Kuwait J. Sci. **41** (1) pp. 93-106, 2014

Triangular fuzzy sub Γ-semihypergroups in Γ-semihypergroups

BIJAN DAVVAZ* AND VIOLETA LEOREANU-FOTEA**

*Department of Mathematics, Yazd University, Yazd, Iran
*Corresponding author, E-mail: davvaz@yazd.ac.ir
**Faculty of Mathematics, Al.I. Cuza University, Iasi, Romania

ABSTRACT

A Γ-semihypergroup is a generalization of a semigroup, a generalization of a semihypergroup and a generalization of a Γ-semigroup. In this paper, by using the notion of triangular norms, we define the concept of triangular fuzzy sub Γ-semihypergroups of a Γ-semihypergroup, and we study a few results in this respect.

2000 Mathematics Subject Classification: 20N20, 20N25.

Keywords: Semihypergroup; Γ-semigroup; Γ-semihypergroup; fuzzy sub Γ-semihypergroup.

1. A BRIEF EXCURSION INTO BASIC DEFINITIONS

1.1. Γ-semigroups

The concept of Γ-semigroups was introduced in Sen & Saha (1986) and Saha, (1987) as a generalization of semigroups and ternary semigroups. Many classical notions related to semigroups have been extended to Γ-semigroups and a lot of results on Γ-semigroups are published by a lot of mathematicians, for instance see (Hila, 2008; Sen & Saha, 1990). We recall the following definition from Sen & Saha (1986). Let $M = \{a, b, c, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. Then M is called a Γ-*semigroup* if there exists a mapping $M \times \Gamma \times M \to M$ written as $(a, \gamma, b) \mapsto a\gamma b$ satisfying the following identity $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. Let K be a non-empty subset of M. Then, K is called a *sub* Γ-*semigroup* of M if $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma$.

1.2. Semihypergroups

Hyperstructures, in particular semihypergroups, were introduced in 1934 by a French mathematician, Marty, at the 8th Congress of Scandinavian Mathematicians (Marty, 1934). Since then, hundreds of papers and several books have been written on this topic, see (Corsini, 1993; Corsini & Leoreanu, 2003; Davvaz & Leoreanu-Fotea, 2007; Vougiouklis, 1994). Let S be a non-empty set and let $\wp^*(S)$ be the set of all non-empty subsets of S. A *hyperoperation* on S is a map $\circ : S \times S \to \wp^*(S)$ and the couple $(S, \circ)$ is called a *hypergroupoid*. If A and B are non-empty subsets of S, then we denote $A \circ B = \bigcup_{a \in A, b \in B}, a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$. A hypergroupoid $(S, \circ)$ is called a *semihypergroup* if for all x, y, z of S we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that $\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$. A semihypergroup $(S, \circ)$ is called a *hypergroup* if for all $x \in S$, we have $x \circ S = S \circ x = S$. Many authors studied different aspects of semihypergroups, for instance (Bonansinga & Corsini, 1982; Davvaz, 2000a; Davvaz & Poursalavati, 2000; Fasino & Freni 2007; Leoreanu, 2000).

1.3 Γ-semihypergroups

Davvaz and his coauthors (Anvariyeh *et al.*, 2010a; Anvariyeh *et al.,* 2010b; Hedayati & Davvaz, 2011; Heidari & Davvaz, 2011; Heidari et al., 2010; Hila *et al.*, 2012) studied the concept of Γ-semihypergroups. Let S and Γ be two non-empty sets. S is called a Γ-*semihypergroup* if every $\gamma \in \Gamma$ is a hyperoperation on S, i.e, $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in S$ we have $x\alpha(y\beta z) = (x\alpha y)\beta z$. Note that this definition is a generalization of the definition of Γ-semigroups.

Example 1 Let S be a semigroup and Γ be a non-empty subset of S. We define the map $S \times \Gamma \times S \to \wp^*(S)$ by $(x, \gamma, y) \mapsto \{a \in S \mid a \in x\gamma y\}$. Then, S is a Γ-semihypergroup.

Example 2 (Anvariyeh et al., 2010b). Let $S = [0, 1]$ and $\Gamma = N^*$. For every $x, y \in S$ and $\gamma \in \Gamma$, we define $\gamma : S \times S \to \wp^*(S)$ by $x\gamma y = [0, \frac{xy}{\gamma}]$. Then, S is a Γ-semihypergroup.

1.4 Fuzzy sets

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets have been introduced by Zadeh as an extension of the classical notion of sets (Zadeh, 1965). Let X be a set. A *fuzzy subset* A of X is characterized by a membership function $\mu_A : X \to [0, 1]$ which associates with each point $x \in X$ its *grade* or *degree of membership* $\mu_A(x) \in [0, 1]$. Fuzzy sets generalize classical sets, since the characteristic functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Let A and B be fuzzy subsets of X. Then, (1) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, for all $x \in X$; (2) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, for all $x \in X$; (3) $C = A \cup B$ if and only if

$\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$, for all $x \in X$; (4) $D = A \cap B$ if and only if $\mu_D(x) = \min\{\mu_A(x), \mu_B(x)\}$, for all $x \in X$. The *complement* of A, denoted by A^c, is defined by $\mu_{A^c}(x) = 1 - \mu_A(x)$, *for all* $x \in X$. Let f be a mapping from a set X to a set Y. Let μ be a fuzzy subset of X and λ be a fuzzy subset of Y. Then the *inverse image* $f^{-1}(\lambda)$ of λ is the fuzzy subset of X defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$. The *image* $f(\mu)$ of μ is the fuzzy subset of Y that, for all $y \in Y$, is defined by $f(\mu)(y) = \{\sup\{\mu(t) \mid t \in f^{-1}(y)\}$ if $f^{-1}(y) \neq \emptyset$ and $f(\mu)(y) = 0$ otherwise. It is not difficult to see that the following assertions hold: (1) If $\{\lambda_i\}_{i\in I}$ is a family of fuzzy subsets of Y, then $f^{-1}\left(\bigcup_{i\in I}\lambda_i\right) = \bigcup_{i\in I} f^{-1}(\lambda_i)$ and $f^{-1}\left(\bigcap_{i\in I}\lambda_i\right) = \bigcap_{i\in I} f^{-1}(\lambda_i)$; (2) If μ is a fuzzy subset of X, then $\mu \subseteq f^{-1}(f(\mu))$.

Moreover, if f is one to one, then $f^{-1}(f(\mu)) = \mu$; (3) If λ is a fuzzy subset of Y,

then $f\left(f^{-1}(\lambda)\right) \subseteq \lambda$.

Moreover, if f is onto, then $f\left(f^{-1}(\lambda)\right) = \lambda$.

1.5 Fuzzy Γ-hyperideals

After the introduction of fuzzy sets by Zadeh, reconsideration of concepts of classical mathematics began. Rosenfeld introduced fuzzy sets in the context of group theory and formulated the concept of a fuzzy subgroup of a group (Rosenfeld, 1971). Since then, many researchers are engaged in extending the concepts of abstract algebra to the framework of the fuzzy setting. Sardar and his coauthors (Sardar & Majumdar, 2009; Sardar *et al.*, 2010) studied the notion of fuzzy ideals of a Γ-semigroup and investigated some of their properties. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. Davvaz introduced the notion of fuzzy subhypergroups as a generalization of fuzzy subgroups in Davvaz (1999) and this topic was continued by himself and others. Davvaz studied the notion of fuzzy ideals (sub-semihypergroups) of a semihypergroup (Davvaz, 2000b; Davvaz, 2005; Davvaz, 2006) and investigated some of their properties. Davvaz and Leoreanu-Fotea defined the notion of a fuzzy Γ-hyperideal of a Γ-semihypergroup and study some properties of it. We recall the following definition from Davvaz & Leoreanu-Fotea, (2012). Let S be a Γ-semihypergroup and μ be a fuzzy subset of S. Then, (1) μ is called a *fuzzy left* Γ-*hyperideal* of S if $\mu(y) \leq \inf_{z\in x\gamma y}\{\mu(z)\}$, for all $x, y \in S$ and $\gamma \in \Gamma$; (2) μ is called a *fuzzy right* Γ-*hyperideal* of S if $\mu(x) \leq \inf_{z\in x\gamma y}\{\mu(z)\}$, for all $x, y \in S$ and $\gamma \in \Gamma$; (3) μ is called a *fuzzy* Γ-*hyperideal* of S if it is both a fuzzy left Γ-hyperideal and a fuzzy right Γ-hyperideal of S.

1.6 Triangular norms

In mathematics, a t-norm (or, triangular norm) is a kind of binary operation used in the framework of probabilistic metric spaces and in multi-valued logic, specifically in fuzzy logic. A t-norm generalizes intersection in a lattice and conjunction in logic. The name triangular norm refers to the fact that in the framework of probabilistic metric spaces t-norms are used to generalize triangle inequality of ordinary metric spaces. The concept of a triangular norm was introduced in Menger (1942) in order to generalize the triangular inequality of a metric. The current notion of a t-norm and its dual operation is due to Schweizer & Sklar (1960). Anthony and Sherwood redefined a fuzzy subgroup of a group by using the notion of t-norm (Anthony & Sherwood, 1979), also see (Jun & Hong, 2001). By a *t-norm* T, we mean a function $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following conditions:

- T(x,1) = x,
- $T(x, y) \le T(x, z)$ if $y \le z$,
- T(x,y) = T(y,x),
- T(x,T(y,z)) = T(T(x,y),z)

for all $x, y, z \in R$. Here are some examples of t-norms:

$$* \quad T_0(x, y) = \begin{cases} x & \textit{if}\ y = 1, \\ y & \textit{if}\ x = 1, \\ 0 & \textit{otherwise}, \end{cases}$$

$$* \quad T_1(x, y) = \max\{0, x + y - 1\},$$

$$* \quad T_2(x, y) = \frac{xy}{2 - (x + y - xy)},$$

$$* \quad T_3(x, y) = xy,$$

$$* \quad T_4(x, y) = \frac{xy}{x + y - xy},$$

$$* \quad T_5(x, y) = \min\{x, y\}.$$

For every t-norm T, we set $\Delta_T = \{x \in [0, 1] \mid T(x, x) = x\}$. A t-norm on $[0, 1]$ is called a *continuous t-norm* if T is a continuous function from $[0, 1] \times [0, 1]$ to $[0, 1]$ with respect to the usual topology. Note that the t-norm “Min" is a continuous t-norm.

2. *T*-FUZZY SUB Γ-SEMIHYPERGROUPS

In this section, we define the notions of *T*-fuzzy sub Γ-semihypergroups and *T*-fuzzy Γ-hyperideals of a Γ-semihypergroup and study some properties of them.

Let T be a t-norm and μ be a fuzzy set of Γ-semihypergroup S. Then, we say μ has *imaginable property* if $Im\mu \subseteq \Delta_T$.

Definition 2.1 Let S be a Γ-semihypergroup, T be a t-norm and μ be a fuzzy subset of S. Then, μ is called a *T*-fuzzy sub Γ-semihypergroup of S if

$$T(\mu(x), \mu(y)) \leq \inf_{z \in x\gamma y}\{\mu(z)\},\ \forall x, y \in S,\ \forall \gamma \in \Gamma.$$

A *T*-fuzzy sub Γ-semihypergroup of S is said to be *imaginable* if it satisfies the imaginable property.

Clearly, if S is a Γ-semigroup, then, μ is a *T-fuzzy sub* Γ*-semigroup* of S when

$$T(\mu(x), \mu(y)) \leq \mu(x\gamma y),\ \forall x, y \in S,\ \forall \gamma \in \Gamma.$$

Example 3 Suppose that S is a semihyper group and Γ is a non-empty subset of S. For any $x, y \in S$ and $\gamma \in \Gamma$, we define $x\gamma y = \{x, \gamma, y\}$. Then, S is a Γ-semihyper group. We define the fuzzy subset μ of S by

$$\mu(x) = \begin{cases} \frac{3}{4} & \textit{if } x \in \Gamma \\ \frac{5}{9} & \textit{otherwise} \end{cases}$$

and we consider the t-norm $T(r, s) = \dfrac{rs}{2 - (r + s - rs)}$, where $r, s \in [0, 1]$. Then, for any $x, y \in S$ and $\gamma \in \Gamma$, we have

$$\inf_{z \in x\gamma y}\{\mu(z)\} = \min\{\mu(x), \mu(\gamma), \mu(y)\} = \frac{5}{9}.$$

On the other hand, we have the following cases:

- $x, y \in \Gamma$,
- $x \notin \Gamma$ and $y \in \Gamma$ (or, $x \in \Gamma$ and $y \notin \Gamma$),
- $x, y \notin \Gamma$.

Regarding the above cases, we have:

- $$T(\frac{3}{4},\frac{3}{4}) = \frac{\frac{3}{4}\cdot\frac{3}{4}}{2-(\frac{3}{4}+\frac{3}{4}-\frac{9}{16})} = \frac{9}{17},$$

- $$T(\frac{3}{4},\frac{5}{9}) = \frac{\frac{3}{4}\cdot\frac{5}{9}}{2-(\frac{3}{4}+\frac{5}{9}-\frac{15}{36})} = \frac{15}{40},$$

- $$T(\frac{5}{9},\frac{5}{9}) = \frac{\frac{5}{9}\cdot\frac{5}{9}}{2-(\frac{5}{9}+\frac{5}{9}-\frac{25}{81})} = \frac{25}{81}.$$

Thus, in every case, we obtain

$$T(\mu(x),\mu(y)) \leq \inf_{z\in x\gamma y}\{\mu(z)\}.$$

Therefore, μ is a T-fuzzy sub Γ-semihypergroup of S.

Example 4 Let S be the set of all non-positive integers and Γ be the set of all non-positive even integers. For every $x, y \in S$ and $\gamma \in \Gamma$, we define $x\gamma y$ =usual multiplication of integers. Then, S is a Γ-semigroup. We define the fuzzy subset μ of S by

$$\mu(x) = \begin{cases} 0.8 & f\,x = 0 \\ 0.2 & if\,x \in \{-1,-2\} \\ 0.5 & if\,x < -2 \end{cases}$$

and we consider the t-norm $Min(r,s) = \min\{r,s\}$. Then, it is easy to see that μ is a Min-fuzzy sub Γ-semigroup of S.

Lemma 2.2 *Let S be a Γ-semihypergroup, T be a t-norm and μ be a T-fuzzy sub Γ-semihypergroup of S. Then,*

$$T_n(\mu(x_1),\ldots,\mu(x_n)) \leq \inf_{z\in x_1\gamma\ldots\gamma x_n}\{\mu(z)\},\ \forall x_1,\ldots,x_n \in S,\ \forall\gamma\in\Gamma,$$

where

$$T_n(t_1,\ldots,t_n) = \begin{cases} t_1 & \text{if } n = 1 \\ T(t_1,t_2) & \text{if } n = 2 \\ T(t_i, T_{n-1}(t_1,\ldots,t_{i-1},t_{i+1},\ldots,t_n)) & \text{if } n > 2. \end{cases}$$

Proof. The proof is straightforward by mathematical induction.

Lemma 2.3 *Let S be a Γ-semihypergroup, T be a t-norm and μ be a T-fuzzy sub Γ-semihypergroup of S. Let A and B be non-empty subsets of S. Then,*

$$T\left(\inf_{a\in A}\{\mu(a)\}, \inf_{b\in B}\{\mu(b)\}\right) \leq \inf_{z\in A\gamma B}\{\mu(z)\}, \quad \forall\gamma\in\Gamma,$$

Proof. The proof is straightforward.

Theorem 2.4 *Let S be a Γ-semihypergroup, T be a t-norm, μ be a fuzzy subset of S with imaginable property and b be the maximum of $Im\mu$. Then, the following two statements are equivalent:*

- μ is a T-fuzzy sub Γ-semihypergroup of S,
- $\mu^{-1}[a,b]$ is a sub Γ-semihypergroup of S whenever $a \in \Delta_T$ and $0 < a \leq b$.

Proof. (1) $\Rightarrow$ (2): Suppose that $a \in \Delta_T$ and $0 < a \leq b$. If $x, y \in \mu^{-1}[a,b]$ and $\gamma \in \Gamma$, then $\inf_{z\in x\gamma y}\{\mu(z)\} \geq T(\mu(x),\mu(y)) \geq T(a,a) = a$, which implies that $x\gamma y \subseteq \mu^{-1}[a,b]$, and so $\mu^{-1}[a,b]$ is a sub Γ-semihypergroup of S.

(2) $\Rightarrow$ (1): Suppose that $x, y \in S$ and $\gamma \in \Gamma$. Since $Im\mu \subseteq \Delta_T$, both $\mu(x)$ and $\mu(y)$ are in Δ_T. Now, we have

$$T(T(\mu(x),\mu(y)),T(\mu(x),\mu(y))) = T(T(\mu(x),T(\mu(y),\mu(x))),\mu(y))$$
$$= T(T(\mu(x),T(\mu(x),\mu(y))),\mu(y)) = T(T(\mu(x),\mu(x)),T(\mu(y),\mu(y))) = T(\mu(x),\mu(y)),$$

and so $T(\mu(x),\mu(y)) \in \Delta_T$. Assume that $a = T(\mu(x),\mu(y))$. If $a = 0$, then

$$T(\mu(x),\mu(y)) = 0 \leq \inf_{z\in x\gamma y}\{\mu(z)\}.$$

Now, let $0 < a = T(\mu(x),\mu(y)) \leq \mu(x) \wedge \mu(y) \leq \mu(x) \leq b$. Hence, $x, y \in \mu^{-1}[a,b]$, which implies that $x\gamma y \subseteq \mu^{-1}[a,b]$. Therefore, $T(\mu(x),\mu(y)) \leq \inf_{z\in x\gamma y}\{\mu(z)\}$.

Let μ be a fuzzy subset of S and $t \in [0,1]$. The set $\mu_t = \{x \in S \mid \mu(x) \geq t\}$ is called a *level subset* of μ. So, we obtain the following corollary:

Corollary 2.5 *Let S be a Γ-semihypergroup and μ be a fuzzy subset of S. Then, μ is a Min-fuzzy sub Γ-semihypergroup of S if and only if every non-empty level subset is a sub Γ-semihypergroup of S.*

Let A be a subset of S. Then, the characteristic function χ_A is a T-fuzzy sub Γ-semihypergroup of S if and only if A is a sub Γ-semihypergroup of S.

Theorem 2.6 *Let S be a Γ-semihypergroup and K be a sub Γ-semihypergroup of S. Let T be the t-norm defined by $T(a,b) = \max\{0, a+b-1\}$ and μ be a fuzzy subset of S defined by*

$$\mu(x) = \begin{cases} r & \text{if } x \in K \\ s & \text{otherwise} \end{cases}$$

for all $a, b \in [0,1]$ and $x \in S$, where $r, s \in [0,1]$ such that $s < r$. Then, μ is a T-fuzzy sub Γ-semihypergroup of S. In particular, if $r = 1$ and $s = 0$, then μ is imaginable.

Proof. Suppose that $x, y \in S$ and $\gamma \in \Gamma$. We consider the following cases:

- If $x, y \in K$, then

$$T(\mu(x), \mu(y)) = T(r,r) = \max\{0, 2r-1\} = \begin{cases} 2r-1 & \text{if } r \geq \frac{1}{2} \\ 0 & \text{if } r < \frac{1}{2} \end{cases}$$

$$\leq r = \mu(z), \quad \text{for all } z \in x\gamma y.$$

- If $x \in K$ and $y \notin K$, then

$$T(\mu(x), \mu(y)) = T(r,s) = \max\{0, r+s-1\} = \begin{cases} r+s-1 & \text{if } r+s \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\leq s = \mu(z), \quad \text{for all } z \in x\gamma y.$$

- If $x, y \notin K$, then

$$T(\mu(x), \mu(y)) = T(s,s) = \max\{0, 2s-1\} = \begin{cases} 2s-1 & \text{if } s \geq \frac{1}{2} \\ 0 & \text{if } s < \frac{1}{2} \end{cases}$$

$$\leq s = \mu(z) \quad \text{for all } \; z \in x\gamma y.$$

Therefore,

$$T(\mu(x), \mu(y)) \leq \inf_{z \in x\gamma y} \{\mu(z)\}, \; \forall x, y \in S, \; \forall \gamma \in \Gamma,$$

which implies that μ is a T-fuzzy sub Γ-semihypergroup of S.

Now, suppose that $r = 1$ and $s = 0$. Then, we obtain $T(r, r) = \max\{0, 2r - 1\} = 1 = r$ and $T(s, s) = \max\{0, 2s - 1\} = 0 = s$. So, $r, s \in \Delta_T$ which implies that $Im\mu \subseteq \Delta_T$. Therefore, μ is imaginable.

Let S_1 and S_2 be Γ_1- and Γ_2-semihypergroups respectively. If there exists a map $\varphi : S_1 \to S_2$ and a bijection $f : \Gamma_1 \to \Gamma_2$ such that

$$\varphi(x\gamma y) = \{\varphi(z) \mid z \in x\gamma y\} = \varphi(x)f(\gamma)\varphi(y),$$

for all $x, y \in S_1$ and $\gamma \in \Gamma_1$, then we say that (φ, f) is a *homomorphism* from S_1 to S_2. Also, if φ is a bijection then (φ, f) is called an *isomorphism*, and S_1 and S_2 are *isomorphic*.

Proposition 2.7 *Let* S_1 *and* S_2 *be* Γ_1- *and* Γ_2-*semihypergroups respectively. Let* (φ, f) *be a homomorphism from* S_1 *to* S_2*. If* λ *is a T-fuzzy sub* Γ-*semihypergroup of* S_2*, then* $\varphi^{-1}(\lambda)$ *is a T-fuzzy sub* Γ-*semihypergroup of* S_1*, too.*

Proof. Suppose that $x, y \in S_1$ and $\gamma \in \Gamma_1$. Then, we have

$$\begin{aligned}\inf_{z\in x\gamma y}\{\varphi^{-1}(\lambda)(z)\} &= \inf_{z\in x\gamma y}\{\lambda(\varphi(z))\} \geq \inf_{\varphi(z)\in\varphi(x\gamma y)}\{\lambda(\varphi(z))\} \geq \inf_{\varphi(z)\in\varphi(x)f(\gamma)\varphi(y)}\{\lambda(\varphi(z))\}.1in \\ &\geq T(\lambda(\varphi(x)), \lambda(\varphi(y))) = T(\varphi^{-1}(\lambda)(x), \varphi^{-1}(\lambda)(y)).\end{aligned}$$

Therefore, $\varphi^{-1}(\lambda)$ is a T-fuzzy sub Γ-semihypergroup of S_1.

Let $\{a_i\}_{i\in I}$ and $\{b_j\}_{j\in J}$ be two sets of real numbers in $[0, 1]$. Then, we say that T is *infinitely distributive* if

$$T\left(\sup_{i\in I}\{a_i\}, \sup_{j\in J}\{b_j\}\right) = \sup_{i\in I, j\in J}\{T(a_i, b_j)\}.$$

Lemma 2.8 *If T is continuous, then T is infinitely distributive.*

Proof. See (Zahedi & Mashinchi, 1989).

Lemma 2.9 *Let T be a continuous t-norm and* $\{\mu_i\}_{i\in I}$ *be a family of T-fuzzy sub* Γ-*semihypergroups of S. Then,* $\bigcap_{i\in I}\mu_i$ *is a T-fuzzy sub* Γ-*semihypergroup of S.*

Proof. By Lemma 2.8, for any $x, y \in S$ and $\gamma \in \Gamma$, we have

$$\inf_{z \in x\gamma y}\left\{\left(\bigcap_{i\in I}\mu_i\right)(z)\right\} = \inf_{z\in x\gamma y}\left\{\inf_{i\in I}\{\mu_i(z)\}\right\}$$

$$= \inf_{i\in I}\left\{\inf_{z\in x\gamma y}\{\mu_i(z)\}\right\} \geq \inf_{i\in I}\{T(\mu_i(x), \mu_i(y))\}$$

$$= T\left(\inf_{i\in I}\{\mu_i(x)\}, \inf_{i\in I}\{\mu_i(y)\}\right) = T\left(\left(\bigcap_{i\in I}\mu_i\right)(x), \left(\bigcap_{i\in I}\mu_i\right)(y)\right)$$

Lemma 2.10 *Let S_1 and S_2 be Γ_1- and Γ_2-semihypergroups, respectively and (φ, f) be an onto homomorphism from S_1 to S_2. Then for every $t \in (0, 1]$, we have $\varphi(\mu)_t = \bigcap_{t>\varepsilon>0} \varphi(\mu_{t-\varepsilon})$.*

Proof. The proof is similar to the proof of Lemma 3.5 in (Ajmal, 1994).

Proposition 2.11 *Let S_1 and S_2 be Γ_1- and Γ_2-semihypergroups respectively and μ be a fuzzy subset of S_1. Let (φ, f) be an onto homomorphism from S_1 to S_2. If μ is a Min-fuzzy sub Γ-semihypergroup of S_1, then $\varphi(\mu)$ is a Min-fuzzy sub Γ-semihypergroup of S_2, too.*

Proof. Suppose that μ is a *Min*-fuzzy sub Γ-semihypergroup of S_1. By Corollary 2.5, $\varphi(\mu)$ is a *Min*-fuzzy sub Γ-semihypergroup of S_2 if every nonempty level subset $\varphi(\mu)_t$ is a sub Γ-semihypergroup of S_2. Thus, assume that $\varphi(\mu)_t$ is any nonempty level subset. If $t = 0$, then $\varphi(\mu)_t = S_2$, and if $0 < t \leq 1$, then by Lemma 2.10, we have $\varphi(\mu)_t = \bigcap_{t>\varepsilon>0} \varphi(\mu_{1-\varepsilon})$. By Corollary 2.5, $\mu_{t-\varepsilon}$ for each $t > \varepsilon > 0$ is a sub Γ-semihypergroup of S_1. Hence, $\varphi(\mu_{t-\varepsilon})$ is a sub Γ-semihypergroup of S_2. By Lemma 2.9, $\varphi(\mu)_t$ being an intersection of a family of sub Γ-semihypergroups is also a sub Γ-semihypergroup of S_2 and the proposition is proved.

Let S_1, S_2 be Γ-semihypergroups and let μ, λ be T-fuzzy sub Γ-semihypergroups of S_1, S_2, respectively. The *product* of μ, λ is defined to be the T-fuzzy subset $\mu \times \lambda$ of $S_1 \times S_2$ with $(\mu \times \lambda)(x, y) = T(\mu(x), \lambda(x))$, for all $(x, y) \in S_1 \times S_2$.

Proposition 2.12. *In the above definition, $\mu \times \lambda$ is a T-fuzzy sub Γ-semihypergroup of $S_1 \times S_2$.*

Proof. Suppose that $(x_1, x_2), (y_1, y_2) \in H_1 \times H_2$. For every $(\alpha_1, \alpha_2) \in (x_1, x_2) \circ (y_1, y_2)$ we have

$$(\mu \times \lambda)(\alpha_1, \alpha_2) = T(\mu(\alpha_1), \lambda(\alpha_2)) \geq T(T(\mu(x_1), \mu(y_1)), T(\lambda(x_2), \lambda(y_2))$$

$$= T(T(T(\mu(x_1), \mu(y_1)), \lambda(x_2), \lambda(y_2))) = T(T(\lambda(x_2), T(\mu(x_1), \mu(y_1)), \lambda(y_2)))$$

$$= T(T(T(\lambda(x_2), \mu(x_1)), \mu(y_1), \lambda(y_2))) = T(\lambda(y_2), T(\mu(y_1), T(\lambda(x_2), \mu(x_1)))$$

$$= T(T(\mu(x_1), \lambda(x_2)), T(\mu(y_1), \lambda(y_2)) = T((\mu \times \lambda)(x_1, x_2), (\mu \times \lambda)(y_1, y_2)).$$

Taking the infimum in the complete lattice $([0,1], \leq, \vee, \wedge)$ over all $(\alpha_1, \alpha_2) \in (x_1, x_2)o(y_1, y_2)$ we get

$$\inf_{(\alpha_1,\alpha_2)\in(x_1,x_2)o(y_1,y_2)} \{(\mu \times \lambda)(\alpha_1, \alpha_2)\} \geq T((\mu \times \lambda)(x_1, x_2), (\mu \times \lambda)(y_1, y_2)).$$

CONCLUSION

In this paper, by using the notion of triangular norms, we gave a new definition for fuzzy sub Γ-semihypergroups of a Γ-semihypergroup. Although, we introduced the concept of triangular fuzzy sub Γ-semihypergroups, in fact we gave a generalization of most of the papers regarding to semigroups, Γ-semigroups, semihypergroups and Γ-semihypergroups in fuzzy algebraic structures.

REFERENCES

Ajmal, N. 1994. Homomorphism of fuzzy groups, correspondence theorem and fuzzy quotient groups. Fuzzy Sets and Systems **61**: 329-339.

Anthony, J.M. & Sherwood, H. 1979. Fuzzy groups redefined. J. Math. Anal. Appl. **69**: 124-130.

Anvariyeh, S.M., Mirvakili, S. & Davvaz, B. 2010(a). On Γ-hyperideals in Γ-Semihypergroups. Carpathian Journal of Mathematics **26**: 11-23.

Anvariyeh, S.M., Mirvakili, S. & Davvaz, B. 2010(b). Pawlak's approximations in Γ- semihypergroups. Computers and Mathematics with Applications **60**: 45-53.

Bonansinga, P. & Corsini, P. 1982. On semihypergroup and hypergroup Homomorphisms. Boll. Un. Mat. Ital. B **(6)** 1: 717-727.

Corsini, P. 1993. Prolegomena of Hypergroup Theory. Second edition, Aviani editore, Italy.

Corsini, P. & Leoreanu, V. 2003. Applications of hyperstructure theory. Advances in Mathematics, Kluwer Academic Publishers, Dordrecht.

Davvaz, B. 1999. Fuzzy H_v-groups. Fuzzy Sets and Systems **101**: 191-195.

Davvaz, B. 2000a. Some results on congruences on semihypergroups. Bull. Malays. Math. Sci. Soc. (2) **23**: 53-58.

Davvaz, B. 2000b. Strong regularity and fuzzy strong regularity in semihypergroups. Korean J. Comput. Appl. Math. **7**: 205-213.

Davvaz, B. 2005. Characterizations of sub-semihypergroups by various triangular Norms. Czechoslovak Mathematical Journal **55**: 923-932.

Davvaz, B. 2006. Intuitionistic hyperideals of semihypergroups. Bull. Malaysian Math. Sci. Soc.(2) **29**: 203-207.

Davvaz, B. & Leoreanu-Fotea, V. 2007. Hyperring Theory and Applications. International Academic Press, Inc, 115, Palm Harber, USA.

Davvaz, B. & Leoreanu-Fotea, V. 2012. Structure of fuzzy Γ-hyperideals of Γ-Semihypergroups. J. Mult.-Valued Logic and Soft Computing **19**: 519-535.

Davvaz, B. & Poursalavati, N.S. 2000. Semihypergroups and S-hypersystems. Pure Math. Appl. **11**: 43-49.

Fasino, D. & Freni, D. 2007. Existence of proper semihypergroups of type U on the Right. Discrete Math. **307**: 2826-2836.

Hedayati, H. & Davvaz, B. 2011. Regular relations and hyperideals in $H_v - \Gamma$-Semigroups. Utilitas Mathematica **86**: 169-182.

Heidari, D. & Davvaz, B. 2011. Γ-Semihypergroups associated to binary relations. Iranian Journal of Science and Technology, Transaction A, **A2**: 69-80.

Heidari, D., Dehkordi, S.O. & Davvaz, B. 2010. Γ-Semihypergroups and their properties, U.P.B. Sci. Bull., Series A **72**(1): 195-208.

Hila, K., Davvaz, B. & Dine, J. 2012. Study on the structure of Γ-Semihypergroups. Communications in Algebra **40**: 2932-2948.

Hila, K. 2008. On regular, semiprime and quasi-reflexive Γ-semigroup and minimal quasi-ideals. Lobachevski J. Math. **29**: 141-152.

Jun, Y.B. & Hong, S.M. 2001. On imaginable T-fuzzy subalgebras and imaginable T- fuzzy closed ideals in BCH-algebras. Int. J. Math. Math. Sci. **27**: 269-287.

Leoreanu, V. 2000. About the simplifiable cyclic semihypergroups. Ital. J. Pure Appl. Math. **7**: 69-76.

Marty, F. 1934. Sur une generalization de la notion de groupe. 8^{iem} congres Math. Scandinaves, Stockholm, 45-49.

Menger, K. 1942. Statistical metrics. Proc. Nat. Acad. Sci. U.S.A. **8**: 535-537.

Rosenfeld, A. 1971. Fuzzy groups. J. Math. Anal. Appl. 35: 512-517.

Saha, N.K. 1987. On Γ-semigroup II. Bull. Calcutta Math. Soc. **79**: 331-335.

Sardar, S.K. & Majumder, S.K. 2009. On fuzzy ideals in Γ-semigroups. Int. J. Algebra **3**: 775-784.

Sardar, S.K., Davvaz, B. & Majumder, S.K. 2010. A study on fuzzy interior ideals of Γ− semigroups, Computers and Mathematics with Applications **60**: 90-94.

Schweizer, B. & Sklar, A. 1960. Statistical metric spaces. Pacific J. Math. 10: 313- 334.

Sen, M.K. & Saha, N.K. 1986. On Γ-semigroups I. Bull. Calcutta Math. Soc. **78**: 180-186.

Sen, M.K. & Saha, N.K. 1990. Orthodox Γ-semigroups. Internat. J. Math. Math. Sci. **13**: 527-534.

Vougiouklis, T. 1994. Hyperstructures and their repesentations. Hadronic Press Inc, U.S.A.

Zadeh, L.A. 1965. Fuzzy sets. Information and Control **8**: 338-353.

Zahedi, M.M. & Mashinchi, M. 1989. Some results on redefined fuzzy subgroups. J. Sci. I. R. Iran **6**: 65-67.

Submitted : *08/02/2012*
Revised : *01/11/2012*
Accepted : *05/11/2012*

مثيل فوزمر جزئية مثلثية مشوشة في مثيل الفوزمر

[*]بيجان دافاز و [**] فيوليتا ليورينو – فوتي

[*]قسم الرياضيات – جامعة يزد – يزد – إيران
[**]قسم الرياضيات – جامعة ال . كوزا – ياسي – رومانيا

خلاصة

يعتبر مفهوم الفوزمرة تعميم لمفهوم مثيل الزمرة. نقوم في هذا البحث باستخدام المعايير المثلثية لنعرف مفهوم الفوزمرة الجزئية المثلثية المشوشة ونتوصل لبعض النتائج في هذا المجال.