variable and an $nRFT$ is an $nRFT$.

Proof: $Truth\ (X_i \wedge nRFT) = \min\big(Truth\ (X_i), Truth\ (nRFT)\big) \leq 0.5$, and hence $(X_i \wedge nRFT)$ is an $nRFT$.

Property 2: The disjunction of an RFT. and any IFL variable is an RFT.

Proof: $Truth\ (RFT \vee X_i) = \max\big(Truth\ (RFT), Truth\ (X_i)\big) > 0.5$, and hence $(RFT \vee X_i)$ is an RFT.

Property 3: If a variable is an RFT then its complement is an $nRFT$.

Proof: $\{ X_i = RFT\} \leftrightarrow \{V(X_i) = < G, l >, l < 0.5,\} \leftrightarrow \{V(\bar{X}_i) = < l, G >, l < 0.5,\} \rightarrow \{\bar{X}_i = nRFT\}$.

The converse is not necessarily true. Let $\bar{X}_i = nRFT$ such that $V(\bar{X}_i) = <0.3, 0.4>$. Hence, $V(X_i) = <0.4, 0.3>$, which does not mean that X_i is an RFT. Hence, it is possible that both a variable and its complement are $nRFT$, but it is not possible that both a variable and its complement are RFT.

Property 4: The conjunction of a variable and its complement is an $nRFT$, but their disjunction is not necessarily an RFT.

Proof: There are three possibilities

o The variable is RFT, hence its complement is $nRFT$ and their conjunction is $nRFT$ while their disjunction is RFT.

o The variable is $nRFT$, hence its conjunction with its complement is $nRFT$, while the disjunction of the variable and its complement is the same type as the complement.

o Both the variable and its complement are $nRFT$, hence both their conjunction and disjunction are $nRFT$.

Property 5: A product (term) that has at least one opposition (one pair of complementary literals) is definitely an $nRFT$.

Proof: In the product $P = T \wedge X_i \wedge \bar{X}_i$, the conjunction $(X_i \wedge \bar{X}_i)$ of complementary literals is an $nRFT$, and the conjunction of the IFL variable T and this $nRFT$ is an $nRFT$. The product P is definitely an $nRFT$ irrespective of the validities of T, X_i, and $\bar{X}_i$.

Property 6: The conjunction of some variables is RFT if and only if each of the variables is RFT, namely

$$\text{T(n)}: \{\wedge_{i=1}^{n} X_i = RFT\} \leftrightarrow \{X_k = RFT, 1 \leq k \leq n\}, \; n \geq 1. \quad (17)$$

Proof: This set of theorems {T(n), $n \geq 1$} is proved by mathematical induction via

o Proof of the *base case* T(1) which is trivially true since

$$\{X_1 = RFT\} \leftrightarrow \{X_1 = RFT\}. \quad (18)$$

o Proof of the *inductive case*:

$$T(l) \rightarrow T(l+1), l \geq 1. \quad (19)$$

For convenience, we first prove T(2), namely

$$\{X_1 \wedge X_2 = RFT\} \leftrightarrow \{min\,(a_1, a_2) > 0.5\} \leftrightarrow \{a_1 > 0.5, \; a_2 > 0.5\} \leftrightarrow \{X_1 = RFT, X_2 = RFT\}. \quad (20)$$

Then we prove (19) by introducing $Y_l = \wedge_{i=1}^{l} X_i, \; l \geq 1$,, and noting that $T(l)$ and $T(l+1)$ are

$$\text{T}(l): \; \{Y_l = RFT\} \leftrightarrow \{X_k = RFT, 1 \leq k \leq l\}, l \geq 1 \quad (21)$$

$$\text{T}(l+1): \{Y_{l+1} = RFT\} \leftrightarrow \{X_k = RFT, 1 \leq k \leq (l+1)\}, \quad l \geq 1. \quad (22)$$

Now, since $Y_{l+1} = Y_l \wedge X_{l+1}, l \geq 1$, then by virtue of (20) and (21), the LHS of (22) is given by

$\{Y_{l+1} = RFT\} \leftrightarrow \{Y_l = RFT, \; X_{l+1} = RFT\} \leftrightarrow \{\{X_k = RFT, 1 \leq k \leq l\}, \; X_{l+1} = RFT\} \leftrightarrow$

$\{X_k = RFT, 1 \leq k \leq (l+1)\} \leftrightarrow$ the RHS of (22). Q.E.D.

A corollary of (17) is

$$\{\wedge_{i=1}^{n} X_i = nRFT\} \leftrightarrow \{At\ least\ one\ X_k = nRFT, 1 \leq k \leq n\}, \qquad n \geq 1. \quad (23)$$

Property 7: The disjunction of several variables is $nRFT$ if and only if each of the variables is $nRFT$, namely

$$\text{D(n)}: \quad \{\vee_{i=1}^{n} X_i = nRFT\} \leftrightarrow \{X_k = nRFT, 1 \leq k \leq n\}, \; n \geq 1. \quad (24)$$

Proof: These theorems {D(n), $n \geq 1$} are again proved by mathematical induction in a fashion dual to that of the former theorem. A corollary of (24) is that a DNF is an RFT iff at least one of its terms is an RFT:

$$\{\vee_{i=1}^{n} P_i = RFT\} \leftrightarrow \{At\ least\ one\ P_k = RFT, 1 \leq k \leq n\}, \quad n \geq 1. \quad (25)$$

Property 8: Two disjoint products cannot be both RFT.

Proof: Two products P and T are disjoint iff one of them, say P has a complemented literal X_i, and the other product T has the un-complemented literal $\bar{X}_i$ of the same variable. The conjunction $(P \wedge T)$ is of the form $G \wedge X_i \wedge \bar{X}_i$ and hence is definitely an $nRFT$ (Property 5). Therefore, the two products P and T cannot be both RFT (Property 6).

Property 9: A disjoint DNF has at most one RFT term.

Proof: A disjoint DNF consists of products such that no two of them are simultaneously RFT.

Property 10: If a disjoint DNF is RFT, then it has exactly one RFT term. Moreover, this particular term has a Truth exactly equal to that of the DNF.

Proof: A disjoint DNF has *at most* one RFT term (Property 9). An RFT DNF has *at least* one RFT term (Corollary of Property 7). Therefore, a disjoint RFT DNF has *exactly* one RFT term. This particular term has the maximum Truth among the Truths of the DNF terms, and is the only candidate for matching the Truth of the DNF, being the only term of Truth exceeding 0.5. Falsity of this term is not known exactly, but must be greater than or equal to that of the DNF.

5. Fuzzy satisfiability via Boolean-equation solving

The satisfiability equation (1) is now replaced by the requirement that $f(\mathbf{X})$ be an RFT of a validity

$$V(f(\mathbf{X})) = <a_f, b_f>, \quad (26)$$

with $a_f > 0.5$. When $f(\mathbf{X})$ is written as a disjoint DNF, a specific solution of (26) is that a single term T_i of this disjoint DNF be an RFT of a validity

$$V(T_i) = < a_T, b_T >, \quad (27)$$

with a Truth

$$a_T = a_f, \quad (28)$$

and a Falsity b_T such that

$$b_f \leq b_T \leq 1 - a_f. \quad (29)$$

Now, each of the variables X_k either

· Appears un-complemented in the term T_i, and hence X_k is RFT of validity $< a_k, b_k >$ such that $a_k \geq a_T$ and $b_k \leq b_T$.

· Appears complemented in the term T_i, and hence $\overline{X_k}$ is RFT of validity $< b_k, a_k >$ such that $b_k \geq a_T$ and $a_k \leq b_T$, where the variable X_k is $nRFT$ of validity $< a_k, b_k >$.

· Does not appear at all in the term T_i, and hene it is unspecified beyond being an IFl Variable.

Example 1 (revisited):

The satisfiability equation (1) is now replaced by the requirement that $f(\mathbf{X})$ defined by (2) and (3) be an RFT of validity

$$V(f(\mathbf{X})) = < 0.6, 0.3 >, \quad (30)$$

A solution of (30) is obtained if a term in Table 1 is an RFT of a Truth = 0.6 and Falsity in [0.3, 0.4]. The solution making the term $\overline{X_1}X_2\overline{X_3}$ an RFT means that X_4 and X_5 are unspecified, while each of $\overline{X_1}$, X_2, and $\overline{X_3}$ is an RFT, with $b_1 \geq 0.6$, $a_2 \geq 0.6$, and $b_3 \geq 0.6$, while each of a_1, b_2, and a_3 is in the interval [0.3, 0.4]. Other solutions corresponding to other terms in Table 1 can be deduced similarly.

6. Conclusions

Propositional logic is sound, decidable, compact, consistent and complete under plausible axioms (Paul Williams, 2009). According to the Excluded Middle principle, any variable in this logic is either 1 or 0. A *slightly richer* system applied herein to Boolean satisfiability is *IFL equipped with RFT*, in which any variable is either *RFT* or *nRFT*. The Excluded Middle concept is retained despite the fuzzification characterizing a variable by three entities (two of which are independent), namely Truth, Falsity, and Hesitancy. Further study for the *IFL system equipped with RFT* is warranted to determine its similarities and subtle differences with crisp logic, and to explore its other potential applications.

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المشبعية في منطق ضبابي حدسي ذي تحصيل حاصل واقع

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الملخص

إن أي متغير في المنطق الضبابي الحدسي (IFL) يأتي على إحدى صورتين: تحصيل حاصل ضبابي واقعي (RFT) ذي صدق يتجاوز النصف، أو تحصيل حاصل ضبابي واقعي (nRFT) ذي صدق يساوي أو ينقص عن النصف. يؤدي ذلك إلى انشطار يشابه إلى حد ما مفهوم "النصف المستبعد"، في المنطق الجاسئ أو العادي (OL) وإن كان يسمح بتوفر خاصيتي الزيف والتأرجح فضلا عن خاصية الصدق في متغير RFT. ونتيجة لذلك، يمكننا تضبيب العديد من المسائل (والحلول) في المنطق البولاني دون أي تغيير يذكر في جوهرها. قمنا بتضبيب مسألة من هذه المسائل هي مسألة المشبعية البولانية. وقد حققنا ذلك بتحويل صيغة مضروب مجاميع إلى صيغة مجموع مضاريب متنافية، ثم حل المعادلة البولانية ثنائية القيمة الناتجة. وقد تمت المحافظة على جوهر استراتيجية الحل هذه عند الانتقال من المنطق الجاسئ إلى المنطق الضبابي الحدسي وذلك بفضل مفهوم تحصيل الحاصل الضبابي الواقعي. لقد قمنا بالبرهنة على صحة جميع الخطوات اللازمة في عملية التضبيب، كما قدمنا مثالاً توضيحياً لطريقتنا في حالتي المنطق الجاسئ والمنطق الضبابي الحدسي.