Table 6. Minimum ratio of true mean life to specified life for the acceptability of a lot with producer's risk of 0.05 and under $EE(7,0.3)$

		t/μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	36.658	36.658	48.916	30.568	45.843	61.117	76.411	91.685
	1	7.883	9.154	8.628	10.783	9.293	12.389	15.489	18.585
	2	5.322	5.656	5.981	5.487	8.229	6.825	8.533	10.239
	3	3.857	4.388	3.965	4.955	5.608	4.899	6.125	7.349
	4	3.462	3.249	3.676	3.758	4.344	3.937	4.922	5.906
	5	2.960	2.972	2.967	3.068	3.606	3.360	4.201	5.040
	6	2.633	2.780	2.916	3.139	3.123	4.163	3.719	4.462
	7	2.562	2.637	2.532	2.743	2.782	3.709	3.372	4.046
	8	2.367	2.527	2.536	2.452	2.528	3.370	3.110	3.731
	9	2.217	2.259	2.286	2.546	2.852	3.108	2.904	3.484
	10	2.201	2.208	2.307	2.335	2.640	2.899	2.737	3.285
0.9	0	48.876	54.986	48.916	61.135	45.843	61.117	76.411	91.685
	1	11.427	11.824	12.216	15.267	16.17	12.389	15.489	18.585
	2	6.865	6.822	7.548	7.475	8.229	10.971	8.533	10.239
	3	5.244	5.088	5.855	6.143	5.608	7.477	6.125	7.349
	4	4.104	4.224	4.335	4.594	5.635	5.791	4.922	5.906
	5	3.688	3.709	3.966	3.708	4.601	4.807	6.010	5.040
	6	3.405	3.367	3.314	3.644	3.931	4.163	5.205	4.462
	7	3.040	3.122	3.193	3.165	3.463	3.709	4.637	4.046
	8	2.908	2.938	3.095	3.169	3.118	3.370	4.214	3.731
	9	2.685	2.794	2.774	2.857	2.852	3.108	3.886	3.484
	10	2.613	2.679	2.734	2.611	3.079	2.899	3.624	3.285
0.95	0	61.095	73.314	73.373	61.135	91.683	61.117	76.411	91.685
	1	14.963	14.484	15.777	15.267	16.17	21.558	15.489	18.585
	2	8.404	7.983	9.103	9.433	8.229	10.971	13.716	10.239
	3	5.706	5.785	5.855	6.143	7.431	7.477	9.348	7.349
	4	4.745	4.709	4.988	5.418	5.635	5.791	7.240	5.906
	5	4.171	4.075	4.459	4.336	4.601	4.807	6.010	7.211
	6	3.789	3.658	3.709	4.142	3.931	4.163	5.205	6.245
	7	3.358	3.363	3.519	3.580	4.113	4.617	4.637	5.563
	8	3.177	3.143	3.372	3.520	3.677	4.156	4.214	5.056
	9	2.919	2.972	3.014	3.163	3.343	3.802	3.886	4.663
	10	2.818	2.835	2.946	2.883	3.079	3.520	3.624	4.348
0.99	0	97.752	91.643	97.83	91.702	91.683	122.231	152.817	91.685
	1	20.262	19.793	19.328	19.718	22.895	21.558	26.953	32.34
	2	10.710	11.452	10.652	11.377	11.21	14.945	13.716	16.458
	3	7.548	7.866	7.719	8.485	9.212	9.906	9.348	11.216
	4	6.023	6.156	6.283	6.234	6.889	7.513	7.240	8.687
	5	5.136	5.168	5.438	5.573	5.561	6.134	6.010	7.211
	6	4.557	4.529	4.881	4.636	5.465	5.241	5.205	6.245
	7	4.151	4.082	4.166	4.398	4.746	4.617	5.772	5.563
	8	3.849	3.753	3.920	3.868	4.220	4.902	5.196	5.056
	9	3.501	3.677	3.729	3.767	3.819	4.457	4.753	4.663
	10	3.330	3.456	3.366	3.417	3.916	4.105	4.401	4.348

4. Interpretation and application

In this section, we explain the tabulated values of the minimum sample sizes, operating characteristic values and the minimum ratio of true mean life to specified life for the acceptability of a lot with producer's risk of 0.05. The Subsections (4.1) and (4.2) are devoted for $EE(0.5,5)$ and $EE(7,0.3)$, respectively. In Subsection (4.3) a real data set is considered to illustrate the application of the suggested acceptance sampling plans in the field.

4.1 Results for $EE(0.5,5)$

When the lifetime followed the extended exponential distribution with parameters $\alpha = 0.5$ and $\beta = 5$, the smallest sample sizes necessary to ensure that the mean life exceeds μ_0 with probability at least P^*, with associated acceptance number c are presented in Table 1. For example, assume that the researcher want to assert that the mean life is at least 1000 hours with probability $P^* = 0.75$. For the acceptance number $c = 2$, the life test is at least $t = 1257$ hours $\left(t/\mu_0 = 1.257\right)$. Therefore, the tabulated sample size value is $m = 3$ units, and these units have to be tested. Now, if at most 2 units out of 3 units fail before a specified time t within 1000 hours, the lot is accepted with a confidence of 0.75 and rejected elsewhere. Thus, the process is to truncate the time test with a time of 1.257 times of the determined mean life. That is the average life is more than or equal 1000 hours.

In Table 3, we summarized the values of the operating characteristic function for the proposed acceptance sampling plans adopted from Table 1 for different values of t/μ_0 and various values of P^* when the acceptance number c is 2. Therefore, based on Table 3, the operating characteristic values for the sampling plan $\left(m=3,\ c=2,\ t/\mu_0=1.257\right)$ are:

$\frac{\mu}{\mu_0}$	2	4	6	8	10	12
$L(p)$	0.658140	0.908451	0.964379	0.982719	0.990368	0.994098

This implies that if the true mean life is four times the specified mean life $\left(\mu/\mu_0=4\right)$, the producer's risk is about 0.091549, and for $\mu/\mu_0=6,8,10,12,$ the producer's risk is about $0.035621, 0.017281, 0.009632, 0.005902$, respectively. However, the producer's risk approaches zero the mean life is at least 10000 or $\mu/\mu_0 \geq 10$.

Table 5 can be used to find the minimum ratio of the true average lifetime to the specified mean life (μ/μ_0) for the acceptance of a lot with the producer's risk $\delta=0.05$, for different values of the acceptance number c and t/μ_0. For the above example, the smallest value of μ/μ_0 is 5.212 when $P^*=0.75$, $t/\mu_0=1.257$, and $c=2$, or the consumer's risk is 0.25. Specifically, the product must have a mean life more than 5212 hours, $\mu/\mu_0=5.212$ in order to accept the lot with probability $P^*=0.75$. However, the true mean life time needed to accept 75% of the lots is provided in Table 3 based on the suggested acceptance sampling plan.

4.2 Results for $EE(7,0.3)$

In this subsection, the suggested acceptance sampling plan is considered, when the lifetime follows the extended exponential distribution with parameters $\alpha=7$ and $\beta=0.3$. The process given in the above subsection can be followed here to interpret the results. Therefore, we will not explain the Tables 2, 4 and 6 again; but we will compare the results in both cases $EE(0.5,5)$ and $EE(7,0.3)$. Based on the Tables 1 and 2, it is noted that the sample size necessary to assert the mean life μ to exceed a given value μ_0 for $EE(0.5,5)$ are less than that of $EE(7,0.3)$ in general, while they are equal when $P^*=0.75$ with $t/\mu_0=3.927, 4.712$ and when $P^*=0.90$ with $t/\mu_0=4.712$. Therefore, when $\alpha=0.5$ and $\beta=5$ the operating characteristic values in Table 3 are smaller than their counterparts when $\alpha=7$ and $\beta=0.3$ in Table 4, and hence the minimum ratio of true mean life to specified life for the acceptability of a lot with producer's risk of 0.05 for $EE(0.5,5)$ are larger than their counterparts of $EE(7,0.3)$.

In general, all vales of the sample size m obtained in this paper for the suggested acceptance sampling plan with extended exponential distribution are found to be less than the corresponding values of m tabulated by Baklizi & El Masri (2004) for Birnbaum Saunders model, Kantam *et al*. (2001) for log-logistic model and Gupta & Groll (1961) for gamma distribution.

4.3. An application

Now we consider a data set which was already considered by Wood (1996). The data set represents the failure time in hours of a software, which represents the lifetimes from the starting of the execution of the software until which the failure of the software is experienced. We have the following observations; 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218. The mean and variance for these data are 2624.78 and 2.54×10^6, respectively.

The maximum likelihood estimators for the extended exponential distribution parameters α and β based on the $m=9$ data are $\hat{\alpha}=0.0004$ and $\hat{\beta}=6.8\times10^{-10}$, respectively. For the data of the release of a software, the Kolmogorov-Smirnov (K-S) test, Anderson-Darling (A-D) statistic, Cramér-von Mises (C-M) test, Kuiper statistic, Pearson χ^2, Watson U^2 statistic are considered to fit these real data. Their values are given in Table 7.

It is clear from these statistics that the extended exponential distribution provides good fit to this software data set.

Let the specified mean life is $\mu_0=1000$ hours and the testing time be $t=1257$ hours, that is the ratio will be $\left(t/\mu_0=1.257\right)$. Therefore, for acceptance number $c=2$ with probability of $P^*=0.99$, the required sample size is found from Table 1 to be 9. Thus the acceptance sampling plan from truncated life test based on the $EE(0.5,5)$ is $\left(m=9, c=2, t/\mu_0=1.257\right)$. For the acceptance number $c=2$, the life test is at least $t=1257$ hours.

Table 7. Goodness-of-Fit Statistics values

Statistic	K-S	A-D	C-M	Kuiper	Pearson χ^2	Watson U^2
	0.197824	0.628838	0.106867	0.223686	1.55556	0.0601004
P-Value	0.80865	0.614845	0.552245	0.689428	0.816757	0.652319

Based on the software data, we have to decide whether to reject or accept the lot. The lot will be accepted only if the number of failures before 1257 hours is at most 2. Since there are two failures at 968 and 512 from the 9 software data before the time $t = 1257$ hours, then we will accept the lot, asserting a mean life time 1000 hours with probability $P^* = 0.99$.

5. Conclusions and suggestions

In this paper, acceptance sampling plans for the extended exponential life time distribution based on truncated lifetime tests are proposed. To the best of our knowledge, the acceptance sampling plan for the extended exponential distribution has not been developed in the literature. Various values of the extended exponential distribution parameters are considered and the necessary tables based on the suggested sampling plan are presented. It turns out that the proposed sampling plan can be easy to operate for practitioners. However, the author may be interested in developing sampling plans based on the extended exponential percentiles to satisfy both consumer's risk and producer's risk simultaneously.

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مخططات قبول العينات على اساس اختبارات الحياة المقطوعة للتوزيع الاسي الموسع

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الملخص

في هذا البحث، تم اقتراح مخططات قبول عينات للتوزيع الأسي الموسع عندما يتم اقتطاع وقت اختبار الحياة في وقت ثابت محدد مسبقاً. تم ايجاد الحد الأدنى من حجم العينة المطلوبة لتأكيد متوسط الحياة المحدد المقابل لخطر محدد من قبل العميل. تم توفير قيم لدالة التشغيل المميزة لمخططات قبول العينات المقترح وقيم مخاطر المنتج. تم تزويد بعض الجداول وتوضيح النتائج بأمثلة عددية. وأوضح تطبيق لمجموعة من البيانات الحقيقية أنه يمكن استخدام مخططات قبول العينات المقترحة في الصناعة.