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Acceptance sampling plans based on truncated life tests for extended exponential distribution

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Abstract

In this paper, acceptance sampling plans are suggested for the extended exponential distribution mean, when the life time test is truncated at a pre-fixed time. The minimum sample sizes needed to assert the specified life mean are presented under a given customer's risk. The operating characteristic function values of the proposed sampling plans and producer's risk are provided. Some tables are given and the results are illustrated by numerical examples. An application of a real data set reveals that the proposed acceptance sampling plans can be used in the industry.

Keywords: Acceptance sampling plans; consumer's risk; extended exponential distribution; operating characteristic; producer's risk.

1. Introduction

Acceptance sampling plan is an inspection method used to determine, whether to accept or reject a specific lot of material. The main problem in acceptance sampling plan is the total time spends in testing the lot. Also, finding the smallest sample size necessary to ensure a certain mean life, when the life test is terminated at a pre-assigned time t, and not to number of failures for a given acceptance number c. Therefore, the decision is to accept a lot, if the given life can be established with a pre-determined high probability P^*, where the confidence level P^* is the chance of rejecting a bad lot having mean life time $\mu < \mu_0$ is at least P^*.

In recent statistical literature different acceptance sampling plans based on truncated life tests for the population mean are investigated; for example, by Sobel & Tischendrof (1959) for exponential distribution, Balakrishnan (2007) for generalized Birnbaum–Saunders distribution Al-Nasser & Al-Omari (2013) for the exponentiated Fréchet distribution, Al-Omari (2014) for three parameters Kappa distribution and Al-Omari (2015) for generalized inverted exponential distribution. Al-Omari *et al*. (2016) investigated double acceptance sampling plan for time-truncated life tests based on half normal distribution. Kantam *et al*. (2001) considered truncated life tests for log-logistic distribution Aslam *et al*. (2010) for proposed variables sampling plan for life testing in a continuous process under Weibull distribution, and Baklizi & El Masri (2004) for Birnbaum Saunders model.

Lio *et al*. (2010) investigated the acceptance sampling plans based on truncated life tests for the Burr type XII percentiles Rosaiah & Kantam (2005) introduced acceptance sampling based on the inverse Rayleigh distribution Rao & Kantam (2010) for the percentiles of the log-logistic distribution. Al-Omari (2016) for generalized inverse Weibull distribution. Acceptance sampling plans for percentiles based on the inverse Rayleigh distribution are suggested by Rao *et al*. (2012). Ramaswamy & Jayasri (2013, 2014) suggested time truncated chain sampling plans for Marshall-Olkin extended exponential and generalized Rayleigh distribution. Double acceptance sampling plan based on truncated life tests is another acceptance sampling method, which is considered by Rao (2011) for Marshall–Olkin extended exponential distribution and Aslam & Jun (2010) for generalized log-logistic distribution. Al-Omari & Zamanzade (2017) proposed double acceptance sampling plan for transmuted generalized inverse Weibull distribution and Ramaswamy & Anburajan (2012) for generalized exponential distribution.

The main object of this article is to present yet another acceptance sampling plan that can be used as an alternative to the ones mentioned above. By exploring the literature, we note that there is no study in acceptance sampling plans based on the extended exponential distribution before. Therefore, we suggested new acceptance sampling plan based on truncated life tests for the extended exponential distribution. However, the suggested acceptance sampling plan can be considered using other extensions of the extended exponential distribution.

The rest of the paper is arranged as follows. In the next section, we introduce the extended exponential distribution briefly. In the third section, acceptance sampling plans for the truncated life tests based on the extended exponential distribution are developed. Some illustrated tables of the proposed acceptance sampling plans and their descriptions are given in fourth section as well as a real data set is used to illustrate the performance of the suggested sampling plans. Conclusions and suggestions are summarized in the last section.

2. Extended exponential distribution

Gómez *et al*. (2014) proposed an extension of the exponential distribution based on mixture of positive distributions, namely; extended exponential distribution. A random variable X is said to have an extended exponential distribution with parameters α and β, $EE(\alpha,\beta)$, if its probability density function (pdf) is given by

$$f_X(x)=\frac{\alpha^2(1+\beta x)}{\alpha+\beta}e^{-\alpha x},$$
$$x>0,\alpha>0,\beta\geq 0. \quad (1)$$

The corresponding cumulative distribution function (cdf) of the $EE(\alpha,\beta)$ is defined as

$$F_X(x)=\frac{\alpha+\beta-(\alpha+\beta+\alpha\beta x)e^{-\alpha x}}{\alpha+\beta}. \quad (2)$$

From Gómez *et al*. (2014), the following properties of the extended exponential distribution can be presented in this paper. The mean and variance of the $EE(\alpha,\beta)$, respectively, are

$$E(X)=\frac{\alpha+2\beta}{\alpha(\alpha+\beta)}, \quad (3)$$

and

$$Var(X)=\frac{\alpha^3+5\alpha^2\beta+6\alpha\beta^2+2\beta^2}{\alpha^5+3\alpha^4\beta+3\alpha^3\beta^2+\alpha^2\beta^3}. \quad (4)$$

If the quality parameter $\mu=\mu_0$, then $\mu_0=\dfrac{\alpha_0+2\beta_0}{\alpha_0(\alpha_0+\beta_0)}$,

where μ_0 is the specified mean life time. The moment generating function and the rth moment of the $EE(\alpha,\beta)$ are

$$M_X(t)=\frac{\alpha^2(\alpha+\beta-t)}{(\alpha+\beta)(t-\alpha)^2}, \quad (5)$$

and

$$E(X^r)=\frac{r\Gamma(r)}{\alpha^r(1+\beta)}[\alpha+(1+r)\beta], \quad (6)$$

$r=1,2,\ldots,$

where $\Gamma(.)$ is the gamma function. The hazard rate function $H(x)$ of the $EE(\alpha,\beta)$ random variable is

$$H(x)=\frac{f_X(x)}{1-F_X(x)}=\frac{\alpha^2(1+\beta x)}{\beta+\alpha(1+\beta x)}. \quad (7)$$

The maximum likelihood estimator (MLE) of β is $\hat{\beta}=\dfrac{\hat{\alpha}(1-\hat{\alpha}\bar{x})}{\hat{\alpha}\bar{x}-2}$, while the maximum likelihood estimator of α can be obtained by solving numerically the equation $\sum_{i=1}\dfrac{x_i}{1-(1-\bar{x}\hat{\alpha})(\hat{\alpha}x_i-1)}=\dfrac{n}{\hat{\alpha}}$, where $\hat{\alpha}\in\left(\dfrac{1}{\bar{x}},\dfrac{2}{\bar{x}}\right)$. The moment estimators of the parameters β and α are $\tilde{\beta}=\dfrac{\tilde{\alpha}(1-\tilde{\alpha}\bar{x})}{\tilde{\alpha}\bar{x}-2}$, $\tilde{\alpha}\in\left(\dfrac{1}{\bar{x}},\dfrac{2}{\bar{x}}\right)$, and $\tilde{\alpha}=\dfrac{2\bar{x}\pm\sqrt{4\bar{x}^2-2\overline{x^2}}}{\overline{x^2}}$. For more details about the $EE(\alpha,\beta)$, see Gómez *et al*. (2014).

3. The suggested acceptance sampling plans

We assumed that the life time of a product followed an extended exponential distribution defined in Equations (1) and (2). However, to the best of our knowledge, the acceptance sampling plan based on the extended exponential distribution has not been considered before.

An acceptance sampling plan based on truncated life tests consists of:

(1) The number of units m on test.

(2) The acceptance number c.

(3) The maximum test duration time t.

(4) The ratio, t/μ_0 where μ_0 is the specified average life.

3.1 Minimum sample size

Given P^* and assuming that the lot size is large enough to be considered infinite, then the probability of accepting a lot can be obtained based on the cumulative binomial distribution function up to acceptance number c and a smallest sample size m, to ensure that $\mu>\mu_0$ must satisfy the inequality

$$\sum_{i=0}^{c}\binom{m}{i}p^i(1-p)^{m-i}\leq 1-P^*, \quad (8)$$

where $P^*\in(0,1)$, and $p=F(t;\mu_0)$ is a monotonically increasing function of t/μ_o and known as the probability of a failure observed during the time t.

If the number of observed failures within the time t is at most c, then from Inequality (8) we can confirm with probability P^* that $F(t;\mu)\leq F(t;\mu_0)$, which implies $\mu_0\leq\mu$.

The minimum sample sizes that satisfy above inequality for $t/\mu_0=0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, with P^* = 0.75, 0.9, 0.95, 0.99 and $c=0,1,2,\ldots,10$. The values of t/μ_0 and P^* presented in this work are the same with the corresponding values of Baklizi & El Masri (2004) for Birnbaum Saunders model, Kantam *et al*. (2001) for log-logistic model and Gupta & Groll (1961) for gamma distribution. The minimum sample sizes based on the suggested acceptance sampling plan are presented in Table 1 for $\alpha=0.5$ and $\beta=5$, while for $\alpha=7$ and $\beta=0.3$ they are presented in Table 2.

Table 1. Minimum sample sizes *m* to be tested for a time *t* necessary to assert the mean life μ to exceed a given value μ_0, with probability P^* and acceptance number *c* for $EE(0.5, 5)$

		t/μ_0							
P^*	*c*	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	2	1	1	1	1	1	1	1
	1	3	3	2	2	2	2	2	2
	2	5	4	3	3	3	3	3	3
	3	7	5	5	4	4	4	4	4
	4	8	7	6	5	5	5	5	5
	5	10	8	7	7	6	6	6	6
	6	11	9	8	8	7	7	7	7
	7	13	10	9	9	8	8	8	8
	8	14	12	10	10	9	9	9	9
	9	16	13	12	11	10	10	10	10
	10	17	14	13	12	11	11	11	11
0.90	0	2	2	1	1	1	1	1	1
	1	4	3	3	2	2	2	2	2
	2	6	5	4	4	3	3	3	3
	3	8	6	5	5	4	4	4	4
	4	10	7	6	6	5	5	5	5
	5	11	9	8	7	6	6	6	6
	6	13	10	9	8	7	7	7	7
	7	14	11	10	9	8	8	8	8
	8	16	13	11	10	9	9	9	9
	9	18	14	12	11	11	10	10	10
	10	19	15	14	13	12	11	11	11
0.95	0	3	2	2	1	1	1	1	1
	1	5	4	3	3	2	2	2	2
	2	7	5	4	4	3	3	3	3
	3	9	7	6	5	4	4	4	4
	4	10	8	7	6	6	5	5	5
	5	12	9	8	7	7	6	6	6
	6	14	11	9	9	8	7	7	7
	7	16	12	11	10	9	8	8	8
	8	17	13	12	11	10	9	9	9
	9	19	15	13	12	11	10	10	10
	10	20	16	14	13	12	11	11	11
0.99	0	4	3	2	2	2	1	1	1
	1	7	5	4	3	3	2	2	2
	2	9	6	5	5	4	3	3	3
	3	11	8	7	6	5	4	4	4
	4	12	9	8	7	6	6	5	5
	5	14	11	9	8	7	7	6	6
	6	16	12	10	9	8	8	7	7
	7	18	14	12	11	9	9	8	8
	8	20	15	13	12	10	10	9	9
	9	21	16	14	13	11	11	10	10
	10	23	18	15	14	12	12	11	11

Table 2. Minimum sample sizes m to be tested for a time t necessary to assert the mean life μ to exceed a given value μ_0, with probability P^* and acceptance number c under the $EE(7, 0.3)$

		t/μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	1	1	1	1	1
	1	5	4	3	3	2	2	2	2
	2	8	6	5	4	4	3	3	3
	3	10	8	6	6	5	4	4	4
	4	13	9	8	7	6	5	5	5
	5	15	11	9	8	7	6	6	6
	6	17	13	11	10	8	8	7	7
	7	20	15	12	11	9	9	8	8
	8	22	17	14	12	10	10	9	9
	9	24	18	15	14	12	11	10	10
	10	27	20	17	15	13	12	11	11
0.90	0	4	3	2	2	1	1	1	1
	1	7	5	4	4	3	2	2	2
	2	10	7	6	5	4	4	3	3
	3	13	9	8	7	5	5	4	4
	4	15	11	9	8	7	6	5	5
	5	18	13	11	9	8	7	7	6
	6	21	15	12	11	9	8	8	7
	7	23	17	14	12	10	9	9	8
	8	26	19	16	14	11	10	10	9
	9	28	21	17	15	12	11	11	10
	10	31	23	19	16	14	12	12	11
0.95	0	5	4	3	2	2	1	1	1
	1	9	6	5	4	3	3	2	2
	2	12	8	7	6	4	4	4	3
	3	14	10	8	7	6	5	5	4
	4	17	12	10	9	7	6	6	5
	5	20	14	12	10	8	7	7	7
	6	23	16	13	12	9	8	8	8
	7	25	18	15	13	11	10	9	9
	8	28	20	17	15	12	11	10	10
	9	30	22	18	16	13	12	11	11
	10	33	24	20	17	14	13	12	12
0.99	0	8	5	4	3	2	2	2	1
	1	12	8	6	5	4	3	3	3
	2	15	11	8	7	5	5	4	4
	3	18	13	10	9	7	6	5	5
	4	21	15	12	10	8	7	6	6
	5	24	17	14	12	9	8	7	7
	6	27	19	16	13	11	9	8	8
	7	30	21	17	15	12	10	10	9
	8	33	23	19	16	13	12	11	10
	9	35	26	21	18	14	13	12	11
	10	38	28	22	19	16	14	13	12

3.2 Operating characteristic of the sampling plan $(m,\ c,\ t/\mu_0)$

For the sampling plan $(m,\ c,\ t/\mu_0)$, the operating characteristic (OC) function gives the probability of accepting the lot is given by

$$
\begin{aligned}
OC(p) &= P(\text{Accepting a lot}) \\
&= \sum_{i=0}^{c} \binom{m}{i} p^{i}(1-p)^{m-i} \\
&= 1 - I_p(c+1, m-c),
\end{aligned}
\tag{9}
$$

where $p = F(t;\mu)$ is a function of μ (the lot quality parameter), and $I_p(c+1, m-c)$ is the incomplete beta function defined as $I_p(w,u) = \frac{(w+u-1)!}{(w-1)!(u-1)!}\int_0^p z^{w-1}(1-z)^{u-1}dz$, $w,u > 0$. It is of interest to say that for fixed t, p itself is a monotonically decreasing function of $\mu \geq \mu_0$, while the operating characteristic function is a decreasing function of p. The OC function values as a function of $\mu \geq \mu_0$ for the sampling plan $(m,\ c=2,\ t/\mu_0)$ for the $EE(\alpha,\beta)$ with parameters $\alpha = 0.5$ and $\beta = 5$ are presented in Table 3 and for the parameters $\alpha = 7$ and $\beta = 0.3$ the OC function values are provided in Table 4.

Table 3. Operating characteristic values for the sampling plan $(m, c, t/\mu_0)$ for a given probability P^*, under the $EE(0.5,5)$

P^*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	5	0.628	0.592330	0.887470	0.955908	0.978551	0.988028	0.992659
	4	0.942	0.536713	0.862252	0.944384	0.972509	0.984503	0.990433
	3	1.257	0.658140	0.908451	0.964379	0.982719	0.990368	0.994098
	3	1.571	0.529776	0.853396	0.939381	0.969624	0.982726	0.989271
	3	2.356	0.281214	0.691747	0.853459	0.920735	0.952705	0.969640
	3	3.141	0.139007	0.529970	0.747501	0.853490	0.908547	0.939426
	3	3.927	0.066355	0.390879	0.636187	0.774915	0.853434	0.899986
	3	4.712	0.031168	0.281214	0.529905	0.691747	0.791218	0.853459
0.90	6	0.628	0.440641	0.817121	0.923507	0.961495	0.978053	0.986350
	5	0.942	0.329456	0.746685	0.88747	0.941498	0.965980	0.978551
	4	1.257	0.349670	0.757587	0.892677	0.944279	0.967619	0.979592
	4	1.571	0.216073	0.645492	0.828963	0.906930	0.944300	0.964182
	3	2.356	0.281214	0.691747	0.853459	0.920735	0.952705	0.969640
	3	3.141	0.139007	0.529970	0.747501	0.853490	0.908547	0.939426
	3	3.927	0.066355	0.390879	0.636187	0.774915	0.853434	0.899986
	3	4.712	0.031168	0.281214	0.529905	0.691747	0.791218	0.853459
0.95	7	0.628	0.315597	0.738860	0.883681	0.939461	0.964775	0.977784
	5	0.942	0.329456	0.746685	0.887470	0.941498	0.965980	0.978551
	4	1.257	0.349670	0.757587	0.892677	0.944279	0.967619	0.979592
	4	1.571	0.216073	0.645492	0.828963	0.906930	0.944300	0.964182
	3	2.356	0.281214	0.691747	0.853459	0.920735	0.952705	0.969640
	3	3.141	0.139007	0.529970	0.747501	0.853490	0.908547	0.939426
	3	3.927	0.066355	0.390879	0.636187	0.774915	0.853434	0.899986
	3	4.712	0.031168	0.281214	0.529905	0.691747	0.791218	0.853459
0.99	9	0.628	0.148891	0.577251	0.788079	0.882381	0.928760	0.953822
	6	0.942	0.188896	0.623706	0.817121	0.900189	0.940174	0.961495
	5	1.257	0.164109	0.591847	0.796457	0.887275	0.931801	0.955823
	5	1.571	0.076551	0.448935	0.694878	0.820665	0.887314	0.925073
	4	2.356	0.056472	0.391678	0.645611	0.785087	0.862130	0.906974
	3	3.141	0.139007	0.529970	0.747501	0.853490	0.908547	0.939426
	3	3.927	0.066355	0.390879	0.636187	0.774915	0.853434	0.899986
	3	4.712	0.031168	0.281214	0.529905	0.691747	0.791218	0.853459

Table 4. Operating characteristic values for the sampling plan $(n,c,t/\mu_0)$ for a given probability P^*, with acceptance number $c=2$ under $EE(7,0.3)$

			μ/μ_0					
P^*	m	t/μ_0	2	4	6	8	10	12
0.75	8	0.628	0.630488	0.902940	0.962701	0.982040	0.990039	0.993918
	6	0.942	0.595716	0.889113	0.956668	0.978951	0.988262	0.992807
	5	1.257	0.563313	0.875126	0.950389	0.975689	0.986369	0.991616
	4	1.571	0.619350	0.896698	0.959724	0.980452	0.989103	0.993324
	4	2.356	0.362857	0.765633	0.896746	0.946524	0.968969	0.980463
	3	3.141	0.503471	0.839426	0.932539	0.965899	0.980500	0.987842
	3	3.927	0.364928	0.755937	0.889533	0.941840	0.965883	0.978353
	3	4.712	0.258265	0.668967	0.839392	0.912093	0.947145	0.965890
0.90	10	0.628	0.469135	0.832892	0.931187	0.965650	0.980522	0.987929
	7	0.942	0.474986	0.835258	0.932212	0.966171	0.980821	0.988115
	6	1.257	0.409205	0.798946	0.914507	0.956584	0.975118	0.984467
	5	1.571	0.418578	0.803226	0.916439	0.957587	0.975699	0.984832
	4	2.356	0.362857	0.765633	0.896746	0.946524	0.968969	0.980463
	4	3.141	0.193428	0.619535	0.812543	0.896770	0.937746	0.959756
	3	3.927	0.364928	0.755937	0.889533	0.941840	0.965883	0.978353
	3	4.712	0.258265	0.668967	0.839392	0.912093	0.947145	0.965890
0.95	12	0.628	0.333914	0.752732	0.891178	0.943732	0.967394	0.979493
	8	0.942	0.369374	0.775657	0.902940	0.950263	0.971338	0.982040
	7	1.257	0.285793	0.715447	0.870841	0.932086	0.960231	0.974808
	6	1.571	0.265873	0.697971	0.860889	0.926262	0.956601	0.972413
	4	2.356	0.362857	0.765633	0.896746	0.946524	0.968969	0.980463
	4	3.141	0.193428	0.619535	0.812543	0.896770	0.937746	0.959756
	4	3.927	0.097352	0.481317	0.717144	0.834962	0.896727	0.931512
	3	4.712	0.258265	0.668967	0.839392	0.912093	0.947145	0.965890
0.99	15	0.628	0.188396	0.625896	0.819031	0.901521	0.941089	0.962138
	11	0.942	0.155491	0.585898	0.793693	0.885886	0.931032	0.955360
	8	1.257	0.193553	0.630025	0.821244	0.902768	0.941846	0.962628
	7	1.571	0.161341	0.591300	0.796730	0.887639	0.932111	0.956066
	5	2.356	0.174148	0.602956	0.803307	0.891448	0.934465	0.957609
	5	3.141	0.064363	0.418789	0.670248	0.803347	0.875253	0.916499
	4	3.927	0.097352	0.481317	0.717144	0.834962	0.896727	0.931512
	4	4.712	0.047339	0.362857	0.619473	0.765633	0.848088	0.896746

3.3 Producer's risk

The producer's risk is defined as the probability of rejecting the lot, when $\mu > \mu_0$, and it is given by

$$P(p) = P(\text{Rejecting a lot}) = \sum_{i=c+1}^{m}\binom{m}{i}p^i(1-p)^{m-i}. = I_p(c+1, m-c) \tag{10}$$

For a given value of the producer's risk, say δ , under a given sampling plan, one may be interested in knowing what is the smallest value of $\mu > \mu_0$ that will ensure that the producer's risk is at most δ. The value of $\mu > \mu_0$ is the minimum positive number for which $p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ satisfies the inequality

$$PR(p) = \sum_{i=c+1}^{m}\binom{m}{i}p^i(1-p)^{m-i} \le \delta. \tag{11}$$

For a given acceptance sampling plan $(m, c, t/\mu_0)$ at a given confidence level P^*, the smallest value of μ/μ_0 satisfying Inequality (11) are presented in Table 5 for $\alpha = 0.5$ and $\beta = 5$ and they are presented in Table 6 for $\alpha = 7$ and $\beta = 0.3$.

Table 5. Minimum ratio of true mean life to specified life for the acceptability of a lot with producer's risk of 0.05 under the $EE(0.5,5)$

		t/μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	46.556	34.921	46.599	58.239	87.339	116.44	145.578	174.678
	1	8.215	12.323	9.454	11.815	17.718	23.622	29.533	35.436
	2	5.697	6.274	5.212	6.514	9.769	13.024	16.283	19.538
	3	4.682	4.278	5.709	4.679	7.017	9.354	11.695	14.033
	4	3.503	4.299	4.424	3.762	5.642	7.521	9.404	11.283
	5	3.307	3.511	3.674	4.591	4.817	6.422	8.030	9.634
	6	2.779	3.001	3.183	3.978	4.266	5.688	7.111	8.532
	7	2.731	2.645	2.836	3.545	3.870	5.159	6.450	7.740
	8	2.418	2.807	2.578	3.222	3.570	4.760	5.951	7.140
	9	2.413	2.553	2.907	2.972	3.335	4.446	5.558	6.669
	10	2.200	2.352	2.693	2.773	3.145	4.192	5.241	6.289
0.9	0	46.556	69.833	46.599	58.239	87.339	116.44	145.578	174.678
	1	11.630	12.323	16.443	11.815	17.718	23.622	29.533	35.436
	2	7.188	8.545	8.372	10.464	9.769	13.024	16.283	19.538
	3	5.577	5.666	5.709	7.135	7.017	9.354	11.695	14.033
	4	4.751	4.299	4.424	5.529	5.642	7.521	9.404	11.283
	5	3.779	4.242	4.685	4.591	4.817	6.422	8.030	9.634
	6	3.535	3.592	4.005	3.978	4.266	5.688	7.111	8.532
	7	3.043	3.140	3.529	3.545	3.870	5.159	6.450	7.740
	8	2.950	3.221	3.177	3.222	3.570	4.760	5.951	7.140
	9	2.873	2.915	2.907	2.972	4.457	4.446	5.558	6.669
	10	2.607	2.674	3.138	3.365	4.158	4.192	5.241	6.289
0.95	0	69.830	69.833	93.185	58.239	87.339	116.44	145.578	174.678
	1	15.019	17.444	16.443	20.551	17.718	23.622	29.533	35.436
	2	8.6680	8.545	8.372	10.464	9.769	13.024	16.283	19.538
	3	6.466	7.023	7.561	7.135	7.017	9.354	11.695	14.033
	4	4.751	5.254	5.737	5.529	8.291	7.521	9.404	11.283
	5	4.248	4.242	4.685	4.591	6.885	6.422	8.03	9.634
	6	3.908	4.169	4.005	5.005	5.965	5.688	7.111	8.532
	7	3.662	3.622	4.189	4.41	5.316	5.159	6.45	7.740
	8	3.213	3.221	3.746	3.971	4.832	4.76	5.951	7.140
	9	3.101	3.270	3.407	3.633	4.457	4.446	5.558	6.669
	10	2.808	2.989	3.138	3.365	4.158	4.192	5.241	6.289
0.99	0	93.105	104.745	93.185	116.462	174.656	116.44	145.578	174.678
	1	21.771	22.529	23.277	20.551	30.819	23.622	29.533	35.436
	2	11.613	10.781	11.402	14.25	15.692	13.024	16.283	19.538
	3	8.233	8.365	9.372	9.450	10.700	9.354	11.695	14.033
	4	5.984	6.195	7.011	7.169	8.291	11.054	9.404	11.283
	5	5.180	5.669	5.661	5.855	6.885	9.179	8.030	9.634
	6	4.650	4.738	4.793	5.005	5.965	7.952	7.111	8.532
	7	4.275	4.565	4.832	5.236	5.316	7.087	6.450	7.740
	8	3.995	4.028	4.298	4.682	4.832	6.442	5.951	7.140
	9	3.553	3.620	3.890	4.257	4.457	5.943	5.558	6.669
	10	3.406	3.606	3.568	3.922	4.158	5.543	5.241	6.289