Kuwait J. Sci. **42 (2)** pp. 128-140, 2015

On the quaternionic Mannheim curves of Aw(k)-type in Euclidean space E^3

SEZAI KIZILTUĞ [*] AND YUSUF YAYLI[**]

[*]*Department of Mathematics, Faculty of Science and Arts, Erzincan University, Erzincan, Turkey email: skiziltug24@hotmail.com, skiziltug@erzincan.edu.tr*

[**] *Department of Mathematics, Faculty of Sciences, Ankara University, Ankara, Turkeyemail: yayli@science.ankara.edu.tr*
Corresponding Author: skiziltug24@hotmail.com, skiziltug@erzincan.edu.tr

ABSTRACT

In this paper, we consider that the curvature conditions of Aw(k)-type $(1 \leq k \leq 3)$ quaternionic curves in Euclidean space E^3 and investigates quaternionic Mannheim curves $\alpha : I \rightarrow Q$ with $k \neq 0$ and $r \neq 0$. Besides, we show that quaternionic Mannheim curves are Aw(2) -type and Aw (3)-type quaternionic curves in E^3. But, there is no such a Mannheim curve of Aw(1) -type.

Keywords: Aw(k)-type curve; general helix; mannheim curves ; quaternion algebra.

2000 Mathematics Subject Calssification: Msc:11R52, 53A04

INTRODUCTION

The quaternion was introduced by Hamilton. His initial attempt to generalize the complex numbers by introducing a three-dimensional object failed in the sense that the algebra he constructed for these three-dimensional object did not have the desired properties. On the 16th October 1843 Hamilton discovered that the appropriate generalization is one in which the scalar (real) axis is left unchanged whereas, the vector (imaginary) axis is supplemented by adding two further vector axis.

Besides, there are three different types of quaternions, namely real, complex and dual quaternions. A real quaternion is defined as $q = q_0 + q_1 e_1 + q_2 e_2 + q_3 e_3$ is composed of four units (1, $e_1, e_2, e_3)$ where e_1, e_2, e_3 are orthogonal unit spatial vectors, $q_i (i = 0,1,2,3)$ are real numbers and this quaternion may be written as a linear combination of a real part(scalar) and vectorial part(a spatial vector).

Quaternions uses in both theoretical and applied mathematics, in particular for calculations involving three-dimensional rotations such as in three-dimensional computer graphics and computer vision. They can be used alongside other methods, such as Euler angles and matrices, or as an alternative to them depending on the application. Furthermore, Bharathi & Nagaraj (1985) represented the curves by unit quaternions in E^3 and E^4 and called these curves as quaternionic curves. They studied the differential geometry of space curves and introduced Frenet frames and formulae by using quaternions. After them, Çöken & Tuna (2004) studied quaternionic inclined curves in the semi-Euclidean space E_2^4. Gök *et al.*(2011) have defined Quaternionic B ^{2} -Slant Helices in the Euclidean Space E^4. Karadağ & Sivridağ (1997) have studied quaternionic inclined curves. Many interesting results on curves of Aw(k)-type have been obtained by many mathematicians. Özgür & Gezgin (2005) studied a Bertrand curve of Aw(k)-type and showed that there was no such Bertrand curve of Aw(1)-type and was of Aw(3)-type if and only if it was a right circular helix. In addition they studied weak Aw(2)-type and Aw(3)-type conical geodesic curves in E^3. K z ltuğ & Yayl (2014) investigated curves Aw(k)-type in the equiform geometry of the Galilean space. In this paper, we have done a study on quaternionic Mannheim partner curves of Aw(k)-type. However, to the best of author's knowledge, quaternionic Mannheim partner curves of Aw(k)-type have not been presented in three dimensional Euclidean space E^3. Therefore, this study is proposed to serve such a need.

PRELIMINARIES

In this section, we give the basic elements of the theory of quaternions and quaternionic curves. A more complete elementary treatment of quaternions and quaternionic curves can be found inin (Bharathi & Nagaraj, 1985) and (Karadağ & Sivridağ, 1997), respectively..

A real quaternion q is an expression of the form

$$q = a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4 \tag{1}$$

where a_i, $(1 \le i \le 4)$ are real numbers, and e_i, $(1 \le i \le 4)$ are quaternionic units which satisfy the non-commutative multiplication rules

$$e_i \times e_i = -e_4, (1 \le i \le 3) \tag{2}$$

$$e_i \times e_j = e_k = -e_j \times e_i, (1 \le i, j \le 3)$$

where (ijk) is an even permutation of (123) in the Euclidean space. The algebra of the quaternions is denoted by Q and its natural basis is given by (e_1, e_2, e_3, e_4). A real quaternion can be given by the form

$$q = s_q + v_q \tag{3}$$

where $s_q = a_4$ is scalar part and $v_q = a_1e_1 + a_2e_2 + a_3e_3$ is vector part of q.

The conjugate of $q = s_q + v_q$ is defined by

$$\overline{q} = s_q - v_q. \tag{4}$$

This defines the symmetric real-valued, non-degenerate, bilinear form as follows:

$$h : Q \times Q \to \mathbb{R}, (p,q) \to h(p,q) = \frac{1}{2}(q \times \overline{p} + p \times \overline{q}) \tag{5}$$

which is called the quaternion inner product (Bharathi & Nagaraj, 1985). Then the norm of q is given by

$$\|q\|^2 = h(q,q) = q \times \overline{q} = \overline{q} \times q = a_1^2 + a_2^2 + a_3^2 + a_4^2. \tag{6}$$

If $\|q\| = 1$, then q is called unit quaternion. Then, inverse of the quaternion q is given by

$$q^{-1} \frac{\overline{q}}{\|q\|}. \tag{7}$$

Let $q = s_q + v_q = a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4$ and $p = s_p + v_p = b_1e_1 + b_2e_2 + b_3e_3 + b_4e_4$ two quaternions in Q. Then the quaternion product of q and p is given by

$$q \times p = s_q s_p - \langle v_q, v_p \rangle + s_q v_p + s_p v_q + v_q \wedge v_p \tag{8}$$

where $\langle , \rangle$ and $\wedge$ denote the inner product and vector product in Euclidean 3-space E^3.

q is called a spatial quaternion whenever $q + \overline{q} = 0$ and called a temporal quaternion whenever $q - \overline{q} = 0$. Then a general quaternion q can be given as

$$q = \frac{1}{2}(q + \overline{q}) + \frac{1}{2}(q - \overline{q}).$$

The quaternion $\frac{1}{2}(q - \overline{q})$ is a spatial quaternion and called spatial part of q and the quaternion $\frac{1}{2}(q + \overline{q})$ is a temporal quaternion and called temporal part of q (Bharathi & Nagaraj, 1985).

The three-dimensional Euclidean space E^3 is identified with the space of spatial quaternions $\{q \in Q : q + \overline{q} = 0\}$ in an obvious manner. Let $I = [0,1]$ be an interval in real line R and $s \in I$ be parameter along the regular curve

$$\alpha : I \subset \mathrm{R} \to Q, s \to \alpha(s) = \sum_{i=1}^{3} \alpha_i(s) e_i \tag{9}$$

chosen such that the tangent $\alpha(s) = t$ is unit, i.e., $\|t\| = 1$ for all s. Then $\alpha(s)$ is called spatial quaternionic curve (Bharathi & Nagaraj, 1985). Since $\|t\| = 1$, $t' \times \overline{t} + t \times \overline{t} =' 0$ holds and it means that t is orthogonal to t and moreover $t \times \overline{t}$ is a spatial quaternion.

Since t' is itself a spatial quaternion, we define a spatial quaternion n_1 and non-negative scalar function $k = k(s)$ is called principal curvature. n_1 is orthogonal to t. Then by considering that $t \times \bar{t}$ is a spatial quaternion, there exists a unit spatial quaternion $n_2(s) = t(s) \times n_1(s) = -n_1(s) \times t(s)$. Then the set $\{t(s), n_1(s), n_2(s)\}$ is called Frenet frame along the quaternionic curve $\alpha(s)$, where $t(s)$ is unit tangent, $n_1(s)$ is unit principal normal and $n_2(s)$ is unit binormal of the curve $\alpha(s)$. The Frenet formulae of the quaternionic curve $\alpha(s)$ are

$$\begin{bmatrix} t'(s) \\ n_1'(s) \\ n_2'(s) \end{bmatrix} = \begin{bmatrix} 0 & k(s) & 0 \\ -k(s) & 0 & r(s) \\ 0 & -r(s) & 0 \end{bmatrix} \begin{bmatrix} t(s) \\ n_1(s) \\ n_2(s) \end{bmatrix} \tag{10}$$

where $k = k(s)$ is principal curvature and $r = r(s)$ is torsion of $\alpha(s)$. (For Details (Bharathi & Nagaraj, 1985)).

Theorem 1 (8) (Karadağ & Sivridağ, 1997) Let $\alpha : I \to Q$ be a real spatial quaternionic curve with arc length parameter s and nonzero curvatures $\{k(s), r(s)\}$. If α is general helix if and only if

$$\frac{r(s)}{k(s)} \text{ is constant.} \tag{11}$$

QUATERNIONIC CURVES OF AW(K)-TYPE

Let $\alpha : I \to Q$ be an arc-lenght parametrized unit speed real spatial quaternionic curve in Euclidean 3-space. The curve $\alpha = \alpha(s)$ is called a Frenet curve of osculating order 3 if its derivatives $\alpha'(s), \alpha''(s), \alpha'''(s), \alpha''''(s)$ are linearly dependent and $\alpha'(s), \alpha''(s), \alpha'''(s), \alpha''''(s)$ are no longer linearly independent for all $s \in I$. To each Frenet curve of order 3 one can associate an orthonormal 3-frame $t(s), n_1(s), n_2(s)$ along α such that $(\alpha'(s)) = t$ called the Frenet frame and functions k, $r : I \to \mathbb{R}$ called the Frenet curvatures.

Proposition 2 $\alpha : I \to Q$ be an arc-lenght parametrized unit speed real spatial quaternionic curve in Euclidean 3-space, then we have

$$\alpha'(s) = t(s)$$

$$\alpha''(s) = k(s) n_1(s)$$

$$\alpha'''(s) = -k^2(s) t(s) + k'(s) n_1(s) + k(s) r(s) n_2(s)$$

$$\alpha''''(s) = (-3k(s)k'(s)\ t(s) + (k''(s) - k^3(s) - k(s)r^2(s)\ n_1(s)$$
$$+ (2k'(s)r(s) + k(s)r'(s)) n_2(s).$$

Notation 3 Let us write

$$N_1(s) = k(s)n_1(s), \tag{12}$$

$$N_2(s) = k^{'}(s)n_1(s) + k(s)r(s)n_2(s), \tag{13}$$

$$N_3(s) = (k^{''}(s) - k^3(s) - k(s)r^2(s))n_1(s) + (2k^{'}(s)r(s) + k(s)r^{'}(s))n_2(s). \tag{14}$$

Remark 1. $\alpha^{'}(s)$, $\alpha^{''}(s)$, $\alpha^{'''}(s)$, $\alpha^{''''}(s)$ are linearly dependent if and only if $N_1(s)$, $N_2(s)$, $N_3(s)$ are linearly dependent.

As the definition of Aw(k) type curves in (Arslan & Özgür, 1999), we have

Definition 1. Real spatial quaternionic curves (of osculating order 3) in Euclidean space are (Arslan & Özgür, 1999)

(*i*) of type weak Aw(2) if they satisfy

$$N_3(s) = h(N_3(s), N_2^*(s))N_2^*(s), \tag{15}$$

(*ii*) of type weak Aw(3) if they satisfy

$$N_3(s) = h(N_3(s), N_1^*(s))N_1^*(s) \tag{16}$$

where

$$N_1^*(s) = \frac{N_1(s)}{\|N_1(s)\|}, N_2^*(s) = \frac{N_2(s) - h(N_2(s), N_1^*(s))N_1^*(s)}{\|N_2(s) - h(N_2(s), N_1^*(s))N_1^*(s)\|}.$$

Proposition 2. Let α be a real spatial quaternionic curve (of osculating order 3) in Euclidean space. If α is of type weak Aw (2) then

$$k^3(s) - k^{''}(s) + k(s)r^2(s) = 0. \tag{17}$$

Proposition 3. Let α be a real spatial quaternionic curve (of osculating order 3) in Euclidean space. If α is of type weak Aw (3) then

$$2k^{'}(s)r(s) + k(s)r(s) = 0. \tag{18}$$

Definition 2. Real spatial quaternionic curves (of osculating order 3) in Euclidean space are (*i*) of type Aw (1) if they satisfy $N_3(s)=0$,

(*ii*) of type Aw (2) if they satisfy

$$\left\|N_2(s)\right\|^2 N_3(s)=h(N_3(s),N_2(s))N_2(s), \tag{19}$$

(*iii*) of type Aw(3) if they satisfy

$$\left\|N_1(s)\right\|^2 N_3(s)=h(N_3(s),N_1(s))N_1(s). \tag{20}$$

Theorem 2. Let α be a real spatial quaternionic curve (of osculating order 3) in Euclidean space. Then α is of type Aw(1) if and only if

$$k^3(s)-k''(s)+k(s)r^2(s)=0 \tag{21}$$

and

$$2k'(s)r(s)+k(s)r(s)=0. \tag{22}$$

Proof. Since α is a curve of type Aw(1), we have $N_3(s)=0$. Then from Eq. 14, we have

$$(k^3(s)-k''(s)+k(s)r^2(s)\ n_1(s)+((2k'(s)r(s)+k(s)r(s)\ n_2(s)=0.$$

Furthermore, since $n_1(s)$ and $n_2(s)$ are linearly independent, we get

$$k^3(s)-k''(s)+k(s)r^2(s)=0 \text{ and } k'(s)r(s)+k(s)r(s)=0.$$

The converse statement is trivial. Hence our theorem is proved.

Theorem 3. Let α be a real spatial quaternionic curve (of osculating order 3) in Euclidean space. Then α is of type Aw(2) if and only if

$$\begin{aligned} &2(k'(s))^2 r(s)+k'(s)k(s)r'(s)-\\ &k''(s)k(s)r(s)+k^4(s)r(s)+k^2(s)r^3(s)=0. \end{aligned} \tag{23}$$

Proof. Suppose that α is a Frenet curve of order 3, then from 13 and 14, we can write

$$N_2(s)=\gamma(s)n_1(s)+\beta(s)n_2(s), \tag{24}$$

$$N_3(s)=\eta(s)n_1(s)+\delta(s)n_2(s), \tag{25}$$

where γ, β, η and δ are differentiable functions. Since $N_2(s)$ and $N_3(s)$ are linearly dependent, coefficients determinant is equal to zero and hence one can write

$$\begin{vmatrix} \gamma(s) & \beta(s) \\ \eta(s) & \delta(s) \end{vmatrix}=0. \tag{26}$$

Here,

$$\gamma(s)=k'(s),\beta(s)=k(s)r(s)$$

and

$$\eta(s) = k''(s) - k^3(s) - k(s)r^2(s),$$
$$\delta(s) = 2k'(s)r(s) + k(s)r'(s).$$

Substituting these into (26), we obtain (23).

Conversely if the equation 23 holds, it is easy to show that α is of type Aw(2).

This completes the proof.

Corollary 1. Let a real spatial quaternionic curve (of osculating order 3). If it is quaternionic cylindrical helix and α is of type Aw (2) then

$$3(k'(s))^2 - k''(s)k(s) + k^4(s)\left(1+c^2\right) = 0 \tag{27}$$

where $c = \dfrac{r(s)}{k(s)}$ is constant.

Theorem 4. Let α be a quaternionic general helix in Euclidean 3-space. If α is of type Aw(2), then

$$\kappa(s) = \frac{1}{\sqrt{-As^2 + Bs + C}} \quad \text{and} \quad r(s) = \sqrt{A-1}\,\kappa(s) \tag{28}$$

where $A = 1 + c^2$, B and C are real constants.

Proof. Suppose that α is a general helix of type Aw(2). Then Eq. 27 holds. If we substitute $\kappa(s) = x$ in 27, we get

$$x\frac{d^2x}{ds^2} - 3\left(\frac{dx}{ds}\right)^2 = Ax^4, A = 1 + c^2. \tag{29}$$

Let us take $x = y^p$ and differentiating it twice we obtain

$$\frac{dx}{ds} = py^{p-1}\frac{dy}{ds}, \tag{30}$$

$$\frac{d^2x}{ds^2} = p(p-1)y^{p-2}\left(\frac{dy}{ds}\right)^2 + py^{p-1}\frac{d^2y}{ds^2}. \tag{31}$$

Now, the substitution of 30 and 31 into 29, weget

$$y^p\left[py^{p-1}\frac{d^2y}{ds^2} + p(p-1)y^{p-2}\left(\frac{dy}{ds}\right)^2\right] - 3p^2y^{2p-2}\left(\frac{dy}{ds}\right)^2 = Ay^{4p},$$

$$py^{2p-1}\frac{d^2y}{ds^2} + p(p-1)y^{2p-2}\left(\frac{dy}{ds}\right)^2 - 3p^2y^{2p-2}\left(\frac{dy}{ds}\right)^2 = Ay^{4p}.$$

Putting ($p(p-1)=3p^2$ i.e. $p=-\frac{1}{2}$) into the last equation we get

$$py^{2p-1}\frac{d^2y}{ds^2}=Ay^{4p}.$$

So,

$$\frac{d^2y}{ds^2}=-2A.$$

Now, we solve this last equation. Since $\frac{dy}{ds}=-2As+B$, we get

$$y=-As^2+Bs+C.$$

Furthermore, use of $x=y^{\frac{-1}{2}}$ we obtain

$$x=(-As^2+Bs+C)^{\frac{1}{2}}.$$

Since $c=\frac{r(s)}{k(s)}$, we have the result.

Corollary 2. Let α be a real spatial quaternionic curve (of osculating order 3) in Euclidean space. If α is of type Aw(2), then α can not be a circular helix.

Theorem 5. Let α be a real spatial quaternionic curve (of osculating order 3) in Euclidean space. Then α is of type Aw(3) if and only if

$$r(s)=\frac{c}{k^2(s)}. \tag{32}$$

Proof. Since α is a curve of type Aw (3), then 20 holds on α . Substituting 12 and 14 into 20, we get

$$2k^{'}(s)r(s)+k(s)r(s)=0.$$

If we solve above differential equation, we get

$$r(s)=\frac{c}{k^2(s)}.$$

The converse statement is trivial. Hence our theorem is proved.

Corollary 3. Let α be a general helix of osculating order 3.Then α is of type Aw (3) if and only if α is a circular helix.

Proof. Suppose that α is a general helix of type Aw (3). Combining 11 and 32 we find $k(s)$ and $r(s)$ are nonzero constants. Thus, α is circular helix. The converse statements is trivial.

AW (k)-TYPE QUATERNIONIC MANNHEIM CURVES IN E^3

This section characteries the curvatures of AW(k)-type quaternionic Mannheim curves in E^3 We provided some theorems and conclusion on Aw(k)-type $(k=1,2,3)$ quaternionic Mannheim Curves in E^3.

Definition 3. (Liu & Wang, 2008). A curve $\alpha : I \to Q$ with $k \neq 0$ is called a Mannheim curve if there exist a curve $\widetilde{\alpha} : I \to Q$ such that, at the corrsponding points of curves, the principal normal lines of α coincides with binormal lines of $\widetilde{\alpha}$. In this case $\widetilde{\alpha}$ is called Mannheim partner curve of α the pair $(\alpha, \widetilde{\alpha})$ is said to be a Mannheim pair (Liu, H. & Wang. 2008; F, Orbay, K. & Kasap, E. 2009).

Theorem 6. The distance between the corresponding points of the quaternionic Mannheim curves is constant in E^3.

Proof. Suppose that α is a Mannheim curve. Then by the definition we can assume that

$$\alpha(s) = \widetilde{\alpha}(s) + \lambda(s)\widetilde{n}_2(s) \tag{33}$$

for some function $\lambda(s)$. By taking the derivative of 33 with respect to s and applying Equations 10, we have

$$t(s) = \tilde{t}(s) - \lambda(s)\tilde{r}(s)\tilde{n}_1(s) + \dot{\lambda}(s)\tilde{n}_2(s). \tag{34}$$

Since $\widetilde{n}_2(s)$ is coincident with $n_1(s)$ in direction, we get

$$\dot{\lambda}(s) = 0.$$

This means that $\lambda(s)$ is a nonzero constant. Thus we have

$$t = \widetilde{t}(s) - \lambda(s)\widetilde{r}(s)\widetilde{n}_1(s). \tag{35}$$

On the other hand, from the distance function between two points, we have

$$d\left(\tilde{\alpha}(s), \alpha(s)\right) = \left\|\alpha(s) - \tilde{\alpha}(s)\right\| = \left\|\lambda\tilde{n}_2(s)\right\| = |\lambda|.$$

Namely, $d\left(\tilde{\alpha}(s), \alpha(s)\right) =$ constant. Hence, the proof is completed.

Theorem 7. Let $\widetilde{\alpha}$ be a quaternionic Mannheim partner curve of α. Then there is a following relation between Mannheim curve α and Mannheim partner curve $\widetilde{\alpha}$

$$\widetilde{\alpha}(s) = \alpha(s) - \lambda n_1(s) \tag{36}$$

where λ is nonzero constant.

Proof. Since $n_1(s)$ and $\widetilde{n}_2(s)$ are linearly dependent, Eq. 33 can be written as

$$\widetilde{\alpha}(s) = \alpha(s) - \lambda n_1(s)$$

Now, there is a curve $\tilde{\alpha}$ for all values of nonzero constant λ.

Proposition 4. Let $(\tilde{\alpha},\alpha)$ be a quaternionic Mannheim pair in E^3. The linear relation between the curvature and torsion of the curve α is given as follows:

$$\mu r(s)-\lambda k(s)=1 \tag{37}$$

Proof. Denote the Darboux frames of $\alpha(s)$ and $\tilde{\alpha}(s)$ by $\{t(s),n_1(s),n_2(s)\}$ and $\{\tilde{t}(s),\tilde{n}_1(s),\tilde{n}_2(s)\}$, respectively. Let angle between $t(s)$ and $\tilde{t}(s)$, which is tangent vector of $\tilde{\alpha}(s)$ be θ. Thus, we have

$$\tilde{t}(s)=\cos\theta t(s)-\sin\theta\tilde{n}_2(s). \tag{38}$$

Since $\alpha(s)$ and $\tilde{\alpha}(s)$ are Mannheim curve mate, we have

$$\tilde{\alpha}(s)=\alpha(s)-\lambda n_1(s). \tag{39}$$

If differentiating 39 with respect to s, we get

$$\tilde{t}(\tilde{s})\frac{d\tilde{s}}{ds}=\left(1+\lambda k(s)\right)t(s)-\lambda r(s)n_2(s). \tag{40}$$

Thus, from 38 and 40 we have

$$\frac{(1+\lambda k(s))}{\cos\theta}=\frac{-\lambda r(s)}{\sin\theta}. \tag{41}$$

From Above equation, we obtain

$$-\lambda k(s)+\cot\theta\lambda r(s)=1.$$

If take $\cot\theta\lambda=\mu$, we get

$$\mu r(s)-\lambda k(s)=1.$$

Corollary 4. Suppose that $k(s)\neq 0$ and $r(s)\neq 0$. Then α is a quaternionic Mannheim curve if and only if there exist a nonzero real number λ such that

$$\lambda(k'(s)r(s)-k(s)r'(s))-r'(s)=0. \tag{42}$$

Proof. By the proposition 19, α is a Mannheim curve if and only if there exist real numbers $\lambda\neq 0$ and μ such that $\mu r(s)-\lambda k(s)=1$. This is equivalent to the condition that there exists a real number $\lambda\neq 0$ such that $\frac{1+\lambda k(s)}{r(s)}$ is constant. Differentiating both sides of the last equality, we have

$$\lambda(k'(s)r(s)-k(s)r'(s))-r'(s)=0.$$

Proposition 5. Let $\alpha: I\to Q$ be a quaternionic Mannheim curve with $k(s)\neq 0$ and $r(s)\neq 0$. Then α is of Aw(2)-type if and only if there is a non zero rael number λ such that

$$\lambda k(s)r(s)(k^2(s)-r^2(s))-\lambda r''(s)(\lambda k(s)+1)=0. \tag{43}$$

Proof. Since α is of type Aw (2), Eq.23 holds and since α is a real spatial quaternionic Mannheim curve, Eq. 42 holds. If both of these equations are considered, 43 is obtained.

Theorem 8. Let $\alpha : I \to Q$ be a quaternionic Mannheim curves with $k(s) \neq 0$ and $r(s) \neq 0$. Then α is of Aw(3)-type if and only if α is a right circular helix.

Proof. Now suppose that $\alpha : I \to Q$ is a quaternionic Mannheim curve of Aw(3)-type with $k(s) \neq 0$ and $r(s) \neq 0$. Then the Eqs. 32 and 42 hold on α. Differentiating 32, we have

$$r'(s) = \frac{-2ck'(s)}{k^3(s)}. \tag{44}$$

Substituting 32 and 44 in 42, we get

$$k(s) = \frac{2}{3\lambda} = \text{const.} \tag{45}$$

If substituting 45 in 32, the following equation is obtained,

$$r(s) = \frac{9\lambda^2 c}{4} = \text{const.}$$

Since $k(s)$ and $r(s)$ are nonzero constants, α is a right circular helix.

Theorem 9. Let $\alpha : I \to Q$ be a quaternionic Mannheim curves with $k(s) \neq 0$ and $r(s) \neq 0$. If α is of weak Aw(2)-type, then following equation hold

$$\lambda k(s) r(s)(k^2(s) + r^2(s)) - \lambda r''(s)(1 - \lambda k(s)) = 0. \tag{46}$$

Proof. Since α is of type weak AW(2), Eq. 17 holds and since α is a quaternionic Mannheim curve, Eq. 42 holds. Arranging Eq. 17, we have

$$k''(s) = k^3(s) + k(s) r^2(s). \tag{47}$$

Differentiating 42, we get

$$\lambda(k''(s) r(s) - k(s) r''(s)\ \ - r''(s) = 0. \tag{48}$$

If Eq. 42 is substituted in 48 , then 46 is obtained.

Theorem 10. There is not a quaternionic Mannheim curve $\alpha : I \to Q$ with $k(s) \neq 0$ and $r(s) \neq 0$ of AW(1)-type·

Proof. Since α is of type AW(1), Eq.21 and 22 holds and since α is a quaternionic Mannheim curve, Eq.42 holds. If we solve this differential equation 22, we get

$$r(s) = \frac{c}{k^2(s)}. \tag{49}$$

Differentiating 49, we get

$$r'(s) = \frac{-2c}{k^3(s)}. \tag{50}$$

So substituting 49 and 50 in 42 and making necessary arrangements, we get

$$\lambda^2 = \frac{-2c^2}{72}. \tag{51}$$

Whic given us $\lambda^2 < 0$. Since λ is real number, this isnt possible. So $\alpha : I \to Q$ isnt a AW(1)-type Mannheim curve. This completes proof.

REFERENCES

Arslan, K. & Özgür, C. 1999. Curves and surfaces of AW(k)-type. Geometry and Topology of Submanifolds, IX (Valenciennes/Lyan/Leuven,1997), World Science Publishing River Edge, NJ, pp. 21–26.

Bharathi, K. & Nagaraj, M. 1985. Quaternion valued function of a real Serret-Frenet formulae. Indian Journal of Pure and Applied Mathematics, **16**: 741-756.

Çöken, A. C. & Tuna, A. 2004. On the quaternionic inclined curves in the semi-Euclidean space E_2^4, Applied Mathematics and Computation **155**: 373-389.

Gök, I 307. & Okuyucu, O. Z. & Kahraman, F. & Hacisalihoglu, H. H. 2011. On the Quaternionic B_2-Slant Helices in the Euclidean Space E944, Advances in Applied Clifford Algebras, **21**: 707–719.

Karadağ, M.& Sivridağ, A.İ. 1997. Kuaterniyonik Eğilim Çizgileri için karakterizasyonlar. Erciyes Üniversitesi Fen Bilimleri Enstitüsü Dergisi, **13** 1-2 : 37-53..

Kızıltuğ, S. & Yaylı, Y. 2014. Bertrand curves of AW(k)-type in the equiform geometry of the Galilean Space. Abstract and Applied Analysis, Article ID 402360, pp.6.

Liu, H. & Wang, F. 2008. Mannheim partner curves in 3-space, Journal of Geometry, vol.88: **1**-2, pp.120-126.

Özgür, C. & Gezgin, F. 2005. On some curves of AW(k)-type. Differential Geometry Dynamical Systems, **7**: 74–80. . 7: 74–80.

Submitted **:** 13/01/2014

Revised **:** 20/03/2014

Accepted **:** 23/03/2014

حول منحنيات مانهايم المرباعية من النوع AW (k) في الفضاء الاقليدي E3

*** سيزاي كيزلدج ، **يوسف يالي**

*جامعة ارزينكان – كلية العلوم والآداب – قسم الرياضيات – ارزينكان – تركيا
* جامعة أنقرة – كلية العلوم – قسم الرياضيات – أنقرة – تركيا

خلاصة

نقوم في هذا البحث بدراسة شروط التقوس لمنحنيات من النوع AW (k) ، ($1 \leq k \leq 3$) المرباعية ودراسة منحنيات مانهايم. نثبت إضافة إلى ذلك، أن منحنيات مانهايم المرباعية هي من النوع AW (2) ومن النوع AW (3) في E^3. لكن لا يوجد منحنيات مانهايم مرباعية من النوع AW (1).