

Sensitivity assessment of air and refrigeration

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Abstract

In engineering sector, research on a system is influenced by understanding the relationship between its physical components. Since, air and refrigeration system depends on its four essential physical components, namely, compressor, condenser, expansion device and the evaporator, the researchers adopt an appropriate mathematical model of an air and refrigeration system for measuring the system's performance. In this research work, the reliability measures of a repairable air and refrigeration system is investigated, by which reliability engineers or designers can determine how reliability can be improved using appropriate vicissitudes. The sensitivity analysis for several variations in reliability characteristics along with the modifications in precise values of their input parameters has been done. At last, some numerical examples and their graphical representation have also been taken to highlight the practical utility of the model. Through the overall study, it is concluded that the power supply in air and refrigeration system required extra care, it is the most sensitive part of air and refrigeration system's performance with respect to its other components.

Keywords: Air and refrigeration system; reliability measures; sensitivity analysis; stochastic modeling; transition state probability.

1. Introduction

Due to the progressive development of high-speed, high-precision and intensive technologies, components of automated air and refrigeration system have a great significance in engineering as well as in industries. Not only the cooling depends on this system, it plays an important role in freezing function. The failures of these components or early failures of this system affect its working and reduce or stop the cooling and freezing. These probabilistic failures and repairs are used to cover the performance or working of the system and affect the industries economically as well. It is necessarily required to analyze the reliability of the system before using it (Wu *et al.*, 2009; Yang *et al.*, 2011). High reliability of the system becomes more significant in daily life and necessary for the relation with the machine and the automation of industrial process. Reliability is a key element in performance evaluation and life testing of the system. Many researchers have done a lot of work in the field of system reliability but no one has considered the air and refrigeration system.

Riffat *et al.* (1997) discussed the application of the main natural refrigerants, for refrigeration and air-conditioning systems, as an alternative to synthetic new refrigerants (HFCs). Chrysler *et al.* (1999) studied the system that requires cooling and conveyed the cooling systems that are reliable for mainframe computers or any other electronic system. Spence *et al.* (2005) examined the performance of

an air-cycle refrigeration unit for road transport and studied comprehensively, comparing it with the original model. They found that the turbo-machinery satisfied the original design requirements, while the heat exchangers proved to be a major performance handicap. In the future, risks accompanying with the awareness of the environment using hydro-chloro fluoro carbon (HCFC) and hydro fluoro carbon (HFC) refrigerant fluids has encouraged alternative, natural refrigerant fluids that can deliver safe and sustainable refrigeration. The refrigeration system is the first redundant system containing two discrete coolant loops ephemeral with a single cold plate and in contact of the circuit module. Trutassanawin *et al.* (2006) investigated a refrigeration system that is miniature-scale and suitable for electronics cooling applications. They demonstrate the feasibility of miniature-scale refrigeration with the prototype system for using in cooling condensed electronic devices and also discussed the results of further research on refrigeration systems and system simulation models applicable to electronics cooling. Hajidavalloo & Eghtedari (2010) investigated the application of evaporative cooled air condenser instead of air-cooled condenser and solved the problem of performance of air conditioner in an efficient way.

A mathematical model of the air and refrigeration system has been designed in this study. To derive the probability expression of several reliability measures, Markov process,

supplementary variable technique and Laplace transform have been employed. The generalized expressions of the state transition probabilities have also been derived and investigated. To overcome the effect of each failure on the modeled air and refrigeration system, sensitivity analysis has also been performed. The overall study has been explained by taking some numerical examples.

2. Mathematical model details

2.1. Nomenclature

All the notations used in this work are shown in Table 1.

Table 1. Notations.

Notation	Description
	Time scale.
t	Laplace transform variable.
s	Laplace transformation of $P(t)$.
$\bar{P}(s)$	Failure rates for compressor/ condenser / expansion device / evaporator / power supply.
$\lambda_1 / \lambda_2 / \beta_1 / \beta_2 / \lambda_p$	The probability of the state S_0 .
$P_0(t)$	The probability of the stage S_i at time t when $i=1, 2, 3, \dots, 12$.
$P_i(t)$	The probability density function of the state S_j , when $j=13, 14, 15, 16, 17$.
$P_j(x, t)$	Repair rates for the partially failed system.
$\mu(x)$	Repair rates for the complete failed system.
$\phi(x)$	Good working state of the system.
S_0	Partially failed (degraded) state of system when system is partially failed due to minor failure; $i=1, 2, \dots, 12$.
S_i	The state of the system when system is completely failed due to major failure; $j=13, 14, 15, 16, 17$.
S_j	Up state system probability at time t or availability of the system
$P_{up}(t)$	The reliability of the system at time t .

$RI(t)$

2.2. System description

Air and refrigeration system consists of four important components or equipments, compressor, condenser, expansion device and evaporator. The vapour at low temperature and pressure enters the compressor, where it is compressed and its temperature and pressure rises. The vapour from compressor goes to the condenser where it is

condensed into high pressure liquid and then it passes to the expansion valve, where it is throttled down to low pressure and low temperature. After leaving the expansion valve, it enters the evaporator, where heat is extracted from the surroundings and it vaporizes to low pressure vapour. It is assumed that these components are connected in parallel, which is revealed in Figure 1(a). In this work, we use two types of failures namely minor failure, and major failure. System goes to the partially failed state or degraded state after minor failure but due to major failure, system goes to complete failed state. In this novel research work, the state transition diagram of an air and refrigeration system, which is shown in Figure 1(b), is designed with the consideration of following assumptions-

- (i) The system has three possible modes, namely, normal, partial failure and total failure.
- (ii) Distribution of failure rates and repair rates are general (Singh *et al.*, 2013).
- (iii) A single repair facility is available to attend the partially or totally failed system.
- (iv) Repairs are perfect i.e. repair facility never does any damage to the units of the network.
- (v) The unit recovers its functioning perfectly upon repair.
- (vi) The repair time of the failed system is arbitrarily distributed.

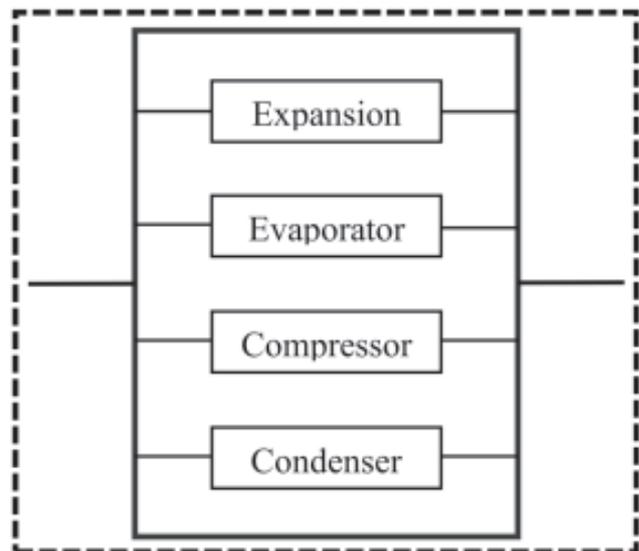


Fig. 1(a). Configuration diagram

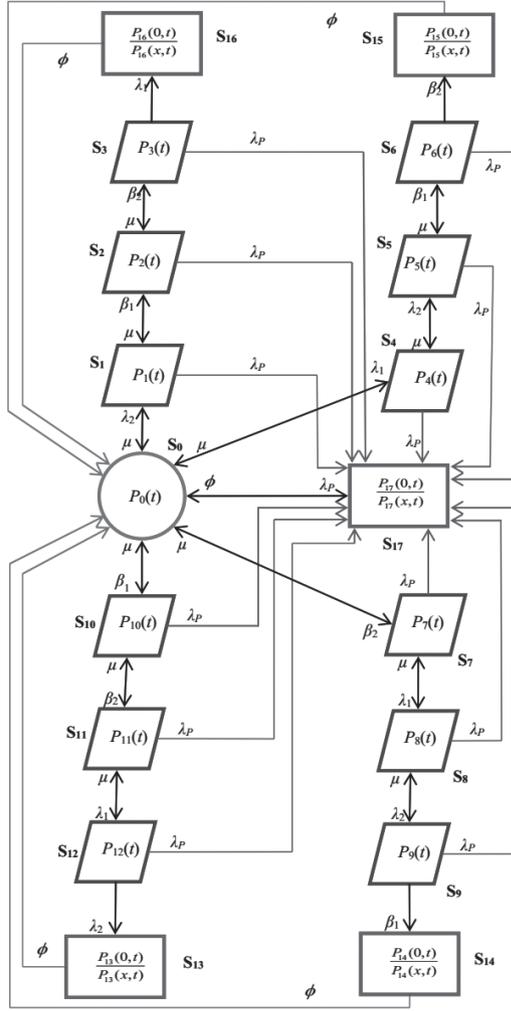


Fig. 1 (b). Transition state diagram

2.3. Formulation of the model

By probability considerations and continuity arguments, the following differential equations overriding the behavior of the system appear appropriate.

$$\left[\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \beta_1 + \beta_2 + \lambda_p \right] P_0(t) = \mu(y) \sum P_i(t) + \int_0^{\infty} \phi(x) \sum_{j=13}^{17} P_j(x,t) dx; \quad i = 1, 4, 7, 10 \quad (1)$$

$$\left[\frac{\partial}{\partial t} + \beta_1 + \lambda_p + \mu(y) \right] P_i(t) = \mu(y) P_{i+1}(t) + \lambda_2 P_{i-1}(t); \quad i = 1, 5 \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \beta_2 + \lambda_p + \mu(y) \right] P_i(t) = \mu(y) P_{i+1}(t) + \beta_1 P_j(t); \quad i = 2, 10; \quad j = 1, 0 \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \xi + \lambda_p + \mu(y) \right] P_i(t) = \psi P_{i-1}(t); \quad i = 3, 6, 9, 12; \quad \xi = \lambda_1, \beta_2, \beta_1, \lambda_2; \quad \psi = \beta_2, \beta_1, \lambda_2, \lambda_1 \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \lambda_2 + \lambda_p + \mu(y) \right] P_i(t) = \mu(y) P_{i+1}(t) + \lambda_1 P_j(t); \quad i = 4, 8; \quad j = 0, 7 \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \lambda_1 + \lambda_p + \mu(y) \right] P_i(t) = \beta_2 P_j(t) + \mu(y) P_{i+1}(t); \quad i = 7, 11; \quad j = 0, 10 \quad (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x) \right] P_i(x,t) = 0; \quad i = 13, 14, 15, 16, 17 \quad (7)$$

Boundary conditions

$$P_i(0,t) = \xi P_j(t); \quad \xi = \lambda_2, \beta_1, \beta_2, \lambda_1; \quad i = 13, 14, 15, 16; \quad j = 12, 9, 6, 3 \quad (8)$$

$$P_{17}(0,t) = \lambda_p \sum_{i=1}^{12} P_i(t) \quad (9)$$

Initial condition

$$P_i(0) = 1 \text{ if } i = 0 \text{ or } 0 \text{ if } i \geq 1 \quad (10)$$

Taking Laplace transformation of Equations (1) to (9) using Equation 10, we have

$$\left[s + \lambda_1 + \lambda_2 + \beta_1 + \beta_2 + \lambda_p \right] \bar{P}_0(s) = \mu(y) \sum \bar{P}_i(s) + \int_0^{\infty} \phi(x) \sum_{j=13}^{17} \bar{P}_j(x,s) dx; \quad i = 1, 4, 7, 10 \quad (11)$$

$$\left[s + \beta_1 + \lambda_p + \mu(y) \right] \bar{P}_i(s) = \mu(y) \bar{P}_{i+1}(s) + \lambda_2 \bar{P}_{i-1}(s); \quad i = 1, 5 \quad (12)$$

$$\left[s + \beta_2 + \lambda_p + \mu(y) \right] \bar{P}_i(s) = \mu(y) \bar{P}_{i+1}(s) + \beta_1 \bar{P}_j(s); \quad i = 2, 10; \quad j = 1, 0 \quad (13)$$

$$\left[s + \xi + \lambda_p + \mu(y) \right] \bar{P}_i(s) = \psi \bar{P}_{i-1}(s); \quad i = 3, 6, 9, 12; \quad \xi = \lambda_1, \beta_2, \beta_1, \lambda_2; \quad \psi = \beta_2, \beta_1, \lambda_2, \lambda_1 \quad (14)$$

$$\left[s + \lambda_2 + \lambda_p + \mu(y) \right] \bar{P}_i(s) = \mu(y) \bar{P}_{i+1}(s) + \lambda_1 \bar{P}_j(s); \quad i = 4, 8; \quad j = 0, 7 \quad (15)$$

$$\left[s + \lambda_1 + \lambda_p + \mu(y) \right] \bar{P}_i(s) = \beta_2 \bar{P}_j(s) + \mu(y) \bar{P}_{i+1}(s); \quad i = 7, 11; \quad j = 0, 10 \quad (16)$$

$$\left[s + \frac{\partial}{\partial x} + \phi(x) \right] \bar{P}_i(x,s) = 0; \quad i = 13, 14, 15, 16, 17 \quad (17)$$

$$\bar{P}_i(0,s) = \xi \bar{P}_j(s); \quad \xi = \lambda_2, \beta_1, \beta_2, \lambda_1; \quad i = 13, 14, 15, 16; \quad j = 12, 9, 6, 3 \quad (18)$$

$$\bar{P}_{17}(0,s) = \lambda_p \sum_{i=1}^{12} \bar{P}_i(s) \quad (19)$$

After solving Equations (11) to (19), one obtain the Laplace transformation of state transition probabilities as

$$\bar{P}_0(s) = \frac{1}{D(s)} \quad (20)$$

$$\bar{P}_1(s) = \frac{\lambda_2 C_7}{C_8} \bar{P}_0(s) \quad (21)$$

$$\bar{P}_2(s) = \frac{\lambda_2 \beta_1 (s + A_3)}{C_8} \bar{P}_0(s) \quad (22)$$

$$\bar{P}_3(s) = \frac{\lambda_2 \beta_1 \beta_2}{C_8} \bar{P}_0(s)$$

$$\bar{P}_4(s) = \frac{\lambda_1 C_5}{C_6} \bar{P}_0(s)$$

$$\bar{P}_5(s) = \frac{\lambda_2 \lambda_1 (s + A_2)}{C_6} \bar{P}_0(s)$$

$$\bar{P}_6(s) = \frac{\lambda_2 \beta_1 \lambda_1}{C_6} \bar{P}_0(s)$$

$$\bar{P}_7(s) = \frac{\beta_2 C_3}{C_4} \bar{P}_0(s)$$

$$\bar{P}_8(s) = \frac{\beta_2 \lambda_1 (s + A_1)}{C_4} \bar{P}_0(s)$$

$$\bar{P}_9(s) = \frac{\lambda_2 \beta_2 \lambda_1}{C_4} \bar{P}_0(s)$$

$$\bar{P}_{10}(s) = \frac{\beta_1 C_1}{C_2} \bar{P}_0(s)$$

$$\bar{P}_{11}(s) = \frac{\beta_2 \beta_1 (s + A_4)}{C_2} \bar{P}_0(s)$$

$$\bar{P}_{12}(s) = \frac{\lambda_1 \beta_2 \beta_1}{C_2} \bar{P}_0(s)$$

$$\bar{P}_{13}(s) = \left(\frac{1 - \bar{S}_\mu(s)}{s} \right) \frac{\lambda_1 \lambda_2 \beta_2 \beta_1}{C_2} \bar{P}_0(s)$$

$$\bar{P}_{14}(s) = \left(\frac{1 - \bar{S}_\mu(s)}{s} \right) \frac{\lambda_1 \lambda_2 \beta_2 \beta_1}{C_4} \bar{P}_0(s)$$

$$\bar{P}_{15}(s) = \left(\frac{1 - \bar{S}_\mu(s)}{s} \right) \frac{\lambda_1 \lambda_2 \beta_2 \beta_1}{C_6} \bar{P}_0(s)$$

$$\bar{P}_{16}(s) = \left(\frac{1 - \bar{S}_\mu(s)}{s} \right) \frac{\lambda_1 \lambda_2 \beta_2 \beta_1}{C_8} \bar{P}_0(s)$$

$$\bar{P}_{17}(s) = \left(\frac{1 - \bar{S}_\mu(s)}{s} \right) \lambda_p \sum_{i=1}^{12} \bar{P}_i(s)$$

Where

$$D(s) = (s + A_5) - \mu H_1 - \bar{S}_\mu(s) \{ \lambda_1 \lambda_2 \beta_1 \beta_2 H_2 + \lambda_p (H_1 + H_3) \}$$

$$A_1 = \beta_1 + \lambda_p + \mu(y); \quad A_2 = \beta_2 + \lambda_p + \mu(y);$$

$$A_3 = \lambda_1 + \lambda_p + \mu(y); \quad A_4 = \lambda_2 + \lambda_p + \mu(y);$$

$$A_5 = \lambda_1 + \lambda_2 + \beta_1 + \beta_2 + \lambda_p$$

$$C_1 = (s + A_3)(s + A_4) - \mu \lambda_1;$$

$$C_2 = (s + A_2)C_1 - \mu \beta_2 (s + A_4);$$

$$C_3 = (s + A_1)(s + A_4) - \mu \lambda_2;$$

$$C_4 = (s + A_3)C_3 - \mu \lambda_1 (s + A_1);$$

$$C_5 = (s + A_1)(s + A_2) - \mu \beta_1;$$

$$C_6 = (s + A_4)C_5 - \mu \lambda_2 (s + A_2);$$

$$C_7 = (s + A_3)(s + A_2) - \mu \beta_2;$$

$$C_8 = (s + A_1)C_7 - \mu \beta_1 (s + A_3);$$

$$H_1 = \frac{C_7 \lambda_2}{C_8} + \frac{C_5 \lambda_1}{C_6} + \frac{C_3 \beta_2}{C_4} + \frac{C_1 \beta_1}{C_2};$$

$$H_2 = \frac{1}{C_8} + \frac{1}{C_6} + \frac{1}{C_4} + \frac{1}{C_2};$$

$$H_3 = \frac{\beta_1 \lambda_2 (s + A_3 + \beta_2)}{C_8} + \frac{\lambda_1 \lambda_2 (s + A_2 + \beta_1)}{C_6}$$

$$+ \frac{\beta_2 \lambda_1 (s + A_1 + \lambda_2)}{C_4} + \frac{\beta_1 \beta_2 (s + A_4 + \lambda_1)}{C_2}$$

(31)

Laplace transformation of transition state probabilities of the upstate (i.e. good or degraded state) system.

(32)

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \sum_{i=1}^{12} \bar{P}_i(s) = \bar{P}_0(s)[1 + H_3] \quad (38)$$

(33)

Laplace transformation of transition state probabilities of the downstate (i.e. failed state) system.

(34)

$$\bar{P}_{down}(s) = \sum_{i=13}^{17} \bar{P}_i(s)$$

(35)

$$= \frac{[1 - \bar{S}_\mu(s)]}{s} [\lambda_1 \lambda_2 \beta_1 \beta_2 H_2 + \lambda_p H_3] \quad (39)$$

3. Particular cases and numerical computation

3.1. Availability analysis

Let us consider that the 100% repair facility is available in the designed air and refrigeration system. Assuming the values of different failure and repair rates as $\lambda_1 = 0.020$, $\lambda_2 = 0.025$, $\beta_1 = 0.015$, $\beta_2 = 0.035$, $\lambda_p = 0.050$, $\phi(x) = 1$, $\mu(y) = 1$ (Ram & Goyal, 2015) and substituting in Equation (38). One can obtain the availability in terms of time t , after taking the inverse Laplace transformation of Equation (38). Varying the time unit t from 0 to 100, we get Table 2 and corresponding Figure 2, which shows the trend of availability. From the graph, one can see that the availability of the air and refrigeration system decreases efficiently as time increases.

Table 2. Availability as function of time.

Time (t)	Availability $P_{up}(t)$
0	1.00000
1	0.95170
2	0.90713
5	0.78988
10	0.62939
15	0.50162
20	0.39978
25	0.31862
30	0.25394
35	0.20239
40	0.16130
45	0.12855
50	0.10246
55	0.08166
60	0.06508
65	0.05187
70	0.04134
75	0.03295
80	0.02626
85	0.02093
90	0.01668
95	0.01329
100	0.01059

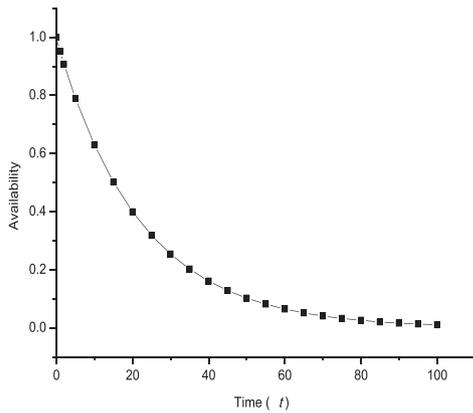


Fig. 2. Availability v/s Time

3.2. Non-availability analysis

Non-availability of the air and refrigeration system can be obtained by substituting the value of failure and repair rates as $\lambda_1 = 0.020$, $\lambda_2 = 0.025$, $\beta_1 = 0.015$, $\beta_2 = 0.035$, $\lambda_p = 0.050$, $\phi(x) = 1$, $\mu(y) = 1$ (Ram & Goyal, 2015) in the inverse Laplace transformation of Equation (39). After varying the time unit t from 0 to 100, we get Table 3 and Figure 3 which shows the behavior of non-

availability of the system. From the critical examination of the graph, one can observe that non-availability of the system increases as time passage.

Table 3. Non-Availability as function of time.

Time (t)	Non-Availability $P_{down}(t)$
0	0.00000
1	0.04831
2	0.09287
5	0.21011
10	0.37061
15	0.49838
20	0.60021
25	0.68137
30	0.74606
35	0.79761
40	0.83870
45	0.87144
50	0.89754
55	0.91834
60	0.93492
65	0.94813
70	0.95866
75	0.96705
80	0.97374
85	0.97907
90	0.98332
95	0.98671
100	0.98940

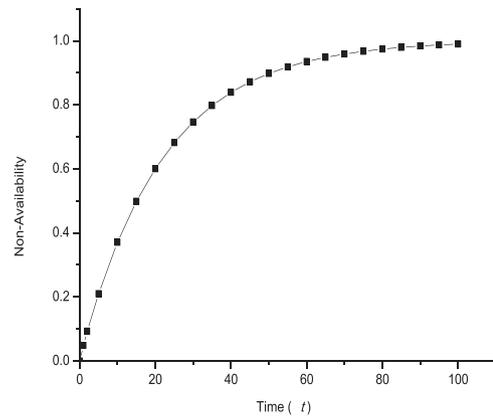


Fig. 3. Non-availability v/s Time

3.3. Reliability analysis

Considered that the repair facility is not available, therefore putting the values of all repair rates equal to zero in Equation (38). Substituting the different failure rates as $\lambda_1 = 0.020$, $\lambda_2 = 0.025$, $\beta_1 = 0.015$,

$\beta_2 = 0.035$, $\lambda_p = 0.050$, after taking the inverse Laplace transformation, one can determine the reliability in terms of time t as

$$RI(t) = 7.65625e^{(-0.065t)} + 1.875e^{(-0.085t)} - 7e^{(-0.07t)} - 1.5e^{(-0.075t)} - 0.03125e^{(-0.145t)} \quad (40)$$

Now, varying the time unit t from 0 to 20 in Equation (40) and acquire the reliability of the designed system, which is revealed in Table 4 and represented graphically in Figure 4.

Table 4. Reliability as function of time

Time (t)	Reliability $RI(t)$
0	1.00000
1	0.95123
2	0.90484
5	0.77878
10	0.60634
15	0.47172
20	0.36651
25	0.28426
30	0.21999
35	0.16984
40	0.13079
45	0.10044
50	0.07693
55	0.05876
60	0.04477
65	0.03402
70	0.02579
75	0.01951
80	0.01472
85	0.01108
90	0.00833
95	0.00625
100	0.00468

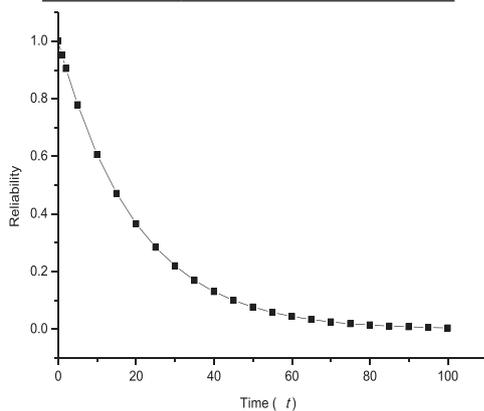


Fig. 4. Reliability v/s Time

The reliability of the complex repairable air and refrigeration system also decreases smoothly with respect to the increment of time.

3.4. Mean time to failure (MTTF)

Suppose that the repair facility is not available, therefore taking each repair rate equal to zero and the limit as s tends to zero in Equation (38),

$$MTTF = \lim_{s \rightarrow 0} \bar{P}_{up}(s) = \frac{1}{(\lambda_1 + \lambda_2 + \lambda_p + \beta_1 + \beta_2)} \left\{ 1 + \frac{\lambda_2[(\beta_2 + \lambda_p)(\lambda_1 + \lambda_p) + \beta_1(\lambda_1 + \lambda_p + \beta_2)]}{(\beta_1 + \lambda_p)(\beta_2 + \lambda_p)(\lambda_1 + \lambda_p)} + \frac{\lambda_1[(\beta_1 + \lambda_p)(\beta_2 + \lambda_p) + \lambda_2(\beta_1 + \lambda_p + \beta_2)]}{(\beta_1 + \lambda_p)(\beta_2 + \lambda_p)(\lambda_2 + \lambda_p)} + \frac{\beta_1[(\lambda_1 + \lambda_p)(\lambda_2 + \lambda_p) + \beta_2(\lambda_1 + \lambda_p + \lambda_2)]}{(\lambda_1 + \lambda_p)(\beta_2 + \lambda_p)(\lambda_2 + \lambda_p)} + \frac{\beta_2[(\beta_1 + \lambda_p)(\lambda_2 + \lambda_p) + \lambda_1(\beta_1 + \lambda_p + \lambda_2)]}{(\lambda_1 + \lambda_p)(\beta_1 + \lambda_p)(\lambda_2 + \lambda_p)} \right\} \quad (41)$$

Now, varying the failure rates one by one as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 respectively and substituting the values of other failure rates as $\lambda_1 = 0.020$, $\lambda_2 = 0.025$, $\beta_1 = 0.015$, $\beta_2 = 0.035$, $\lambda_p = 0.050$ in Equation (41), one can obtain Table 5 and Figure 5.

Table 5. MTTF as function of failure rates

Variation	MTTF				
in $\lambda_1,$	λ_1	λ_2	β_1	β_2	λ_p
$\lambda_2, \beta_1,$					
β_2, λ_p					
0.1	19.29613	19.39120	19.13612	19.49764	9.97139
0.2	19.25931	19.36199	19.08734	19.47820	4.99841
0.3	19.26537	19.36836	19.09322	19.48539	3.33307
0.4	19.27602	19.37812	19.10552	19.49436	2.49993
0.5	19.28589	19.38695	19.11722	19.50214	1.99997
0.6	19.29426	19.39436	19.12725	19.50853	1.66666
0.7	19.30125	19.40051	19.13568	19.51377	1.42857
0.8	19.30709	19.40563	19.14276	19.51810	1.25000
0.9	19.31203	19.40994	19.14876	19.52173	1.11111

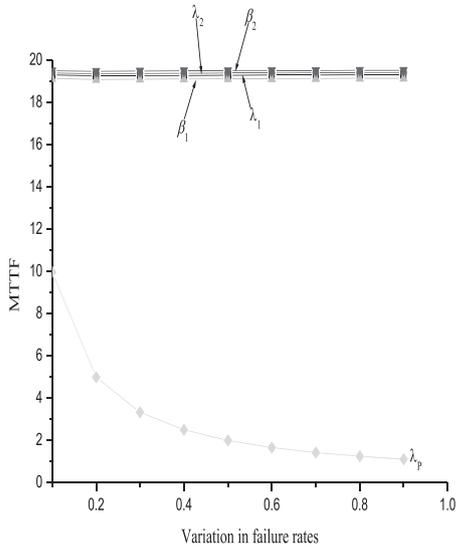


Fig. 5. MTTF v/s Failure rates

The mean time to failure of the designed air and refrigeration system first decreases sharply but after some variation in failure, it decreases smoothly with respect to the failure rate of power supply. With respect to the equipment failure rates, MTTF of the air and refrigeration system is fluctuating with minor difference. It is approximately equal with respect to the failure rate of compressor, condenser, expansion device and evaporator.

3.5. Sensitivity analysis

Sensitivity of a measure is defined as the partial derivative of the function with respect to their input factors (Henley & Kumamoto, 1992; Andrews & Moss, 1993). Here, these input factors are failure rates of the designed system.

3.5.1. Reliability sensitivity

Reliability sensitivity can be obtained by partial differentiation of Equation (38) with respect to each failure rate, after putting the repair rates zero and taking inverse Laplace transform of Equation (38). Now, putting the values of different failure rates as $\lambda_1 = 0.020$, $\lambda_2 = 0.025$, $\beta_1 = 0.015$, $\beta_2 = 0.035$, $\lambda_p = 0.050$ in partial differentiation and varying time t from 0 to 20. One can determine Table 6 and corresponding Figure 6.

Table 6. Reliability sensitivity as a function of time

Time (t)	Reliability Sensitivity				
	$\frac{\partial RI(t)}{\partial \lambda_1}$	$\frac{\partial RI(t)}{\partial \lambda_2}$	$\frac{\partial RI(t)}{\partial \beta_1}$	$\frac{\partial RI(t)}{\partial \beta_2}$	$\frac{\partial RI(t)}{\partial \lambda_p}$
0	0.00000	.,.....	.,.....	0.00000	0.00000
1	0.00000	0.00000	0.00000	0.00000	-0.95123
2	-0.00003	-0.00002	-0.00004	-0.00002	-1.80967
5	-0.00087	-0.00069	-0.00117	-0.00048	-3.89391
10	-0.00896	-0.00704	-0.01215	-0.00485	-6.06339
15	-0.02921	-0.02276	0.03998	-0.01541	-7.07578
20	-0.05973	-0.04612	-0.08246	-0.03065	-7.33019
25	-0.09470	-0.07249	-0.13187	-0.04729	-7.10647
30	-0.12800	-0.09712	-0.17978	-0.06219	-6.59975
35	-0.15510	-0.11664	-0.21972	-0.07330	-5.94449
40	-0.17362	-0.12943	-0.24806	-0.07982	-5.23145
45	-0.18303	-0.13525	-0.26373	-0.08185	-4.51989
50	-0.18411	-0.13487	-0.26755	-0.08009	-3.84650
55	-0.17838	-0.12953	-0.26141	-0.07548	-3.23204
60	-0.16759	-0.12065	-0.24767	-0.06898	-2.68625
65	-0.15348	-0.10954	-0.22873	-0.06146	-2.21150
70	-0.13756	-0.09734	-0.20673	-0.05359	-1.80548
75	-0.12105	-0.08492	-0.18343	-0.04589	-1.46305
80	-0.10483	-0.07293	-0.16019	-0.03867	-1.17768
85	-0.08954	-0.06177	-0.13797	-0.03214	-0.94228
90	-0.07556	-0.05169	-0.11739	-0.02640	-0.74982
95	-0.06308	-0.04279	-0.09882	-0.02145	-0.59369
100	-0.05216	-0.03509	-0.08239	-0.01727	-0.46794

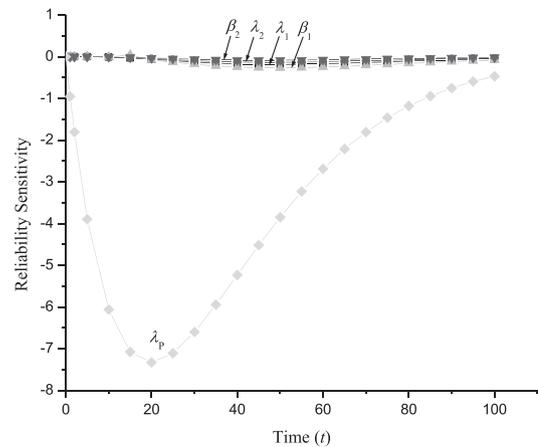


Fig. 6. Sensitivity of reliability v/s Time

3.5.2. MTTF Sensitivity

MTTF sensitivity can be prophesied by partial differentiation of Equation (41) with respect to the input parameters and then setting the values of input parameters as

$\lambda_1 = 0.020$, $\lambda_2 = 0.025$, $\beta_1 = 0.015$, $\beta_2 = 0.035$, $\lambda_p = 0.050$ (Ram & Goyal, 2015) in partial derivatives of Equation 41, one gets the Table 7 and corresponding Figure 7.

Table 7. MTTF sensitivity as a function of input parameters

Variation in $\lambda_1, \lambda_2,$ $\beta_1, \beta_2,$ λ_p	MTTF Sensitivity				
	$\frac{\partial MTTF}{\partial \lambda_1}$	$\frac{\partial MTTF}{\partial \lambda_2}$	$\frac{\partial MTTF}{\partial \beta_1}$	$\frac{\partial MTTF}{\partial \beta_2}$	$\frac{\partial MTTF}{\partial \lambda_p}$
0.1	-1.09491	-0.90747	-1.39697	-0.67778	-98.87021
0.2	-0.02099	-0.00170	-0.04840	0.02708	-24.96526
0.3	0.10110	0.09498	0.11358	0.09115	-11.10715
0.4	0.10534	0.09500	0.12382	0.08487	-6.24919
0.5	0.09130	0.08116	0.10896	0.07056	-3.99977
0.6	0.07639	0.06738	0.09189	0.05772	-2.77769
0.7	0.06375	0.05596	0.07707	0.04749	-2.04078
0.8	0.05357	0.04686	0.06499	0.03950	-1.56248
0.9	0.04545	0.03966	0.05528	0.03326	-1.23456

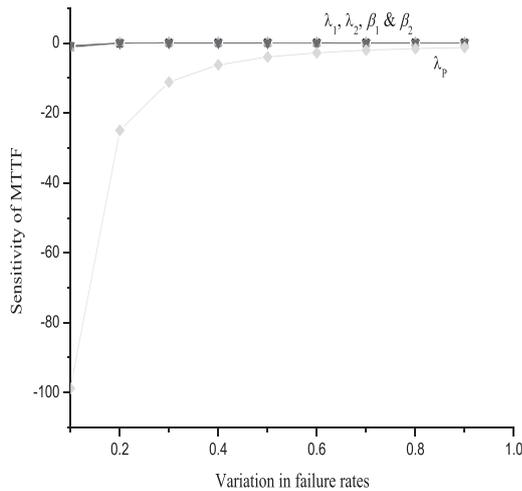


Fig. 7. MTTF sensitivity as a function of input parameters

Sensitivity of MTTF decreases very slightly with respect to the variation in failure rates of compressor, condenser, expansion device and evaporator while it increases sharply with respect to the variation in failure rate of power supply. The precarious examination of Figure 7, resulted that the MTTF sensitivity is first fluctuating and after some variation, it decreases and coincide with each other with respect to the variation in failure rates of compressor, condenser, expansion device and evaporator.

4. Conclusion

An inspection model for an air and refrigeration system is developed in this paper that inspected the performance of the air and refrigeration system in its lifetime with the help of reliability measures. It is concluded that the system is highly sensitive with regard to power supply, as compared to its other components. So, by controlling the failure rate of power supply, one can improve the availability of the system and can achieve high reliability of the air and refrigeration system. Furthermore, it is very convenient in many applications in industries and also convenient for medical sector. Our study on an air and refrigeration system plays an important role in maintaining the system and is beneficial for the engineers and technicians in improving the performance of units and increasing the reliability of an aircraft refrigeration system.

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خلاصة

في القطاع الهندسي، تتأثر البحوث المتعلقة بنظام ما بفهم العلاقة بين مكوناته المادية. وبما أن نظام الهواء والتبريد يعتمدان على المكونات الفيزيائية الأساسية الأربعة، وهي: المكبس والمكثف وجهاز التمديد والمُبخر، يتبنى الباحثون نموذجاً رياضياً مناسباً لنظام الهواء والتبريد لقياس أداء النظام. في هذا البحث، تمت دراسة مقاييس الموثوقية لنظام الهواء والتبريد القابل للإصلاح، والتي من خلالها يستطيع مهندسين أو مصممين الموثوقية تحديد مدى إمكانية تحسين الموثوقية باستخدام التقلبات المناسبة. وقد تم إجراء تحليل الحساسية لعدة اختلافات في خصائص الموثوقية جنباً إلى جنب مع تعديلات القيم الدقيقة لمعاملات المدخلات. وفي النهاية، تم كذلك أخذ بعض الأمثلة العددية وتمثيلها البياني لتسليط الضوء على الاستخدام العملي للنموذج. ومن خلال مجمل الدراسة، نستنتج أن إمداد الطاقة في نظام الهواء والتبريد يتطلب عناية إضافية، فهو الجزء الأكثر حساسية في أداء نظام الهواء والتبريد استناداً إلى مكوناته الأخرى.