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Fixed points of weakly compatible mappings in fuzzy metric spaces

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ABSTRACT

In this paper, we prove some common fixed point theorems for weakly compatible mappings in fuzzy metric spaces with common property (E.A) and give some examples to illustrate our results. As an application to our main result, we present a common fixed point theorem for four finite families of self mappings in fuzzy metric spaces by using the notion of the pairwise commuting mappings. Our results improve and extend some relevant results existing in the literature.

Keywords: Common property (E.A); fixed point; fuzzy metric space; property (E.A); weakly compatible mappings.

INTRODUCTION

Zadeh (1965) investigated the concept of a fuzzy set in his seminal paper. In the last two decades, there has been a tremendous development and growth in fuzzy mathematics. The concept of fuzzy metric spaces was introduced by Kramosil & Michalek (1975), which opened an avenue for further development of analysis in such spaces. Further, George & Veeramani (1994) modified the concept of fuzzy metric space introduced by Kramosil & Michalek (1975) with a view to obtain a Hausdorff topology, which has very important applications in quantum particle physics, particularly, in connection with both string and ε^{∞} theory (El Naschie (1998, 2004, 2007) and references mentioned therein). Fuzzy set theory also has many applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication, etc. Consequently, in due course of time, some metric fixed point results were generalized to fuzzy metric spaces by various authors viz Grabiec (1988); Subrahmanyam (1995); Vasuki (1999); Cho (1993, 1997); Chang *et al*. (1997); Cho *et al*. (1998, 2009, 1995); Jung *et al*. (1994); Jungck *et al*. (1993); Pathak *et al*. (1997, 1998); Sessa & Cho (1993).

In 2002, Aamri & El-Moutawakil (2002) defined the notion of the (E.A) property for self-mappings, which contained the class of non-compatible mappings in metric spaces. It was pointed out that the (E.A) property allows replacing the completeness requirement of the space with a more natural condition of closedness of the range as well as relaxes the completeness of the whole space, the continuity of one or more mappings and the containment of the range of one mapping into the range of other, which is utilized to construct the sequence of joint iterates. Further, Liu *et al*. (2005) defined the notion of the common property (E.A), which contains the (E.A) property and proved some common fixed point theorems under hybrid contractive conditions. Many authors have proved common fixed point theorems in fuzzy metric spaces for many kinds of generalized contractive conditions. For details, we refer to Beg, *et al*. (2014); Fang & Gao (2009); Gopal *et al*. (2011); Imdad & Ali (2006, 2008); Kutukcu *et al*. (2006); Kumar (2011); Kumar & Fisher (2010); LiS *et al*. (2009); Murthy *et al*. (2010); O'Regan & Abbas (2009); Pant & Chauhan (2011); Pant & Pant (2007); Sedghi *et al*. (2009); Shen *et al*. (2012); Singh *et al*. (2012); Singh & Tomar (2004); Tanveer *et al*. (2012).

In the present paper, we prove some common fixed point theorems for weakly compatible mappings by using the common property (E.A) in fuzzy metric spaces. As an application, we present fixed point theorems for six mappings and four finite families of self-mappings in fuzzy metric spaces using the notion of the pairwise commuting mappings due to Imdad *et al*. (2009). Our results improve and extend the corresponding results of Sedghi *et al*. (2010) and some others.

PRELIMINARIES

Definition 1 [Schweizer & Sklar, 1983] A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm, if it satisfies the following conditions:

1. * is associative and commutative,
2. * is continuous,
3. $a*1=a$ for all $a\in[0,1]$,
4. $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$ for all $a,b,c,d\in[0,1]$.

Examples of continuous t-norms are $a*b=\min\{a,b\}$, $a*b=ab$ and $a*b=\max\{a+b-1,0\}$.

Definition 2. [George & Veeramani, 1994] A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space, if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, $t, s > 0$,

1. $M(x, y, t) > 0$,
2. $M(x, y, t) = 1$ if and only if $x = y$,
3. $M(x, y, t) = M(y, x, t)$,
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
5. $M(x, y, \cdot) : (0, \infty) \rightarrow [0,1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Let $(X, M, *)$ be a fuzzy metric space. For any $t > 0$, the open ball $\mathfrak{B}(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$\mathfrak{B}(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}.$$

Now, let $(X, M, *)$ be a fuzzy metric space and τ the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $\mathfrak{B}(x, r, t) \subset A$. Then τ is a topology on X induced by the fuzzy metric M.

George & Veeramani (1994) obtained the following example which showed that every metric induces a fuzzy metric:

Example 1. Let (X, d) be a metric space. Denote $a * b = ab$ (or $a * b = \min\{a, b\}$) for all $a, b \in [0,1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}.$$

Then $(X, M_d, *)$ is a fuzzy metric space and the fuzzy metric M_d induced by the metric d is often referred as the standard fuzzy metric.

Lemma 1. (Grabiec, 1988) Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing for all $x, y \in X$.

Definition 3. (Mishra *et al.*, 1994) Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be compatible, if $M(ASx_n, SAx_n, t) \rightarrow 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that Ax_n, $Sx_n \rightarrow z$ for some $z \in X$ as $n \rightarrow \infty$.

Definition 4. (Jungck, 1996) Two self mappings A and S of a non-empty set X are said to be weakly compatible (or coincidentally commuting), if they commute at their coincidence points, i.e. if $Az = Sz$ some $z \in X$, then $ASz = SAz$.

Remark 1. (Jungck, 1996) Two compatible self mappings are weakly compatible, but the converse is not true. Therefore, the concept of weak compatibility is more general than that of compatibility.

Definition 5. (Abbas *et al*., 2009) A pair of self mappings A and S of a fuzzy metric space $(X,M,*)$ is said to satisfy the property (E.A), if there exists a sequence $\{x_n\}$ in X for some $z \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z .$$

Remark 2. It is noted that the weak compatibility and the $(E.A)$ property are independent to each other (Pathak *et al*., 2007).

Definition 6. (Abbas *et al*. 2009) Two self mappings A and S on a fuzzy metric space $(X,M,*)$ are non-compatible, if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$, but, for some $t > 0$, $\lim_{n\to\infty} M(ASx_n, SAx_n, t)$ is either less than 1 or nonexistent.

Remark 3. From Definition 6, it is easy to see that any non-compatible self-mappings of a fuzzy metric space $(X,M,*)$ satisfy the $(E.A)$ property. But two mappings satisfying the $(E.A)$ property need not be non-compatible (Pathak *et al*. 2007).

Definition 7. (Abbas *et al*., 2009) Two pairs of self mappings (A,S) and (B,T) of a fuzzy metric space $(X,M,*)$ are said to satisfy the common property $(E.A)$ if there exists two sequences $\{x_n\},\{y_n\}$ in X for some z in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z .$$

Example 2. Let (*X*, *M*, *) be a fuzzy metric space with $X = [-1,1]$ and $M(x,y,t) = \dfrac{t}{t + |x-y|}$ for all $x, y \in X$ and $t > 0$. Define the self-mappings A, B, S and T on X as $Ax=x$, $Bx=-x$, $Sx = \dfrac{x}{5}$ and $Tx = -\dfrac{x}{5}$ for all $x \in X$. Then, from the sequences $\{x_n\} = \left\{\dfrac{1}{n}\right\}$ and $\{y_n\} = \left\{-\dfrac{1}{n}\right\}$ in X, we can easily verify that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 0 \in X .$$

This shows that the pairs (A,S) and (B,T) share the common property $(E.A)$.

Definition 8 (Imdad *et al*., 2009) Two families of self mappings $\{A_i\}_{i=1}^{m}$ and $\{S_k\}_{k=1}^{n}$ are said to be pairwise commuting if

1. $A_i A_j = A_j A_i$ for all $i, j \in \{1,2,\ldots,m\}$,

2. $S_k S_l = S_l S_k$ for all $k,l \in \{1,2,\dots,n\}$,
3. $A_i S_k = S_k A_i$ for all $i \in \{1,2,\dots,m\}$ and $k \in \{1,2,\dots,n\}$.

Throughout this paper, $(X,M,*)$ is considered to be a fuzzy metric space with condition $\lim_{t\to\infty} M(x,y,t)=1$ for all $x,y \in X$.

RESULTS

In 2010, Sedghi *et al*. (2010) proved a common fixed point theorem for a pair of weakly compatible mappings with the $(E.A)$ property in fuzzy metric spaces by using the following function:

Let Φ is a set of all increasing and continuous functions $\phi:(0,1]\to(0,1]$ such that $\phi(t)>t$ for all $t \in (0,1)$.

Example 3. [Sedghi, *et al*., 2010] A function $\phi:(0,1]\to(0,1]$ defined by $\phi(t)=t^{\frac{1}{2}}$ belongs to the set Φ.

Theorem 1. (Theorem 1, Sedghi *et al*., 2010) Let $(X,M,*)$ be a fuzzy metric space and A, S be self-mappings of X satisfying the following conditions:

1. $A(X) \subseteq S(X)$ and $A(X)$ or $S(X)$ is a closed subset of X,
2. there exists k, $1 \le k < 2$, such that

$$M(Ax,Ay,t) \ge \varphi\left(\min\left\{\begin{array}{l} M(Sx,Sy,t), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t} \min\left\{\begin{array}{l} M(Sx,Ax,t_1), \\ M(Sy,Ay,t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t} \max\left\{\begin{array}{l} M(Sx,Ay,t_3), \\ M(Sy,Ax,t_4) \end{array}\right\} \end{array}\right\}\right) \tag{1}$$

for all $x,y \in X, t>0$. Suppose that the pair (A,S) satisfies the $(E.A)$ property and (A,S) is weakly compatible. Then A and S have a unique common fixed point in X.

Before proving our main theorems, we begin with the following observation.

Lemma 2. Let A,B,S and T be self mappings of a fuzzy metric space $(X,M,*)$, where $*$ is a continuous t-norm. Suppose that

1. $A(X) \subset T(X)$ (or $B(X) \subset S(X)$),
2. the pair (A,S) (or (B,T)) satisfies the $(E.A)$ property,

3. $\{B(y_n)\}$ converges for any sequence $\{y_n\}$ in X whenever $\{T(y_n)\}$ converges or $\{A(x_n)\}$ converges for any sequence $\{x_n\}$ in X whenever $\{S(x_n)\}$ converges,
4. there exist $\phi \in \Phi$ and k, $1 \le k < 2$, such that

$$M(Ax,By,t) \ge \varphi\left(\min\left\{\begin{array}{c} M(Sx,Ty,t), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t} \min\left\{\begin{array}{l} M(Sx,Ax,t_1), \\ M(Ty,By,t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t} \max\left\{\begin{array}{l} M(Sx,By,t_3), \\ M(Ty,Ax,t_4) \end{array}\right\} \end{array}\right\}\right) \tag{2}$$

for all $x, y \in X$ and $t > 0$. Then the pairs (A,S) and (B,T) share the common property $(E.A)$.

Proof. Suppose the pair (A,S) satisfies the (E.A) property, then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z \tag{3}$$

for some $z \in X$. Since $A(X) \subset T(X)$, it follows that, for each $\{x_n\} \subset X$, there corresponds a sequence $\{y_n\} \subset X$ such that $Ax_n = Ty_n$. Therefore, we have

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ax_n = z. \tag{4}$$

Thus also we have $Ax_n \to z$, $Sx_n \to z$ and $Ty_n \to z$.

Now, we show that $By_n \to z$. Let $By_n \to l$ for all $t > 0$ as $n \to \infty$. Then it is enough to show that $z = l$. Suppose that $z \ne l$. Then there exists $t_0 > 0$ such that

$$M\left(z,l,\frac{2}{k}t_0\right) > M(z,l,t_0), \tag{5}$$

In order to establish the claim embodied in (5), let us assume that (5) does not hold. Then we have $M\left(z,l,\frac{2}{k}t\right) = M(z,l,t)$ for all $t > 0$. Using this equality repeatedly, we obtain

$$M(z,l,t) = M\left(z,l,\frac{2}{k}t\right) = \ldots = M\left(z,l,\left(\frac{2}{k}\right)^n t\right) \to 1$$

as $n \to \infty$. This shows that $M(z,l,t)=1$ for all $t>0$, which contradicts $z \neq l$ and hence (5) is proved. Using the inequality (2) with $x = x_n$ and $y = y_n$, we get

$$M(Ax_n, By_n, t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(Sx_n, Ty_n, t_0), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{l} M(Sx_n, Ax_n, t_1), \\ M(Ty_n, By_n, t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{l} M(Sx_n, By_n, t_3), \\ M(Ty_n, Ax_n, t_4) \end{array}\right\} \end{array}\right\}\right)$$

$$\geq \varphi\left(\min\left\{\begin{array}{c} M(Sx_n, Ty_n, t_0), \\ \min\left\{M(Sx_n, Ax_n, \varepsilon), M\left(Ty_n, By_n, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \\ \max\left\{M\left(Sx_n, By_n, \frac{2}{k}t_0 - \varepsilon\right), M(Ty_n, Ax_n, \varepsilon)\right\} \end{array}\right\}\right)$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $n \to \infty$, it follows that

$$M(z, l, t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(z, z, t_0), \\ \min\left\{M(z, z, \varepsilon), M\left(z, l, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \\ \max\left\{M\left(z, l, \frac{2}{k}t_0 - \varepsilon\right), M(z, z, \varepsilon)\right\} \end{array}\right\}\right)$$

$$= \phi\left(M\left(z, l, \frac{2}{k}t_0 - \varepsilon\right)\right)$$

$$> M\left(z, l, \frac{2}{k}t_0 - \varepsilon\right)$$

as $\varepsilon \to 0$ and so we have

$$M(z,l,t_0) \geq M\left(z, l, \frac{2}{k}t_0\right),$$

which contradicts (5). Therefore, $z = l$. Hence the pairs (A,S) and (B,T) share the common property (E.A). This completes the proof.

Remark 4. In general, the converse of Lemma 2 is not true (see Example 3.1, Ali, *et al*. 2010).

Now we prove a common fixed point theorem for two pairs of mappings in fuzzy metric space.

Theorem 2. Let A, B, S and T be self-mappings of a fuzzy metric space $(X, M, *)$, where $*$ is a continuous t-norm satisfying the inequality (2) of Lemma 2. Suppose that

1. the pairs (A, S) and (B, T) share the common property $(E.A)$,
2. $S(X)$ and $T(X)$ are closed subsets of X.

Then the pairs (A, S) and (B, T) have a coincidence point each other. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof. If the pairs (A, S) and (B, T) share the common property (E.A), then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z, \tag{6}$$

for some $z \in X$. Since $S(X)$ is a closed subset of X, we have $\lim_{n\to\infty} Sx_n = z \in S(X)$. Therefore, there exists a point $u \in X$ such that $Su = z$.

Now, we assert that $Au=Su$. Suppose that $Au \neq Su$. Then there exists $t_0 > 0$ such that

$$M\left(Au, Su, \frac{2}{k}t_0\right) > M(Au, Su, t_0). \tag{7}$$

In order to establish the claim embodied in (7), let us assume that (7) does not hold. Then we have $M\left(Au, Su, \frac{2}{k}t\right) = M(Au, Su, t)$ for all $t > 0$. Using this equality repeatedly, we obtain

$$M(Au, Su, t) = M\left(Au, Su, \frac{2}{k}t\right) = \ldots = M\left(Au, Su, \left(\frac{2}{k}\right)^n t\right) \to 1$$

as $n \to \infty$. This shows that $M(Au, Su, t)=1$ for all $t > 0$, which contradicts $Au \neq Su$ and hence (7) is proved. Using the inequality (2) with $x = u$ and $y = y_n$, we get

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$$M(Au, By_n, t_0) \geq \varphi\left(\min\left\{\begin{array}{l} M(Su, Ty_n, t_0), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{l} M(Su, Au, t_1), \\ M(Ty_n, By_n, t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{l} M(Su, By_n, t_3), \\ M(Ty_n, Au, t_4) \end{array}\right\} \end{array}\right\}\right)$$

$$\geq \varphi\left(\min\left\{\begin{array}{l} M(z, Ty_n, t_0), \\ \min\left\{M\left(z, Au, \frac{2}{k}t_0 - \varepsilon\right), M(By_n, Ty_n, \varepsilon)\right\}, \\ \max\left\{M(z, By_n, \varepsilon), M\left(Ty_n, z, \frac{2}{k}t_0 - \varepsilon\right)\right\} \end{array}\right\}\right)$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $n \to \infty$, it follows that

$$M(Au, z, t_0) \geq \varphi\left(\min\left\{\begin{array}{l} M(z, z, t_0), \\ \min\left\{M\left(z, Au, \frac{2}{k}t_0 - \varepsilon\right), M(z, z, \varepsilon)\right\}, \\ \max\left\{M(z, z, \varepsilon), M\left(z, z, \frac{2}{k}t_0 - \varepsilon\right)\right\} \end{array}\right\}\right)$$

$$= \varphi\left(M\left(z, Au, \frac{2}{k}t_0 - \varepsilon\right)\right)$$

$$> M\left(Au, z, \frac{2}{k}t_0 - \varepsilon\right)$$

as $\varepsilon \to 0$ and so we have

$$M(Au, z, t_0) \geq M\left(Au, z, \frac{2}{k}t_0\right),$$

which contradicts (7). Therefore, we have $Au = Su = z$ and hence u is a coincidence point of (A, S).

Since $T(X)$ is a closed subset of X, there exists a point $v \in X$ such that $Tv = z$.

Now, we show that $Bv = Tv = z$. Suppose that $Bv \neq Tv$. Then there exists $t_0 > 0$ such that

$$M\left(Bv, Tv, \frac{2}{k}t_0\right) > M(Bv, Tv, t_0). \tag{8}$$

To support the claim, let it be untrue. Then we have $M\left(Bv, Tv, \frac{2}{k}t\right) = M(Bv, Tv, t)$ for all $t > 0$. Using this equality repeatedly, we obtain

$$M(Bv, Tv, t) = M\left(Bv, Tv, \frac{2}{k}t\right) = \ldots = M\left(Bv, Tv, \left(\frac{2}{k}\right)^n t\right) \to 1$$

as $n \to \infty$. This shows that $M(Bv, Tv, t) = 1$ for all $t > 0$, which contradicts $Bv \neq Tv$ and hence (8) is proved. Using the inequality (2) with $x = x_n$ and $y = v$, we get

$$M(Ax_n, Bv, t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(Sx_n, Tv, t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{l} M(Sx_n, Ax_n, t_1), \\ M(Tv, Bv, t_2) \end{array}\right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{l} M(Sx_n, Bv, t_3), \\ M(Tv, Ax_n, t_4) \end{array}\right\} \end{array}\right\}\right)$$

$$\geq \varphi\left(\min\left\{\begin{array}{c} M(Sx_n, z, t_0), \\ \min\left\{M(Sx_n, Ax_n, \varepsilon), M\left(z, Bv, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \\ \max\left\{M\left(Sx_n, Bv, \frac{2}{k}t_0 - \varepsilon\right), M(z, Ax_n, \varepsilon)\right\} \end{array}\right\}\right)$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $n \to \infty$, it follows that

$$M(z,Bv,t_0)\geq\varphi\left(\min\left\{\begin{array}{c}M(z,z,t_0),\\ \min\left\{M(z,z,\varepsilon),M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\},\\ \max\left\{M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right),M(z,z,\varepsilon)\right\}\end{array}\right\}\right)$$

$$=\varphi\left(M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right)$$

$$>M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)$$

as $\varepsilon\to 0$ and so we have

$$M(z,Bv,t_0)\geq M\left(z,Bv,\frac{2}{k}t_0\right),$$

which contradicts (8). Therefore, we have $Bv = Tv = z$, which shows that v is a coincidence point of the pair (B,T). Since the pair (A,S) is weakly compatible, it follows that $Az = Asu = Sau = Sz$.

Now, we assert that z is a common fixed point of (A,S). If $z\neq Az$, then, by using (2) with $x=z$ and $y=v$, we get, for some $t_0>0$,

$$M(Az,Bv,t_0)\geq\varphi\left(\min\left\{\begin{array}{c}M(Sz,Tv,t_0),\\ \sup\limits_{t_1+t_2=\frac{2}{k}t_0}\min\left\{\begin{array}{l}M(Sz,Az,t_1),\\ M(Tv,Bv,t_2)\end{array}\right\},\\ \sup\limits_{t_3+t_4=\frac{2}{k}t_0}\max\left\{\begin{array}{l}M(Sz,Bv,t_3),\\ M(Tv,Az,t_4)\end{array}\right\}\end{array}\right\}\right)$$

$$M(Az,z,t_0)\geq\varphi\left(\min\left\{\begin{array}{c}M(Az,z,t_0),\\ \min\left\{M(Az,Az,\varepsilon),M\left(z,z,\frac{2}{k}t_0-\varepsilon\right)\right\},\\ \max\left\{M(Az,z,\varepsilon),M\left(z,Az,\frac{2}{k}t_0-\varepsilon\right)\right\}\end{array}\right\}\right)$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $\varepsilon \to 0$, we have

$$M(Az, z, t_0) \geq \varphi\left(\min\left\{M(Az, z, t_0), M\left(z, Az, \frac{2}{k}t_0\right)\right\}\right)$$
$$= \varphi\left(M(Az, z, t_0)\right)$$
$$> M(Az, z, t_0),$$

which is a contradiction. Hence $Az = Sz = z$, i.e., z is a common fixed point of (A, S). Also, the pair (B, T) is weakly compatible and so $Bz = BTv = TBv = Tz$.

Now, we show that z is also a common fixed point of (B, T). If $z \neq Bz$, then, by using (2) with $x = u$ and $y = z$, we get, for some $t_0 > 0$,

$$M(Au, Bz, t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(Su, Tz, t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{l} M(Su, Au, t_1), \\ M(Tz, Bz, t_2) \end{array}\right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{l} M(Su, Bz, t_3), \\ M(Tz, Au, t_4) \end{array}\right\} \end{array}\right\}\right)$$

$$M(z, Bz, t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(z, Bz, t_0), \\ \min\left\{M(z, z, \varepsilon), M\left(Bz, Bz, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \\ \max\left\{M(z, Bz, \varepsilon), M\left(Bz, z, \frac{2}{k}t_0 - \varepsilon\right)\right\} \end{array}\right\}\right)$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $\varepsilon \to 0$, we have

$$M(z, Bz, t_0) \geq \varphi\left(\min\left\{M(z, Bz, t_0), M\left(Bz, z, \frac{2}{k}t_0\right)\right\}\right)$$
$$= \varphi\left(M(z, Bz, t_0)\right)$$
$$> M(z, Bz, t_0),$$

which is a contradiction. Therefore, we have $Bz = z = Tz$, which shows that z is a common fixed point of the pair (B,T). Therefore, z is a common fixed point of both the pairs (A,S) and (B,T). The uniqueness of common fixed point is an easy consequence of the inequality (2). This completes the proof.

Remark 5. Theorem 2 improves the result of Sedghi *et al*. (2010) for two pairs of self mappings without any requirement on containment of the ranges amongst the involved mappings.

The following examples illustrates Theorem 2.

Example 4. Let $(X,M,*)$ be a fuzzy metric space, where $X=[2,19)$, with t-norm $*$ is defined by $a*b=ab$ for all $a,b\in[0,1]$ and $M(x,y,t)=\dfrac{t}{t+|x-y|}$ for all $x,y\in X$ and $t>0$. Let $\phi:(0,1]\to(0,1]$ be a function defined by $\phi(t)=t^{\frac{1}{2}}$ and A,B,S,T be self-mappings defined by

$$A(x)=\begin{cases}2, & if\ x\in\{2\}\cup(3,19);\\ 15, & if\ x\in(2,3],\end{cases}\qquad B(x)=\begin{cases}2, & if\ x\in\{2\}\cup(3,19);\\ 12, & if\ x\in(2,3],\end{cases}$$

$$S(x)=\begin{cases}2, & if\ x=2;\\ 10, & if\ x\in(2,3];\\ \dfrac{x+1}{2}, & if\ x\in(3,19),\end{cases}\qquad T(x)=\begin{cases}2, & if\ x=2;\\ 11+x, & if\ x\in(2,3];\\ \dfrac{3x-1}{4}, & if\ x\in(3,19).\end{cases}$$

Taking $\{x_n\}=\left\{3+\dfrac{1}{n}\right\}$, $\{y_n\}=\{2\}$ or $\{x_n\}=\{2\}$, $\{y_n\}=\left\{3+\dfrac{1}{n}\right\}$, it is clear that both the pairs (A,S), (B,T) satisfy the common property (E.A) and

$$\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}By_n=\lim_{n\to\infty}Ty_n=2\in X.$$

It is noted that

$$A(X)=\{2,15\}\not\subset[2,14]=T(X),\quad B(X)=\{2,12\}\not\subset[2,10]=S(X).$$

On the other hand, $S(X)$ and $T(X)$ are closed subsets of X. Thus all the conditions of Theorem 2 are satisfied and 2 is a unique common fixed point of the pairs (A,S) and (B,T) which also remains a point of coincidence as well. Also, all the involved mappings are even discontinuous at their unique common fixed point 2.

Theorem 3. The conclusion of Theorem 2 remains true if the condition (2) of Theorem 1 is replaced by the following:

(2)' $\overline{A(X)} \subset T(X)$ and $\overline{B(X)} \subset S(X)$, where $\overline{A(X)}$ is the closure range of A and $\overline{B(X)}$ is the closure range of B.

Proof. Since the pairs (A,S) and (B,T) satisfy the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$. Then, since $z \in \overline{A(X)}$ and $\overline{A(X)} \subset T(X)$, there exists a point $v \in X$ such that $z = Tv$. By the proof of Theorem 3.1, we can show that the pair (B,T) has a coincidence point, call it v, i.e., $Bv = Tv$. Since $z \in \overline{B(X)}$ and $\overline{B(X)} \subset S(X)$, there exists a point $u \in X$ such that $z = Su$.

Similarly, we can also prove that the pair (A,S) has a coincidence point, call it u, i.e., $Au = Su$. The rest of the proof is on the lines of the proof of Theorem 2 and hence it is omitted. This completes the proof.

Corollary 1. The conclusions of above proved theorems remain true, if the condition (b) of Theorem 2 and the condition (2)' of Theorem 3 are replaced by the following:

(2)'' $A(X)$ and $B(X)$ are closed subsets of X if $A(X) \subset T(X)$ and $B(X) \subset S(X)$.

Theorem 4. Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Let A, B, S and T be mappings from X into itself and satisfying the conditions (1)-(4) of Lemma 2. Suppose that

(e) $S(X)$ (or $T(X)$ is a closed subset of X.

Then the pairs (A,S) and (B,T) have a coincidence point each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A,S) and (B,T) are weakly compatible.

Proof. In view of Lemma 2, the pairs (A,S) and (B,T) share the common property $(E.A)$, i.e., there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$$

for some $z \in X$. If $S(X)$ is a closed subset of X, then, from the lines of Theorem 2, we can show that the pair (A,S) has coincidence point, say u, i.e., $Au = Su = z$. Since $A(X) \subset T(X)$ and $Au \in A(X)$, there exists $v \in X$ such that $Au = Tv$. The rest of the proof runs on the lines of the proof of Theorem 2 and so details are omitted.

Remark 6. Theorem 2 extends the result of Sedghi, *et al*. (2010) for two pairs of self mappings. Theorem 4 is also a partial improvement of Theorem 2 besides relaxing the closedness of one of the subspaces.

Example 5. In the setting of Example 4, replace the self-mappings A, B, S and T by the following, besides retaining the rest:

$$A(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3,19); \\ 5, & \text{if } x \in (2,3], \end{cases} \qquad B(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3,19); \\ 7, & \text{if } x \in (2,3], \end{cases}$$

$$S(x) = \begin{cases} 2, & \text{if } x = 2; \\ 14, & \text{if } x \in (2,3]; \\ \dfrac{x+1}{2}, & \text{if } x \in (3,19), \end{cases} \qquad T(x) = \begin{cases} 2, & \text{if } x = 2; \\ 11+x, & \text{if } x \in (2,3]; \\ \dfrac{3x-1}{4}, & \text{if } x \in (3,19). \end{cases}$$

It is noted that

$$A(X) = \{2,5\} \subset [2,14] = T(X) \quad B(X) = \{2,7\} \subset [2,10) \cup \{14\} = S(X)$$

Also, the pairs (A,S) and (B,T) are commuting at 2, which is their common coincidence point. Thus all the conditions of Theorems 3, 4 and Corollary 1 are satisfied and 2 is a unique common fixed point of A, B, S and T. Here, it may be pointed out that Theorem 2 is not applicable to this example as $S(X)$ is not a closed subset of X. Also, notice that all the mappings in this example are even discontinuous at their unique common fixed point 2.

By choosing A, B, S and T suitably, we can drive a multitude of common fixed point theorems for a pair or triod of self mappings. If we take $A = B$ and $S = T$ in Theorem 2 then we get Theorem 1 due to Sedghi, *et al*. (2010). Our next theorem is proved for six self mappings in fuzzy metric space, which extends earlier proved Theorem 2.

Theorem 5. Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Let A, B, R, S, H and T be mappings from X into itself and satisfying the following conditions:

1. The pairs (*A, SR*) and (*B, TH*) share the common property (E.A),
2. *Sr(X)* and *TH (X*) are closed subsets of X ,
3. there exist $\phi \in \Phi$ and k , $1 \le k < 2$, such that

$$M(Ax,By,t) \geq \varphi\left(\min\left\{\begin{array}{c} M(SRx,THy,t), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t} \min\left\{\begin{array}{l} M(SRx,Ax,t_1), \\ M(THy,By,t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t} \max\left\{\begin{array}{l} M(SRx,By,t_3), \\ M(THy,Ax,t_4) \end{array}\right\} \end{array}\right\}\right) \quad (9)$$

for all $x, y \in X$ and $t > 0$. Then the pairs (*A, SR*) and (*B, TH*) have a coincidence point each other. Moreover, A, B, R, S, H and T have a unique common fixed point provided the pairs (*A, SR*) and (*B, TH*) are commuting pairwise (i.e., *AS* = *SA, AR* = *RA, SR* = *RS, BT, TB, BH* = *BH* = *HB , and TH* = *HT*).

Proof. Since the pairs (*A, SR*) and (*B, TH*) are commuting pairwise, obviously, both the pairs are weakly compatible. By Theorem 2, *A, B, SR* and *TH* have a unique common fixed point $z \in X$.

Now, we show that z is a unique common fixed point of the self mappings A, R and S . If $z \neq Rz$, then, by using (9) with $x = Rz$ and $y = z$, we get, for some $t_0 > 0$,

$$M(A(Rz),Bz,t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(SR(Rz),THz,t_0), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{c} M(SR(Rz),A(Rz),t_1), \\ M(THz,Bz,t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{c} M(SR(Rz),Bz,t_3) \\ M(THz,A(Rz),t_4), \end{array}\right\}, \end{array}\right\}\right)$$

$$M(Rz,z,t_0) \geq \varphi\left(\min\left\{\begin{array}{c} M(Rz,z,t_0), \\ \min\left\{M(Rz,Rz,\varepsilon), M\left(z,z,\frac{2}{k}t_0-\varepsilon\right)\right\}, \\ \max\left\{M(Rz,z,\varepsilon), M\left(z,Rz,\frac{2}{k}t_0-\varepsilon\right)\right\} \end{array}\right\}\right)$$

for all $\varepsilon \in \left(0, \frac{2}{k} t_0\right)$. As $\varepsilon \to 0$, we have

$$M(Rz, z, t_0) \geq \varphi\left(\min\left\{M(Rz, z, t_0), M\left(z, Rz, \frac{2}{k} t_0 - \varepsilon\right)\right\}\right)$$

$$= \varphi(M(Rz, z, t_0))$$

$$> M(Rz, z, t_0),$$

which is a contradiction. Therefore, $Rz = z$ and so $S(Rz) = S(z) = z$. Similarly, we get $Tz = Hz = z$. Hence z is a unique common fixed point of self mappings A, B, R, S, H and T in X. This completes the proof.

Corollary 2. Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Let $\{A_i\}_{i=1}^m, \{B_r\}_{r=1}^n, \{S_k\}_{k=1}^p$ and $\{T_g\}_{g=1}^q$ be four finite families of self mappings from X into itself such that $A = A_1 A_2 \ldots A_m, B = B_1 B_2 \ldots B_n, S = S_1 S_2 \ldots S_p$ and $T = T_1 T_2 \ldots T_q$ satisfying the inequality (2). If the pairs (A, S) and (B, T) share the common property (E.A) along with the closedness of $S(X)$ and $T(X)$, then (A, S) and (B, T) have a point of coincidence each other. Moreover, $\{A_i\}_{i=1}^m, \{B_r\}_{r=1}^n, \{S_k\}_{k=1}^p$ and $\{T_g\}_{g=1}^q$ have a unique common fixed point provided the pairs of families $(\{A_i\}, \{S_k\})$ and $(\{B_r\}, \{T_g\})$ are commuting pairwise, where $i \in \{1, 2, \ldots, m\}, k \in \{1, 2, \ldots, p\}, r \in \{1, 2, \ldots, n\}$ and $g \in \{1, 2, \ldots, q\}$.

Proof. The proof of this theorem is similar to that of Theorem 3.1 contained in Imdad, *et al*. (2009) and hence details are avoided.

Remark 7. Corollary 2 extends the result of Sedghi *et al*., (2010) to four finite families of self mappings.

By setting $A_1 = A_2 = \ldots = A_m = A$, $B_1 = B_2 = \ldots = B_n = B$, $S_1 = S_2 = \ldots = S_p = S$ and $T_1 = T_2 = \ldots = T_q = T$ in Corollary 2, we deduce the following:

Corollary 3. Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Let A, B, S and T be mappings from X into itself such that the pairs (A^m, S^p) and (B^n, T^q) share the common property (E.A). Then there exist $\phi \in \Phi$ and k, $1 \leq k < 2$, such that

$$M(A^m x, B^n y, t) \geq \varphi\left(\min\left\{\begin{array}{c} M(S^p x, T^q y, t), \\ \sup\limits_{t_1+t_2=\frac{2}{k}t} \min\left\{\begin{array}{l} M(S^p x, A^m x, t_1), \\ M(T^q y, B^n y, t_2) \end{array}\right\}, \\ \sup\limits_{t_3+t_4=\frac{2}{k}t} \max\left\{\begin{array}{l} M(S^p x, B^n y, t_3), \\ M(T^q y, A^m x, t_4) \end{array}\right\} \end{array}\right\}\right) \tag{10}$$

for all $x, y \in X$, $t > 0$ and m, n, p, q are fixed positive integers. If $S^p(X)$ and $T^q(X)$ are closed subsets of , then the pairs (A, S) and (B, T) have a point of coincidence each other. Further, A, B, S and T have a unique common fixed point provided both the pairs (A^m, S^p) and (B^n, T^q) are commuting pairwise.

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النقاط الصامدة للتطبيقات الضعيفة الانسجام في الفضاءات المترية المشوشة

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خلاصة

نقوم في هذا البحث بإثبات بعض مبرهنات النقطة الصامدة وذلك للتطبيقات الضعيفة الانسجام في الفضاءات المترية المشوشة ذات الخاصية المشتركة (EA). ثم نعطي بعد ذلك بعض الأمثلة لإيضاح نتائجنا. وكتطبيق لنتيجتنا الرئيسية نقوم بعرض مبرهنة شائعة لأربعة مجموعات منتهية من التطبيقات الذاتية في فضاءات مترية مشوشة وذلك باستخدام مفهوم التطبيقات الثنائية التبادل حسب عماد (2009). وتعتبر نتائجنا هنا امتداداً وتحسيناً لنتائج سدغي (2010).