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Finite difference methods for the generalized Huxley and Burgers-Huxley equations

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Abstract

In this paper, an explicit exponential finite difference method is presented to solve the generalized forms of Huxley and Burgers-Huxley equations. These schemes allows to handle any values of δ. The accuracy of the numerical solutions indicates that the present method is well suited for the solution of the generalized Huxley and the generalized Burgers-Huxley equations.

Keywords: Explicit exponential finite difference method; finite difference method; generalized Burgers-Huxley equation; generalized Huxley equation.

1. Introduction

Most of the problems in various field as physics, chemistry, biology, mathematics and engineering are modeled by nonlinear partial differential equations. Two of the nonlinear partial differential equations are the generalized Huxley equation and the generalized Burgers-Huxley equation. Firstly, we consider the generalized Huxley equation

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}=\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right), 0\leq x\leq 1,\ t\geq 0 \qquad (1)$$

with the initial condition

$$u(x,0)=f_1(x)$$

and the boundary conditions

$u(0,t)=g_1(t)$ and $u(1,t)=g_2(t)$.

The equation describes nerve pulse propagation in nerve fibres and wall motion in liquid crystals. Where δ, $\beta\geq 0$ and $\gamma\epsilon(0,1)$ are arbitrary constant depending on the parameters.

Various methods for obtaining numerical solutions to the generalized Huxley equation have been proposed. Hashim *et al.* (2006a) used the Adomian decomposition method for the numerical solution of the mentioned equation. The solution of the generalized Huxley equation was obtained using homotopy perturbation method and Adomian decomposition method by Hashemi *et al.* (2007). Batiha *et al.* (2007) proposed variational iteration method for the solution of the equation. Based on the homotopy analysis method, a scheme was developed to obtain approximation solution of the generalized Huxley equation by Hemida & Mohamed (2012).

Secondly, we consider the generalized Burgers-Huxley equation. The generalized Burgers-Huxley equation of the form;

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}=\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right),$$
$$0\leq x\leq 1,\ t\geq 0 \qquad (2)$$

with the following initial conditions taken from

$$u(x,0)=f_2(x)$$

and the boundary conditions

$u(0,t)=g_3(t)$ and $u(1,t)=g_4(t)$

where $\alpha,\ \beta,\ \gamma$ and δ are parameters that $\beta\geq 0$, $\delta>0$, $\gamma\epsilon(0,1)$. When $\alpha=0$, Equation (2) is reduced to the generalized Huxley equation. If we take $\delta=1$ and $\alpha\neq 0$, $\beta\neq 0$, Equation (2) becomes the following Burgers-Huxley equation:

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u(1-u)(u-\gamma). \qquad (3)$$

Equation (3) shows a prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transport. This equation was investigated by Satsuma in 1986 (Wang *et al*. 1990).

In literature, many numerical methods have been proposed for approximating solution of the generalized

Burgers-Huxley equation. Adomian decomposition method was applied to the generalized Burgers-Huxley equation by Hashim *et al*. (2006b). Javidi (2006a, 2006b) presented the collocation method for solving the equation. Spectral collocation method and Darvishi's preconditionings to solve the generalized Burgers-Huxley equation was used by Darvishi *et al*. (2008). Batiha *et al*. (2008) used the variational iteration method, which was based on the incorporation of a general Lagrange multiplier in the construction of correction functional for the equation. Numerical solutions of the equation was obtained using a polynomial differential quadrature method by Sari & Gürarslan (2009). A numerical solution of the equation, based on collocation method using Radial basis functions, called Kansa's approach was presented by Khattak (2009). Javidi & Golbabai (2009) used the spectral collocation method using Chebyshev polynomials for spatial derivatives and fourth order Runge-Kutta method for time integration to solve the generalized Burgers-Huxley equation. Biazar & Mohammadi (2010) used the differential transform method for solution of the equation. A fourth order finite-difference scheme in a two-time level recurrence relation was proposed for the equation by Bratsos (2011). Çelik (2012) used Haar wavelet method for solving the generalized Burgers-Huxley equation. El-Kady *et al*. (2013) presented based on cardinal Chebyshev and Legendre basis functions with Galerkin method for solution of the equation. The discrete Adomian decomposition method was applied to a fully implicit scheme of the generalized Burgers-Huxley equation by Al-Rozbayani (2013). Also, some Burgers-type equations were solved with many numerical techniques by many authors such as Bhrawy (2013, 2014, 2015), Bhrawy & Abdelkawy (2015), Bhrawy *et al.* (2015a, 2015b), Doha *et al.* (2014) and Başhan *et al.* (2015).

The explicit exponential finite difference method was originally developed by Bhattacharya (1985) for solving of the heat equation. Bhattacharya (1990) and Handschuh & Keith (1992) used exponential finite difference method for the solution of Burgers equation. Bahadır (2005) solved the KdV equation by using the exponential finite difference technique. İnan & Bahadır (2013) solved the linearized Burgers equation by Hopf-Cole transformation using an explicit exponential finite difference method.

In this paper, we design new scheme for solving the generalized forms of Huxley and Burgers-Huxley equations. Tables are presented for the ability of the method to solve the equations for different values of δ. It is clearly seen that the numerical results are reasonably in good agreement with the exact solutions.

2. Explicit exponential finite difference method

In this section, we obtain numerical solutions of the equations by explicit exponential finite difference method. The solution domains are discretized into cells as (x_i, t_n) in which $x_i = ih$, $(i = 0, 1, 2 \cdots, N)$ and $t_n = nk, (n = 0, 1, 2, \cdots)$, $h = \Delta x = \frac{b-a}{N}$ is the spatial mesh size and $k = \Delta t$ is the time step. Explicit exponential finite difference method for Equation (1) and Equation (2) take the following linear forms, which are valid for values of i lying in the interval $1 \leq i \leq N-1$. Also, where U_i^n denotes the explicit exponential finite difference approximation and $u(x,t)$ denotes exact solution. All numerical computations are performed with the space step $h = 0.01$ and the time step $k = 0.00001$. The accuracy of the proposed method is measured using the absolute error, which is defined as

$$|u(x_i, t_n) - U(x_i, t_n)|.$$

2.1. The generalized Huxley equation

We rearrange Equation (1) to obtain

$$\frac{\partial u}{\partial t} = \beta u\left(1 - u^{\delta}\right)\left(u^{\delta} - \gamma\right) + \frac{\partial^2 u}{\partial x^2}. \tag{4}$$

Dividing by u

$$\frac{\partial lnu}{\partial t} = \frac{1}{u}\left(\beta u\left(1 - u^{\delta}\right)\left(u^{\delta} - \gamma\right) + \frac{\partial^2 u}{\partial x^2}\right). \tag{5}$$

Using the finite difference approximations for derivatives Equation (5) have been taken following form

$$U_i^{n+1} = U_i^n exp\left\{\frac{k}{U_i^n}\left[\beta U_i^n\left(1 - (U_i^n)^{\delta}\right)\left((U_i^n)^{\delta} - \gamma\right) + \left(\frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{h^2}\right)\right]\right\} \tag{6}$$

or Equation (6) can be written as

$$U_i^{n+1} = U_i^n exp\left\{k\left[\beta\left(1 - (U_i^n)^{\delta}\right)\left((U_i^n)^{\delta} - \gamma\right) + \frac{1}{h^2}\left(\frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{U_i^n}\right)\right]\right\}. \tag{7}$$

2.2. Numerical example

In this section, numerical solutions for the generalized Huxley equation are presented to demonstrate the accuracy of the proposed method.

Consider the generalized Huxley equation of the form;

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}=\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right),\ 0\le x\le 1,\ t\ge 0 \quad (8)$$

with the initial condition

$$u(x,0)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}tanh(\sigma\gamma x)\right]^{1/\delta}$$

and the boundary conditions

$$u(0,t)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}tanh\left\{\sigma\gamma\left\{\frac{(1+\delta-\gamma)\rho}{2(1+\delta)}\right\}t\right\}\right]^{1/\delta}$$

and

$$u(1,t)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}tanh\left\{\sigma\gamma\left(1+\left\{\frac{(1+\delta-\gamma)\rho}{2(1+\delta)}\right\}t\right)\right\}\right]^{1/\delta}.$$

The exact solution of this equation was derived by Wang *et al*. (1990) using nonlinear transformations and is given by

$$u(x,t)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}tanh\left\{\sigma\gamma\left(x+\left\{\frac{(1+\delta-\gamma)\rho}{2(1+\delta)}\right\}t\right)\right\}\right]^{1/\delta}.$$

where $\sigma=\delta\rho/4(1+\delta)$ and $\rho=\sqrt{4\beta(1+\delta)}$.

For the numerical computation, we use the parameters $\beta=1,\ \gamma=0.001$ and $\delta=1,\ 2,\ 3$ and 5. Numerical results, exact solutions and absolute errors at different values of t are given in Table 1-4. Table 5 shows comparison of the present method with $\emptyset_5$ (Hashim *et al.* 2006a), HPM (Hashemi *et al.* 2007) and VIM (Batiha *et al.* 2007) methods for $\delta=3$. From these tables, it is obvious that numerical solutions are in excellent agreement with the exact solutions.

Table 1. Numerical solutions for $\delta=1$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.000500020	0.000500030	1.030307E-08
	0.1	0.000500028	0.000500043	1.506294E-08
	1	0.000500245	0.000500268	2.248771E-08
	2	0.000500495	0.000500517	2.248873E-08
	5	0.000501244	0.000501267	2.248861E-08
0.5	0.05	0.000500078	0.000500101	2.313697E-08
	0.1	0.000500075	0.000500113	3.843952E-08
	1	0.000500276	0.000500338	6.246539E-08
	2	0.000500526	0.000500588	6.246868E-08
	5	0.000501275	0.000501338	6.246834E-08
0.9	0.05	0.000500161	0.000500172	1.030307E-08
	0.1	0.000500169	0.000500184	1.506294E-08
	1	0.000500386	0.000500409	2.248771E-08
	2	0.000500636	0.000500659	2.248872E-08
	5	0.000501386	0.000501408	2.248859E-08

Table 2. Numerical solutions for $\delta=2$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.022361423	0.022361884	4.608094E-07
	0.1	0.022361769	0.022362443	6.736791E-07
	1	0.022371493	0.022372499	1.005330E-06
0.5	0.05	0.022363431	0.022364466	1.034739E-06
	0.1	0.022363305	0.022365024	1.719095E-06
	1	0.022372287	0.022375079	2.792493E-06
0.9	0.05	0.022366586	0.022367047	4.607405E-07
	0.1	0.022366932	0.022367606	6.736028E-07
	1	0.022376654	0.022377659	1.005253E-06

Table 3. Numerical solutions for $\delta = 3$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.079372385	0.079374020	1.635704E-06
	0.1	0.079373613	0.079376004	2.391262E-06
	1	0.079408123	0.079411690	3.567016E-06
0.5	0.05	0.079378283	0.079381956	3.672757E-06
	0.1	0.079377837	0.079383939	6.101814E-06
	1	0.079409710	0.079419618	9.907906E-06
0.9	0.05	0.079388254	0.079389889	1.635280E-06
	0.1	0.079389482	0.079391872	2.390793E-06
	1	0.079423978	0.079427544	3.566539E-06

Table 4. Numerical solutions for $\delta = 5$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.218677837	0.218682343	4.506511E-06
	0.1	0.218681220	0.218687808	6.587901E-06
	1	0.218776259	0.218786078	9.819106E-06
0.5	0.05	0.218690074	0.218700192	1.011803E-05
	0.1	0.218688845	0.218705655	1.680963E-05
	1	0.218776619	0.218803892	2.727357E-05
0.9	0.05	0.218713530	0.218718034	4.504603E-06
	0.1	0.218716909	0.218723495	6.585790E-06
	1	0.218811884	0.218821701	9.816960E-06

Table 5. Comparisons of the numerical solutions for $\delta = 3$.

x	t	Exact	Present Method	$\emptyset_5$	HPM	VIM
0.1	0.05	0.079374020	0.079372385	0.079370053	0.079370053	0.079370053
	0.1	0.079376004	0.079373613	0.079368069	0.079368069	0.079368070
	1	0.079411690	0.079408123	0.079332345	0.079332344	0.079332364
0.5	0.05	0.079381956	0.079378283	0.079377989	0.079377989	0.079377989
	0.1	0.079383939	0.079377837	0.079376006	0.079376006	0.079376006
	1	0.079419618	0.079409710	0.079340288	0.079340288	0.079340307
0.9	0.05	0.079389890	0.079388254	0.079385924	0.079385924	0.079385924
	0.1	0.079391872	0.079389482	0.079383941	0.079383941	0.079383941
	1	0.079427544	0.079423978	0.079348230	0.079348230	0.079348250

2.3. Generalized Burgers-Huxley equation

When Equation (2) is rearranged, following equation is obtained

$$\frac{\partial u}{\partial t} = \beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right) - \alpha u^{\delta}\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} \quad (9)$$

Dividing by u

$$\frac{\partial lnu}{\partial t} = \frac{1}{u}\left(\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right) - \alpha u^{\delta}\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}\right) \quad (10)$$

Using the finite difference approximations for derivatives Equation (10) have been taken following form

$$U_i^{n+1} = U_i^n exp\left\{\frac{k}{U_i^n}\left[\beta U_i^n\left(1-(U_i^n)^{\delta}\right)\left((U_i^n)^{\delta}-\gamma\right) - \alpha (U_i^n)^{\delta}\left(\frac{U_{i+1}^n - U_{i-1}^n}{2h}\right) + \left(\frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{h^2}\right)\right]\right\}. \quad (11)$$

2.4. Numerical example

Then, we consider the following generalized Burgers-Huxley equation,

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right),$$
$$0\le x\le 1,\ t\ge 0 \qquad (12)$$

with the following initial conditions taken from

$$u(x,0)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}tanh(A_1x)\right]^{1/\delta}$$

and the boundary conditions

$$u(0,t)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}tanh(-A_1A_2t)\right]^{1/\delta}$$

and

$$u(1,t)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}tanh[A_1(1-A_2t)]\right)^{1/\delta}.$$

The exact solution of Equation (2) is

$$u(x,t)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}tanh[A_1(x-A_2t)]\right)^{1/\delta}$$

where

$$A_1=\frac{-\alpha\delta+\delta\sqrt{\alpha^2+4\beta(1+\delta)}}{4(1+\delta)}\gamma,$$

$$A_2=\frac{\gamma\alpha}{1+\delta}-\frac{(1+\delta-\gamma)\left(-\alpha+\sqrt{\alpha^2+4\beta(1+\delta)}\right)}{2(1+\delta)}.$$

In Tables 6-8, we present numerical and exact solutions for various values of x, t and δ with $\alpha=1$, $\beta=1$, $\gamma=0.001$. Absolute errors for various values of x, t and δ with $\alpha=0.1, \beta=0.001, \gamma=0.0001$ showed in Table 9. Table 10 shows comparison of the present method with Ismail *et al*. (2004), Hashim *et al*. (2006b), Batiha *et al*. (2008), Biazar & Mohammadi (2010) and Al-Rozbayani (2013) for $\alpha=\beta=\delta=1\gamma=0.001$. From Table 6-10, it can be observed that the computed results show excellent agreement with the exact solutions.

Table 6. Numerical solutions for $\delta=1$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.000500022	0.000500037	1.545406E-08
	0.1	0.000500040	0.000500062	2.259326E-08
	1	0.000500478	0.000500512	3.372930E-08
	2	0.000500978	0.000501012	3.373074E-08
	5	0.000502477	0.000502511	3.373004E-08
0.5	0.05	0.000500053	0.000500087	3.470545E-08
	0.1	0.000500055	0.000500112	5.765928E-08
	1	0.000500468	0.000500562	9.369803E-08
	2	0.000500968	0.000501061	9.370276E-08
	5	0.000502467	0.000502561	9.370081E-08
0.9	0.05	0.000500122	0.000500137	1.545515E-08
	0.1	0.000500140	0.000500162	2.259559E-08
	1	0.000500578	0.000500612	3.373379E-08
	2	0.000501078	0.000501112	3.373524E-08
	5	0.000502577	0.000502611	3.373453E-08

Table 7. Numerical solutions for $\delta=2$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.022362607	0.022364010	1.403437E-06
	0.1	0.022364803	0.022366855	2.051589E-06
	1	0.022414941	0.022417997	3.056211E-06
0.5	0.05	0.022362800	0.022365952	3.151589E-06
	0.1	0.022363561	0.022368796	5.235663E-06
	1	0.022411444	0.022419934	8.490079E-06
0.9	0.05	0.022366490	0.022367893	1.403378E-06
	0.1	0.022368686	0.022370738	2.051624E-06
	1	0.022418814	0.022421871	3.056441E-06

Table 8. Numerical solutions for $\delta = 3$.

x	t	Present Method	Exact	Absolute Error
0.1	0.05	0.079382153	0.079390945	8.792581E-06
	0.1	0.079397429	0.079410279	1.285017E-05
	1	0.079737642	0.079756666	1.902468E-05
0.5	0.05	0.079377393	0.079397138	1.974459E-05
	0.1	0.079383675	0.079416470	3.279434E-05
	1	0.079709947	0.079762802	5.285446E-05
0.9	0.05	0.079394539	0.079403331	8.791420E-06
	0.1	0.079409809	0.079422659	1.284951E-05
	1	0.079749911	0.079768936	1.902522E-05

Table 9. Absolute errors for various values of x, t and δ.

x	t	$\delta = 1$	$\delta = 2$	$\delta = 4$	$\delta = 8$
0.1	0.2	1.562197E-13	3.838587E-11	1.277322E-09	1.277322E-09
	0.5	1.770078E-13	4.349372E-11	1.447292E-09	1.447292E-09
	0.8	1.780854E-13	4.375841E-11	1.456095E-09	1.456095E-09
0.5	0.2	4.238971E-13	1.041588E-11	3.465972E-09	3.465972E-09
	0.5	4.911674E-13	1.206880E-11	4.016006E-09	4.016006E-09
	0.8	4.946513E-13	1.215443E-11	4.044490E-09	4.044490E-09
0.9	0.2	1.697482E-13	3.838586E-11	1.277324E-09	1.277324E-09
	0.5	1.770078E-13	4.349372E-11	1.447294E-09	1.447294E-09
	0.8	1.780854E-13	4.375840E-11	1.456098E-09	1.456097E-09

Table 10. Comparisons of the absolute for $\delta = 1$.

x	t	Present Method	Ismail et al. (2004)	Hashim et al. (2006b)	Batiha et al. (2008)	Biazar & Mohammadi (2010)	Al-Rozbayani (2013)
0.1	0.05	1.545406E-08	1.93715E-07	1.87406E-08	1.87405E-08	1.87406E-08	1.87406E-08
	0.1	2.259326E-08	3.87434E-07	3.74812E-08	3.74813E-08	3.74813E-08	3.74812E-08
	1	3.372930E-08	3.87501E-06	3.74812E-07	3.74812E-07	3.748125E-07	3.74812E-07
0.5	0.05	3.470545E-08	1.9373E-07	1.87406E-08	1.87405E-08	1.87406E-08	1.87406E-08
	0.1	5.765928E-08	3.87464E-07	3.74812E-08	1.37481E-08	3.74813E-08	3.74812E-08
	1	9.369803E-08	3.87531E-06	3.74812E-07	3.74813E-07	3.748125E-07	3.74812E-07
0.9	0.05	3.470545E-08	1.93745E-07	1.87406E-08	1.87405E-08	1.87406E-08	1.87406E-08
	0.1	5.765928E-08	3.87494E-07	3.74812E-08	3.74813E-08	3.74813E-08	3.74812E-08
	1	9.369803E-08	3.87561E-06	3.74812E-07	3.74813E-07	3.748125E-07	3.74812E-07

3. Conclusion

In this paper, explicit form for the numerical solutions obtained by applied exponential finite difference method in the space and the time has been presented. Numerical solutions for different values of δ are given using tables for both equations. According to the results presented in these tables, the present method offer high accuracy for the numerical solutions of the nonlinear generalized forms of Huxley and Burgers-Huxley equations. Table 5 and Table 10 clearly show that explicit exponential finite difference method is more efficient than the other methods.

References

Al-Rozbayani, A.M. (2013). Discrete Adomian decomposition method for solving Burger's-Huxley Equation, International Journal of Contemporary Mathematical Sciences, **8**(13)**:**623-631.

Bahadır, A.R. (2005). Exponential finite-difference method applied to Korteweg-de Vries equation for small times, Applied Mathematics and Computations, **160**:675-682.

Başhan, A., Karakoç, S.B.G. & Geyikli, T. (2015). Approximation of the KdVB equation by the quintic B-spline differential quadrature method, Kuwait Journal of Science, **42**(2)**:**67-92.

Batiha, B., Noorani, M.S.M. & Hashim, I. (2007). Numerical simulation of the generalized Huxley equation by He's variational iteration method, Applied Mathematics and Computations, **186**:1322-1325.

Batiha, B., Noorani, M.S.M. & Hashim, I. (2008). Application of variational iteration method to the generalized Burgers-Huxley equation, Chaos, Solitons & Fractals, **36**:660-663.

Bhattacharya, M.C. (1985). An explicit conditionally stable finite difference equation for heat conduction problems, International Journal for Numerical Methods in Engineering, **21**:239-265.

Bhattacharya, M.C. (1990). Finite difference solutions of partial differential equations, Communications in Applied Numerical Methods, **6**:173-184.

Bhrawy, A.H. (2013). A Jacobi-Gauss-Lobatto collocation method for solving generalized Fitzhugh-Nagumo equation with time-dependent coefficients, Applied Mathematics and Computations, **222**:255-264.

Bhrawy, A.H. (2014). An efficient Jacobi pseudospectral approximation for nonlinear complex generalized Zakharov system, Applied Mathematics and Computations, **247**:30-46.

Bhrawy, A.H. (2015). A highly accurate collocation algorithm for 1+1 and 2+1 fractional percolation equations, Journal of Vibration and Control, **22**(9):2288-2310.

Bhrawy, A.H. & Abdelkawy, M.A. (2015). A fully spectral collocation approximation for multi-dimensional fractional Schrodinger equations, Journal of Computational Physics, **294**:462-483.

Bhrawy, A.H., Zaky, M.A. & Baleanu, D. (2015a). New numerical approximations for space-time fractional Burgers' Equations via a Legendre Spectral-Collocation method, Romanian Reports of Physics, **67**(2):340-349.

Bhrawy, A.H., Taha, T.M. & Machado, J.A.T. (2015b). A review of operational matrices and spectral techniques for Fractional Calculus, Nonlinear Dynamics, **81**:1023-1052.

Biazar, J. & Mohammadi, F. (2010). Application of differential transform method to the generalized Burgers-Huxley equation, Applications and Applied Mathematics: An International Journal, **5**(10):1726-1740.

Bratsos, A.G. (2011). A fourth order improved numerical scheme for the generalized Burgers-Huxley equation, American Journal of Computational Mathematics, **1**:152-158.

Çelik, İ. (2012). Haar wavelet method for solving generalized Burgers-Huxley equation, Arab Journal of Mathematical Sciences, **18**:25-37.

Darvishi, M.T., Kheybari, S. & Khani, F. (2008). Spectral collocation method and Darvishi's preconditionings to solve the generalized Burgers-Huxley equation, Communications in Nonlinear Science and Numerical Simulation, **13**:2091-2103.

Doha, E.H., Bhrawy, A.H., Abdelkawy, M.A. & Hafez, R.M. (2014). A Jacobi collocation approximation for nonlinear coupled viscous Burgers' equation, Central European Journal of Physics, **12**(2):111-122.

El-Kady, M., El-Sayed, S.M. & Fathy, H.E. (2013). Development of Galerkin method for solving the generalized Burger's Huxley equation, Mathematical Problems in Engineering, doi: 10.1155/2013/165492.

Handschuh, R.F. & Keith, T.G. (1992). Applications of an exponential finite-difference technique, Numerical Heat Transfer, **22**:363-378.

Hashemi, S.H., Daniali, H.R.M. & Ganji, D.D. (2007). Numerical simulation of the generalized Huxley equation by He's homotopy perturbation method, Applied Mathematics and Computations, **192**:157-161.

Hashim, I., Noorani, M.S.M. & Batiha, B. (2006a). A note on the Adomian decomposition method for the generalized Huxley Equation, Applied Mathematics and Computations, **181**:1439-1445.

Hashim, I., Noorani, M.S.M. & Al-Hadidi, M.R.S. (2006b). Solving the generalized Burgers-Huxley Equation using the Adomian decomposition method, Mathematical and Computer Modelling, **43**:1404-1411.

Hemida, K. & Mohamed, M.S. (2012). Application of homotopy analysisi method to fractional order generalized Huxley equation, Journal of Applied Functional Analysis, **7**(4):367-372.

İnan, B., & Bahadır, A.R. (2013b). An explicit exponential finite difference method for the Burgers' equation, European International Journal of Science and Technology, **2**(10):61-72.

Ismail, H.N.A., Raslan, K. & Rabboh, A.A.A. (2004). Adomian decomposition method for Burger's-Huxley and Burger's-Fisher equations, Applied Mathematics and Computations, **159**:291-301.

Javidi, M. (2006a). A numerical solution of the generalized Burger's-Huxley equation by pseudospectral method and Darvishi's preconditioning, Applied Mathematics and Computations, **175**:1619-1628.

Javidi, M. (2006b). A numerical solution of the generalized Burger's-Huxley equation by spectral collocation method, Applied Mathematics and Computations, **178**:338-344.

Javidi, M. & Golbabai, A. (2009). A new domain decomposition algorithm for generalized Burger's-Huxley equation based on Chebyshev polynomials and preconditioning, Chaos, Solitons & Fractals, **39**(2):849-857.

Khattak, A.J. (2009). A computational meshless method for the generalized Burger's-Huxley equation, Applied Mathematical Modelling, **33**:218-3729.

Sari, M. & Gürarslan, G. (2009). Numerical solutions of the generalized Burgers-Huxley equation by a differential quadrature method, Mathematical Problems in Engineering, doi: 10.1155/2009/370765.

Wang, X.Y., Zhu, Z.S. & Lu, Y.K. (1990). Solitary wave solutions of the generalized Burgers-Huxley equation, Journal of Physics A: Mathematical and General, **23**:271-274.

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طرق الفروق المحدودة لمعادلات هوكسلي وبرجرز-هوكسلي (Huxley and Burgers-Huxley) المُعَممة

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ملخص

في هذا البحث، تم عرض طريقة فروق محدودة أسية واضحة لحل الصيغ المُعَممة من معادلات هوكسلي وبرجرز هوكسلي (Huxley and Burgers-Huxley). وتتيح هذه المخططات معالجة أي قيم ل δ. وتشير دقة الحلول العددية إلى أن الطريقة الحالية مناسبة تماماً لحل معادلات هوكسلي وبرجرز هوكسلي (Huxley and Burgers-Huxley) المُعَممة.