$$u(0,y) = b(y). \tag{45}$$

From (43) and (44) by integrating from 0to y, we get

$$u_x(x,y) - u_x(x,0) = \int_0^y \frac{f(x)\, g(t)}{x^\beta + t^\beta}\, dt\,.$$

Thus

$$u_x(x,y) = a(x\,) + \int_0^y \frac{f(x)\, g(t)}{x^\beta + t^\beta}\, dt\,. \tag{46}$$

From (45), (46) by integrating from 0to x, we have

$$u(x,y) - u(0,y) = \int_0^x a(s\,)\, ds + \int_0^x \int_0^y \frac{f(s)\, g(t)}{s^\beta + t^\beta}\, ds\, dt$$

hence

$$u(x,y) = b(y) + \int_0^x a(s\,)\, ds + \int_0^x \int_0^y \frac{f(s)\, g(t)}{s^\beta + t^\beta}\, ds\, dt. \tag{47}$$

Suppose that

$$\int_0^x \int_0^y \frac{f(s)\, g(t)}{s^\beta + t^\beta}\, ds\, dt \le \int_0^\infty \int_0^\infty \frac{f(s)\, g(t)}{s^\beta + t^\beta}\, ds\, dt\,, \tag{48}$$

it follows from (47) and (48) that

$$u(x,y) \le b(y) + \int_0^x a(s\,)\, ds + \int_0^\infty \int_0^\infty \frac{f(s)\, g(t)}{s^\beta + t^\beta}\, ds\, dt$$

$$\le b(y) + \int_0^x a(s\,)\, ds + \frac{1}{\beta}\left[B\left(\frac{1}{\beta p}\,, 1 - \frac{1}{\beta p}\right)\right]^{\frac{1}{p}} \left[B\left(\frac{1}{\beta q}\,, 1 - \frac{1}{\beta q}\right)\right]^{\frac{1}{q}} \times$$

$$\left\{\int_0^\infty s^{1-\beta} f^p(s) ds\right\}^{\frac{1}{p}} \left\{\int_0^\infty t^{1-\beta} g^q(t)\, dt\right\}^{\frac{1}{q}}, \tag{49}$$

which is application of Hilbert's integral inequality.

Example 3.Consider the following partial differential equation

$$u_{xy} = \frac{f(x)\, g(y)}{(x^2 + xy)^\propto}\,, \tag{50}$$

where

$$u_x(x,0) = a(x\,)\,, \tag{51}$$

$$u(0,y) = b(y). \tag{52}$$

From (50) and (51) by integrating from 0 to y, we get

$$u_x(x,y) - u_x(x,0) = \int_0^y \frac{f(x)\, g(t)}{(x^2 + xt)^\propto}\, dt\,,$$

hence

$$u_x(x,y) = a(x) + \int_0^y \frac{f(x)\, g(t)}{(x^2+xt)^{\propto}}\, dt\,. \tag{53}$$

From (52) and (53) by integrating from 0 to x, we have

$$u(x,y) - u(0,y) = \int_0^x a(s)\, ds + \int_0^x \int_0^y \frac{f(s)\, g(t)}{(s^2+st)^{\propto}}\; ds\, dt,$$

hence

$$u(x,y) = b(y) + \int_0^x a(s)\, ds + \int_0^x \int_0^y \frac{f(s)\, g(t)}{(s^2+st)^{\propto}}\; ds\, dt. \tag{54}$$

Suppose that

$$\int_0^x \int_0^y \frac{f(s)\, g(t)}{(s^2+st)^{\propto}}\; ds\, dt \le \int_0^{\infty} \int_0^{\infty} \frac{f(s)\, g(t)}{(s^2+st)^{\propto}}\; ds\, dt\,, \tag{55}$$

it follows from (54) and (55) that

$$u(x,y) \le b(y) + \int_0^x a(s)\, ds + \int_0^{\infty} \int_0^{\infty} \frac{f(s)\, g(t)}{(s^2+st)^{\propto}}\; ds\, dt$$

$$\le b(y) + \int_0^x a(s)\, ds + \left\{B\left(\frac{1}{p}\,, \propto - \frac{1}{p}\right) \int_0^{\infty} s^{1-2\propto} f^p(s)\, ds\right\}^{\frac{1}{p}} \times$$

$$\left\{B\left(\frac{1}{q} - \propto\,, 2\propto - \frac{1}{q}\right) \int_0^{\infty} t^{1-2\propto} g^q(t) dt\right\}^{\frac{1}{q}},$$

which is application of Hilbert's integral inequality.

5. Acknowledgment

This work is supported by Deanship of Scientific Research, Taibah University, Kingdom of Saudi Arabia.

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Submitted: 15/11/2015
Revised : 10/02/2016
Accepted : 16/02/2016

تعميم للمتباينات التكاملية من نوع هيلبرت – هاردي

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خلاصة

في هذا البحث، تمت دراسة المتباينات التكاملية لهيلبرت مع بعض المعلمات، وذلك باستخدام أساليب جديدة في البرهان. وتُعتبر جميع نتائج هاردي ويانغ حالات خاصة من المتباينات الجديدة المُقدمة. وكتطبيق لنتائجنا، نقدم بعض الأمثلة التطبيقية التي تدعم نتائجنا.