

## Ion temperature gradient mode linear and nonlinear structures in electron-ion plasma, with stationary charged dust grain

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### Abstract

Ion temperature gradient mode nonlinear structures are studied with stationary charged dust. The dynamical role here is played by heavy ions. The small amplitude limit influence of various plasma parameters shows that the phase velocity of the mode modifies the overall plasma dynamics. Further, from the solution of the model equations we obtain Kortweg-de-Vries (KdV) and Korteweg-de-Vries Burger (KdVB) like equations, the solution of which gives solitary and shock wave nonlinear structures. Numerical analysis of these nonlinear structures shows that dust number density and polarity have a substantial impact on these structures. The present observation may be beneficial in space and laboratory plasma investigations.

**Keywords:** Electron-ion plasma; KdV and KdVB like equations; linear and nonlinear structures; stationary charged dust.

### 1. Introduction

Researchers have taken an interest in ion temperature gradient driven modes, linear and nonlinear structures, and the confinement of the plasma in devices for the last few decades. Poloidal flows and related radial electric fields are of wide importance in confinement and transport, as is the high temperature gradient in auxiliary devices. The ion temperature gradient (ITG), which is represented by  $\eta_i = d \ln T_i / d \ln n_i$  in the pellet injection, is discovered to be a critical parameter in the experiment. Here  $T_e$ ,  $T_i$ ,  $n_e$ , and  $n_i$  are the electron and ion temperatures and its number densities (Benz, 2002). Waves arise when there are variations in the density, temperature, and pressure of the species in the plasma which cause instabilities in the medium. The  $\eta_i$  effect occurs when the energy confinement time reaches a saturation point due to density. Many tokamaks have been found to feature ITG-induced electrostatic instabilities and drift waves. It is a good candidate for strange behaviour in an ion energy conduction (Deeba *et al.*, 2012). The contribution of non-adiabatic electrons is overlooked in the ITG model, which assumes that the ion temperature gradient is the main cause of turbulence. The  $\eta_e$ -mode of the electron temperature gradient (ETG) is due to the electron number density and its temperature gradients that cause microinstabilities in cross-field of the fluid. The  $\eta_e$  and  $\eta_i$  modes are both thought to be greater than collision-driven fluxes. The ion temperature gradient can cause a significant amount of ion energy loss when compared to the electron gyroradius  $\rho_e$  (Mamun & Shukla, 2002).

To analyse a small amplitude perturbation, the set of the magnetohydrodynamic equations can be linearized using Fourier analysis, and the dispersion relation for the ITG linear mode can be obtained by connecting these equations. That relationship can be used to determine the linear characteristics of plasma species such as frequency, phase velocity, and growth rate, among other things (Mamun, 2019). The researcher shows that fluid variables such as species temperatures, densities, and so on affect the linear behaviour of the plasma species (Deeba *et al.*, 2011). Nonlinear research is concerned with the structures that explain the plasma's nonlinear behavior (Gary & Tokar, 1985). The solitary wave was

introduced by John Scott Russell. In a fluid, these waves have a finite amplitude and a defined shape over a wide distance. The unbalance in the fluid species densities causes these structures. This is a sort of nonlinear instability in plasma that allows heat, mass, and momentum to be transported through the fluid (Malik *et al.*, 2008). The solution of a nonlinear partial differential equation, often known as the Korteweg de Vries equation, yields solitary waves. Solitary waves nonlinearly superpose with each other, and each soliton goes with its original shape after the interaction. Because of its particle behaviour, Zabusky and Kruskal termed the solitary wave the soliton (Singh & Malik, 2007).

Shocks are created when a wave in a fluid moves faster than the speed of sound. Shock waves, like other waves, are progressive waves that can convey energy from one point in a medium to another. In comparison to the solitary wave, shocks dissipate their energy more quickly (Rizzato, 1988). The nonlinear differential equation, also known as the Korteweg de Vries Burger equation, yields the shock wave solution. Shock waves' role in the generation of, e.g., cosmic rays during supernova explosions and strong field generation in bow shocks makes it a field of interest (Berezhiani *et al.*, 1992; Berezhiani & Mahajan, 1994).

Nonlinear waves have been studied in dusty multi-component plasma using large amplitude approximations e.g., vortices, solitons and shocks etc. Soliton which are solutions of the Korteweg-de-Vries (KdV) equation (Michel, 1982; Thorne *et al.*, 2000) may either be compressive or refractive type. These nonlinear structure have been studied both in homogeneous and inhomogeneous plasma. These studies reveals as reported in various studies that the speed, amplitude and width modify, which either increase/decrease the transportation property of these nonlinear structures (Hoshino *et al.*, 1992; Murad *et al.*, 2021; Zakir U & ul Haque, 2022). Ion temperature gradient which was first reported by Rudakov and later studied by number of researchers is considered as a main character in the confinement devices such as in tokamak (Khan, Zakir, ul Haque & Qamar, 2021; Khan *et al.*, 2020; Khan, Ullah & Haque, 2021; Khan *et al.*, 2022, 2023).

To the best of our knowledge, have reported the study of ITG mode driven solitary and shock waves with stationary charged dust in magnetoplasma. These investigations are important because immersion of charged dust modify the existing plasma modes, resulting new effects for example confinements issues. This article is organized as: § 1: a short introduction to the background of the importance of this study, § 2: model and basic formalism to the present study, § 3: linear (small amplitude) analysis, § 4: nonlinear (large amplitude) analysis, § 5: numerical analysis and finally in § 6: the concluding remarks.

## 2. Formalism

Assuming a plasma made of electron, ion and dust in which electrons are subjected to the well known Maxwellian distribution, dust is stationary and positively charged and ions are dynamic. Further, to investigate the impartial role of the dust charged state in present case, negative polarity dust is also examined. Also, we assume that the temperature and density perturbations are along the  $x$ -directions i.e.,  $T_i(x)$  and  $n_i(x)$ . To get a valid result of the proposed model we make use of the hydrodynamic equations for ion species in electron-ion-dust (e-i-d) plasma for which the momentum equation given by:

$$(\partial t + \mathbf{v}_i \cdot \nabla) \mathbf{v}_{iz} = -\frac{eE_z}{m_i} - \frac{\nabla p_i}{m_i n_i}, \quad (1)$$

where  $E_z = -\nabla \phi$  the electric field, and  $\phi$  denotes the electric potential in the plasma. By expanding the ion numbered density and temperature to the first order, as  $n_i = n_{i0} + n_{i1}$ ,  $T_i = T_{i0} + T_{i1}$ , where  $n_{i1} \ll n_{i0}$  and  $T_{i0} \ll T_{i1}$ , respectively, as we've mentioned  $n_0$  and  $T_0$ , in which, "0" refers to unperturbed values, while "1" refers to perturbed quantities.  $\omega \ll \omega_{ci}$  (where  $\omega_{ci}$  is the ion cyclotron frequency). As the ion velocity as the superposition of numerous drifts, i.e.,

$$\mathbf{v}_i = v_{EB} \hat{z} + v_{Di} + v_{iz} \hat{z} \quad (2)$$

where  $v_{EB} = e/B_0 (\hat{z} \times \nabla \phi)$  is the  $E \times B$  drift,  $v_{Di} = e/B_0 n_i (\hat{z} \times \nabla p_i)$ , is the diamagnetic drift and  $v_{pi} = -e/B_0 \omega_{ci} (\partial_t + \mathbf{v}_i \cdot \nabla) \hat{z} \times \mathbf{v}_i$ , is the ion polarization drift. The different symbol  $\phi_i$  and  $p_i$  used in these equations referred to the electrostatic potential and pressure, where  $v_{iz}$  is the ion fluid velocity

along the z-axis. Here  $p_i = n_i T_i$ , where  $n_i, T_i$  represent the number density and temperature of the ion. The continuity equation for the ion species of the mode is

$$\partial_t n_i + \nabla \cdot (n_i \mathbf{v}_i) = 0, \quad (3)$$

and energy balancing equation of the ion species is

$$\frac{3}{2} (\partial_t + \mathbf{v}_i \cdot \nabla) T_i + T_i (\nabla \cdot \mathbf{n}_i) = \frac{1}{n_i} \nabla \cdot [(5cT_i/2eB_0)\hat{z} \times \nabla T_i] \quad (4)$$

where  $1/n_i \nabla \cdot [(5cT_i/2eB_0)\hat{z} \times \nabla T_i]$  is the Righi-Leduc heat flux term in the energy balancing equation for the ion. Now the first order perturbed form of the quasi-neutrality condition is written as

$$n_{i1} = n_{e1} \mp n_{d1} \quad (5)$$

In equation (5), the minus sign is for the positive dust grains and vice versa. To write equations. (1)-(4) in the drift approximation, i.e the continuity equation as

$$D_t^i N_1 + \tau_1 (\mathbf{v}_{ni} \cdot \nabla \Phi) - \frac{1}{2} \rho_i^2 \tau_1^{-1} \partial_t \nabla^2 (\Phi + T_1 + N_1) + \partial_z v_{iz} = 0 \quad (6)$$

Different terms used in equation (6) are  $D_t^i = (\partial_t + \mathbf{v}_E \cdot \nabla)$ ,  $\mathbf{v}_{ni} = (T_{i0}/eB_0 \nabla \ln n_{i0} \times \hat{z})$ ,  $\tau_1 = T_{e0}/T_{i0}$ ,  $\Phi = e\phi/T_{e0}$ ,  $T_1 = T_{i1}/T_{i0}$  and  $N_1 = n_{i1}/n_i$  are the doppler shift in the time rate, ion number density drift, normalized electron to ion temperature ratio and normalized plasma potential. The ion momentum equation for the ion dynamics as

$$D_t^i v_{iz} = -c_s^2 \partial_z \phi - \tau_1^{-1} c_s^2 \partial_z (T_1 + N - 1) - \mathbf{v}_{iz} \partial_z v_{iz}, \quad (7)$$

while the energy balancing equation for the ion species in the e-i-d plasma is

$$D_t^i T_1 = \frac{2}{3} D_t^i N_1 + \tau_1 \left( \eta_i - \frac{2}{3} \right) \mathbf{v}_{ni} \cdot \nabla \Phi. \quad (8)$$

Because electrons in our calculations follow the Maxwellian distribution, their number density will be expressed as  $n_e = n_{e0} \frac{e\phi}{T_e}$ , where  $n_e$  represents the total electron number density and  $n_{e0}$  represents the electron number density at the furthest location from the perturbation center. We use Fourier analysis to obtain the simplified version of equations. (6)–(8) due to the small amplitude of the perturbation. All variables in the plasma are supposed to change exponentially as  $\exp(\iota k_y y + \iota k_z z - \omega t)$ , where  $k_y$  and  $k_z$  are wave numbers along the  $y$  and  $z$ -axes, respectively, and  $\omega$  is the wave frequency. The linearized form of the continuity equation is as follows:

$$\omega N_1 - \tau_1 \omega_{ni} \Phi + \frac{1}{2} \tau_1^{-1} \omega k^2 \Phi + \frac{1}{2} \rho_i^2 \tau_1^{-1} \omega k^2 (N_1 + T_1) - K_z v_{iz} = 0, \quad (9)$$

and the momentum equation as

$$v_{iz} = \frac{c_s^2 k_z}{\omega} (\Phi + \tau_1^{-1} (N_1 + T_1)), \quad (10)$$

while the energy balancing equation in its linear form is

$$T_1 = \frac{2}{3} N_1 - \tau \frac{(\eta_i - \frac{2}{3})}{\omega} \omega_{ni} \Phi. \quad (11)$$

Where  $\omega_{ni}$  is the frequency generated due to the ion number density drift. The Normalized ions density expression for the e-i-d plasma as

$$N_1 = A_1 \Phi. \quad (12)$$

Here  $A_1 = (n_{e0}/n_{i0} \pm z_d^2 n_{d0} T_e / m_d n_{i0})$ . To get the linear root and its different features that show the dynamics of the linear properties of the plasma, the plus sign is for the positive charge of the dust, and the minus sign is for the negative charge of the dust particles.

### 3. Linear analysis

To get linear dispersion relation we couple equations (9)-(12) along with equation (12) and get a cubic root equation which is given by:

$$\begin{aligned} \omega^3 A_1 + \frac{1}{2} \rho_i^2 \tau_1^{-1} k^2 \left( 1 + \frac{5}{3} A_1 \right) - \tau_1 + \frac{1}{2} \rho_i^2 k^2 \left( \eta_i - \frac{2}{3} \right) \omega_{ni} \\ - cs^2 k_z^2 \left( 1 + \frac{5}{3} \tau_1^{-1} A_1 \right) \omega + cs^2 k_z^2 \left( \eta_i - \frac{2}{3} \right) \omega_{ni} = 0. \end{aligned} \quad (13)$$

By neglecting the larmor radius effect and taking  $\eta_i > 1$  we can write equation. (13) as

$$\omega^3 A_1 - \tau_1 \omega_{ni} - cs^2 k_z^2 \left( 1 + \frac{5}{3} \tau_1^{-1} A_1 \right) \omega + cs^2 k_z^2 \eta_i \omega_{ni} = 0. \quad (14)$$

We can see that plasma characteristics like dust number density, dust charge number, dust mass, and other well-known parameters like  $\eta_i$ , temperature, and the density of the different species in the plasma all have an impact on  $A_1$ . As a result, for the studied model of e-i-d plasma, this is a completely different dispersion relation in which dust particles, whether stationary positively charged or static negatively charged, have an impact on the linear properties of the e-i-d magnetoplasma in both cases. We'll go on to the following part to acquire the nonlinear ITG-driven mode.

### 4. Nonlinear analysis

#### 4.1 Solitons

In this part, we shall derive a nonlinear equation in the form of Kortewage-de-Varies (KdV), with a solitary wave potential as the solution. To simplify our computation, we write equations (6)-(8) in new coordinates (the new coordinate system is  $\xi = y + \alpha z - ut$ , where  $\alpha$  and  $u$  are the smallest angles with the  $z$ -axis and wave velocity, respectively). We'll write the equation of continuity in these new coordinates as

$$\partial_\xi N_1 - \frac{\tau_1}{u} \mathbf{V}_{ni} \partial_\xi \Phi - \frac{1}{2} \rho_i^2 \tau_1^{-1} \partial_\xi^3 (\Phi + N_1 + T_1) - \frac{\alpha}{u} \partial_\xi V_{iz} = 0. \quad (15)$$

The ion momentum equation as

$$V_{iz} = \frac{cs^2 \alpha}{u} \left[ \Phi + \tau_1^{-1} (N_1 + T_1) - \frac{1}{2} \frac{cs^2 \alpha^2}{u^2} \Phi + \tau_1^{-1} (N_1 + T_1)^2 \right] \quad (16)$$

while the energy balancing equation for the ion species in the fluid is

$$T_1 = B_1 \Phi \quad (17)$$

where the coefficient in equation (17) is given as  $B_1 = 2/3M - \tau/u (\eta_i - 2/3) v_{ni}$ . By combining equations (16)-(17), we can get the nonlinear partial differential equation as

$$M_1 \partial_\xi \Phi + M_2 \Phi \partial_\xi \Phi + M_3 \partial_\xi^3 \Phi = 0 \quad (18)$$

where the coefficients in the equation (19) are

$$M_1 = M - \frac{\tau v_{ni}}{u} - \frac{cs^2 \alpha^2}{u^2} 1 + \tau_1^{-1} (M + B_1)$$

$$M_2 = \frac{cs^4 \alpha^4}{u^4} 1 + \tau_1^{-1} (M + B_1)^2$$

$$M_3 = \frac{1}{2} \rho_i^2 \tau_1^{-1} (1 + M + B_1)$$

The nonlinear partial differential principal coefficients are  $M_2$  and  $M_3$ ; the first is called the nonlinear coefficient, while the second is called the dispersion coefficient. These coefficients reveal that plasma properties, notably dust concentration and charge polarity, modify nonlinear structures.

$$\Phi + G_1 \Phi \partial_\xi \Phi + H_1 \partial_\xi^3 \Phi = 0 \quad (19)$$

Here  $G_1 = M_2/M_1$  and  $H_1 = M_3/M_1$  are now the compact form of the nonlinear and dispersion coefficients. By simple integration we can get the solution of the equation (20) in the form of solitary wave potential

$$\Phi = \Phi_0 \sec h \left( \frac{\xi}{w} \right)^2 \quad (20)$$

$\Phi_0 = 3u/G_1$  is the amplitude of the solitary wave potential, and  $w = \sqrt{4H_1/u}$  is the normalisation factor for the phase of the soliton. First, we note that the amplitude of the soliton is dependent on the nonlinear coefficient and phase speed of the waves, i.e., we may create both slow and rapid-speed solitary waves in the ITG nonlinear mode of electron-ion-dust plasma. Second, both the dispersion coefficient and the phase speed of the solitary wave affect the wave's phase.

#### 4.2 Shocks

It is one of the nonlinear structures in fluids where the perturbed amplitude is large and the medium dissipation is in the form of neutral and charged ion collisions as  $V_n u_i$ . The collision drift caused by this term is  $V_c = c/B_0 \omega_{ci} \nabla \Phi$  in the momentum equation. This term is added to the ion momentum equation to get the ions' dynamics in the form of a nonlinear structure (shock) and explains its nonlinear properties. We can derive the KdV-Burger equation in a co-moving coordinate system by combining equations (15) and (17) with the new momentum equation.

$$\partial_\xi \Phi + G_1 \Phi \partial_\xi \Phi - J_1 \partial_\xi^2 \Phi = 0 \quad (21)$$

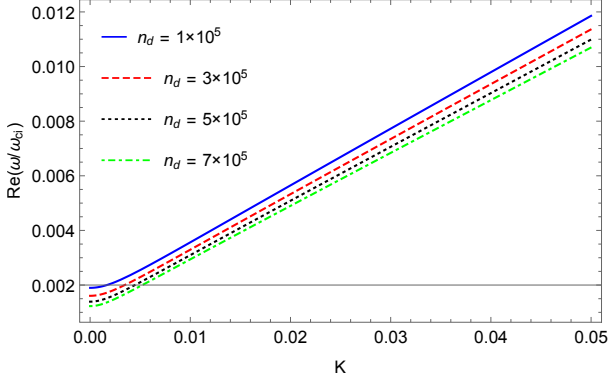
where  $J_1 = \rho_i^2 V_n \alpha / u$ , here we can observe the dissipation coefficient depends on the collision frequency which is the result of the neutral ion collision in the plasma and viscosity of the medium. Here we can observe also two type of the shock wave i.e the compressive shock and refractive shock depends on the magnitude and sign of the nonlinear and dissipation coefficients. Equation (21) denotes Burger equation which has solution of the form

$$\Phi = \frac{2J_1}{G_1} [1 - \tanh \xi], \quad (22)$$

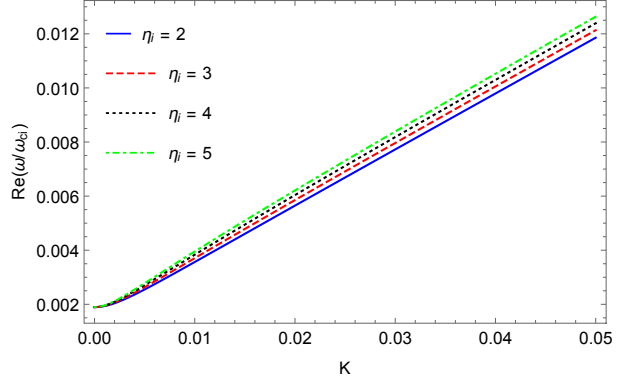
where  $G_1 = M_3/M_1$ . The profile of the shock wave strictly depends on the sign and magnitude of  $J_1/G_1$  ratio. Looking at the values of  $G_1$ ,  $M_1$  and  $M_3$  one may observe the effect of dust polarity and its concentration on the shock structures.

### 5. Results and Discussion

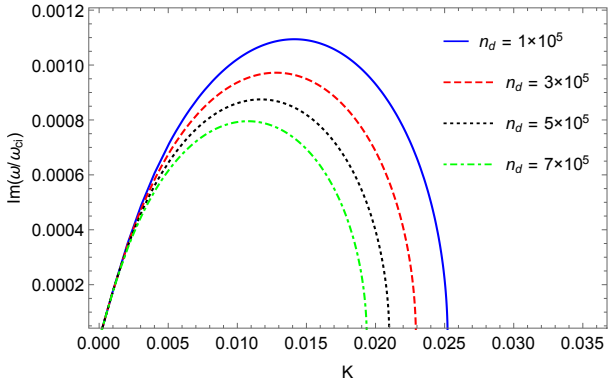
We plotted some of the ITG mode roots and nonlinear structures against the other physical factors to present a clear image of what is happening in the atmosphere of the present model. For numerical analysis, we used data from (Weiland, 2000), where some of the parameters are:  $n_i = 10^8$ ,  $n_e = 10^9$ ,  $c = 3 \times 10^{10}$ ,  $e = 4.8 \times 10^{-10}$ ,  $m_d = 10^{-6}$ ,  $m_i = 1.67 \times 10^{-24}$  and  $m_e = 9.11 \times 10^{-28}$ . In figure 1, the phase velocity of the ITG linear mode is shown against the wave number  $k$  for various dust density. When comparing the slopes of each plot in the same figure, the phase velocity changes slightly with the dust grain densities. The phase velocity decreases with dust concentrations. As a result, linear waves in electron-ion plasma with dust grains transmitted information at a slower rate than linear waves in plasma without dust particles. In figure 2, we show one of the real roots for the ITG mode versus  $k$  with different values of the ion temperature gradient  $\eta_i$ . As the dust is positively charged, the phase velocity of the ITG mode rises as  $\eta_i$  value increases, that is shown in the graph. In figure 3, we displayed one of the imaginary roots for the ITG mode against the wave number  $k$  with various values of dust number density while keeping the other plasma parameters constant as stated in the text. According to the graph, the growth rate of the ITG mode decreases with dust density. A detailed examination of the graph reveals that the growth rate increases as ion temperature gradient coefficient  $\eta_i$  increases. We demonstrated solitary wave potential for ion temperature gradient (ITG) nonlinear mode as opposed to  $\xi$  in figure 5, for various values of dust number density. The graph illustrates that when dust number



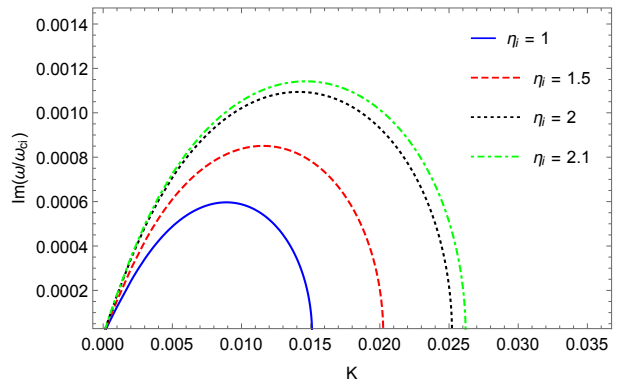
**Fig. 1.** The plot of real root against wave number  $k$  for positive dust based on Eq. (14), keep constant all the parameters as mention in the text while changing the dust number density.



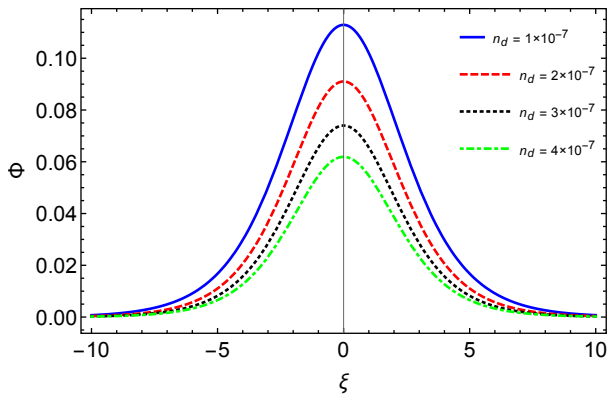
**Fig. 2.** Real root against  $k$  with positively charge stationary dust for different value of  $\eta_i$ , keep other plasma parameters constant as given in the text.



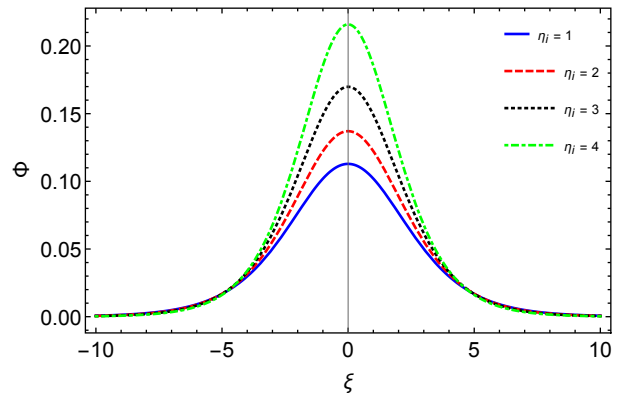
**Fig. 3.** Imaginary root against  $k$  with positively charge stationary dust for different value of dust number density, keep other plasma parameters constant as given in the text.



**Fig. 4.** Imaginary root against  $k$  with positively charge stationary dust for different value of  $\eta_i$ , keep other plasma parameters constant as given in the text.

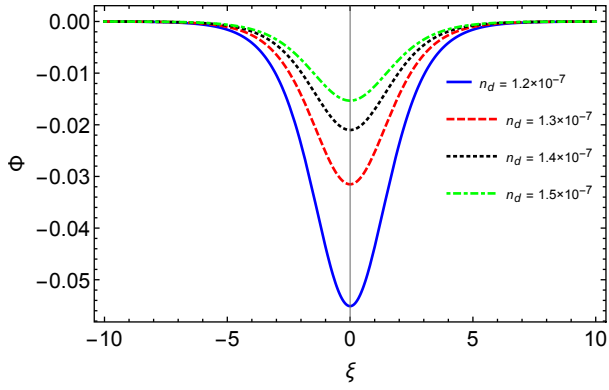


**Fig. 5.** Solitary potential waves against  $\xi$  with positively charge stationary dust for different value of dust number density, keep other plasma parameters constant as given in the text.

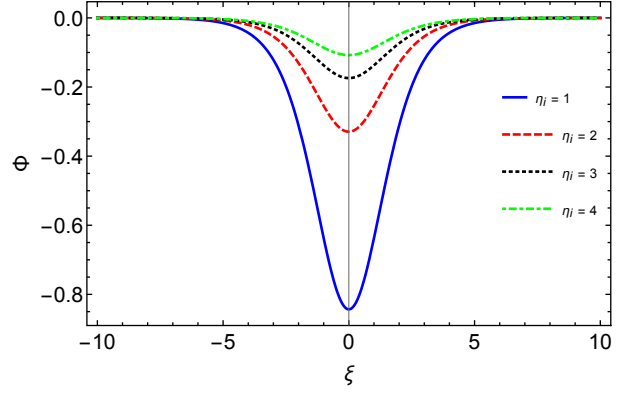


**Fig. 6.** Solitary potential waves against  $\xi$  with positively charge stationary dust for different value of  $\eta_i$ , keep other plasma parameters constant as given in the text.

density increases, soliton amplitude decreases and the wave's dispersion property changes somewhat owing to dust concentration in the electron-ion-dust plasma. In figure 6, we plotted ITG nonlinear mode

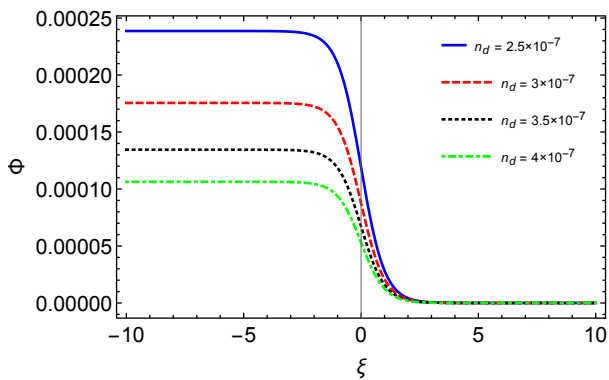


**Fig. 7.** Solitary potential waves against  $\xi$  with negatively charge stationary dust for different value of dust number density , keep other plasma parameters constant as given in the text.

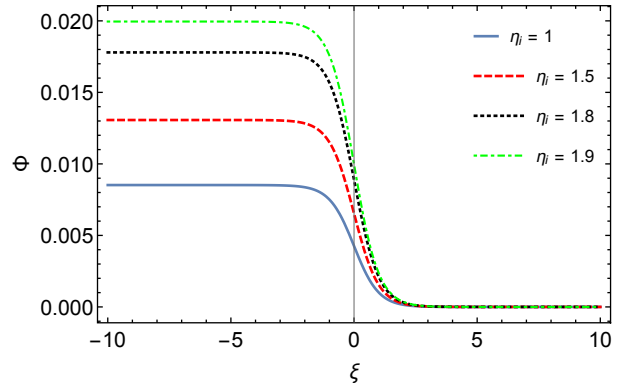


**Fig. 8.** Solitary potential waves against  $\xi$  with negatively charge stationary dust for different value of  $\eta_i$  , keep other plasma parameters constant as given in the text.

solitary wave potentials versus  $\xi$  for various values of  $\eta_i$ , and we can see that the soliton amplitude increases as  $\eta_i$  increases. In figure 7, we show the solitary wave potential for the ITG nonlinear mode as opposed to  $\xi$  for various values of the dust number density with negatively charged dust consideration. When dust is taken negative, we notice a rarefactive type of solitary wave and soliton amplitude falloff with dust concentration, as seen in the same plot. Ion temperature gradient coefficient  $\eta_i$  lowers the amplitude of rarefactive type solitary waves in figure 8 and also affects the dispersion properties of the waves. We plotted shock wave potential versus  $\xi$  in figure 9, for various values of positively charged dust density. The shock amplitude decreases as the dust density increases, demonstrating that positive dust works as a shock absorber in the plasma medium. The shock wave potential for the ion temperature gradient nonlinear mode against the phase  $\xi$  for various values of  $\eta_i$  is shown in figure 10. The shock amplitude grows as  $\eta_i$  increases, destabilising the nonlinear mode of the e-i-d plasma and generating greater dissipation in the plasma medium, leading the shock wave to expand in amplitude.

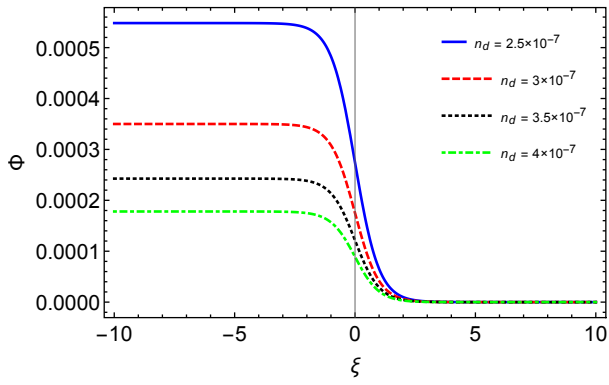


**Fig. 9.** Shock wave potential against  $\xi$  with positively charge stationary dust for different value of dust number density , keep other plasma parameters constant as given in the text.

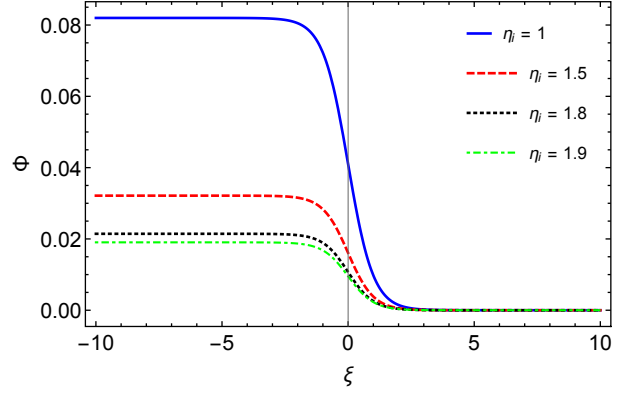


**Fig. 10.** Shock wave potential against  $\xi$  with positively charge stationary dust for different value of  $\eta_i$  , keep other plasma parameters constant as given in the text.

We plotted the shock wave potential for the ITG nonlinear mode against the  $\xi$  for various values of the dust number density. The dust is negatively charged in figure 11. Aging, this plot gives the same result of decreasing shock amplitude, but the amplitude decrease rate is smaller than that for the positive dust grain consideration. Figure 12 depicts the effect of  $\eta_i$  on shock, which shows a different effect of shock wave amplitude than that shown in figure 10, indicating that the negative polarity of the dust can change the entire physics in the plasma medium. The following linear findings can be used to understand



**Fig. 11.** Shock wave potential against  $\xi$  with negatively charge stationary dust for different value of dust number density , keep other plasma parameters constant as given in the text.



**Fig. 12.** Shock wave potential against  $\xi$  with negatively charge stationary dust for different value of  $\eta_i$  , keep other plasma parameters constant as given in the text.

the behaviour of plasma as: 1. Dust density, since dust is present in both the laboratory and the stellar and interstellar environments. 2. the polarity of the dust, or whether it is positively or negatively charged It may have an impact on the plasma's linear properties in either scenario. 3. As one of the fictitious roots of the linear dispersion is displayed, dust concentration can alter the rate at which the plasma grows. 4. The ion temperature gradient coefficient has an impact on phase velocity and growth rate.

The structures that account for the nonlinear behaviour of the plasma are the focus of nonlinear study. The following features we have obtained from the nonlinear study: 1. The soliton's amplitude, width, and energy all decrease. It implies that a solitary wave's spikiness can be reduced by the charge dust concentration. 2. that the wave's dispersion feature likewise reduces as the wave's strength does. 3. Also, the rarefactive solitary wave's amplitude drops for negatively charged dust grains, along with its energy and width. 5. The intensity of the compressive soliton is highest, i.e., for positively charged dust, as contrasted to negatively charged dust. 6. Similarly, the nonlinear wave shock is also affected by the dust grain concentration as well as by its polarity. 7. These structures are also affected by the ion temperature gradient coefficient.

Due to the reflection of ions and positively charged dust particles from the electrostatic field created inside the plasma, a decreasing soliton amplitude may therefore be linked to a decreasing soliton energy. The wave particle energy exchange mechanism, whose calculations are possible from kinetic theory but beyond the domain of the present research, may also be a cause of loss in the soliton amplitude. Here, the nonlinear structure of the inhomogeneous plasma has been explored. These studied characteristics can shed more light on the movement of mass, energy, and momentum of dynamical species in fluids. This could explain the confinement of the species in the tokamak plasma for the thermonuclear reaction. Our findings demonstrate that variations in speed, amplitude, and width have an impact on the transportation properties of these nonlinear structures. We can apply these theoretical approaches to the space plasma like the stellar, interstellar spaces, to the planetary spaces etc., where the electron-ion plasma is incorporated by the dust grain.

## 6. Conclusion

We investigated electron-ion plasma in the present study by including the stationary charged dust grain concentration, where electrons have the lightest inertia and can follow a Maxwellian distribution in ion temperature gradient driven solitons and shocks. Plasma ion particles are heavy and considered dynamic. Our theoretical calculation was based on one of the roots of the ITG mode dispersion relation, which characterised the fluid's linear features and their influence due to various plasma parameters such as ion temperature, density, the ion temperature gradient coefficient,  $\eta_i$ , dust number density, polarity, and so on. In small amplitude limits, we found that the linear properties of the mode in the presence of stationary and charged dust particles in electron-ion plasma can alter the plasma linear behavior. Step-



ping towards the large amplitude limit, we solved the mode equations and derived the Korteweg-de-Vries equation, which is solved using a simple algebraic approach. This result reveals a significant modification of the soliton structure owing to the increased dust number density in the electron-ion magneto-plasma. An important role is noted when the dust polarity is changed from positive to negative, giving the formation of the compressive type of solitons as shown numerically. The other type of nonlinear structure known as shock in the electron-ion plasma with the additional component dust is found to be modified due to charge polarity and density. That finding revealed that dust in plasma may alter the physical environment, resulting in the appearance of distinct qualities in the same waves because dust is the main component in laboratory and astrophysical plasma.

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