Modulational Stability Analysis of Ion Temperature Gradient Mode in Electron-ion Plasma

K. Aziz*[,1,2], U. Zakir[1], Q. Haque[3], I. Haq[2], F. Khan, [2]
[1]*Dept. of Physics, University of Malakand, Malakand, 18000, Pakistan*
[2]*Dept. of Physics, Government Degree College, Swat, 18000, KP, Pakistan*
[3]*Theoretical Plasma Physics Division PINSTECH, P.O. Nilore, Islamabad, 610101, Pakistan*
**Corresponding author: azizkhanphysics@gmail.com*

Abstract

A theoretical investigation of the modulational stability of the ion temperature gradient (ITG) mode in electron–ion plasma is presented. To examine linear features of dynamic species in plasma, we used reduction perturbation to solve Braginskii's transportation equations and get phase and group velocities. The results reveal that ion species velocities are affected by plasma factors such as ion temperature, density, and ion temperature gradient coefficient, among others. We also find a nonlinear Schrodinger equation. This equation shows that the plasma dynamics depends on the coefficients of nonlinearity and dissipation of the nonlinear Schrodinger equation. These coefficients are strongly related to the plasma variables. The present investigation may be helpful in space and laboratory plasma, e.g., fusion confinement devices.

Keywords: Electron-ion plasma; ion temperature gradient mode; reduction perturbation; stability analysis.

1. Introduction

Plasmas are multi-component fluids that are characterised by space and time scales. Understanding turbulence is one of the most complicated issues for theoretical plasma physicists. Turbulent motion may be found in a number of plasmas such as in the lab and in space and so on. In recent years, a sufficiently large number of studies have focused on a specific class of plasma dynamic models in which the ion temperature gradient (ITG) plays a significant role, such as in confinement devices where the temperature gradient is larger than the density gradient, resulting in serious diffusion and thermal leakage (Ruderman & Sutherland, 1975). The ion temperature gradient mode is described as a kind of plasma turbulence created and sustained by an ion temperature gradient coefficient i.e., $\eta_i = L_n/L_T$, where $L_T = 1/(\partial_x$ ln $T_{i0}(x))$ and $L_n = 1/(\partial_x ln n_{i0}(x))$ are the ion temperature and density scale lengths (Rogister *et al.*, 1988), respectively. Rudakov and Sagdeev (Rudakov & Sagdeev, 1961) initially established such a mode in slab geometry and it was further extended by incorporating nonuniform density with a shear magnetic field, an external magnetic field, and pressure effect (Coppi *et al.*, 1967) among other things. Using the trapped electron mode (Hahm & Tang, 1989), a heat flow effect is introduced in the same mode. Shukla and Weiland used an ion temperature gradient mode to generate nonlinear structures in the form of dipolar vortices (Shukla, 1990; Weiland, 2000). The electron temperature gradient mode denoted as "η_e" is the counterpart of the ion temperature gradient and accounts for micro instabilities (Strintzi & Jenko, 2007).

Relatively larger amplitude waves in plasma such as vortices, solitons, and shocks are significant because they play a major role in the movement of heat, mass, and momentum (Temerin *et al.*, 1982; Block & Fälthammar, 1990; Nielsen *et al.*, 1996). A number of writers investigated the aforementioned structures under various situations. This sort of plasma is thought to exist both in space and in laboratories (Ginzburg, 1971; Manchester & Taylor, 1977; Michel, 1982). In the presence of ions, several low frequency waves (Begelman *et al.*, 1984; Helander & Ward, 2003; Liang *et al.*, 1998; Gahn *et al.*, 2000) can be generated, which are important not only from a cosmological and astrophysical perspective, but also in the context of laboratory plasma (Lominadze *et al.*, 1982; Nejoh, 1996; Mahmood & Akhtar, 2008). The modulational instabilities of different wave modes in plasma have received a lot of interest in recent years due to their importance in stable wave propagation (Dubinov & Sazonkin, 2009; Salahuddin *et al.*, 2002; Kourakis it et al., 2006). Some of the authors discussed the modulational instability in electron-positron–ion plasma as well as pair ion plasma (Esfandyari-Kalejahi, Kourakis, Mehdipoor & Shukla, 2006; Esfandyari-Kalejahi, Kourakis & Shukla, 2006). The authors of (Khan *et al.*, 2020; Khan, Ullah & Haque, 2021) investigated the ion temperature gradient (ITG) mode driven soliton and shock wave structure in an electron-ion plasma. Further they extended their findings by including the heat flux effect and ion entropy in the energy balance equation of the mode (Khan, Zakir, ul Haque & Qamar, 2021; Zakir U & ul Haque, 2022). Linear and nonlinear ion temperature gradient small amplitude structures were studied by (Khan, Zakir, Rahman, Ali & Haque, 2021) in electron-ion magnetized plasma with stationary charged dust grains. Ion temperature gradient driven mode under the pressure plays a role in the formation of nonlinearity in the plasma (Murad *et al.*, 2021). Based on the importance of ion temperature gradient modes in plasma confinement devices, the nonlinear component in the momentum equation generate soliton like structures with the smaller frequency limits, these nonlinear structures were studied by (Horton *et al.*, 2003; Khan *et al.*, 2022). No one has yet investigated the stability analysis for the same mode and frequency. We conducted the first ever stability analysis of the ITG mode in electron-ion plasma. The study can explain some of the instability that occurs in the tokomak or plasma confinement devices. The manuscript of our study is divided into the following sections: In section 2, we discuss the model and the basic model equations. In section 3, we apply the reductive perturbation technique and obtain an expression of the phase velocity of the mode. The derivation of the nonlinear Schrodinger equation is discussed in section 4. The stability of the present mode is presented through simulation in section 5, and summary of the present work is discussed in section 6.

2. Theoretical model

We consider the simplest electron-ion plasma, where the electron is taken to be inertialess because of its smaller mass as compared to the ion species that are dynamic. Further, electrons are subjected to obey Maxwellian distribution. The variation in the temperature of ions and its number density along x-axis i.e., $T_i(x)$ and $n_i(x)$ and low frequency of the ITG mode $\partial_t \ll \omega_{ci}$ where $\omega_{ci} = eB_0/m_i c$ is the ion cyclotron frequency, e is the charge on the ion, B_0 is the background magnetic field to the plasma, m_i is mass for ion species and c is the speed of light. To set the mode dispersion relation we used continuity, momentum, energy balance and Poisson's equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + e\frac{\partial \psi}{\partial x} + \tau_0 \frac{\partial (T+n)}{\partial x} = 0. \tag{1}$$

The continuity equation for the ion species is

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0, \tag{2}$$

and the energy balancing equation is

$$\frac{\partial T}{\partial t} - \frac{2}{3}\frac{\partial n}{\partial t} - \sigma_1 \frac{\partial \psi}{\partial x} = 0, \tag{3}$$

with $\sigma_1 = \tau_0(\eta_i - 2/3)$. Finally, we apply Poission's equation to get potential which always to be there due to quasineutrality of the plasma

$$\frac{\partial^2 \psi}{\partial x^2} = (n_e - n) \tag{4}$$

where n_e is the electron number density, n_i is the ion number density in the electron-ion plasma, and ψ refer to the potential of plasma. Since we considered Maxwellian distribution for our model, thus we can write the density expression as

$$n_e = n_{e0} \exp\left(\frac{e\psi}{T_e}\right). \tag{5}$$

At this point, it should be mentioned that in equations. (1)-(4), the normalized quantities are $T = T_{i1}/T_{i0}$, $n = n_{i1}/n_{i0}$ and $\tau_0 = T_{e0}/T_{i0}$, while T_{i0}, T_{e0}, n_{i0} are the unperturbed ion, electron temperature and ion number density where as T_{i1}, n_{i1} are the ion perturbed temperature and ion number density, ψ is the potential due to the perturbation in plasma. Equations (1)-(4) can be coupled with equation (5) to get the deserved result for the stability and instability of the ion temperature gradient modulational mode in electron-ion magneto plasma.

3. Reductive perturbation method

To obtain a nonlinear partial differential Schrodinger equation and to investigate the modulational stability of the ion-temperature gradient mode, we can follow any method but one method which is simplest and gives a reliable result is the technique of reductive perturbation method. To simplify our calculation we considered the co-moving coordinates as used in (Washimi & Taniuti, 1966), like

$$\begin{pmatrix} \xi \\ \tau \end{pmatrix} = \epsilon \begin{pmatrix} x - v_0 t \\ \epsilon t \end{pmatrix}, \tag{6}$$

where ϵ is a small positive parameter $(0 < \epsilon < 1)$ that gives the weakness of the amplitude and v_0 phase speed of the ion temperature gradient mode in electron-ion plasma. In above stretch, the wave number is taken of the order of smallness parameter and the different variable in the plasma system that we have considered can be taken in the power series. One can apply power expansion to expand the variables n, v, T and ψ about the unperturbed state as:

$$\begin{pmatrix} n \\ v \\ T \\ \psi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \sum_{j=1}^{\infty} \epsilon^j \sum_{l=1}^{\infty} \begin{pmatrix} n_l^{(j)}(\xi,\tau) \\ v_l^{(j)}(\xi,\tau) \\ T_l^{(j)}(\xi,\tau) \\ \psi_l^{(j)}(\xi,\tau) \end{pmatrix} e^{\iota(kr-\omega t)l}, \tag{7}$$

here $(l, j = 0, 1, 2, 3, 4, ...)$ j refers to the order while l gives the harmonics of the corresponding variables in the power series. In terms of stretching coordinates and smallness parameters the given plasma magnetohydrodynamic equations will be given as:

$$\frac{\partial v}{\partial t} - \epsilon v_0 \frac{\partial v}{\partial \xi} + \epsilon^2 \frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial x} + \epsilon v \frac{\partial v}{\partial \xi} = -\frac{\partial \psi}{\partial x} - \epsilon \frac{\partial \psi}{\partial \xi} - \tau^{-1}(T+n). \tag{8}$$

Equation (8) is the ion momentum equation for the plasma. Now the continuity equation is

$$\frac{\partial n}{\partial t} - \epsilon v_0 \frac{\partial n}{\partial \xi} + \epsilon^2 \frac{\partial n}{\partial \tau} - \frac{2}{3}\frac{\partial n}{\partial t} + \frac{2}{3}\epsilon v_0 \frac{\partial n}{\partial \xi} - \frac{2}{3}\epsilon^2 \frac{\partial n}{\partial \tau} - \sigma_1 \frac{\partial \psi}{\partial x} - \sigma_1 \epsilon \frac{\partial \psi}{\partial \xi} = 0, \tag{9}$$

the energy balancing equation is

$$\frac{\partial T}{\partial t} - \epsilon v_0 \frac{\partial T}{\partial \xi} + \epsilon^2 \frac{\partial T}{\partial \tau} + n\frac{\partial v}{\partial x} + \epsilon n \frac{\partial v}{\partial \xi} + v \frac{\partial n}{\partial x} + \epsilon v \frac{\partial n}{\partial \xi} = 0, \tag{10}$$

and the Poisson's equation as

$$\frac{\partial^2 \psi}{\partial x^2} + 2\epsilon \frac{\partial}{\partial x}\frac{\partial \psi}{\partial \xi} + \epsilon^2 \frac{\partial^2 \psi}{\partial \xi^2} = n_e - n. \tag{11}$$

Collecting different terms of these equations in powers of ϵ, we can get the j-th order reduced equations. The first order $j = 1$ equation we obtained as follow;

$$n_l^{(1)} = \frac{k}{\omega} v_l^{(1)}, \tag{12}$$

$$v_l^{(1)} = \frac{k}{\omega}\psi_l^{(1)} + \tau^{-1}\frac{k}{\omega}\left(T_l^{(1)} + n_l^{(1)}\right), \tag{13}$$

$$T_l^{(1)} = \frac{2}{3\omega} n_l^{(1)} + \sigma_1 \frac{k}{\omega} \psi_l^{(1)}, \tag{14}$$

$$n_l^{(1)} = \left(k^2 + \alpha\right). \tag{15}$$

Here the constants $\alpha = en_{e0}/T_{e0}$ with T_{e0}, is the unperturbed temperature of electron. Collecting equations (12)-(15), for $l = 1$, we can get the following dispersion relation of the ITG mode in electron-ion plasma

$$\frac{\omega}{k} = \sqrt{\frac{1}{k^2 + \alpha}\left(1 + \tau^{-1}\sigma_1 \frac{k}{\omega}\right) + \tau^{-1}\left(\frac{2}{3\omega} + 1\right)}. \tag{16}$$

From equations (12)-(15) we can expressed all the first order variables in terms of $\psi_1^{(1)}$ as:

$$\begin{aligned} n_1^{(1)} &= \Delta_1 \psi_1^{(1)}, \\ v_1^{(1)} &= \Delta_2 \psi_1^{(1)}, \\ T_1^{(1)} &= \Delta_3 \psi_1^{(1)}. \end{aligned} \tag{17}$$

The different coefficients in equation (17) are $\Delta_1 = \left(k^2 + \alpha\right)$, $\Delta_2 = \omega/k\left(k^2 + \alpha\right)$ and $\Delta_3 = 2/3\omega\left(k^2 + \alpha\right) - \sigma_1 k/\omega$. Now for the second order $j = 2$, the reduced expressions with $l = 1$ are

$$\iota\omega n_1^{(2)} - v_0 \frac{\partial n_1^{(1)}}{\partial \xi} + \frac{\partial v_1^{(1)}}{\partial \xi} + \iota k v_1^{(2)} = 0, \tag{18}$$

$$\iota\omega v_1^{(2)} + v_0 \frac{\partial v_1^{(1)}}{\partial \xi} - \frac{\partial \psi_1^{(1)}}{\partial \xi} - \tau^{-1}\iota k\left(T_1^{(2)} + n_1^{(2)}\right) = 0, \tag{19}$$

$$\iota\omega T_1^{(2)} + v_0 \frac{\partial T_1^{(1)}}{\partial \xi} + \sigma_1 \frac{\partial \psi_1^{(1)}}{\partial \xi} - \frac{2}{3}\iota\omega k n_1^{(2)} - \frac{2}{3} v_0 \frac{\partial n_1^{(1)}}{\partial \xi} = 0, \tag{20}$$

$$n_1^{(2)} = 2\iota k \frac{\partial \psi_1^{(1)}}{\partial \xi} \tag{21}$$

where equations (18)-(21) are the corresponding reduced equation of continuity, ion momentum, energy balance and Poisson's. Coupling equations (17)-(21) the following compatibility condition for the ion temperature gradient modulational mode in electron-ion magneto-plasma gives relation of a group velocity as:

$$v_0 = \frac{-3\frac{k^3}{\omega^3}\sigma_1 + \frac{k^2}{\omega^2}\left(7k^2 + 3\tau\right) + 3\frac{k}{\omega}\sigma_1\left(1 + \tau\right) + 3\left(k^2 + \alpha\right)\tau}{2\frac{k^2+\alpha}{\omega}\left(\frac{k^3}{\omega^3} + \frac{k^2}{\omega^2} - \frac{3}{2}\tau\right) - 3\sigma_1\frac{k^2}{\omega^2}}. \tag{22}$$

The phase velocity equation (16) and group velocity equation (22), of the waves passing through the electron-ion plasma, gives a clear dependency of the plasma parameters on the phase as well as on the group velocity. Also one can demonstrate its graphical variation on Mathematica with different parameter of plasma. We have shown in figure 1, the variation of group to phase velocity ratio with the mode parameter η_i. For graphical discussion we used parameters as mention in the (Weiland, 2000). The second order perturbed quantities can be express in terms of $\partial\psi_1^{(1)}/\partial\xi$ as

$$\begin{aligned} n_1^{(2)} &= \Delta_4 \frac{\partial \psi_1^{(1)}}{\partial \xi}, \\ v_1^{(2)} &= \Delta_5 \frac{\partial \psi_1^{(1)}}{\partial \xi}, \\ T_1^{(2)} &= \Delta_6 \frac{\partial \psi_1^{(1)}}{\partial \xi}. \end{aligned} \tag{23}$$

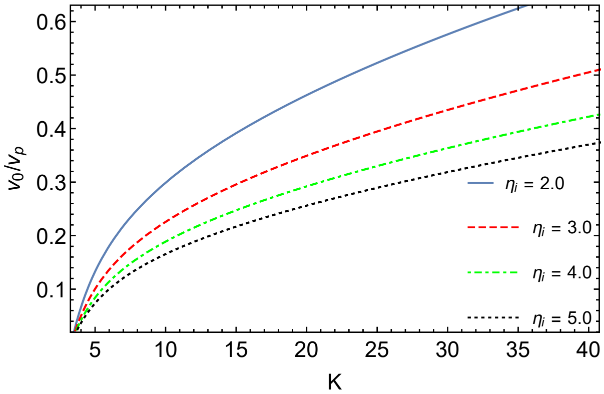

Fig. 1. Plot of v_0/v_p against k, for different value of ion temperature gradient coefficient η_i and all the other parameters keep constant as mention in the text.

We have neglected the second order perturbed potential in the equation (23) i.e., $\psi_1^{(2)} = 0$ and the coefficients are defined as $\Delta_4 = -2\iota k$, $\Delta_5 = 1/\iota k\,(2\omega k + v_0\Delta_1 - \Delta_2)$ and $\Delta_6 = 1/\iota k\,(v_0\Delta_3 + 2/3(v_0\Delta_1) + \sigma_1) - 4/3\iota k(\omega k)$. Now to obtain second harmonic of the carrier wave we can obtain this in terms of $\mid \psi_1^{(1)} \mid^2$, that shows the nonlinear interaction of the mode. Fixing $j, l = 2$, we can get relation of the second harmonic variables as:

$$\begin{aligned} n_2^{(2)} &= \Delta_7 \mid \psi_1^{(1)} \mid^2, \\ v_2^{(2)} &= \Delta_8 \mid \psi_1^{(1)} \mid^2, \\ T_2^{(2)} &= \Delta_9 \mid \psi_1^{(1)} \mid^2, \\ \psi_2^{(2)} &= \Delta_{10} \mid \psi_1^{(1)} \mid^2, \end{aligned} \tag{24}$$

with $\Delta_7 = \left(k^2\Delta_2^2 + \omega k\Delta_1\Delta_2\right) / \left(\omega^2\left(4k^2+\alpha\right) - k\Delta_6\right)$, $\Delta_8 = k + k/\tau(2/3(\alpha) - \sigma_1 k) + k/\tau(\alpha)$, $\Delta_9 = \left(2k\omega\Delta_1\Delta_2 + k\Delta_2^2\right) / \left(k\Delta_8 - \omega\left(\alpha\right)\right)$ and $\Delta_{10} = \iota(2k\Delta_1\Delta_2 + \omega\left(\alpha\right)\Delta_9)\Delta_1 + (\Delta_6\Delta_7 + k\Delta_2^2)\Delta_1/\omega + \left(4k^2+\alpha\right)\Delta_7\Delta_2$. Here the zeroth-harmonic mode that we have expressed in terms of the $\mid \psi_1^{(1)} \mid^2$, that type of harmonic modes are found to be obtained from the self interaction of the carrier waves.

$$\begin{aligned} n_0^{(2)} &= \Delta_{11} \mid \psi_1^{(1)} \mid^2, \\ u_0^{(2)} &= \Delta_{12} \mid \psi_1^{(1)} \mid^2, \\ T_0^{(2)} &= \Delta_{13} \mid \psi_1^{(1)} \mid^2, \\ \psi_0^{(2)} &= \Delta_{14} \mid \psi_1^{(1)} \mid^2, \end{aligned} \tag{25}$$

with $\Delta_{11} = \iota(2k\Delta_1\Delta_2 + \omega\left(\alpha\right)\Delta_9)\Delta_2 + (\Delta_6\Delta_7 + k\Delta_2^2)\Delta_2/\omega$, $\Delta_{12} = 1/\tau\left(\Delta_5 - 2\iota k\right) - v_0\Delta_2$, $\Delta_{13} = -\left(2\iota\omega^2 k + \Delta_{12}\omega - \left(v_0\Delta_5 + 4/3(v_0\iota k)\right)k^2/\tau\right) / \left(\iota\left(k^2/\tau - \omega^2 + 2k^2/3\tau(\omega)\right)\right)$ and $\Delta_{14} = \left(\left(\Delta_2 k + \Delta_1\omega\right)\omega + \Delta_3 - 2k^2/3\tau\right) / \left(\iota\left(k^2/\tau - \omega^2 + 2k^2/3\tau(\omega)\right)\right)$. Equations (23)-(25) show different plasma parameters with its first, second and zeroth harmonics. First harmonic is linear with respect to the first order perturbed normalized potential of the plasma. The second and zeroth harmonic varies with the second power of the perturbed potential.

4. Derivation of nonlinear Schrodinger's equation

To get the nonlinear Schrodinger equation of our assumed plasma, we obtain first the reduced equations for third order, $j = 3$, $l = 1$ component, of the model equations, so the continuity equation takes the form

$$-\iota\omega n_1^{(3)} - v_0\frac{\partial n_1^{(2)}}{\partial\xi} + \frac{\partial n_1^{(1)}}{\partial\tau} + \iota k v_1^{(3)} + \iota k\left(n_1^{(1)}v_0^{(2)} + n_0^{(2)}v_1^{(1)} + n_{-1}^{1}v_2^{(2)} + n_2^{(2)}v_{-1}^{(1)}\right) = 0. \tag{26}$$

The energy balance equation is

$$-\iota\omega T_1^{(3)} - v_0\frac{\partial T_1^{(2)}}{\partial\xi} - \frac{2}{3}\iota\omega n_1^{(3)} + \frac{2}{3}v_0\frac{\partial n_1^{(2)}}{\partial\xi} - \frac{2}{3}\frac{\partial n_1^{(1)}}{\partial\tau} = 0, \tag{27}$$

and momentum equation is

$$- \iota\omega v_1^{(3)} - v_0\frac{\partial v_1^{(2)}}{\partial\xi} + \frac{\partial v_1^{(1)}}{\partial\tau} + \iota k\left(v_0^{(2)}v_1^{(1)} + v_2^{(2)}v_{-1}^{(1)}\right) + \tau^{-1}\left(T_1^3 + n_1^{(3)}\right) \tag{28}$$

$$+ \tau^{-1}\frac{\partial v_1^{(1)}}{\partial\xi}\left(T_1^{(2)} + n_1^{(2)}\right) = 0.$$

Combining equations (26)-(28), finally we can write the NSLE equation as

$$\iota\frac{\partial\psi_1^{(1)}}{\partial\tau} + L\frac{\partial^2\psi_1^{(1)}}{\partial\xi^2} + M\psi_1^{(1)} \mid \psi_1^{(1)} \mid^2 = 0 \tag{29}$$

where $L = Re(\iota B_2/B_1)$ and $M = Re(\iota B_3/B_1)$, while $B_1 = -(\iota\omega\Delta_{14} - \Delta_1)\,\omega/k + (\Delta_3)\,k/\tau\omega - 2/3(\Delta_1)k/\tau\omega - (\Delta_{14})k/\tau\omega + \iota\Delta_{14}k/\tau + \Delta_2$, $B_2 = -\iota\,(\omega\Delta_{13} - 2kv_0)\,\omega/k - (2\iota/3(\omega\Delta_{13}))\,k/\tau\omega + (v_0\Delta_5 + 4\iota/3v_0k)\,k/\tau\omega + \iota\Delta_{13}k/\tau + \Delta_{12}$, $B_3 = -(\iota\omega\Delta_{15} - \Delta_{10})\,\omega/k - \Delta_{15}k/\tau\omega + \iota k/\tau\Delta_{15} + \Delta_{11}$, and $\Delta_{15} = \iota\,(\Delta_{11}k + \Delta_{10}\omega)\,\omega/\left(k^2/\tau - \omega^2 + 2k^2/3\tau\omega\right)$, From equation (29), the stability and instability of the ITG mode is totally dependent on whether the coefficients positive or negative. L is the dissipation coefficient and M is a nonlinear coefficient of the corresponding NLSE. These coefficients are also dependent on the plasma variable and η_i coefficient.

5. Results and discussion

We illuminate here the problem through graphical simulation by taking data as used in (Weiland, 2000), where some of these data are: $m_i = 1.67\times10^{-24}$g, $n_e = 10^{14}\text{cm}^{-3}$, $T_{e0} = 10$kev, $T_{i0} = 0.1T_{e0}$ and $\eta_i = 2$. In figure 1, we have plotted the variations of the group to phase velocities (ratio) versus the wave number k of the modulational wave. This observation clearly shows that the ratio is step-down with a η_i ion temperature gradient coefficient. As we know, the medium in which the group velocity of the wave depends on the wave number is a dispersive medium. As our η_i coefficient is proportional to the ion temperature variation with respect to the ion density, the opposition to the group velocity may be due to the drag forces enhanced in plasma. Further, hot places will have high temperatures and will be more viscous as compared to cold places in a fluid. To discuss the modulational stability or instability of the ion temperature gradient mode in electron-ion plasma, we know from previous observations (Ghosh & Banerjee, 2014; Chowdhury *et al.*, 2018, 2019; Sultana & Kourakis, 2011) that the product of LM can provide information about modulational stability and instability for the mode. There are three main conditions for the modulational waves.

1. A positive LM product indicates a modulationally unstable ion temperature gradient mode in which modulational waves can file up in one region of the plasma while wave intensities become lower in other areas, causing the pendermotive forces created to destabilise the plasma and move the dynamic species from the highly intense region to the lower.

2. For negative LM, we obtain modulationally stable ion temperature gradient mode. Here for that condition, the modulational wave can move to a specific region with no disturbance, so we can say that the modulational ion temperature gradient mode is stable in this condition.

3. For LM equal to zero, we obtain the modulational critical condition. Here on this condition the modulational waves move from stability to instability.

We discussed the product of the dispersion and nonlinear coefficient, i.e., LM versus wave number k with different ion to electron temperature ratios in figure 2. We can see from the plot for $T_i = 0.1T_e$,

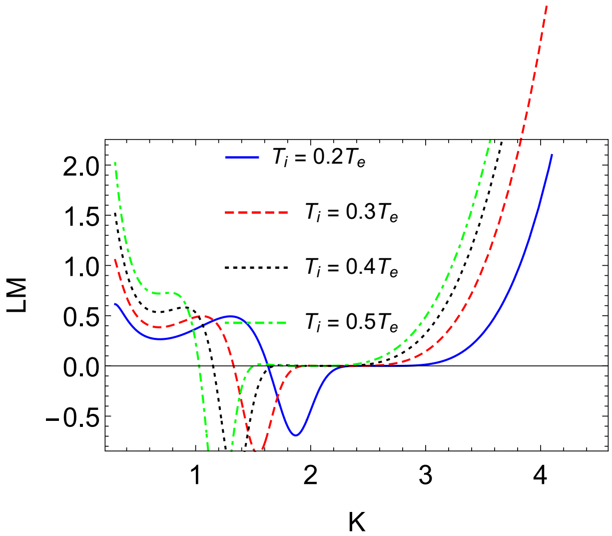

Fig. 2. Plot of LM (stability/instability) against wave number k, based on equation (29), for different value of T_i and all the other parameters keep constant as mention in the text.

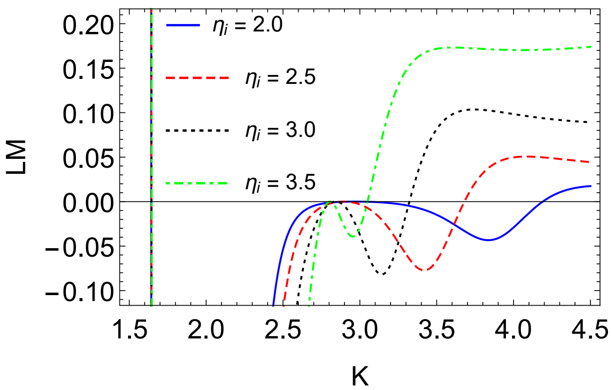

Fig. 3. Plot of LM (stability/instability) against wave number k, based on equation (29), for different value of ion temperature gradient coefficient η_i and all the other parameters keep constant as mention in the text.

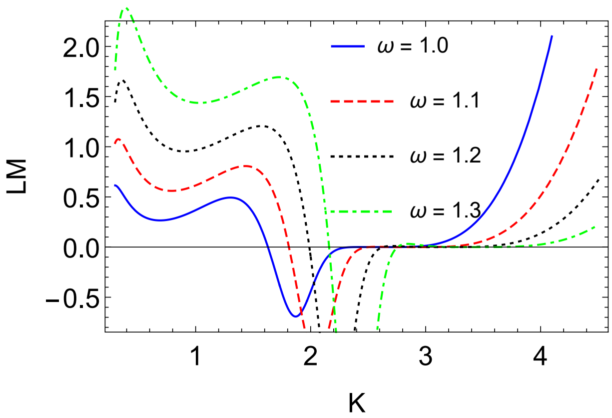

Fig. 4. Plot of LM (stability/instability) against wave number k, based on equation (29), for different value of ω and all the other parameters keep constant as mention in the text.

all the three conditions for modulational waves through electron-ion plasma i.e., LM is positive for wave numbers $k < 2.3$ and $k > 3.8$, but negative for $2.3 < k < 3.1$, indicating that the ITG mode modulational waves are stable in this range, $LM = 0$ for $3.1 \leq k \leq 3.8$. We demonstrated the effect of the mode parameter η_i on the modulational stability and instability of ITG in electron-ion plasma in figure 3, for $\eta_i = 2$, we found LM positive for the wave numbers $k < 1.6$ and $k > 4.2$, its value is zero for $2.8 < k < 3.2$ and negative for the rest within the given range. This region of instability, stability, and critical conditions shifts to a lower wave numbers as the ion to electron temperature ratio increases, which could occur due to the η_i coefficient varies directly with ion temperature with respect to ion density, changing the value and sign of the product. Figure 4 shows the influence of the mode frequency on the LM versus k, observation of the plot shows that for $\omega = 1$ rad/sec, we observe LM positive for the wave number $k < 1.6$ and $k > 3$, so the ITG modulational waves are unstable for this range. The product has a negative value for $1.6 < k < 2.2$, showing that the modulational mode is stable. These regions of instability, stability, and critical conditions are shifted to higher wave numbers with the increase of modulational mode frequency, so that can change the strength and polarity of the product. The shifting of these regions is in contrast compared to figures. 2 and 3. Figure 5 is the contour plot of the stability (i.e., LM product) against ion temperature gradient coefficient η_i and frequency ω, here we can see positive, negative and zero points for the product LM in the plot. We can also find the location of maximum modulational stability and instability of the ITG mode. In figure 5, we can get the point of maximum modulational stable point (-0.04) and the maximum instable point (0.04). In figure 6, we

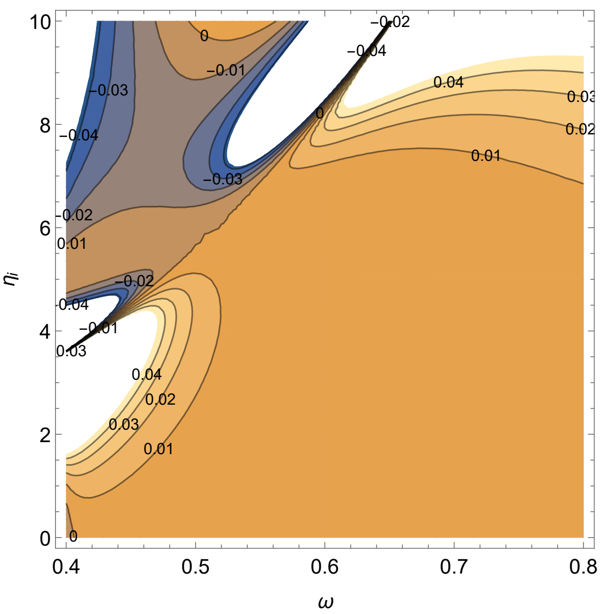

Fig. 5. LM contour plot against ion temperature gradient coefficient η_i and ω, Shows different stable, unstable and neutral points.

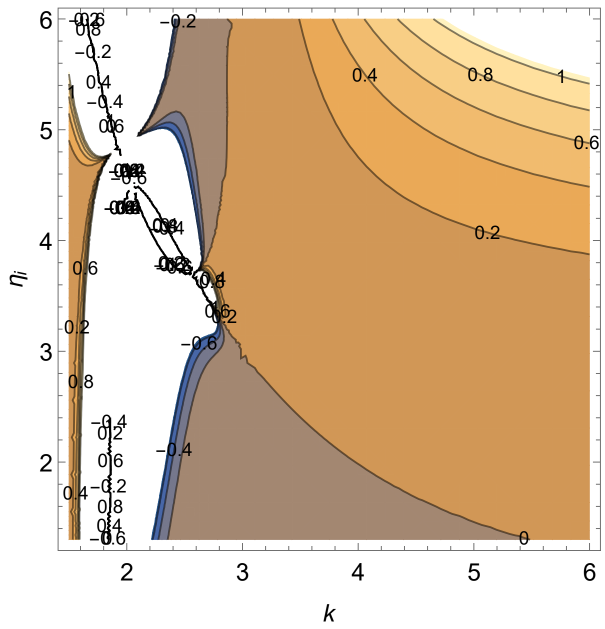

Fig. 6. LM contour plot against ion temperature gradient coefficient η_i and k, Shows different stable, unstable and neutral points.

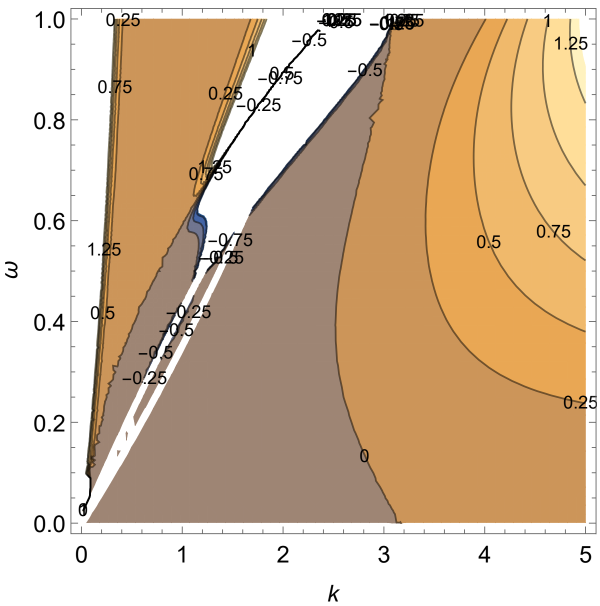

Fig. 7. LM contour plot against ω and k, based on equation (29), Shows different stable, unstable and neutral points.

have shown the instability of the e-i magnetized plasma against ion temperature gradient coefficient η_i and wave number k, where we can see positive, negative and zeros of the product LM in the contour plot. Also the points for the intense stability and instability for the ITG mode are shown in the same plot. In figure 6, the point of maximum modulational stable point is (-0.6) and the maximum unstable point is (0.8). In figure 7, we have discussed the stability versus ω and k, here the maximum modulational stability point is (-0.75), and the maximum unstable point is (1.25), for the given range of the plasma variables as shown in the plot.

6. Conclusion

This study focuses on the modulational stability and instability of the ITG mode in electron-ion plasma, where the set of magnetohydrodynamic (MHD) equations are used for the ion species while electrons in the plasma are taken to be Maxwellian. By the reduction perturbation technique, we get the phase as well as group velocities of the modulated ion temperature gradient mode, which are dependent on various plasma parameters such as electron and ion number densities, their temperatures, and especially on the ion temperature gradient coefficient. The study was extended to the derivation of nonlinear Schrodinger's equation from the set of magnetohydrodynamic (MHD) equations for the ion species in the plasma and obtained two very useful coefficients, i.e., L the dissipation coefficient, and M the nonlinear coefficient. These are also strongly related to plasma variables, and their product LM tells us about the modulational stability and instability of the plasma. The wave number in critical conditions is k_c, on which the product of M and L is zero. For $k_c = 0$, the ion temperature gradient modulational mode goes from stability to instability. Since the modulational instability of the mode occurs when the product of these coefficients is positive, the modulational wave speed is reduced. The crests of the waves are concentrated in one direction with varied intensities. A ponderomotive force is observed in such scenarios, where dynamic plasma species move from higher density to lower density, destabilising the entire plasma, which is a severe concern in tokamak fusion reactors. We have shown in various plots that the product can be changed from positive to negative and vice versa by changing the ion to electron temperature ratios, density ratios, and ion temperature gradient coefficient. Moreover, the contour plots also show the location of different stable and unstable points for the ITG modulational mode electron-ion plasma. The maximum stability and instability points are recognised from the same plots within the given range of various plasma variables clarified that the stability of the madulational waves also changes with the ion temperature gradient mode frequency and wave number. Therefore, if we manage this stability under the effect of the dynamic ion temperature gradient mode, we can confine the total plasma to the region of interest.

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