Method of Quartile for Determination of Weibull Parameters and Assessment of Wind Potential

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Abstract

Weibull Distribution is the most widely used distribution in wind power assessment. Two parameters Weibull distribution is commonly used for wind distribution modeling. The wind turbine converts wind energy into electrical energy. According to Betz law, No wind turbine can convert more than 59% of the available wind energy into electrical energy. The available method to find the parameters, e.g., Empirical Method (EM), Method of Moment (MoM), Energy Pattern Factor Method (EPFM), Maximum Likelihood Method (MLM), Modified Maximum Likelihood Method (MMLM), measure an overestimated value of wind power. An attempt has been made to develop a new method to evaluate Weibull parameters to measure wind potential close to the actual one. The new methods depend on the Quartiles of the wind distribution, Method of Quartile. Wind speed data for twelve months, January to December of 2016, for the cities Hyderabad, Karachi, and Quetta is used in this study. The new method results are compared with the five methods of Weibull parameter determination, EM, MoM, EPFM, MLM, and MMLM. The new method's average wind speed is closer to the actual average wind speed than those measured by other methods. The Root Mean Square Error (RMSE), Mean Absolute Error (MABE), and chi-square statistic calculated by all methods are close. The Akaike Information Criterion (AIC) model selection criterion was used for each method and month. It is found that the AIC values for every month and every city are the lowest for MoQ. It also suggests that the new method, MoQ, is the best among the existing method.

Keywords: Empirical method, energy pattern factor method, maximum likelihood, method of moments, method of quartile, weibull distribution.

1. Introduction

The majority of the countries in the world utilize fossil fuels as the primary source of energy generation. Fossil fuel resources are depleting rapidly; most countries face immense energy crises. Renewable energies have huge potential and are environmentally friendly (Saeed *et al*., 2019); they are the best alternative to conventional resources. Wind energy is one of the most important renewable resources (Farivar *et al*., 2017; Rafique and Rehman, 2017). In recent years, many public and private sectors in different countries have invested in wind energy projects at large and small scales (Keyhani *et al*., 2010; World Wind Energy Report, 2017). Researchers are also playing their role in extracting the most energy from available wind power. The wind energy distribution is bell-shaped and can be modeled by any statistical distribution having the same nature. The various researchers have employed Gaussian (Crutcher and Baer, 1962), Rayleigh (Kaminsky, 1977; Kiss Péter and Jánosi Imre, 2008), gamma (Kaminsky,1977), long normal (Bardsley, 1980; Wakeby and Kappa 2015; Soukissian, 2013), Weibull (Morgan, 2011; Van der A *et. al*., 1980; Sadiq, 2018; Khan, *et al*., 2015), etc., to model wind speed distribution. Among all these distributions, the Weibull function is the most prominent and frequently used to model and estimate wind potentials (Ahmed *et al*., 2021; Weisser, 2003; Wen *et al.* 2009; Bekele and Palm, 2009; Kwon, 2010; Shu *et al*., 2015; Lai and Lin, 2006; Zhou *et al*., 2006). Weibull distribution function with two and three parameters has been used for modeling and was developed by Waloddi Weibull. The modeling requires to have exact knowledge of the parameters of the Weibull distribution. Various numerical approaches have been employed to estimate the parameters. Some of the most frequent methods used for the estimation of the parameters are Empirical Method (EM), Method of Moment (MoM), Energy Pattern Factor Method (EPFM), Maximum Likelihood Method (MLM), and Modified Maximum Likelihood Method (MMLM). Ivana Pobočíková and Zuzana Sedliačková compared four methods the least square method, the weighted least square method, the maximum likelihood method, and the method of moments for estimating the Weibull distribution parameters (Pobočíková and Sedliačková, 2014). Teimouri *et al*., (2013) compared the method of maximum likelihood estimation (MLE), the method of logarithmic moments, the percentile method, the method of moments, and the method of L-moments for Weibull parameter estimation. Munkhammar *et al*., (2017) used the method of moments for nth degree polynomial as an approximation to unknown pdf and Weibull, normal and log-normal distributions. Nwobi and Ugomma, (2014) used numerical simulation to compare different estimation methods and showed maximum likelihood method is the best. Mohammadi *et al*., (2016) used the empirical method, maximum likelihood method, energy pattern factor method, and graphical method to estimate wind power density. Ijjou Tizgui *et al*., (2017), compared seven methods: the empirical method of Lysen, the empirical method of Justus, the maximum likelihood method, the graphical method, and the energy pattern factor method, the method of Mabchour, and the method of moments for wind energy applications. Kundu and Raqab (2009) used a modified maximum likelihood method to estimate three parameters of Weibull distribution. Akdağa and Dinler (2009) proposed a new method for wind energy assessment and estimation of Weibull distribution. Saleh *et al*., (2012) compared MLM, MMLM, GM, MWSM, and PDM methods, among which MWSM and MLM methods were found the most suitable for Suez Gulf, Egypt. Another study for Jubail, Saudi Arabia found that MLM was more effective (Abdul *et al*,. 2017). Narayanan *et al*., (2021) compared nine probability distributions to precisely assess wind potential. They found that no single distribution produces the best fit.

1.1. Objectives of the Study

There are many methods for modeling wind data; one of the most frequently used methods employs Weibull distribution. The Weibull distribution with two parameters is frequently used for wind modeling. The recorded wind data is used to find Weibull parameters, which are used to evaluate wind potential. Researchers have reported that less than 50% of available wind potential is utilized for electricity generation. We thought we should find a new method that only targets the wind speed region most useful in generating electrical energy. The new method, the method of quartile, covers the wind speeds of higher frequencies and is, therefore, more realistic than other methods.

2. Materials and Methods

Weibull distribution has been used to assess wind potential in Hyderabad, Karachi, and Quetta. A Weibull distribution with two parameters is well suited for modeling wind speed data. The recorded wind speed data is employed to determine the Weibull parameters and assess wind potential. Various methods are available for the estimation of these parameters.

2.1. Mathematical Method

Six methods, including a new method (MoQ) together with known methods (EM, MoM, EPFM, MLM, and MMLM) [35], have been used to calculate the shape and scale parameters of the Weibull distribution.

2.1.1. Empirical Method

Justus (1977) gave this method which utilizes average wind speed and the standard deviation of the wind distribution to calculate the shape parameter 'k'. The formula is given by Tizgui, *et al*., (2017) and Rocha *et al*., (2012)

$$k = \left(\frac{\sigma}{\overline{v}}\right)^{-1.086} \quad (1)$$

The scale parameter can be obtained with the help of the shape parameter and average wind speed by the following formula;

$$c = \frac{\overline{v}}{\Gamma\left(1 + \frac{1}{k}\right)} \quad (2)$$

2.1.2. Method of Moments

The nth moment about the origin for Weibull distribution is given by

$$\mu_n^{'} = \int_0^\infty x^n \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} \exp\left(-\left(\frac{v}{c}\right)^k\right) dx \quad (3)$$

$$\mu_n^{'} = c^n\, \Gamma\left(1 + \frac{n}{k}\right) \quad (4)$$

The second moment about the mean is given by

$$\sigma^2 = \mu_2^{'} - \mu_1^{'\,2} = c^2 \left\{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right\} \quad (5)$$

The first moment about the origin is the average wind speed

$$\overline{v} = c\Gamma\left(1 + \frac{2}{k}\right) \quad (6)$$

The ratio of standard deviation and average wind speed is the function of the shape parameter. This equation is used to calculate the value of ‘k’, and the expression of average wind speed is used to find the value of scale parameter ‘c’ (Sukkiramathi *et al.*, 2020; Chang, 2011).

2.1.3. Energy Pattern Factor Method

This method is simple, like the empirical method (Akdag and Dinler, 2009; Rocha *et al.*, 2012), which depends on the ratio of the average cube of wind speed and the cube of average wind speed. This ratio (E_{pf}) is used to find the value of shape parameter ‘k.’

$$E_{pf} = \frac{\overline{v^3}}{\bar{v}^3} \tag{7}$$

$$k = 1 + \frac{3.69}{{E_{pf}}^2} \tag{8}$$

The scale parameter is found in the same way as in the method of moments.

2.1.4. Maximum Likelihood Method

The Maximum Likelihood Method (Rocha *et al.*, 2012) relies on the most likely values of the parameters of the Weibull distribution. The value of the parameters is obtained by maximizing the likelihood function.

The likelihood function of the Weibull distribution is given by

$$L = \prod_{i=1}^{n} f(v_i, k, c) \tag{9}$$

It is equivalent to finding the maximum value of the log of the likelihood function.

$$\ln(L) = \sum_{i=1}^{n} f(v_i, k, c) \tag{10}$$

The value of shape and scale parameters are found by maximizing ln(L) concerning the corresponding parameter, i.e., by finding $\frac{\partial \ln{(L)}}{\partial k}\, and\, \frac{\partial \ln{(L)}}{\partial c}$.

$$k = \left[\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n}\right]^{-1} \tag{11}$$

$$c = \left(\frac{1}{n}\sum\nolimits_{i=1}^{n} v_i^k\right)^{\frac{1}{k}} \tag{12}$$

An important point that must be taken into account to find the shape parameter is to remove all velocity values equal to zero due to the log function.

2.1.5. Modified Maximum Likelihood Method

The modified maximum likelihood method requires wind data in frequency distribution format. The value of shape parameter ‘k’ is found by the iterative method by the formula (Rocha *et al.*, 2012);

$$k = \left[\frac{\sum_{i=1}^{n} v_{\mathrm{i}}^{k} \ln(v_{\mathrm{i}}) f(v_{\mathrm{i}})}{\sum_{\mathrm{i}=0}^{\mathrm{n}} v_{\mathrm{i}}^{\mathrm{k}} f(v_{\mathrm{i}})} - \frac{\sum_{i=1}^{n} \ln(v_{\mathrm{i}}) f(v_{\mathrm{i}})}{f(v \geq 0)}\right]^{-1} \tag{13}$$

The scale parameters are found by the equation similar to that in the maximum likelihood method;

$$c = \left[\frac{1}{f(v \geq 0)} \sum_{i=1}^{n} v_i^k f(v_i)\right]^{\frac{1}{k}} \tag{14}$$

Here $f(v_i)$ is the frequency of wind speed v_i i^{th} bin and $f(v \geq 0)$ is the probability for all $v \geq 0$.

2.1.6. New Method (Method of Quartile- (MoQ))

Weibull parameters 'k' and 'c' are found by Q_1 and Q_3 of the wind distribution, and the CDF is utilized to estimate these parameters. The CDF of Weibull distribution is given by;

$$F(v) = 1 - \exp\left(-\left(\frac{v}{c}\right)^k\right) \tag{15}$$

The first quartile of the distribution Q_1 is defined by

$$F(Q_1) = 0.25 \tag{16}$$

$$1 - \exp\left(-\left(\frac{Q_1}{c}\right)^k\right) = 0.25 \tag{17}$$

$$\exp\left(-\left(\frac{Q_1}{c}\right)^k\right) = 0.75 \tag{18}$$

$$-\left(\frac{Q_1}{c}\right)^k = \ln(0.75) \tag{19}$$

$$\frac{Q_1}{c} = [-\ln(0.75)]^{\frac{1}{k}} \tag{20}$$

Similarly, the third quartile satisfies the equation

$$\frac{Q_3}{c} = [-\ln(0.25)]^{\frac{1}{k}} \tag{21}$$

and

$$\frac{Q_1}{Q_3} = \left[\frac{\ln(0.75)}{\ln(0.25)}\right]^{\frac{1}{k}} \tag{22}$$

The shape parameter 'k' can be found by finding the values of Q_1 and Q_3. The value of 'c' is found by substituting the value of 'k' in equation (6) or (7).

2.1.7. Statistical Errors

The efficiency of a model is measured through the fitting errors; scientists have employed many different types of errors in measuring model efficiency. In this study, the following four errors, RMSE, MABE, and χ2 have been calculated while fitting wind speed data and evaluating Weibull parameters.

$$RMSE = \sqrt{\frac{\sum_1^n [f(v_i) - P(v_i)]^2}{n}} \tag{23}$$

$$MABE = \frac{\sum_1^n |f(v_i) - P(v_i)|}{n} \tag{24}$$

$$\chi 2 = \frac{1}{n} \sum_1^n \frac{(f(v_i) - P(v_i))^2}{P(v_i)} \tag{25}$$

Where $\boldsymbol{P(v_i)}$ is the relative probability and $\boldsymbol{f(v_i)}$ is the Weibull probability density value of wind speed $\boldsymbol{v_i}$ and *n* is the total number of recorded wind speeds. AIC values are calculated to decide the method selection. It compares the relative quality of statistical models for a given data set. It allocates a single score to each model; based on these scores, the best model selection is carried out for a given dataset. The basic formula to calculate AIC depends on log-likelihood and is given by Cavanaugh *et al.*, 2019,

$$AIC = -2(log - likelihood) + 2K \tag{26}$$

Where K is the number of parameters.

2.2. Data Collection

The wind speed data of three cities of Pakistan, Hyderabad, Karachi, and Quetta, utilized in the study, has been obtained from the archives of the Pakistan Meteorological Department.

2.3. Data Analysis

The Weibull parameters were calculated by the new method i.e. (MoQ). The results were compared with the corresponding values calculated by five methods mentioned above. To check the validity various statistical errors were also estimated.

3. Results and Discussion

A new method, namely, Method of Quartile (MoQ), has been developed and implemented to find the shape and scale parameters of the Weibull distribution. Two parameters of Weibull distribution have been employed to model wind speed data of three cities of Pakistan, namely, Karachi, Hyderabad, and Quetta. Five different methods also calculate the Weibull parameters; Empirical Method, Moment of Method, Energy Pattern Factor Method, Maximum Likelihood Method, and Modified Maximum Likelihood Method, to compare the fitting of wind speed data by the newly developed method. Wind speed data recorded at 10 m has been used for modeling. Three error methods (Root Mean square Error, Mean Absolute Error, and Chi-square statistic) have been used to check the reliability of the fit. All error values for January to

December wind data of Karachi, Quetta, and Hyderabad are comparable for EM, MoM, EPFM, MLM, and MMLM (see Supplementary file). In addition to the errors mentioned above, Akaike Information Criterion has been calculated for each city's data. AIC is used for model validation. The lowest value of the AIC validates the best fit model. It has been observed that the AIC values are the lowest for the new method, i.e., all 36 values of the AIC for Method of Quartile are lower than the corresponding values obtained by EM, MoM, EPFM, MLM, and MMLM. The lowest values for the new method make the Method of Quartile, the best method to predict the values of Weibull parameters.

Figures. 1-3 give the AIC values for six model methods to evaluate Weibull parameters. In all three figures, the lowest value of AIC is for MoQ. These two findings also conclude that the best model method to evaluate Weibull parameters is the new method (MoQ).

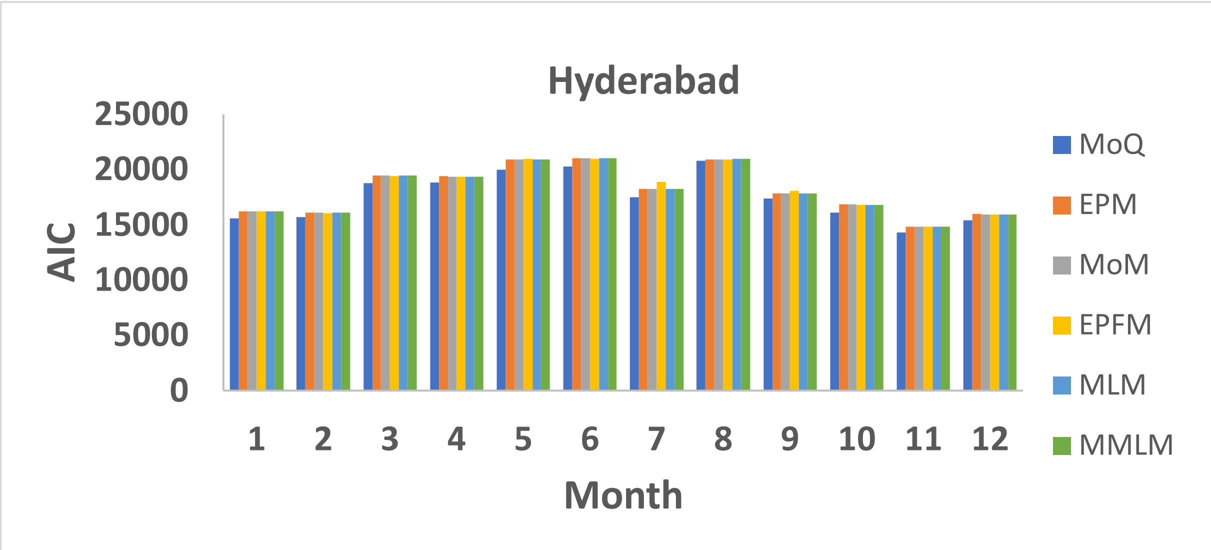

Fig. 1. Akaike Information Criterion for six methods of Weibull parameter evaluation for wind speed distribution of Hyderabad city.

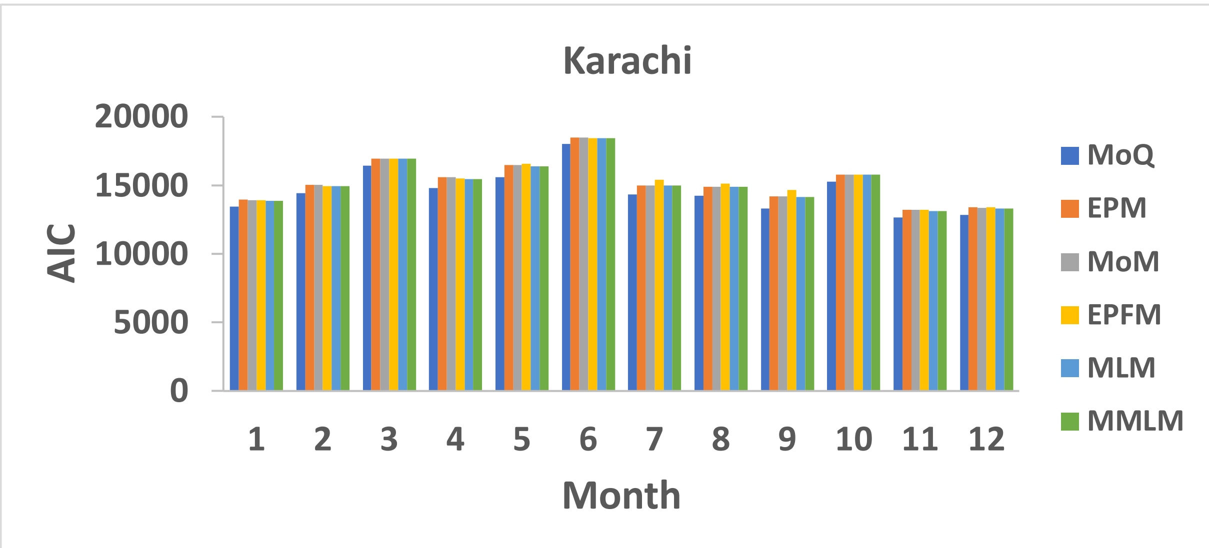

Fig. 2. Akaike Information Criterion for six methods of Weibull parameter evaluation for Karachi city wind speed distribution.

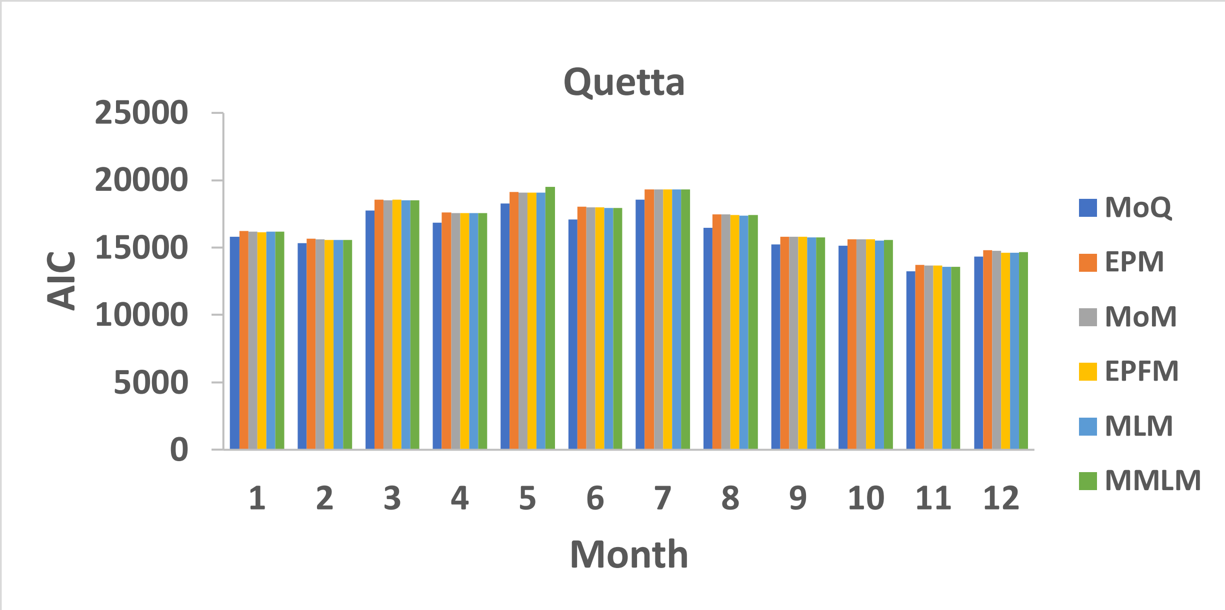

Fig. 3. Akaike Information Criterion for six methods of Weibull parameter evaluation for Quetta city wind speed distribution.

3.1. Average Wind speed

Figures. 4-6. give the average wind speed values for three cities, Hyderabad, Karachi, and Quetta, respectively, calculated by six different methods and recorded wind speed. In all figures, it can be seen that the average speed calculated by the new method (MoQ) is close to the recorded average wind speed for each city and each month.

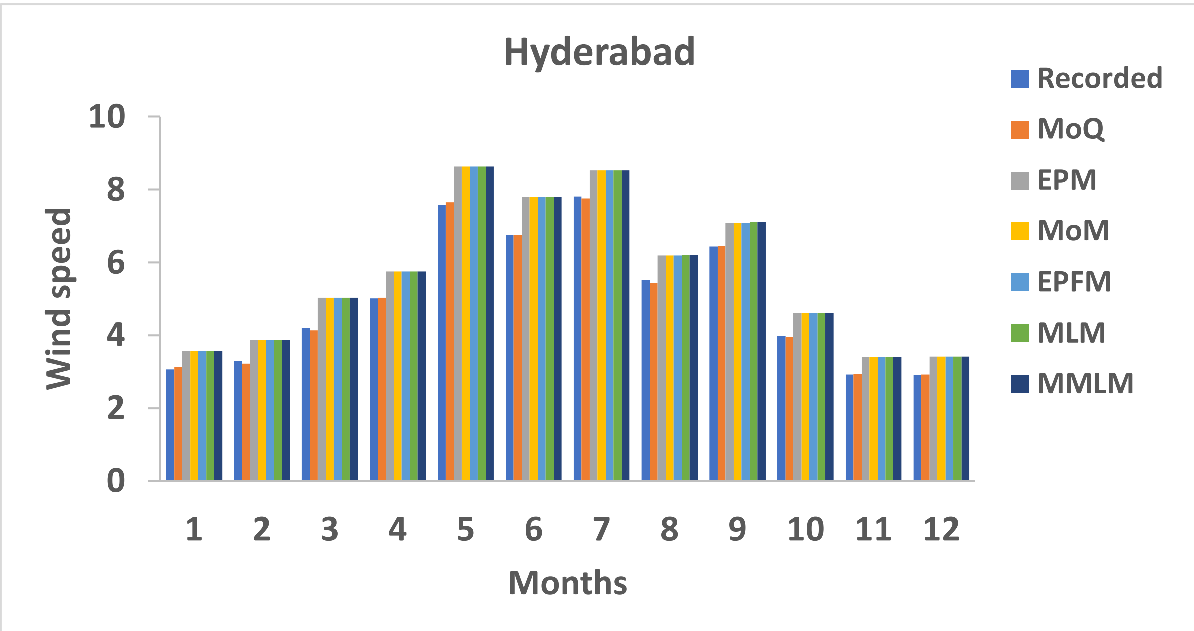

Fig. 4. Average wind speed for the months Jan.-Dec. in Hyderabad city

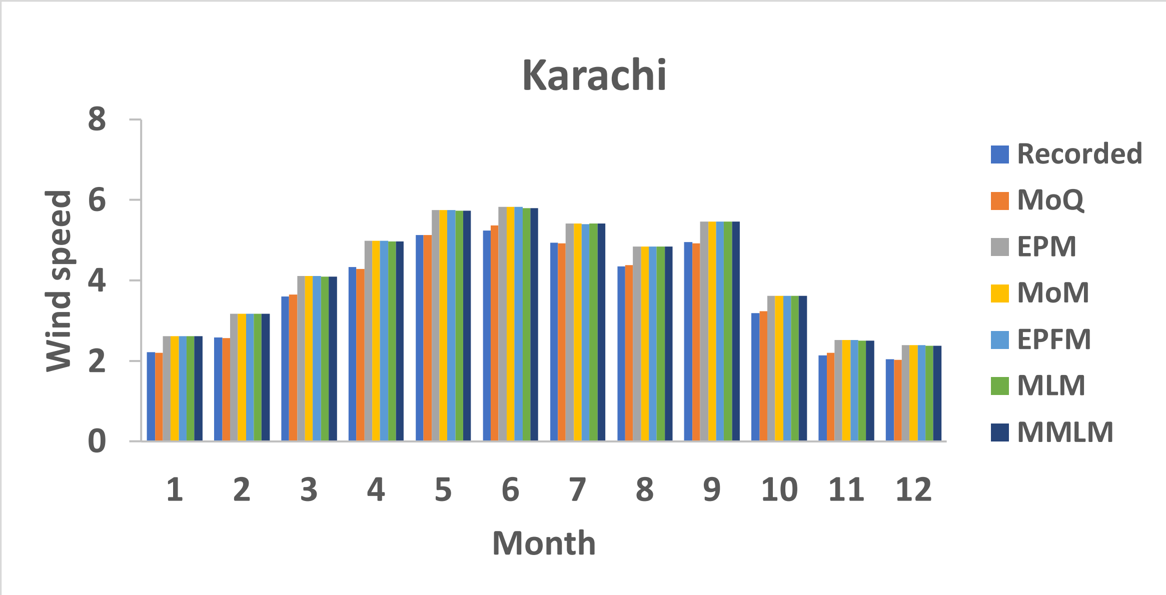

Fig. 5. Average wind speed for the months Jan.-Dec. in Karachi city

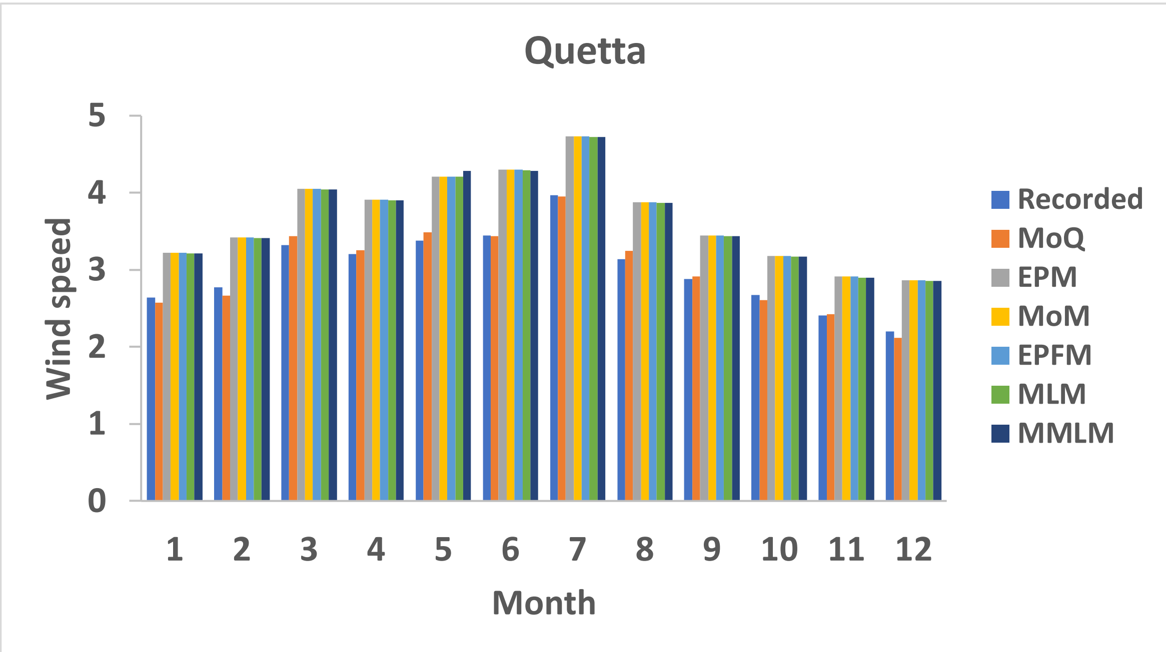

Fig. 6. Average wind speed for the months Jan.-Dec. in Quetta city

The average wind speed measured by any method of evaluation of Weibull parameters depends on the number of bins. As the number of bins increases, the average speed measured by EM, MoM, EPFM, MLM, and MMLM decreases and approaches actual wind speed. The average speed measured by MoQ is close to the average actual wind speed. The figure 7 shows that MoQ measures the same wind speed close to the average wind speed. For all other methods, the wind speed decreases with an increasing number of bins.

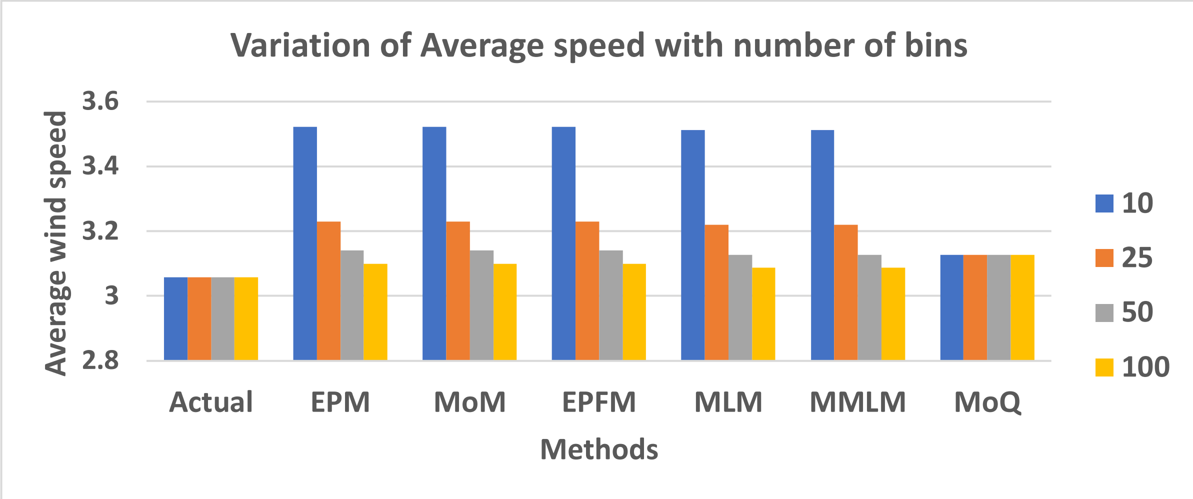

Fig. 7. Variation of Average speed measured by EM, MoM, EPFM, MLM, MMLM, and MoQ with increasing bins.

3.2. Weibull Pdfs

Figures 8-10 show monthly Weibull pdfs and recorded histograms of Hyderabad, Karachi, and Quetta wind speed data, respectively. Each of the three figures has 12 pdfs for January to December. The green circles on each fig. are the pdf points by MoQ. The area under pdfs of MoQ is the least compared to pdfs generated by other methods. All pdfs overlap well on the recorded wind speed histogram, but the pdf obtained by MoQ gives an excellent fit on recorded data for each month and city. The values of shape and scale parameters are given in the supplementary file. The total 36 values of shape and scale parameters for three cities and 12 months by Method of Quartile have the lowest values compared to corresponding values obtained by five other methods. The lower values of Weibull parameters by MoQ make pdf closer to the recorded wind speed data histogram. These lowest values are responsible for lower values of AIC by MoQ.

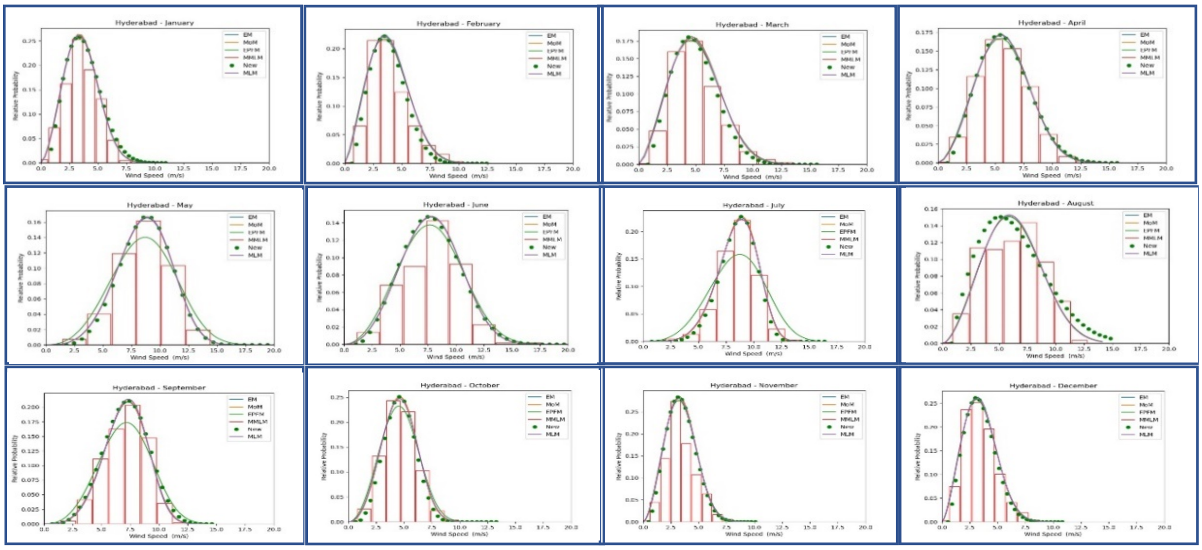

Fig. 8. Weibull pdfs of wind speed distribution of Hyderabad, obtained by six different methods overlapping on the histogram of recorded

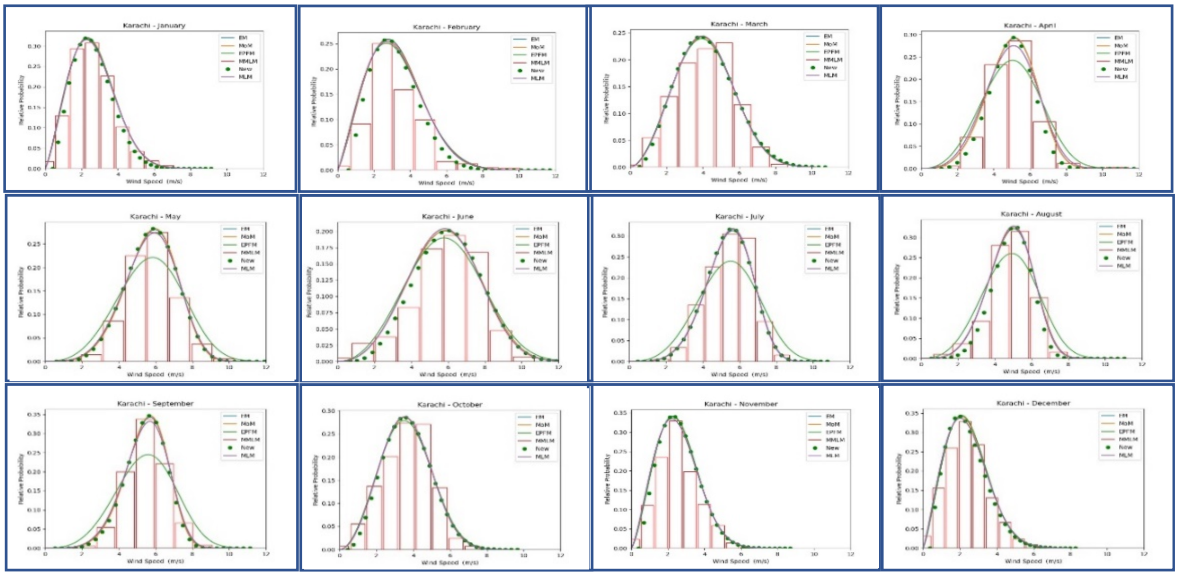

Fig. 9. Weibull pdfs of wind speed distribution of Karachi, obtained by six different methods overlapping on the histogram of recorded data.

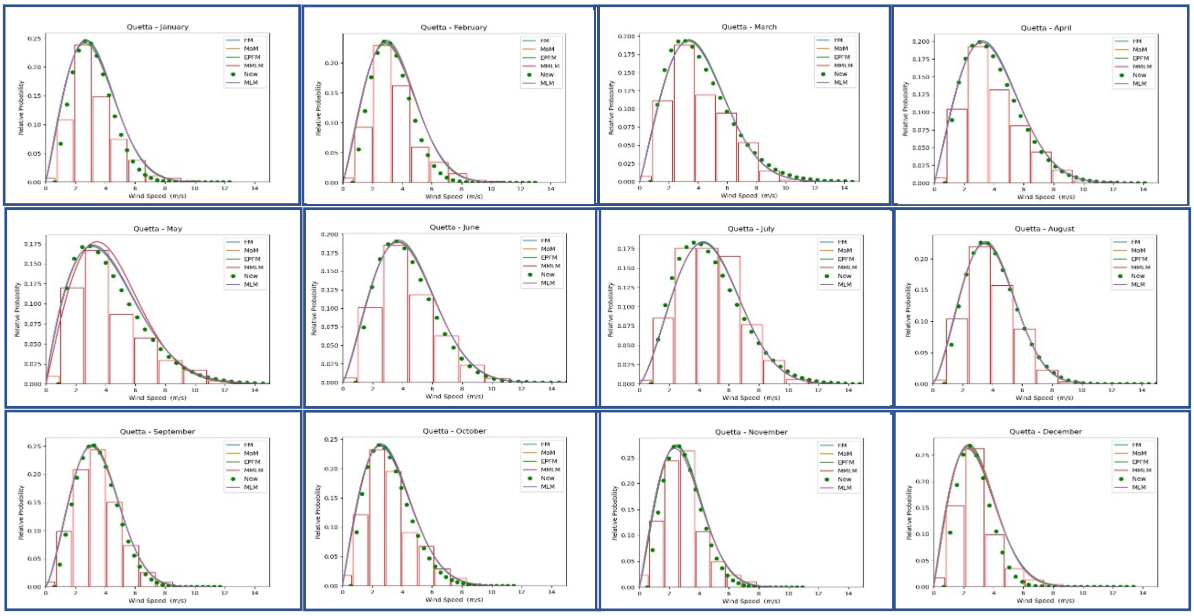

Fig. 10. Weibull pdfs of wind speed distribution of Quetta, obtained by six different methods overlapping on the histogram of recorded data.

3.3. Average Wind Power Density

The wind data used in this study was recorded at an anemometer height of 10m above ground level., The wind shear formula given by following equation Allnoch (1992) is used to find the wind speed at hub height. If v_h and v_H are the wind speed at anemometer height $'h'$ and hub height $'h'$, respectively, then

$$\frac{v_H}{v_h} = \left(\frac{H}{h}\right)^{\alpha}$$

α is Hellman exponent. The average wind power density is given by Gugliani *et al.*, (2021)

$$\overline{WPD} = \frac{1}{2}\rho c^3 \Gamma\left(1 + \frac{3}{k}\right)$$

Where scale and shape parameters' k' and 'c' are calculated at hub height.

Fig. 11 shows the average wind power densities for three cities, Karachi, Hyderabad, and Quetta. The $\overline{WPD}$ is calculated by six methods employed in this study to calculate Weibull parameters, including the new method (Method of Quartile). The $\overline{WPD}$ calculated by five methods EM, MoM, EPFM, MLM, and MMLM calculate the almost same value of $\overline{WPD}$. Therefore, an average value of $\overline{WPD}$ is shown for these five methods labeled as all methods. Each month has three bars for all methods, method of quartile, and actual wind power density measured by recorded wind speed. It can be seen that all other methods measure an overestimated value of $\overline{WPD}$. The $\overline{WPD}$ measured by new methods (MoQ) is relatively closer to the actual $\overline{WPD}$. The fig. also shows that Hyderabad has the highest wind potential (central part of the fig.) compared to Karachi and Quetta.

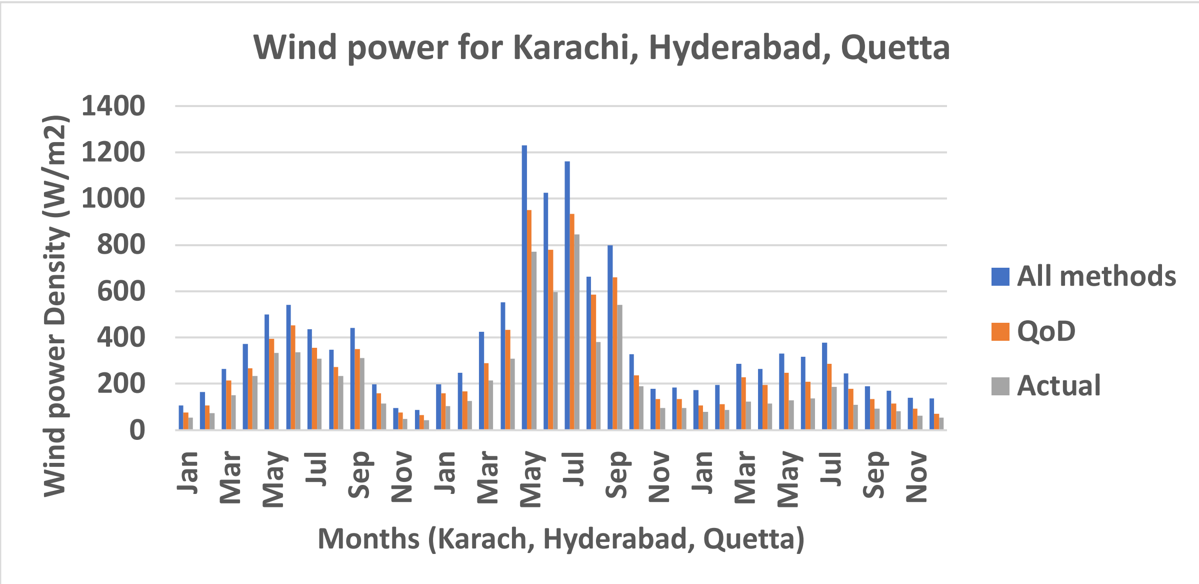

Fig. 11. The average wind power density of Karachi, Hyderabad, and Quetta from Jan.-Dec.

4. Conclusion

A new method has been suggested to evaluate the parameters of Weibull distribution. The new method is based on calculating the first and third quartiles. The wind speed data of Karachi, Hyderabad, and Quetta for the months Jan-Dec are used for the best model selection. The AIC values, average wind speed, and pdfs obtained by different methods have been plotted in figures 1-3, 4-6, and 8-10, respectively. The new method (Method of Quartile) results are compared with those of EM, MoM, EPFM, MLM, and MMLM. Following are the main features of this study;

- The values of Weibull parameters obtained by MoQ are lower than obtained by five other methods.
- The area under pdf by MoQ is the lowest; it indicates that only the region mainly contributes to the wind potential targeted by MoQ.

- The RMSE, MABE, and Chi-square for all six methods are comparable.
- Average wind speeds calculated by MoQ are very close to the average wind speed found by recorded data.
- Akaike Information Criterion is used to select the best method for calculating Weibull parameters.
- From January to December, the AIC values for all three cities (Karachi, Hyderabad, and Quetta) are the lowest for MoQ. The MoQ is the best among the six methods employed to calculate Weibull parameters.
- All other methods measure an overestimated value of average wind power density ($\overline{WPD}$). The $\overline{WPD}$ measured by new methods (MoQ) is relatively closer to the actual $\overline{WPD}$.
- Among the three cities of Pakistan, Hyderabad has the highest average wind power density.

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