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Nonlinear vibration of mechanical systems by means of Homotopy perturbation method

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ABSTRACT

In this study, it has been tried to present a new approximate method by using Homotopy Perturbation Method (HPM) for high nonlinear problems. Three different examples are considered and the application of the Homotopy perturbation method is studied. Runge-Kutta algorithm is used to obtain numerical results. Another analytical method called Energy Balance Method (EBM) is applied to compare the results of HPM and Runge-Kutta algorithm. It has been shown that only one iteration of the method prepares high accurate solution for whole domain. It has been established that Homotopy perturbation method does not need any linearization and overcome the limitations of the perturbation methods.

Keywords: Energy balance method; Homotopy perturbation method; nonlinear oscillators; Runge-Kutta algorithm.

INTRODUCTION

Dynamical models of the problems are usually presented by differential equations. Differential equations are linear and nonlinear. Linear differential equations have exact solutions but when they are nonlinear, it is really hard to find an exact solution for the problem. Therefore, in recent years, finding an exact and analytical solution for the nonlinear differential equations is very important. The effects of important parameters on the nonlinear response of the problems can be easily considered, when we have its analytical solution. The traditional analytical methods have lots of limitations. To overcome these shortcomings, some new approximate methods have been presented to analyze high nonlinear problems. Recently, some researchers have worked on the numerical and analytical methods such as: Homotopy perturbation method (Bayat *et al.*, 2012 ; He,1999) , Hamiltonian approach (Bayat & Pakar., 2011b, 2013b; Bayat *et al.*, 2014a,b), energy balance method (Bayat & Pakar, 2011a; Mehdipour *et al.*, 2010), Variational iteration method (Dehghan & Tatari, 2008; Pakar *et al.*, 2012), amplitude frequency formulation (He, 2008; Bayat *et al.*, 2011; Pakar & Bayat., 2013b), max-min approach (Shen & Mo, 2009; Pakar & Bayat, 2013a), Variational approach method (He, 2007; Xu & Zhang, 2009) and the other analytical and numerical (Cordero *et al.*, 2010; Bayat & Pakar, 2012; 2013a; Bayat *et al.*, 2014; Bor-Lih & Cheng-Ying, 2009; Odibat *et al.*, 2008; Pakar & Bayat, 2012, 2013b; Wu, 2011; Nayfeh & Mook,1973).

Among of these new approximate methods; Homotopy perturbation method is used in this study. Only one iteration of this method leads us to a high accurate solution. Three nonlinear mechanical systems are presented to apply the homotopy perturbation method. The results of homotopy perturbation method are compared with Runge-Kutta's algorithm and energy balance method; it has been demonstrated that the Homotopy perturbation method can be a strong mathematical tool for conservative nonlinear problems.

CONCEPT OF HOMOTOPY PERTURBATION

To explain the basic idea of the Homotopy perturbation method for solving nonlinear differential equations, one may consider the following nonlinear differential equation (He, 1999):

$$A(u)-f(r)=0 \quad r\in\Omega \tag{1}$$

That is subjected to the following boundary condition:

$$B\left(u,\frac{\partial u}{\partial t}\right)=0 \quad r\in\Gamma \tag{2}$$

Where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function, Γ is the boundary of the solution domain (Ω), and $\partial u/\partial t$ denotes differentiation along the outwards normal to Γ. Generally, the operator A may be divided into two parts: a linear part L and a nonlinear part N. Therefore, Equation (3) can be rewritten as follows:

$$L(u)+N(u)-f(r)=0 \quad r\in\Omega \tag{3}$$

By the construct of Homotopy technique, $\nu(r,p)$: $\Omega\times[0,1]\rightarrow R$, which satisfies

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0 \tag{4}$$

Or

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[N(v)-f(r)\right] \tag{5}$$

In Equation (4), $p \in [0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. One may assume that solution of Equation (6) may be written as a power series in *p*, as the following:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \quad (6)$$

The Homotopy parameter *p* is also used to expand the square of the unknown angular frequency ω as follows:

$$\omega_0 = \omega^2 - p\omega_1 - p^2\omega_2 - \ldots \quad (7)$$

Or

$$\omega^2 = \omega_0 + p\omega_1 + p^2\omega_2 + \ldots \quad (8)$$

where ω_0 is the coefficient of $u(r)$ in Equation(3) and should be substituted by the right hand side of Equation(4). Besides, $\omega_i\ (i = 1,2,\ldots)$ are arbitrary parameters that have to be determined.

The best approximations for the solution and the angular frequency ω are

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \quad (9)$$

$$\omega^2 = \omega_0 + \omega_1 + \omega_2 + \ldots \quad (10)$$

APPLICATION

In order to assess the advantages and the accuracy of the Homotopy perturbation method, we will consider the following two examples:

Example 1

In this example we have Duffing equation with constant coefficient that presents in Figure 1 (Mehdipour *et al.*, 2010):

$$\ddot{u} + \frac{k_1}{m}u + \frac{k_2}{2mh^2}u^3 = \frac{F_0}{m}\sin\omega_0 t, \qquad u(0) = A, \qquad \dot{u}(0) = 0 \quad (11)$$

In which *u* and *t* are generalized dimensionless displacements and time variables, respectively.

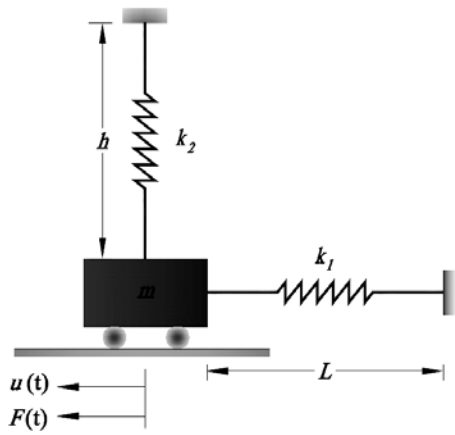

Fig. 1. The physical model of Duffing equation with constant coefficient

Here is the application of Homotopy perturbation to Equation (11). We construct a Homotopy in the following form:

$$H(u,p)=(1-p)\left[\ddot{u}+\frac{k_1}{m}u\right]+p\left[\ddot{u}+\frac{k_1 u}{m}+\frac{1}{2}\frac{k_2 u^3}{mh^2}-\frac{F_0 sin(\omega_0 t)}{m}\right]=0 \tag{12}$$

According to HPM, we assume that the solution of Equation (12) can be expressed in a series of p.

$$u(t)=u_0(t)+pu_1(t)+p^2u_2(t)+..... \tag{13}$$

The coefficients $k_1/m=\Omega$ be, respectively, expanded into a series in p in a similar way,

$$\Omega=\omega^2-p\omega_1-p^2\omega_2+... \tag{14}$$

Substituting Equations.(13) and (14) into Equation (12) after some simplification and substitution and rearranging based on powers of p-terms, we have:

$$p^0:\ddot{u}_0+\omega^2 u_0=0 \tag{15}$$

And,

$$p^1:\ddot{u}_1+\omega^2 u_1=\omega_1 u_0-\frac{1}{2}\frac{k_2 u_0^3}{mh^2}+\frac{F_0\sin(\omega_0 t)}{m} \tag{16}$$

Considering the initial conditions $u_0(0)=A$ and $\dot{u}_0(0)=0$ the solution of Equation (15) is $u_0=A\cos(\omega t)$. Substituting the result into Equation (16), we have:

$$p^1 : \ddot{u} + \omega^2 u_1 = \omega_1 A \cos(\omega t) - \frac{1}{2}\frac{k_2 (A\cos(\omega t))^3}{mh^2} + \frac{F_0 \sin(\omega_0 t)}{m} \tag{17}$$

For achieving the secular term, we use Fourier expansion series as follows:

$$\begin{aligned}\Phi(\omega,t) &= \omega_1 A \cos(\omega t) - \frac{1}{2}\frac{k_2 (A\cos(\omega t))^3}{mh^2} + \frac{F_0 \sin(\omega_0 t)}{m} \\ &= \sum_{n=0}^{\infty} b_{2n+1} \cos\left[(2n+1)\omega t\right] \\ &= b_1 \cos(\omega t) + b_3 \cos(3\omega t) + ... \\ &\approx \frac{1}{8mh^2}\left(3k_2 A^3 - 8\omega_1 A\, m\, h^2\right)\cos(\omega t) + \frac{1}{8mh^2} k_2 A^3 \cos(3\omega t) + \frac{F_0 \sin(\omega_0 t)}{m}\end{aligned} \tag{18}$$

Substituting Equation (18) into Equation (17) yields:

$$p^1 : \ddot{u}_1 + \omega^2 u_1 = \frac{1}{8mh^2}\left(3k_2 A^3 - 8\omega_1 A m h^2\right)\cos(\omega t) + \frac{1}{8mh^2} k_2 A^3 \cos(3\omega t) + \frac{F_0 \sin(\omega_0 t)}{m} \tag{19}$$

Avoiding secular term in $u_1(t)$ gives:

$$\omega_1 = \frac{3}{8}\frac{k_2 A^2}{mh^2} \tag{20}$$

From Equation (14) and setting $p = 1$, we have:

$$\Omega = \omega^2 - \omega_1 \tag{21}$$

Substituting Equation (20) in to Equation (21) and $\Omega = k_1/m$ we can obtain the frequency of the nonlinear oscillator as follows:

$$\omega_{HPM} = \sqrt{\frac{3}{8}\frac{k_2 A^2}{mh^2} + \frac{k_1}{m}} \tag{22}$$

Solving Equation (19) without secular term we obtain,

$$\begin{aligned}u_1(t) = &\frac{\sin(\omega t) F_0 \omega_0}{\omega m\left(\omega_0^2 - \omega^2\right)} + \frac{1}{64}\frac{k_2 A^3 \cos(\omega t)}{\left(\omega^2 h^2\right)} \\ &+ \frac{1}{64}\frac{k_2 A^3 (\omega_0^2 - \omega^2)\cos(3\omega t) - 64 F_0 h^2 \omega^2 \sin(\omega_0 t)}{m\omega^2 h^2\left(\omega_0^2 - \omega^2\right)}\end{aligned} \tag{23}$$

Hence, we can obtain the following approximate solution,

$$u(t) = A\cos(\omega t) + \frac{\sin(\omega t) F_0 \omega_0}{\omega m\left(\omega_0{}^2 - \omega^2\right)} + \frac{1}{64}\frac{k_2 A^3 \cos(\omega t)}{\left(\omega^2 h^2\right)} + \frac{1}{64}\frac{k_2 A^3(\omega_0{}^2 - \omega^2)\cos(3\omega t) - 64 F_0 h^2 \omega^2 \sin(\omega_0 t)}{m\omega^2 h^2\left(\omega_0{}^2 - \omega^2\right)} \tag{24}$$

For comparison of the approximate solution, frequency obtained from solution of nonlinear equation with the energy balance method (Appendix A) is (Mehdipour *et al.*, 2010):

$$\omega_{EBM} = \frac{2}{A}\sqrt{\frac{k_1 A^2}{4m} + \frac{3k_2 A^4}{32mh^2} + \left(\frac{\sqrt{2}}{2} - 1\right)\frac{F_0}{m} A\sin(\omega_0 t)} \tag{25}$$

The numerical solution by with 4th order Runge-Kutta method (Appendix B) for nonlinear equation is:

$$\begin{aligned} \dot{u}_1 &= u_2, & u_1(0) &= A, \\ \dot{u}_2 &= -\frac{k_1}{m}u_1 - \frac{k_2}{2mh^2}u_1^3 + \frac{F_0}{m}\sin\omega_0 t, & u_2(0) &= 0. \end{aligned} \tag{26}$$

Example 2

Consider the motion of a mass *m* moving without friction along a circle of radius *R* that is rotating with a constant angular velocity Ω about its vertical diameter as shown in figure 2. The forces acting on the mass are gravitational force *mg*, the centrifugal of the circle O and the reaction force. The following governing equation has been obtained (Nayfeh & Mook,1973):

$$m R^2\ddot{\theta} - m R^2\Omega^2 \sin(\theta)\cos(\theta) + mgR\sin(\theta) = 0, \qquad \theta(0) = A, \quad \dot{\theta}(0) = 0 \tag{27}$$

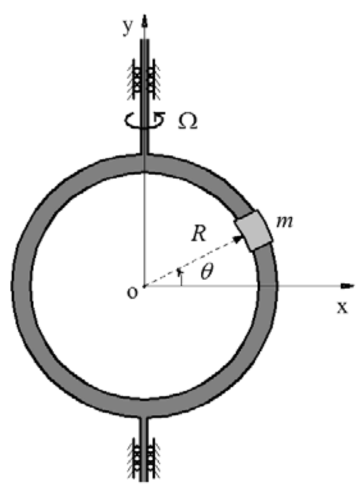

Fig. 2. Particle moving without friction on a rotating circular

$$\ddot{\theta} - \Omega^2 \sin(\theta)\cos(\theta) + \frac{g}{R}\sin(\theta) = 0, \qquad \theta(0) = A, \quad \dot{\theta}(0) = 0 \tag{28}$$

By using the Taylor's series expansion for $\cos(\theta(t))$, $\sin(\theta(t))$ and by some manipulation in Equation (19) we can re-write Equation (19) in the following form:

$$\ddot{\theta} - \Omega^2\left(\theta - \tfrac{1}{6}\theta^3\right)\left(1 - \tfrac{1}{2}\theta^2 + \tfrac{1}{24}\theta^4\right) + \tfrac{g}{R}\left(\theta - \tfrac{1}{6}\theta^3\right) = 0, \tag{29}$$

Now applying Homotopy perturbation to Equation (28) and construct a Homotopy in the following form:

$$H(\theta,p) = (1-p)\left[\ddot{\theta} - \Omega^2\theta + \tfrac{g}{R}\theta\right] + p\left[\ddot{\theta} - \Omega^2\left(\theta - \tfrac{1}{6}\theta^3\right)\left(1 - \tfrac{1}{2}\theta^2 + \tfrac{1}{24}\theta^4\right) + \tfrac{g}{R}\left(\theta - \tfrac{1}{6}\theta^3\right)\right] = 0 \tag{30}$$

According to HPM, we assume that the solution of Equation (28) can be expressed in a series of *p:*

$$\theta(t) = \theta_0(t) + p\theta_1(t) + p^2\theta_2(t) + \ldots.. \tag{31}$$

The coefficients $\Delta = \frac{g}{R} - \Omega^2$ be, respectively, expanded into a series in p in a similar way,

$$\Delta = \omega^2 - p\omega_1 - p^2\omega_2 + \ldots \tag{32}$$

Substituting Equation (31) and Equation (32) into Equation (30) after some simplification and substitution and rearranging based on powers of p-terms, we have:

$$p^0 : \ddot{\theta}_0 + \omega^2\theta_0 = 0 \tag{33}$$

$$p^1 : \ddot{\theta}_1 + \omega^2\theta_1 = \omega_1\theta_0 - \frac{2}{3}\Omega^2\theta_0^3 + \frac{1}{8}\Omega^2\theta_0^5 - \frac{1}{144}\Omega^2\theta_0^7 + \frac{1}{6}\frac{g}{R}\theta_0^3 \tag{34}$$

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Considering the initial conditions $\theta_0(0) = A$ and $\dot{\theta}_0(0) = 0$ the solution of Equation (33) is $\theta_0 = A\cos(\omega t)$ substituting the result into Equation (34), we have:

$$\begin{aligned} p^1 : \ddot{\theta}_1 + \omega^2\theta_1 &= \omega_1 A\cos(\omega t) - \frac{2}{3}\Omega^2 A^3\cos^3(\omega t) + \frac{1}{6}\frac{g}{R}A^3\cos^3(\omega t) \\ &+ \frac{1}{8}\Omega^2 A^5\cos(\omega t)^5 - \frac{1}{144}\Omega^2 A^7\cos^7(\omega t) \end{aligned} \tag{35}$$

For achieving the secular term, we use Fourier expansion series as follows:

$$\begin{aligned}\Phi(\omega,t) &= \omega_1 A\cos(\omega t) - \tfrac{2}{3}\Omega^2 A^3\cos^3(\omega t) + \tfrac{1}{6}\tfrac{g}{R}A^3\cos^3(\omega t) \\ &\quad + \tfrac{1}{8}\Omega^2 A^5\cos(\omega t)^5 - \tfrac{1}{144}\Omega^2 A^7\cos^7(\omega t) \\ &= \sum_{n=0}^{\infty} b_{2n+1}\cos\left[(2n+1)\omega t\right] \\ &= b_1\cos(\omega t) + b_3\cos(3\omega t) + \ldots \\ &\approx \left(\omega_1 A - \tfrac{1}{2}\Omega^2 A^3 + \tfrac{5}{64}\Omega^2 A^5 + \tfrac{1}{8}\tfrac{g}{R}A^3 - \tfrac{35}{9216}\Omega^2 A^7\right)\cos(\omega t) - \tfrac{1}{6}\Omega^2 A^3\cos(3\omega t) \\ &\quad + \tfrac{5}{128}\Omega^2 A^5\cos(3\omega t) + \tfrac{1}{24}\tfrac{g}{R}A^3\cos(3\omega t) + \; - \tfrac{7}{3072}\Omega^2 A^7\cos(3\omega t) \\ &\quad - \tfrac{7}{9216}\Omega^2 A^7\cos(5\omega t)\tfrac{1}{128}\Omega^2 A^5\cos(5\omega t) - \tfrac{1}{9216}\Omega^2 A^7\cos(7\omega t)\end{aligned} \tag{36}$$

Substituting Equation (36) into right hand of Equation (35) yields:

$$\begin{aligned}p^1: \ddot{\theta}_1 + \omega^2\theta_1 &= \left(\omega_1 A - \tfrac{1}{2}\Omega^2 A^3 + \tfrac{5}{64}\Omega^2 A^5 + \tfrac{1}{8}\tfrac{g}{R}A^3 - \tfrac{35}{9216}\Omega^2 A^7\right)\cos(\omega t) \\ &\quad + \sum_{n=0}^{\infty} b_{2n+1}\cos\left[(2n+1)\omega t\right]\end{aligned} \tag{37}$$

Avoiding secular term, gives:

$$\omega_1 = \frac{35}{9216}A^6\Omega^2 + \frac{1}{2}A^2\Omega^2 - \frac{5}{64}A^4\Omega^2 - \frac{1}{8}\frac{g}{R}A^2 \tag{38}$$

From Equation (32) and setting $p = 1$, we have:

$$\Delta = \omega^2 - \omega_1 \tag{39}$$

Substituting Equation (38) in to Equation (39) and $\Delta = \frac{g}{R} - \Omega^2$ we can obtain the frequency of the nonlinear oscillator as follows:

$$\omega_{HPM} = \sqrt{\frac{g}{R} - \Omega^2 + \frac{35}{9216}A^6\Omega^2 + \frac{1}{2}A^2\Omega^2 - \frac{5}{64}A^4\Omega^2 - \frac{1}{8}\frac{g}{R}A^2} \tag{40}$$

Solving Equation (37) without secular term we obtain,

$$\begin{aligned}\theta_1(t) &= \frac{1}{\omega^2}\Big(\tfrac{47}{147456}\Omega^2 A^7\cos(\omega t) + \tfrac{1}{192}\Omega^2 A^5\cos(\omega t) - \tfrac{1}{48}\Omega^2 A^3\cos(\omega t) + \tfrac{1}{192}\tfrac{g}{R}A^3\cos(\omega t) \\ &\quad + \tfrac{7}{24576}\Omega^2 A^7\cos(3\omega t) - \tfrac{5}{1024}\Omega^2 A^5\cos(3\omega t) + \tfrac{1}{48}\Omega^2 A^3\cos(3\omega t) - \tfrac{1}{192}\tfrac{g}{R}A^3\cos(3\omega t) \\ &\quad - \tfrac{1}{3072}\Omega^2 A^5\cos(5\omega t) + \tfrac{7}{221184}\Omega^2 A^7\cos(5\omega t) + \tfrac{1}{442368}\Omega^2 A^7\cos(7\omega t)\Big)\end{aligned} \tag{41}$$

Hence, we can obtain the following approximate solution,

$$\theta(t) = A\cos(\omega t) + \frac{1}{\omega^2}\Big(\tfrac{47}{147456}\Omega^2 A^7\cos(\omega t) + \tfrac{1}{192}\Omega^2 A^5\cos(\omega t) - \tfrac{1}{48}\Omega^2 A^3\cos(\omega t)$$
$$+\tfrac{1}{192}\tfrac{g}{R}A^3\cos(\omega t) + \tfrac{7}{24576}\Omega^2 A^7\cos(3\omega t) - \tfrac{5}{1024}\Omega^2 A^5\cos(3\omega t) + \tfrac{1}{48}\Omega^2 A^3\cos(3\omega t) \quad (42)$$
$$-\tfrac{1}{192}\tfrac{g}{R}A^3\cos(3\omega t) - \tfrac{1}{3072}\Omega^2 A^5\cos(5\omega t) + \tfrac{7}{221184}\Omega^2 A^7\cos(5\omega t) + \tfrac{1}{442368}\Omega^2 A^7\cos(7\omega t)\Big)$$

For comparison of the approximate solution, frequency obtained from solution of nonlinear equation with the energy balance method (Appendix A) is:

$$\omega_{EBM} = \frac{\sqrt{4R\left(-R\Omega^2\cos^2(\frac{\sqrt{2}}{2}A) + 2g\cos(\frac{\sqrt{2}}{2}A) + R\,\Omega^2\cos^2(A) - 2g\cos(A)\right)}}{RA} \quad (43)$$

The numerical solution by with 4th order Runge-Kutta method (Appendix B) for nonlinear equation is:

$$\dot{\theta}_1 = \theta_2, \qquad \theta_1(0) = A,$$
$$\dot{\theta}_2 = \Omega^2\sin(\theta_1)\cos(\theta_1) - \frac{g}{R}\sin(\theta_1), \qquad \theta_2(0) = 0 \quad (44)$$

Example 3

We consider the physical model of nonlinear equation in the following figure with $F(t) = F_0\sin\omega_0 t$, indicated in Figure 3 (Mehdipour *et al*,2010).

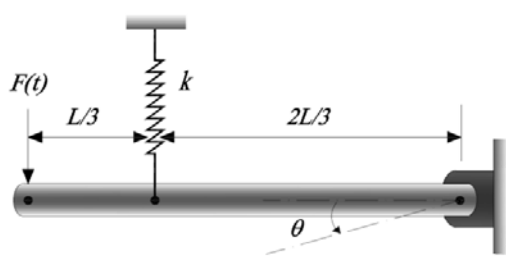

Fig. 3. The physical model of nonlinear equation

The motion equation is:

$$\ddot{\theta} + \frac{4k}{3m}\sin\theta - \frac{3F_0}{ml}\sin\omega_0 t = 0, \qquad \theta(0) = A, \quad \dot{\theta}(0) = 0 \quad (45)$$

This equation is as known as Mathieu equation or the system with dependent coefficients to time.

In which θ and t are generalized dimensionless displacements and time variables, respectively. And consider $\lambda = \frac{4}{3}\frac{k}{m}$ as constant.

The approximation $sin(\theta)=\theta-(1/6)\theta^3+(1/120)\theta^5$ is used.

Now applying Homotopy perturbation to Equation (45) and construct a Homotopy in the following form:

$$H(\theta,p)=(1-p)\left[\ddot{\theta}+\lambda\theta\right]+p\left[\ddot{\theta}+\lambda\left(\theta-\tfrac{1}{6}\theta^3+\tfrac{1}{120}\theta^5\right)-\frac{3F_0}{ml}\sin\omega_0 t\right]=0 \quad (46)$$

According to HPM, we assume that the solution of Equation (45) can be expressed in a series of *p:*

$$\theta(t)=\theta_0(t)+p\theta_1(t)+p^2\theta_2(t)+..... \quad (47)$$

The coefficients λ be, respectively, expanded into a series in *p* in a similar way,

$$\lambda=\omega^2-p\omega_1-p^2\omega_2+... \quad (48)$$

Substituting Equation (47) and Equation (48) into Equation (46) after some simplification and substitution and rearranging based on powers of *p*-terms, we have:

$$p^0:\ \ddot{\theta}_0+\omega^2\theta_0=0 \quad (49)$$

$$p^1:\ \ddot{\theta}_1+\omega^2\theta_1=\omega_1\theta+\tfrac{1}{6}\omega^2\theta^3-\tfrac{1}{120}\omega^2\theta^5+\frac{3F_0\sin(\omega_0 t)}{ml} \quad (50)$$

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Considering the initial conditions $\theta_0(0)=A$ and $\dot{\theta}_0(0)=0$ the solution of Equation (49) is $\theta_0=A\cos(\omega t)$ substituting the result in to Equation (50), we have:

$$p^1:\ \ddot{\theta}_1+\omega^2\theta_1=\omega_1 A\cos(\omega t)+\tfrac{1}{6}\omega^2A^3\cos^3(\omega t)-\tfrac{1}{120}\omega^2A^5\cos^5(\omega t)+\frac{3F_0\sin(\omega_0 t)}{ml} \quad (51)$$

For achieving the secular term, we use Fourier expansion series as follows:

$$\begin{aligned}\Phi(\omega,t)&=\omega_1 A\cos(\omega t)+\tfrac{1}{6}\omega^2A^3\cos^3(\omega t)-\tfrac{1}{120}\omega^2A^5\cos^5(\omega t)+\frac{3F_0\sin(\omega_0 t)}{ml}\\&=\sum_{n=0}^{\infty}b_{2n+1}\cos\left[(2n+1)\omega t\right]\\&=b_1\cos(\omega t)+b_3\cos(3\omega t)+...\\&\approx\left(\omega_1 A-\tfrac{1}{8}\omega^2A^3+\tfrac{1}{192}\omega^2A^5\right)cos(\omega t)-\tfrac{1}{24}\omega^2A^3cos(3\omega t)+...\end{aligned} \quad (52)$$

Substituting Equation (52) into right hand of Equation (51) yields:

$$p^1 : \ddot{\theta}_1 + \omega^2\theta_1 = \left(\omega_1 A - \tfrac{1}{8}\omega^2 A^3 + \tfrac{1}{192}\omega^2 A^5\right)cos(\omega t) + \sum_{n=0}^{\infty} b_{2n+1}\cos\left[(2n+1)\omega t\right] \quad (53)$$

Avoiding secular term, gives:

$$\omega_1 = \tfrac{1}{8}\omega^2 A^2 - \tfrac{1}{192}\omega^2 A^4 \quad (54)$$

From Equation (48) and setting $p = 1$, we have:

$$\lambda = \omega^2 - \omega_1 \quad (55)$$

Substituting Equation (54) in to Equation (55) and $\lambda = 4k/3m$ we can obtain the frequency of the nonlinear oscillator as follows:

$$\omega_{HFM} = -\frac{16\sqrt{k/m}}{\sqrt{-A^4 + 24A^2 + 192}} \quad (56)$$

Solving Equation (53) without secular term we obtain,

$$\begin{aligned}\theta_1 = {} & \frac{-3F_0\omega_0 \sin(\omega t)}{ml\,\omega\left(\omega^2 - \omega_0^2\right)} + \left(\tfrac{1}{192}A^3 - \tfrac{1}{2880}A^5\right)\cos(\omega t) \\ & + \tfrac{1}{3072}A^5\cos(3\omega t) - \tfrac{1}{192}A^3\cos(3\omega t) + \tfrac{1}{46080}A^5\cos(5\omega t) + \frac{3F_0\sin(\omega_0 t)}{\left(\omega^2 - \omega_0^2\right)}\end{aligned} \quad (57)$$

Hence, we can obtain the following approximate solution,

$$\begin{aligned}\theta(t) = {} & A\cos(\omega t) - \frac{3F_0\omega_0 \sin(\omega t)}{ml\,\omega\left(\omega^2 - \omega_0^2\right)} + \left(\tfrac{1}{192}A^3 - \tfrac{1}{2880}A^5\right)\cos(\omega t) \\ & + \tfrac{1}{3072}A^5\cos(3\omega t) - \tfrac{1}{192}A^3\cos(3\omega t) + \tfrac{1}{46080}A^5\cos(5\omega t) + \frac{3F_0\sin(\omega_0 t)}{\left(\omega^2 - \omega_0^2\right)}\end{aligned} \quad (58)$$

For comparison of the approximate solution, frequency obtained from solution of nonlinear equation with the energy balance method (Appendix A) is (Mehdipour *et al.*, 2010):

$$\omega_{EBM} = \frac{2}{A}\sqrt{\frac{4k}{3m}\left(\cos\frac{\sqrt{2}}{2}A - \cos A\right) + \frac{3F_0}{ml}\left(\frac{\sqrt{2}}{2} - 1\right)A\sin(\omega_0 t)} \quad (59)$$

The numerical solution by with 4^{th} order Runge-Kutta method (Appendix B) for nonlinear equation is:

$$\begin{aligned} &\dot{\theta}_1 = \theta_2, && \theta_1(0) = A, \\ &\dot{\theta}_2 = -\frac{4k}{3m}\sin\theta_1 + \frac{3F_0}{ml}\sin\omega_0 t && \theta_2(0) = 0. \end{aligned} \tag{60}$$

RESULT AND DISCUTION

In this part, to verify the results of the new applied method, we have prepared some comparisons between Homotopy perturbation method, energy balance method and numerical solution.

In example 1: Table 1 represent the comparison of time history displacement for two different cases :

Case1: $L=1, h=0.5, m=10, k_1=1000, k_2=1500, F_0=1, \omega_0=2, A=0.8\tan(\pi/6)$

Case2: $L=1, h=1, m=15, k_1=1800, k_2=900, F_0=2, \omega_0=3, A=0.9\tan(\pi/12)$

The results show the high accuracy of the Homotopy perturbation method in comparison of Runge-Kutta's algorithm and energy balance method. Figure 4 shows the comparison of homotopy perturbation method time history displacement diagram with Runge-Kutta and energy balance method for two different amplitudes.

(a): $A=0.9\tan(\pi/12)$ (b): $A=0.9\tan(\pi/18)$.

Figure 5 is shown the Influence of springs stiffness (k_1) and (k_2) on nonlinear frequency. It can be seen from the figure that the increases of the spring stiffness causes the increases in nonlinear frequency.

Table 1 . Comparison of time history response of HPM with EBM and Runge-Kutta (Example 1)

Case1					Case 2				
Time	**HPM**	**EBM**	**RK4**	**Error %**	**Time**	**HPM**	**EBM**	**RK4**	**Error %**
0	0.4619	0.4619	0.4619	0.0043	0	0.2412	0.2412	0.24115	0.0041
0.025	0.4386	0.4407	0.4386	0.0008	0.05	0.2054	0.2055	0.20541	0.0017
0.05	0.3724	0.3791	0.3725	0.0489	0.1	0.1090	0.1093	0.10899	0.0043
0.075	0.2731	0.2826	0.2737	0.2334	0.15	-0.0194	-0.0191	-0.01936	0.0027
0.1	0.1526	0.1603	0.1538	0.7561	0.2	-0.1419	-0.1418	-0.14189	0.0015
0.125	0.0219	0.0232	0.0234	6.3983	0.25	-0.2225	-0.2228	-0.22248	0.0005
0.15	-0.1101	-0.1159	-0.1084	1.5840	0.3	-0.2370	-0.2382	-0.23699	0.0020
0.175	-0.2350	-0.2444	-0.2331	0.8219	0.35	-0.1810	-0.1838	-0.181	0.0089
0.2	-0.3425	-0.3505	-0.3407	0.5575	0.4	-0.0713	-0.0754	-0.07133	0.0403
0.225	-0.4209	-0.4245	-0.4195	0.3249	0.45	0.0596	0.0552	0.059581	0.0549
0.25	-0.4590	-0.4595	-0.4587	0.0803	0.5	0.1733	0.1696	0.17332	0.0165
0.275	-0.4509	-0.4524	-0.4517	0.1690	0.55	0.2363	0.2341	0.23628	0.0042
0.3	-0.3979	-0.4039	-0.3998	0.4772	0.6	0.2295	0.2298	0.22952	0.0043
0.325	-0.3081	-0.3183	-0.3112	0.9870	0.65	0.1551	0.1577	0.1551	0.0027
0.35	-0.1933	-0.2036	-0.1974	2.0837	0.7	0.0353	0.0389	0.035285	0.0264
0.375	-0.0649	-0.0703	-0.0696	6.8419	0.75	-0.0946	-0.0915	-0.09459	0.0057
0.4	0.0677	0.0695	0.0627	7.9761	0.8	-0.1964	-0.1949	-0.19644	0.0013
0.425	0.1960	0.2030	0.1911	2.6011	0.85	-0.2400	-0.2404	-0.24004	0.0000
0.45	0.3106	0.3178	0.3060	1.4912	0.9	-0.2123	-0.2143	-0.21229	0.0014
0.475	0.3999	0.4035	0.3965	0.8670	0.95	-0.1216	-0.1241	-0.12161	0.0049
0.5	0.4523	0.4523	0.4508	0.3472	1	0.0049	0.0034	0.004911	0.3171

Case1: $L=1$, $h=0.5$, $m=10$, $k_1=1000$, $k_2=1500$, $F_0=1$, $\omega_0=2$, $A=0.8\tan(\pi/6)$

Case2: $L=1$, $h=1$, $m=15$, $k_1=1800$, $k_2=900$, $F_0=2$, $\omega_0=3$, $A=0.9\tan(\pi/12)$

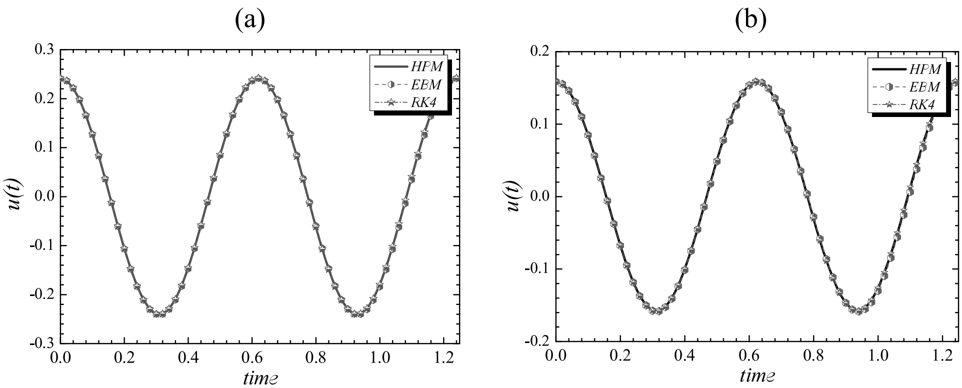

Fig. 4. Comparison of time history response of homotopy perturbation method and energy balance method with the numerical solution for $L=1\ m$, $h=0.9\ m$, $m=10\ kg$, $k_1=1000\ N/m$, $k_2=1100\ N/m$, $F_0=1N$, $\omega_0=1\ rad/sec$ (a): $A=0.9\tan(\pi/12)$ (b): $A=0.9\tan(\pi/18)$

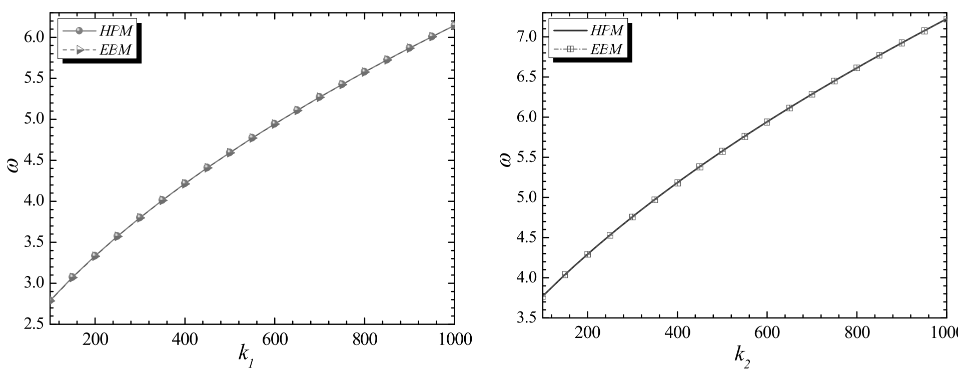

Fig. 5. Influence of springs stiffness (k_1) and (k_2) on nonlinear frequency

In example 2, Table 2 is the comparison of the three applied method to the governing equation of the problem for different important time value and an excellent agreement can be seen.

The cases in table 2 are:

Case1: $A=\pi/4,\ R=1.5,\ \Omega=1.5,\ g=10$

Case2: $A=\pi/2,\ R=0.5,\ \Omega=2,\ g=10$

Figure 6 is comparison of time history displacement response of Homotopy perturbation method and energy balance method with the numerical solution for two different cases:

(i): $A=\pi/6,\ R=0.8,\ \Omega=1.5,\ g=10$ (ii) $A=\pi/3,\ R=1.2,\ \Omega=2.5,\ g=10$

The motion of the problem is periodic.

To see the effects of important parameters on the frequency of the system, we have considered the effect of angular velocity (Ω) and radius(R) on nonlinear frequency in figure 7.

Table 2. Comparison of time history response of HPM with EBM and Runge-Kutta (Example 2).

	Case1					Case2			
Time	**HPM**	**EBM**	**RK4**	**Error %**	**Time**	**HPM**	**EBM**	**RK4**	**Error %**
0	0.7854	0.7854	0.7854	0	0	1.5708	1.5708	1.5708	0
0.2	0.7151	0.7151	0.7148	0.0349	0.1	1.4640	1.4640	1.4652	0.0805
0.4	0.5167	0.5167	0.5158	0.1761	0.2	1.1581	1.1581	1.1625	0.3782
0.6	0.2258	0.2258	0.2241	0.7732	0.3	0.6947	0.6947	0.7034	1.2456
0.8	-0.1055	-0.1055	-0.1079	2.2135	0.4	0.1368	0.1368	0.1498	8.6744
1	-0.4180	-0.4180	-0.4205	0.6061	0.5	-0.4397	-0.4397	-0.4240	3.6972
1.2	-0.6556	-0.6556	-0.6575	0.3019	0.6	-0.9564	-0.9564	-0.9408	1.6577
1.4	-0.7758	-0.7758	-0.7764	0.0835	0.7	-1.3430	-1.3430	-1.3310	0.9001
1.6	-0.7570	-0.7570	-0.7557	0.1718	0.8	-1.5470	-1.5470	-1.5422	0.3073
1.8	-0.6028	-0.6028	-0.5993	0.5828	0.9	-1.5406	-1.5406	-1.5460	0.3533
2	-0.3405	-0.3405	-0.3351	1.6239	1	-1.3246	-1.3246	-1.3419	1.2849
2.2	-0.0173	-0.0173	-0.0167	3.7660	1.1	-0.9285	-0.9285	-0.9572	2.9973
2.4	0.3090	0.3090	0.3156	2.1030	1.2	-0.4062	-0.4062	-0.4438	8.4884
2.6	0.5799	0.5799	0.5852	0.8981	1.3	0.1715	0.1715	0.1793	4.3514
2.8	0.7471	0.7471	0.7496	0.3416	1.4	0.7257	0.7257	0.6849	5.9556
3	0.7804	0.7804	0.7793	0.1371	1.5	1.1813	1.1813	1.1485	2.8567

Case1: $A = \pi / 4,\ R = 1.5,\ \Omega = 1.5,\ g = 10$

Case2: $A = \pi / 2,\ R = 0.5,\ \Omega = 2,\ g = 10$

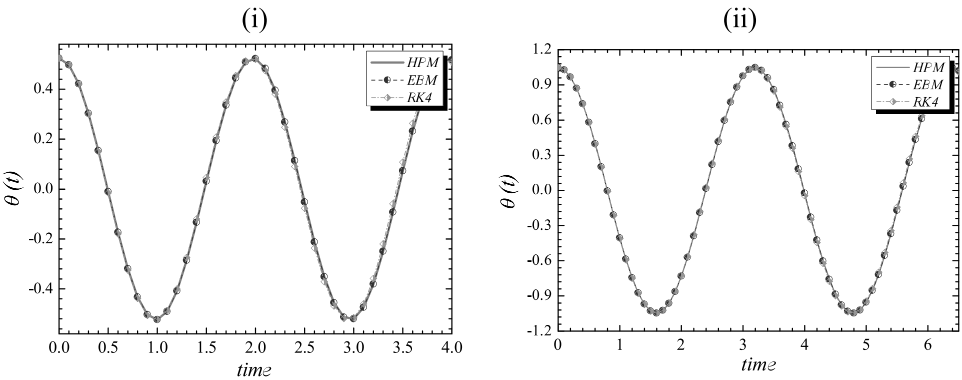

Fig. 6. Comparison of time history response of Homotopy perturbation method and energy balance method with the numerical solution for (i): $A = \pi / 6,\ R = 0.8,\ \Omega = 1.5,\ g = 10$
(ii): $A = \pi / 3,\ R = 1.2,\ \Omega = 2.5,\ g = 10$

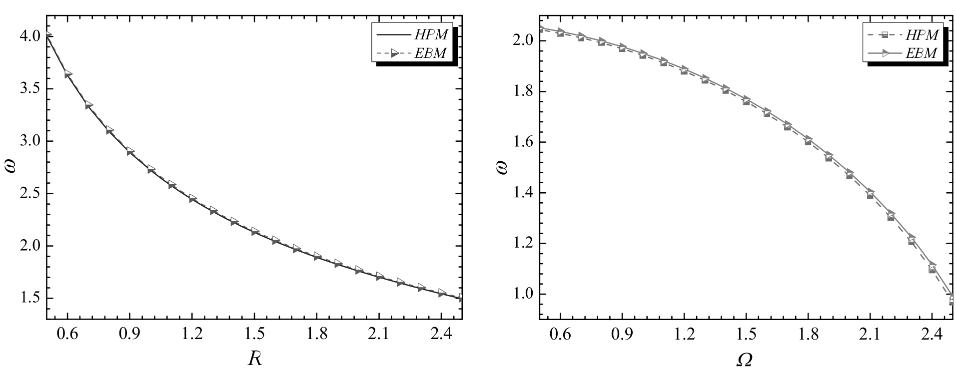

Fig.7. Effect of angular velocity(Ω) and radius(R) on nonlinear frequency

In example 3, again to compare the results of HPM and EBM and numerical solution, a complete comparison has been done for time point values to see the agreement of the methods. As it is shown they are in high agreement. The diagram of the time history displacements is shown in figure 8 for two different amplitudes (I): A=π/12 (II): A=π/3.

The effects of important parameters such as: spring stiffness and amplitude are studied in figure 9. As it is shown in figure 9, by increasing the spring stuffiness, the frequency of the vibration increase and by increasing the amplitude the frequency of the system is decreased.

Table 3. Comparison of time history response of HPM with EBM and Runge-Kutta (Example 3).

Case1					Case 2				
Time	**HPM**	**EBM**	**RK4**	**Error %**	**Time**	**HPM**	**EBM**	**RK4**	**Error %**
0	0.5236	0.5236	0.5236	0.0012	0	0.3491	0.3491	0.3491	0.0028
0.025	0.4988	0.4985	0.4988	0.0056	0.05	0.2616	0.2615	0.2617	0.0138
0.05	0.4264	0.4257	0.4265	0.0313	0.1	0.0426	0.0432	0.0427	0.1858
0.075	0.3130	0.3122	0.3132	0.0834	0.15	-0.1975	-0.1963	-0.1974	0.0423
0.1	0.1691	0.1689	0.1695	0.2332	0.2	-0.3368	-0.3379	-0.3368	0.0112
0.125	0.0087	0.0095	0.0091	5.2831	0.25	-0.3062	-0.3119	-0.3062	0.0164
0.15	-0.1526	-0.1507	-0.1520	0.3450	0.3	-0.1203	-0.1319	-0.1204	0.0958
0.175	-0.2986	-0.2965	-0.2981	0.1735	0.35	0.1282	0.1129	0.1281	0.1075
0.2	-0.4154	-0.4140	-0.4150	0.1045	0.4	0.3136	0.3020	0.3135	0.0296
0.225	-0.4920	-0.4920	-0.4917	0.0575	0.45	0.3434	0.3428	0.3436	0.0339
0.25	-0.5215	-0.5232	-0.5214	0.0115	0.5	0.2032	0.2155	0.2037	0.2031
0.275	-0.5013	-0.5046	-0.5015	0.0464	0.55	-0.0373	-0.0174	-0.0367	1.4509
0.3	-0.4331	-0.4381	-0.4337	0.1272	0.6	-0.2563	-0.2417	-0.2559	0.1423
0.325	-0.3231	-0.3300	-0.3240	0.2619	0.65	-0.3435	-0.3476	-0.3434	0.0066
0.35	-0.1814	-0.1906	-0.1825	0.5947	0.7	-0.2557	-0.2830	-0.2560	0.1066
0.375	-0.0216	-0.0332	-0.0228	5.3405	0.75	-0.0363	-0.0792	-0.0367	1.1241
0.4	0.1406	0.1274	0.1394	0.8915	0.8	0.2045	0.1639	0.2041	0.1840
0.425	0.2892	0.2758	0.2881	0.4016	0.85	0.3447	0.3263	0.3446	0.0328
0.45	0.4099	0.3981	0.4089	0.2303	0.9	0.3149	0.3271	0.3153	0.1211
0.475	0.4912	0.4827	0.4906	0.1185	0.95	0.1296	0.1651	0.1305	0.6254
0.5	0.5260	0.5215	0.5259	0.0156	1	-0.1186	-0.0798	-0.1178	0.7137

Case1: $L=0.5$, $m=10$, $k=1200$, $F_0=1$, $\omega_0=2$, $A=\pi/6$

Case2: $L=1.5$, $m=5$, $k=800$, $F_0=3$, $\omega_0=2$, $A=\pi/9$

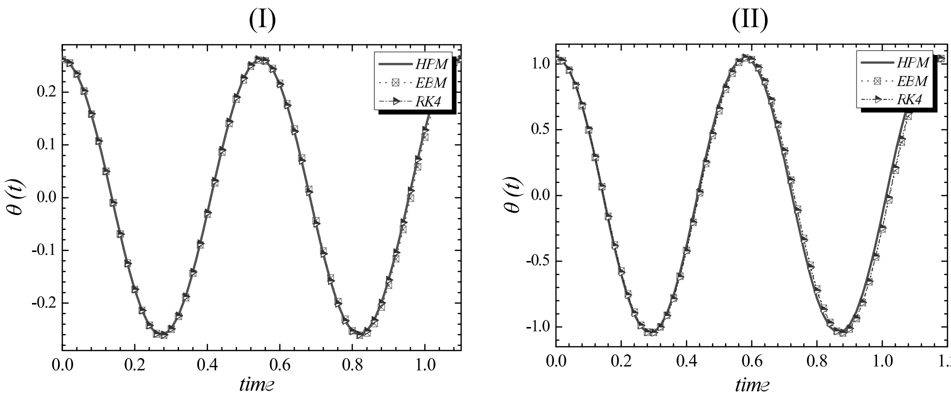

Fig. 8. Comparison of time history response of Homotopy perturbation method and energy balance method with the numerical solution for $L=1\ m$, $m=10\ kg$, $k=1000\ N/m$, $F_0=1N$, $\omega_0=1\ rad/sec$ (I): $A=\pi/12$ (II): $A=\pi/3$

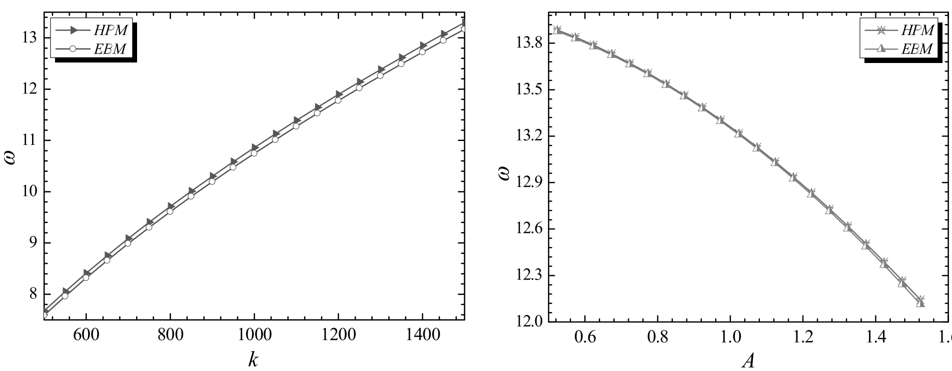

Fig. 9. Influence of spring stiffness (k) and amplitude (A) on nonlinear frequency

CONCLUSION

In this study, we tried to apply a new approximate analytical method to high nonlinear mechanical systems. Homotopy perturbation method has been successfully applied for three different examples. Some patterns and tables have been presented to show the accuracy of the method. The method can converge rapidly to a high accurate solution in comparison to energy balance method and Runge-Kutta'a algorithm. Form the examples, it can be seen that only the first iteration of the problem can lead us to a high accurate solution for whole domain as indicated in this study.

APPENDIX A: BASIC IDEA OF ENERGY BALANCE METHOD

Consider a general nonlinear oscillator in the form (He,2008);

$$\ddot{u} + f\left(u\left(t\right)\right) = 0 \tag{A.1}$$

In which u and t are generalized dimensionless displacement and time variables, respectively. Its variational principle can be easily obtained:

$$J(u)=\int_0^t\left(-\frac{1}{2}\dot{u}^2+F(u)\right)dt \tag{A.2}$$

Where $T=2\pi/\omega$ is period of the nonlinear oscillator, $F(u)=\int f(u)du$.

Its Hamiltonian, therefore, can be written in the form;

$$H=\frac{1}{2}\dot{u}^2+F(u)=F(A) \tag{A.3}$$

Or

$$\Re(t)=\frac{1}{2}\dot{u}^2+F(u)-F(A)=0 \tag{A.4}$$

Oscillatory systems contain two important physical parameters, i.e. The frequency ω and the amplitude of oscillation. A . So let us consider such initial conditions:

$$u(0)=A,\quad \dot{u}(0)=0 \tag{A.5}$$

We use the following trial function to determine the angular frequency ω

$$u(t)=A\cos(\omega t) \tag{A.6}$$

Substituting (A.6) into u term of (A.4), yield:

$$\Re(t)=\frac{1}{2}\omega^2A^2\sin^2(\omega t)+F(A\cos(\omega t))-F(A)=0 \tag{A.7}$$

If, by chance, the exact solution had been chosen as the trial function, then it would be possible to make $\Re$ zero for all values of t by appropriate choice of ω. Since Equation (A.6) is only an approximation to the exact solution, $\Re$ cannot be made zero everywhere. Collocation at $\omega t=\pi/4$ gives:

$$\omega=\sqrt{\frac{2\left(F(A)\right)-F\left(A\cos(\omega t)\right)}{A^2\sin^2(\omega t)}} \tag{A.8}$$

APPENDIX B: BASIC IDEA OF RUNGE-KUTTA (RK)

The most often used method of the Runge-Kutta family is the Fourth-Order one, which extends the idea of the mid-point method, by jumping 1/4th of the way first, then going half-way, using the mid-point method, then going 3/4th of the way and finally jumping all the way.

Consider an initial value problem be specified as follows:

$$\dot{u}=f(t,u),\quad u(t_0)=u_0 \tag{B.1}$$

θ is an unknown function of time t which we would like to approximate. Then RK4 method is given for this problem as below:

$$u_{n+1} = u_n + \frac{1}{6}h\left(k_1 + 2k_2 + 2k_3 + k_4\right), \qquad \text{(B.2)}$$
$$t_{n+1} = t_n + h.$$

for n = 0, 1, 2, 3, . . . , using

$$\begin{aligned} k_1 &= f\left(t_n, u_n\right), \\ k_2 &= f\left(t_n + \frac{1}{2}h,\ u_n + \frac{1}{2}hk_1\right), \\ k_3 &= f\left(t_n + \frac{1}{2}h,\ u_n + \frac{1}{2}hk_3\right), \\ k_4 &= f\left(t_n + h,\ u_n + hk_3\right)^{\theta}. \end{aligned} \qquad \text{(B.3)}$$

Where u_{n+1} is the RK4 approximation of $u\left(t_{n+1}\right)$. and the next value $\left(u_{n+1}\right)$ is determined by the present value $\left(u_n\right)$ plus the weighted average of four increments, where each increment is the product of the size of the interval, *h*, and an estimated slope specified by function *f* on the right-hand side of the differential equation.

- k_1 is the increment based on the slope at the beginning of the interval, using $\dot{u}$,
- k_2 is the increment based on the slope at the midpoint of the interval, using $\dot{u} + \frac{1}{2}hk_1$;
- k_3 is again the increment based on the slope at the midpoint, but now using $\dot{u} + \frac{1}{2}hk_2$;
- k_4 is the increment based on the slope at the end of the interval, using $\dot{u} + hk_3$.

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خلاصة

نحاول في هذا البحث تقديم طريقة تقريب جديدة بواسطة استخدام طريقة الرجفان التحاولي لمسائل عالية غير الخطية. نأخذ ثلاث أمثلة مختلفة و ندرس تطبيق هذه الطريقة عليها . نستخدم خوارزمية رونج – كوتا للحصول على نتائج عددية. كما نطبق طريقة تحليلية أخرى وتسمى طريقة توازن الطاقة، لمقارنة نتائج هذه الطريقة بنتائج خوارزمية رونج – كوتا. يتبين لنا أنه بعد تكرار واحد للطريقة يتبلور حل عالي الدقة على المجال كله. كما نثبت أن طريقة الرجفان التحاولي لا تحتاج أي إخطاط كما أنها تتغلب على معوقات طرق الرجفان جميعها.