Kuwait J. Sci. **45 (1)** pp 20-28, 2018

Electrohydrodynamic flow solution in ion drag in a circular cylindrical conduit using hybrid neural network and genetic algorithm

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Abstract

In this work, we consider the Electrohydrodynamic (EHD) flow equation in ion drag in a circular cylindrical conduit using artificial neural network (ANN) optimized with genetic algorithm (GA). The governing equation is highly nonlinear of degree two and its nonlinearity is based on parameter. We proposed the solution of EHD using three layer ANN with ten neurons in each layer and optimized with GA. The results of the proposed algorithm are compared with numerical solution obtained through MATLAB and least square method (LSM) and are in good agreement with these two methods reported in literature.

Keywords: Artificial neural network modelling; electrohydrodynamic flow analysis; genetic algorithm optimization.

1. Introduction

Over the years there is a great development anda lot of research efforts are being devoted in the field of electrohydrodynamic (EHD) systems. In EHD, electric field is exploited for flow control of dielectric fluids through circular conduits and its prominent application areas are biotechnology, flow meters, accelerators, and magneto-hydrodynamic generators (Subha et al., 2014). The EHD flow equation through circular cylindrical conduit in ion-drag configuration is governed by the following nonlinear singular boundary value problem (McKee et al., 1997):

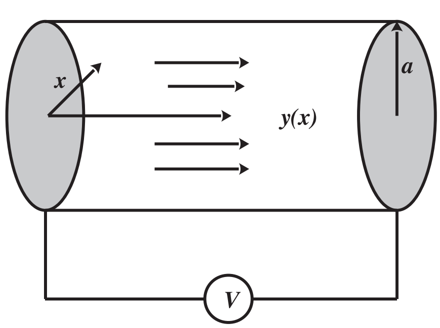

Fig. 1. Ion-drag flow in a circular cylindrical conduit

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+H^2\left(1-\frac{y}{1-\alpha y}\right)=0, \qquad x\epsilon(0,1) \qquad (1)$$

Subject to boundary conditions

$$\frac{dy}{dx}(0)=0, \quad y(1)=0 \qquad (2)$$

Here, $y(x)$ is the velocity of the fluid and is the radial distance from the cylindrical conduit centre.In Equation (1)H is the Hartmann electric number, and is the measure of the strength of non-linearity. Due to this non-linearity, obtaining analytical solutions is not easy and some analytical approaches adopted by many researchers are introduced next.

A Runge-Kutta shooting method was used by McKee et al. (1997) for solution of electrohydrodynamic ion drag flow problem for various values of non-linearity. (Paullet, 1999) presented the existence and uniqueness of a solution of boundary value problem (BVP) and also showed that the numerical results given by McKee et al. (1997) for electrohydrodynamicion drag flow problem are not accurate. Mastroberardino (2011) solved electrohydrodynamic flow BVP by using the homotopyanalysis method (HAM) and showed the strength of presented techniques over the homotopy perturbation method (HPM). It comprehensively proved that the solution obtained by HAM is not divergent as compared to HPM, which yields divergent solutions over the wide range of parameters. In Moghtadaei et al. (2015) used a hybrid technique of spectral collocation and spectral Homotopy analysis method (SHAM) for solution of electrohydrodynamic flow BVP in an ion drag configuration. Also in this method an optimal convergence-control parameter was found by using averaged residual error method and also (Moghtadaei et al., 2015) showed that SHAM is far better than HAM as it convergence behaviour is superior than HAM. In Ghasemi et al. (2014) used a semi-exact weighted residual method which is called least square method (LSM) to form the solution of electrohydrodynamic flow BVP in ion drag configuration. In this study the results obtained are compared with numerical solution and obtained residuals are compared with residuals as found by Mastroberardino,(2011). These comparison showed that results of LSM are in excellent agreement with numerical solution and also residuals obtained are more stable as compared to residuals obtained by HAM. In Bég et al.(2013) presented a new methodology of Chebyshev spectral collocation method (CSCM) for solution of two non-linear BVP of electrohydrodynamic flow. It is shown that the technique presented is in excellent agreement with numerical methods and are useful for the solution of electrohydrodynamic flow problems. In Pandey et al. (2012)used optimal homotopy asymptotic method (OHAM) and Optimal Homotopy Analysis Method for the solution of EHD. In this study it is shown that results obtained are qualitatively similar with (Paullet, 1999).

Over the last decade, neural network and swarm intelligence based hybrid techniques have been applied by many researchers in various fields of engineering and applied sciences. These methods have powerful universal capabilities to solve dynamical non-linear and singular value problems and results obtained by these methods are continuous and convergent over the problem domain (Raja et al., 2011). In Khan et al.(2012) used the evolutionary computational technique using genetic algorithm for solving non-linear singular system arising in polytrophic and isothermal sphere and compared the results with optimal homotopy asymptotic method (OHAM). The proposed algorithm showed excellent agreement with numerical and exact solver. A swarm intelligence based scheme was used by Raja et al. (2011) to solve fractional order linear and complex non-linear systems. The results for linear fractional order systems were in excellent agreement with existing numerical and analytical techniques. However, for complex non-linear systems accuracy was not much promising. In Assareh et al. (2011) used multilayer neural network hybrid with particle swarm optimization (PSO) for the solution of natural convection of Darcian fluid about a vertical full cone embedded in porous media problem. The accuracy of results was excellent as compared to numerical techniques. A Bratu equation was solved (Raja, 2014) by using artificial neural network (ANN) hybridized with PSO and sequential quadratic programming (SQP). A thorough statistical analysis of the technique was presented, which showed its accuracy and convergence behaviour far better than existing methodologies used for the solution of Bratu equation. In Malik et al.(2014) a hybrid heuristic technique using genetic algorithm (GA) and interior point algorithm (IPA) is demonstrated for the solution of non-linear and singular boundary value problems (BVP) arising in physiology. The results obtained were in excellent agreement compared with numerical and exact solution techniques.

Motivated by above, in this research work a neural network hybridized scheme with genetic algorithm (GA) is presented to solve the EHD flow BVP in a circular cylindrical conduit in ion drag configuration. The problem is modelled using neural network with log sigmoid as the activation function. The weights optimization of the modelled network is performed by using GA. The proposed algorithm runs for 100 independent runs for getting comprehensive statistical analysis based on the minimum value achieved by the fitness function and absolute error (AE) in comparison to numerical solution obtained through MATLAB. According to best of author knowledge, this is the first research work in which EHD flow BVP in a circular cylindrical conduit in ion drag configuration is solved using GA optimized artificial neural network (ANN).

The rest of the paper is organized in the following way. In section 2 mathematical modelling of the problem using neural network is presented. In section 3learning procedure is presented and in section 4 results are compared with numerical solution and a comprehensive statistical analysis is presented. Finally in section 5, conclusions regarding the proposed scheme are presented.

2. Problem statement and mathematical modelling

Solution of EHD flow equation through circular cylindrical conduit in ion drag configuration is non-linear problem for which no analytical techniques are known. In the present work its solution through hybrid neural network and genetic algorithm is presented and is compared with few existing numerical techniques.

The EHD flow equation through circular cylindrical conduit in ion drag configuration is governed by the following nonlinear singular boundary value problem repeated here as:

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + H^2\left(1 - \frac{y}{1-\alpha y}\right) = 0, \qquad x\epsilon(0,1)$$

Subject to boundary conditions

$$\frac{dy}{dx}(0) = 0, \quad y(1) = 0$$

In Malek & Shekari Beidokhti (2006), Parisi et al.(2003), Sadoghi Yazdi et al. (2011), Yazdi et al. (2012) modelling of differential equations using (ANN) is presented. The feed-forward neural networks along with optimization techniques are presented for modelling second order and higher order differential equations. According to the proposed schemes, problem is transformed into an unconstrained optimization problem, which is easy to solve. Also a trial function is defined as (Assareh et al., 2011; Raja, 2014).

$$y_T(x,P) = A(x) + B[x, N(x,P)] \qquad (3)$$

In above equation, first term satisfies the boundary conditions and does not contain the adjustable parameters and the second term is a multilayer neural network with adjustable parameters vector P.

The mathematical model of the problem is now formulated with the following trial function:

$$y_T(x,P) = a - ax^2 + x(x-1)N(x,P) \tag{4}$$

Here, a comes from boundary conditions and $N(x,P)$ represents the neural network structure with log sigmoid activation function (Assareh et al. ,2011; Raja, 2014). Now for our problem, first and second derivative of trial function will be of the form respectively.

$$\frac{dy_T(x,P)}{dx} = -2ax + (2x-1)N(x,P) + x(x-1)\frac{dN(x,P)}{dx} \tag{5}$$

$$\frac{d^2y_T(x,P)}{dx^2} = -2a + x(x-1)\frac{d^2N(x,P)}{dx^2} + 2(2x-1)\frac{dN(x,P)}{dx} + 2N(x,P) \tag{6}$$

Where

$$N(x,P) = \sum_{i=1}^{m} \gamma_i \left(\frac{1}{1+e^{-(w_i x+\beta_i)}}\right) \tag{7}$$

$$\frac{dN(x,P)}{dx} = \sum_{i=1}^{m} \gamma_i\, w_i \left(\frac{e^{-(w_i x+\beta_i)}}{1+e^{-(w_i x+\beta_i)}}\right) \tag{8}$$

$$\frac{d^2N(x,P)}{dx^2} = \sum_{i=1}^{m} \gamma_i w_i^2 \left(\frac{2e^{-2(w_i x+\beta_i)}}{(1+e^{-(w_i x+\beta_i)})^3} - \frac{e^{-(w_i x+\beta_i)}}{(1+e^{-(w_i x+\beta_i)})^2}\right) \tag{9}$$

Here, m is the number of neurons and in our case m= 10 also y, w and β are adjustable parameters. By using Equations (1), (2) and Equations (4)—(6) our fitness function will be

$$\varepsilon(x,P) = \sum_{i=1}^{10} \left\{\frac{d^2y_T(x,P)}{dx^2} + \frac{1}{x}\frac{dy_T(x,P)}{dx} + H^2\left(1 - \frac{y_T(x,P)}{1-\alpha y_T(x,P)}\right)\right\}^2, x\epsilon(0,1) \tag{10}$$

The architecture of neural network model for EHD problem is shown in Fig. 1. This fitness function is formed as an optimization problem and its optimum solution is found using GA.

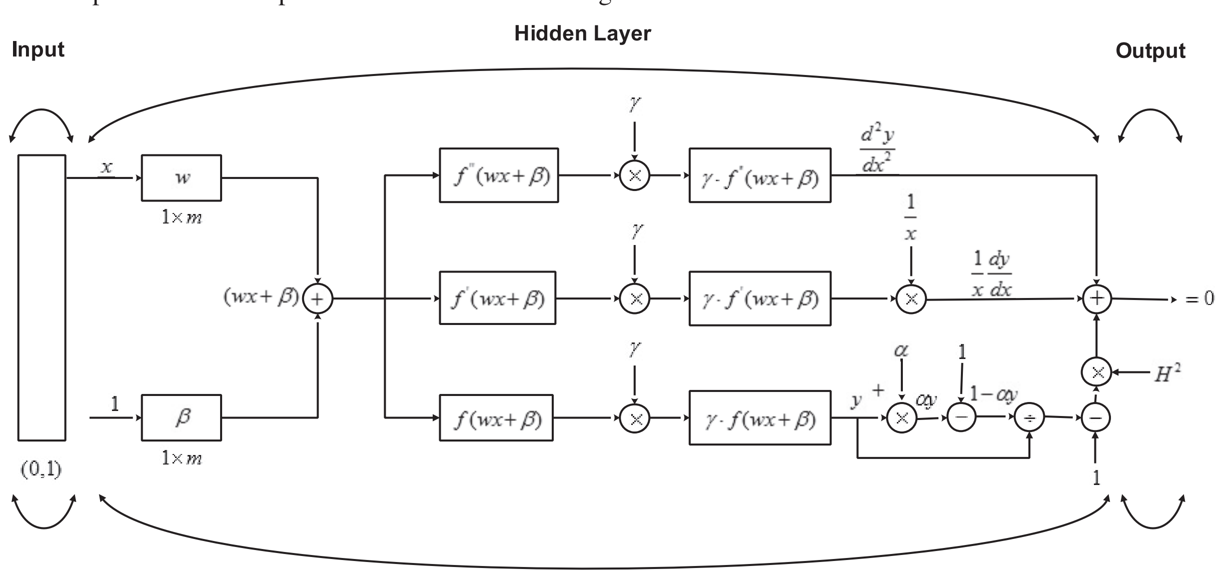

Fig. 2. Architecture of neural network model for EHD

3. Learning procedure

There are certain types of algorithms called evolutionary algorithms (EAs) which are global, parallel and metaheuristic optimization techniques, which mainly are based on the principle of natural selection and genetics. The evolutionary algorithms are based on four approaches: evolutionary programming (EP), evolution strategies (ES), genetic algorithm (GA) and genetic programming (GP). In essence, genetic algorithm was proposed by Goldberg in 1989 and since then it has been used in almost every type of optimization problem.

t:= 0

$\boldsymbol{f}: \boldsymbol{\theta} \to \boldsymbol{\delta}$

FORi :=TOπDOJ $: \pi \to P(\delta)$ [Population Initialization $(P_0, (p_0, p_1, p_\pi))$];

FORi :=**TO**π**DO**$F: \theta \to Q +$ [Fitness (p_i)];

WHILER (termination condition not fulfilled) **DO**

BEGIN

t := t+1

FORi := 1 **TO**π**DO**T [Selection of Individuals $((p_{i,t}))$ from P_{t-1}];

FORi :=**TO**$\pi - 1$**DOSTEP** 2 **DO**

IF Random ([0, 1] $\le P_c$) **THEN**ω_c [Cross $(p_{i,t}, p_{i+1,t})$];

FORi :=**TO**π**DO**

IF Random ([0, 1] $\le P_m$) **THEN** ω_m [Mutate $(p_{i,t})$];

FORi :=**TO**π**DO**$F: \theta \to Q +$ [Fitness $(p_{i,t+1})$];

FORi :=**TO**π**DO**$\beta[P_{i.t+1:=} P_{i.t}]$;

Fig. 3. GA pseudo code

In GA most, suitable chromosomes are searched the population, which forms the search space. This search tries to maintain two things: searching for the best solution out of available solutions, this is called exploitation and at the same time expanding the search space by exploration. Initially GA starts searching the space in which the solution gives poor results. With passage of time it improves the solution by operations of mutation and crossover and ultimately reaching the optimal or global solution. Any GA could be defined by the following 11 parameters given below (Espinosa & Vandewalle, 2000):

$$\theta, \delta, f, F, \pi, J, T, \beta, \omega, Q, R$$

Where, θ is search space, δ is the coding space, f is function for coding, F is fitness function, π is population size, J is function for initialization, T is type of selection, β is the percentage of population that transferred form old population the present population, ω is genetic operator, Q is replacement scheme and R is the termination criterion. The GA on the basis of above said parameters is given in the Fig. 3(Espinosa &Vandewalle, 1998):

The difference between the working methodology of GA and other similar type of methods is that GA use parameters which are coded, whereas other methods use parameters themselves. Coding methodology has great effect on the performance of GA so this is why that the first step of GA is to select some appropriate coding scheme. Mostly GA uses binary coding scheme however, real number coding symbolic coding and tree coding are also used in various variants of GA. The parameters setting for the problem under study for GA are given in Table 1.

Table 1. Parameters setting for GA

Parameter	Value
Population size	200
Fitness scaling	Rank
Selection function	Stochastic uniform
Crossover fraction	0.8
Migration fraction	0.2
Generations	3000
Stall generations limit	50
Function tolerance	1e-06
Constraint tolerance	1e-06

4. Results and simulations

We consider in this section the detailed analysis of the BVP of electrohydrodynamic for proposed scheme. The problem is singular and its non-linearity is highly dependent on value of α. The nonlinearity increases by increasing value of α. The other parameter in the problem is Hartman electric number. We change the values of H^2 by keeping constant and study the non-linearity behaviour on solution. For each observation we run 100 independent runs so that a thorough statistical analysis could be performed. Also, the results obtained are compared with numerical results obtained th

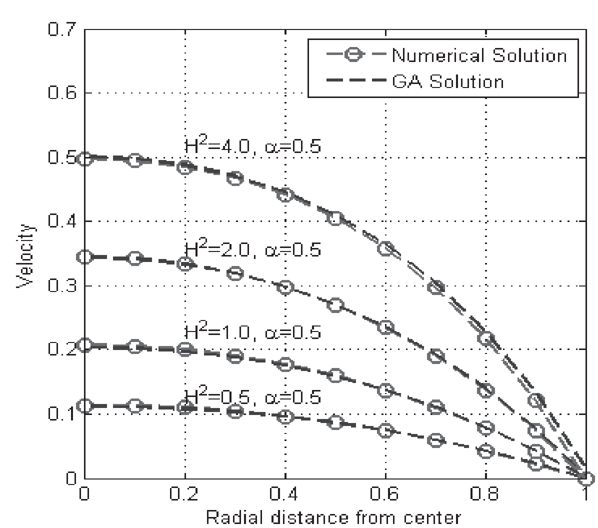

Fig. 4. Effect of Hartman electric number and comparison of numerical and GA solution for

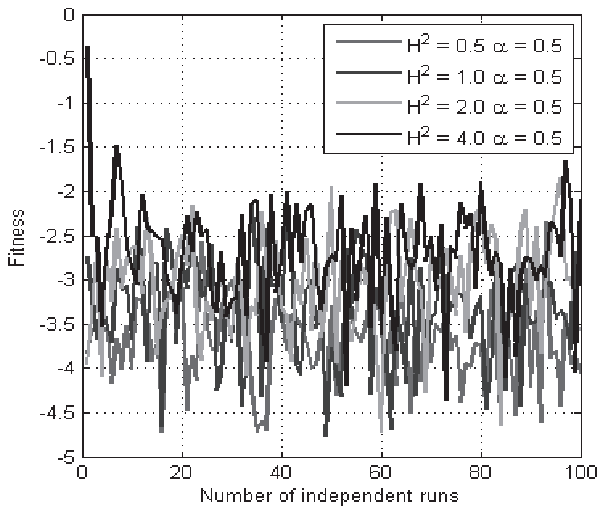

Fig. 5. Fitness value variation with number of independent runs

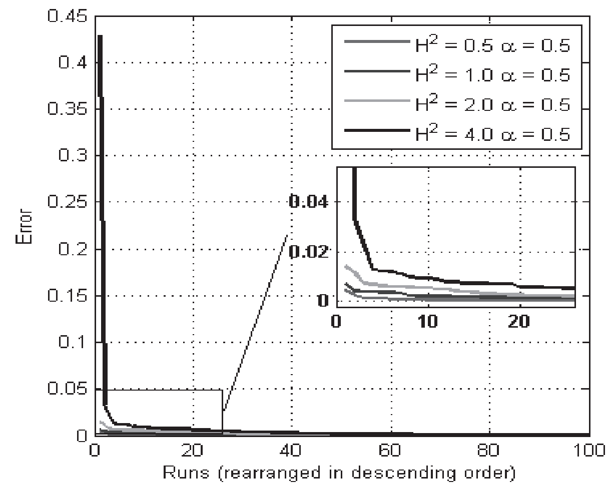

Fig. 6. Convergence behaviour of GA optimized with ANN

Table 2. Comparison of numerical and GA optimized ANN solution

x	$\alpha = 0.5$											
	$H^2 = 0.5$			$H^2 = 1.0$			$H^2 = 2.0$			$H^2 = 4.0$		
	Num.	G.A	A.E	Num.	G.A	A.E	Num.	G.A	A.E	Num.	G.A	A.E
0	0.1137461284	0.1124069421	1.3392e-03	0.2070098492	0.2037138398	3.2960e-03	0.3447324830	0.3440264787	7.0600e-04	0.4975777625	0.5006454772	3.0677e-03
0.1	0.1128373948	0.1113506853	1.4867e-03	0.2054197828	0.2022269126	3.1929e-03	0.3423177450	0.3414388092	8.7894e-04	0.4947770179	0.4975235482	2.7465e-03
0.2	0.1096920313	0.1083755967	1.3164e-03	0.1999075323	0.1971672411	2.7403e-03	0.3339144717	0.3333108764	6.0360e-04	0.4849373328	0.4879872397	3.0499e-03
0.3	0.1042853331	0.1033368503	9.4848e-04	0.1904012749	0.1883432230	2.0581e-03	0.3193066197	0.3190908635	2.1576e-04	0.4674941763	0.4710357173	3.5415e-03
0.4	0.0965942225	0.0960716915	5.2253e-04	0.1768117523	0.1755509737	1.2608e-03	0.2981748320	0.2983906541	2.1582e-04	0.4415233424	0.4455638439	4.0405e-03
0.5	0.0865861073	0.0864493062	1.3680e-04	0.1590130392	0.1585169419	4.9610e-04	0.2700669180	0.2708016050	7.3469e-04	0.4056848919	0.4103914649	4.7066e-03
0.6	0.0742189871	0.0743729020	1.5391e-04	0.1368431219	0.1369879421	1.4482e-04	0.2343987784	0.2357473936	1.3486e-03	0.3581986734	0.3640380805	5.8394e-03
0.7	0.0594415935	0.0597787794	3.3719e-04	0.1101046959	0.1107313391	6.2664e-04	0.1904565414	0.1924704544	2.0139e-03	0.2968234937	0.3044726915	7.6492e-03
0.8	0.0421935650	0.0426351009	4.4154e-04	0.0785662151	0.0795362595	9.7004e-04	0.1374005073	0.1401090164	2.7085e-03	0.2188481234	0.2289919629	1.0144e-02
0.9	0.0224056586	0.0229424175	5.3676e-04	0.0419632237	0.0432271410	1.2639e-03	0.0742714853	0.0777635924	3.4921e-03	0.1211029623	0.1342940187	1.3191e-02
1.0	0.0000000000	0.0007347281	7.3473e-04	0.0000000000	0.0016993051	1.6993e-03	0.0000000000	0.0045466714	4.5467e-03	0.0000000000	0.0168660921	1.6866e-02

Table 3. Parameter values for proposed model

i	$\alpha = 0.5$											
	$H^2 = 0.5$			$H^2 = 1.0$			$H^2 = 2.0$			$H^2 = 4.0$		
	w_i	β_i	γ_i	w_i	β_i	γ_i	w_i	β_i	γ_i	w_i	β_i	γ_i
1	1.0460	0.8236	0.1722	0.8036	0.2141	-0.0709	0.7738	0.5306	0.2701	1.4356	0.7550	-0.2216
2	0.5039	-0.0215	0.4228	0.8246	0.9714	0.2062	0.1592	0.1248	0.0694	1.0558	1.3439	0.4697
3	0.5585	-0.2756	0.5330	0.8057	1.0106	0.8407	1.0935	0.2257	0.0092	2.2976	1.0930	-0.5309
4	0.9447	0.5158	0.4749	1.6798	-0.3542	-0.5200	0.8038	0.3664	-0.0825	1.8084	0.6435	0.6419
5	0.7320	0.6146	-0.1101	1.0259	-0.0253	-0.4711	1.3952	0.5388	-0.5831	1.2059	0.4950	1.1340
6	0.5603	-0.2880	-0.0046	0.0063	-0.2626	-0.4014	0.8880	-0.7236	0.5287	2.0532	-1.0933	-0.0149
7	1.0391	-0.5227	-0.2539	1.3328	0.4222	0.4146	1.1636	0.6057	0.3826	1.3921	0.4406	-0.6566
8	0.3244	1.0256	-0.4542	0.7929	-0.2122	0.3247	1.1338	-0.7054	0.2232	0.6666	0.0549	0.3709
9	0.1350	-0.3296	0.6492	-0.0561	-0.0307	0.0160	1.2838	1.0217	-0.2949	2.0612	1.8134	-0.6304
10	1.3004	0.3687	-0.7960	0.7147	0.4259	-0.0134	1.7578	0.8861	-0.0296	1.4110	1.2463	0.5511

We took 10 neurons in one layer with three optimizable parameters with each neuron with log-sigmoid activation function for the proposed model and with 30 unknown weight parameters to be determined. We have used the feedforward neural network with adjustable weights optimized by GA. The simulation was run from 0 to 1 with a step size of 0.1 and with total grid points of 11. An absolute error (AE) between the numerical solution and the solution obtained by proposed methodology is also calculated and is shown in Table 2. The GA was used to stochastically select the synaptic weights of the neurons such that the fitness function .

First we fixed the nonlinearity parameter and varied the Hartman electric number by selecting values 0.5, 1.0, 2.0 and 4.0 as shown in Figure 4. For 100 independent runs for each value of and the fitness function behaviour is studied as shown in Figure 5. The logarithm of fitness function against the number of runs is plotted to elaborate the differences. As it could be seen from Figure 5that as the value of increases, the convergence of fitness function about the optimum value is engulfed with high peaks which shows that the strength of nonlinearity increases by increasing value of. However, the solution always converges as it is clear from Figure 4despite of the increasing nonlinearity, which shows the stability and convergence behaviour of the solution by ANN. The convergence behaviour of the solution is also plotted in Figure 6 by rearranging the fitness function value and it could be seen in inset zoomed image that by increasing the convergence towards optimum value is rather slow as compared to small value of .

In Figure 4the results obtained by GA optimized ANN are in good agreement with the numerical solution. In Table 2, results obtained through numerical technique and GA optimized ANN are given. An absolute error (AE) is also calculated which is of the

order of 102- to 104-. The optimized weights obtained for hidden neuron are also shown in Table 3. In Ghasemi et al. (2014) least square method (LSM) solution of the problem is presented. We compare our results for, and with LSM in Table 4 the results obtained by proposed methodology are in good agreement with LSM. The analysis is done with Intel (R) Core (TM) i34005-U CPU @1.70 GHz, 4.00 GB RAM, and MATLAB version 2014a.

Table 4. Comparison of least square method solution with GA solution

x	$\alpha = 0.5, H^2 = 0.5$			$\alpha = 1.0, H^2 = 1.0$		
	LSM	G.A	**A.E** ($\|\|y_{GA} - y_{LSM}\|\|$)	LSM	G.A	**A.E** ($\|\|y_{GA} - y_{LSM}\|\|$)
0	0.1137460000	0.1124069421	1.3390e-03	0.20343243	0.2073413745	3.9089e-03
0.1	0.1126461771	0.1113506853	1.2954e-03	0.20156532	0.2051191107	3.5538e-03
0.2	0.1093423619	0.1083755967	9.6676e-04	0.19594756	0.1987440721	2.7965e-03
0.3	0.1038208809	0.1033368503	4.8403e-04	0.18651760	0.1883619592	1.8444e-03
0.4	0.0960586656	0.0960716915	1.3025e-05	0.17316801	0.1740654673	8.9746e-04
0.5	0.0860234312	0.0864493062	4.2587e-04	0.15574958	0.1558547308	1.0515e-04
0.6	0.0736738556	0.0743729020	6.9904e-04	0.13407547	0.1336139033	4.6157e-04
0.7	0.0589597580	0.0597787794	8.1902e-04	0.10792535	0.1071136649	8.1169e-04
0.8	0.0418222777	0.0426351009	8.1282e-04	0.07704953	0.0760277270	1.0218e-03
0.9	0.0221940530	0.0229424175	7.4836e-04	0.04117309	0.0399603384	1.2128e-03
1.0	0.0000006000	0.0007347281	7.3532e-04	0.00000000	0.0015095243	1.5095e-03

The results for other values of and are shown in Figures 7 & 8.

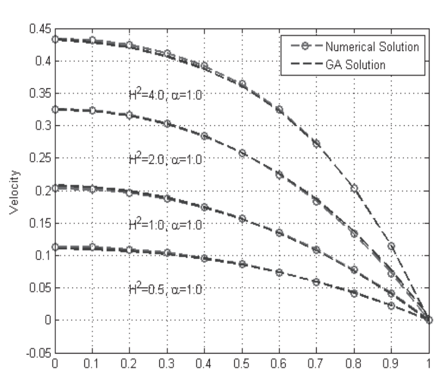

Fig. 7. Effect of Hartman electric number and comparison of numerical and GA solution for α=1.0

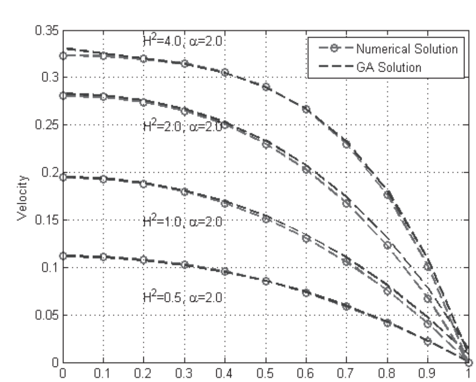

Fig. 8. Effect of Hartman electric number and comparison of numerical and GA solution for

5. Conclusions

Artificial neural network (ANN) having 10 neurons and with three adjustable weights for each neuron and optimized with GA can effectively solve the electrohydrodynamic flow problem in circular cylindrical conduit.

The proposed solution through ANN-GA is in good agreement with numerical solution obtained through MATLAB and error lies in the range of 1002- to 1004-. Also the results obtained through proposed methodology are in good agreement with one of reported technique of LSM. The adopted methodology according to the best of author's knowledge has not been applied before for the solution of electrohydrodynamic flow problem and this shows the versatility of the ANN-GA technique for solution of nonlinear problems.

The convergence, effectiveness, accuracy and reliability of ANN optimized with GA is verified through large number of independent runs of the algorithm and their detailed statistical analysis is performed. The proposed algorithm is seen to converge in every iteration and the results obtained are accurate and in good agreement with algorithms proposed in literature.

In the future, similar investigations could be made for singular BVPs with ANN optimized with particle swarm optimization (PSO), ant colony optimization (ACO), firefly algorithm (FFA) and cuckoo search algorithm (CSA). Also hybrid techniques of global and local optimizers could be employed to investigate this problem.

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Submitted: 06/03/2016
Revised : 05/06/2016
Accepted : 09/06/2016

حل التدفق من نوع الكتروهايدروديناميك لحركة الأيون في دائرة أسطوانية باستخدام شبكة عصبية مهجنة ونمذجة وراثية

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خـلاصـة

ندرس في هذا البحث في معادلة التدفق لحركة الأيون من النوع الالكتروهايدروديناميك (EHD) في دائرة اسطوانية باستخدام شبكة عصبية صناعية (ANN) تم جعلها مُثلى باستخدام نمذجة وراثية (GA). المعادلة الحاكمة غير خطية من الدرجة الثانية وعدم الخطية يعتمد على معلمة a. نقترح حل EDD باستخدام ثلاث طبقات ANN مع عشر خلايا عصبية في كل طبقة تم جعلها مُثلى باستخدام GA. تتفق نتائج النموذج المُقترح مع الحلول الحسابية باستخدام MATLAB وباستخدام طريقة المربعات الصغرى (LSM).