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# N-Legendre and N-slant curves in the unit tangent bundle of surfaces

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**Abstract**

Let $(T_1\mathbb{M}, g_1)$ be a unit tangent bundle of some surface $(\mathbb{M}, g)$ endowed with the induced Sasaki metric. In this paper, we define two kinds of curves called N-legendre and N-slant curves in which the inner product of its normal vector and Reeb vector is, respectively, equal to zero and to non-zero constant in $(T_1\mathbb{M}, g_1)$. Some several important characterizations of these curves are also obtained.

**Keywords:** N-legendre; N-slant; Sasaki metric; sectional curvature; unit tangent bundle.

**AMS Subject Classication:** 53B30, 53C50.

## 1. Introduction

In classical differential geometry of curves, a slant curve is a class of curves such that the angle between the curve's tangent vector field and the Reeb vector field $\xi$ is constant of a given contact metric manifold $(M, g, \phi, \xi, \eta)$. This class of curves is studied by many authors (Călin & Crasmareanu, 2013; Cho *et al*., 2006; Fukunaga & Takahashi, 2015; Janssens & Vanhecke, 1981; Zhong & Lei, 2013).

As an extension, in contact metric manifold $(M, g, \phi, \xi, \eta)$, we define two new kinds of curves called N-legendre and its generalization N-slant curves, which are the classes of curves whose normal vectors and Reeb vectors $\xi$ make an angle $\pi/2$ and a constant angle different from $\pi/2$, respectively.

In Sasaki (1962), Sasaki studied the geometry of the tangent bundle $TM$ endowed with the Sasaki metric $g_s$ of a Riemannian manifold $(M, g)$ and he showed that there exists an almost complex structure $J$ compatible with $g_s$ in $TM$. The unit tangent $T_1M$ is a hypersurface of $TM$. In Tashiro (1963), Tashiro constructed an almost contact metric structure $(g_1^s, \phi', \xi', \eta')$ on $T_1M$ which is endowed with the induced Sasaki metric $g_s$.

In Cho *et al*. (2006), it is proved that in Sasakian 3-manifold $M$, a curve $\gamma$ will be slant if and only if the ratio

$$\frac{\tau \pm 1}{\kappa} \tag{1}$$

is a non-zero constant. Here, $\tau$ and $\kappa$ are the torsion and curvature of $\gamma$, respectively.

In Zhong & Lei (2013), the slant curves in a unit tangent bundle $(T_1\mathbb{M}, g_1, \phi, \xi, \eta)$ of some surface $\mathbb{M}$ are studied only when the curves are geodesic. How about of a nongeodesic case?

In this paper, we study the non-geodesic case by defining two new kinds of curves called N-legendre and N-slant curves in any contact metric manifold. We obtain some results in $(T_1\mathbb{M}, g_1, \phi, \xi, \eta)$ and give some examples in $\mathbb{S}^2$, $\mathbb{R}^2$ and "non-constant sectional curvature of a surface".

## 2. Preliminaries

Let $(M, g)$ be a $n$-dimensional Riemannian manifold and $\nabla$ be the Levi-Civita connection of $g$. The tangent bundle $TM$ over $M$ consists of all pairs $p = (x, u)$, where $x$ is a point of $M$ and $u$ is a vector from the tangent space $T_xM$. We denote the natural projection of $TM$ to $M$ by $\pi(p) = \pi(x, u) = x$.

The tangent space $T_pTM$ of $TM$ at $p$ splits into the horizontal and vertical subspaces $H_p$ and $V_p$ with respect to $\nabla$ as

$$T_pTM = H_p \oplus V_p.$$

For a vector field $X$ on $M$, the horizontal lift of $X$ to a point $p$ in $TM$ is the unique vector $X^h \in H_p$ given by

$$\pi^*(X^h) = X$$

The vertical lift of $X$ to $p$ is the unique vector $X^v \in V_p$ such that

$$X^v(df) = X(f)$$

for all smooth functions $f$ on $M$.

The map

$$X \to X^h$$

is an isomorphism between the vector spaces $T_xM$ and $H_{(x,u)}$. Similarly, the map

$$X \to X^v$$

is an isomorphism between $T_xM$ and $V_{(p,u)}$. Each tangent vector $\tilde{X} \in T_{(p,u)}(TM)$ can be written in the form

$$\tilde{X} = X^h + Y^v$$

where $X, Y \in \mathcal{X}(M)$ are uniquely determined vectors.

For each system of local coordinates $(x_1, x_2, ..., x_n)$ in $M$, one defines, in the standard way, the system of local coordinates $(x_1, x_2, ..., x_n, u_1, u_2, ..., u_n)$ in $TM$. Let

$$X = X^i \frac{\partial}{\partial x_i}$$

be a local vector field on $M$. The vertical and the horizontal lifts of $X$ are defined, respectively, by

$$X^v = X^i \frac{\partial}{\partial u_i} \ ,$$

$$X^h = X^i \frac{\partial}{\partial x_i} - X^i u^j \Gamma_{ij}^k \frac{\partial}{\partial u_k}.$$

The canonical vertical vector field $\mathcal{U} = u^v$ on $TM$ does not depend of the choice of local coordinates and it is defined globally on $TM$. We define a Sasaki metric $g^S$ on a tangent bundle $TM$ of Riemannian manifold $(M, g)$ by

$$\begin{cases} i) \ g_p^s(X^h, Y^h) = g_p^s(X^v, Y^v) = g_x(X, Y) \\ ii) \ g_p^s(X^h, Y^v) = 0 \end{cases}$$

for all vector fields $X$ and $Y$ on $M$ and for all $p = (x, u) \in TM$. For more detail Hathout & Dida (2006) and Sasaki (1962).

The vertical and the horizontal lifts induce on $TM$ an almost complex structure $J$, which is compatible with $g^S$, that is, respectively,

$$JX^v = -X^h, \ JX^h = X^v.$$

## 3. Unit tangent bundle

The unit tangent bundle of a Riemannian manifold $(M, g)$ is the $(2n - 1)$-dimensional hypersurface

$$T_1M = \{p = (x, u) \in TM : g_x(u, u) = 1\}$$

where the canonical vertical vector field $\mathcal{U} = u^v$ is normal to $T_1M$.

Definition 1. The tangential lift of a vector field $X$ on $M$ is a vector field $X^t$ tangent to $T_1M$ defined by

$$X^t = X^v - g(X^v, \mathcal{U})\mathcal{U}.$$

The induced Riemannian metric $g_1^S$ on $T_1M$ is uniquely determined by

$$\begin{cases} i) \ g_1^s(X^h, Y^h) = g^s(X^h, Y^h) \\ ii) \ g_1^s(X^h, Y^t) = 0 \\ iii) \ g_1^s(X^t, Y^t) = g^s(X^v, Y^v) \\ \qquad -g^s(X^v, \mathcal{U})g^s(Y^v, \mathcal{U}) \end{cases} \tag{2}$$

where $X$ and $Y$ are vector fields on $(M, g)$.

Any vector $\tilde{X} \in T_{(x,u)}(T_1M)$ decomposes in

$$\tilde{X} = X^h + Y^t$$

where $X, Y \in \mathcal{X}(M)$ are uniquely determined vectors. The Lie algebra of $C^\infty$ vector fields on $T_1M$ is the set

$$\mathcal{X}(TM) = \left\{X^h + Y^t : X, Y \in \mathcal{X}(M)\right\}.$$

In Tashiro (1963), it is proved that there is a contact structure $(g_1^s, \phi', \xi', \eta')$ in $T_1M$. And in Blair *et al.* (2002) it is given that at any point $(x, u)$ we have

$$\begin{gathered} \xi' = u^h, \ \eta'(\tilde{X}) = g_1^s(\tilde{X}, \xi') \\ \phi'(\tilde{X}, \tilde{Y}) = g_1^s(\tilde{X}, \phi'\tilde{Y}) = 2d\eta'(\tilde{X}, \tilde{Y}) \\ \phi'(X^h) = X^t, \ \phi'(X^t) = -X^h + \eta'(X^h)\xi' \end{gathered} \tag{3}$$

where $X \in \mathcal{X}(M)$ and $\tilde{X}, \tilde{Y} \in \mathcal{X}(TM)$.

Hence, we know that there is a contact metric structure denoted by $(g_1, \phi_1, \xi_1, \eta_1)$ in $T_1M$ such that

$$\eta_1 = \frac{1}{2}\eta', \ \xi_1 = \frac{1}{2}\xi', \ \phi_1 = \phi', \ \ g_1 = \frac{1}{4}g_1^s \tag{4}$$

The Levi-Civita connection $\nabla_1$ of $(T_1M, g_1)$ is described completely by

$$\begin{cases} i)\ \nabla_{1\ X^h} Y^h = \left(\nabla^s_{1\ X^h} Y^h\right)^\top = (\nabla_X Y)^h - \frac{1}{2}(R(X,Y)u)^t \\ ii)\ \nabla_{1\ X^h} Y^t = \left(\nabla^s_{1\ X^h} Y^t\right)^\top = (\nabla_X Y)^t + \frac{1}{2}(R(u,Y)X)^h \\ iii)\ \nabla_{1\ X^t} Y^h = \left(\nabla^s_{1\ X^t} Y^h\right)^\top = \frac{1}{2}(R(u,X)Y)^h \\ iv)\ \nabla_{1\ X^t} Y^t = \left(\nabla^s_{1\ X^t} Y^t\right)^\top = -g(Y,u)X^t. \end{cases} \quad (5)$$

For more information see Blair (2002) and Zhong & Lei (2013).

## 4. N-Legendre and N-slant curves

In this section, we suppose that $(\mathbb{M}, g)$ is a surface with sectional curvature $\mathbb{K}$, $\gamma : I \subset R \to \mathbb{M}$ is a curve and $\tilde{\gamma}(t) = (\gamma(t), X(t))$ is also a curve in almost contact metric structure $(T_1\mathbb{M}, g_1, \phi, \xi, \eta)$ given in Equation (4).

Definition 2. (Zhong & Lei, 2013). The curve $\gamma$ is a slant curve in $(M, g, \phi, \xi, \eta)$ if the angle between the tangent vector field $T$ of $\gamma$ and $\xi$ is constant.

Now, we define two new kinds of curves called N-legendre and N-slant curves by the following definition.

Definition 3. Let $\gamma$ be a curve in an almost contact metric structure manifold $(M, g, \phi, \xi, \eta)$. The curve $\gamma$ is called a N-legendre (resp. N-slant) curve if the angle between the normal vector field $N$ of $\gamma$ and $\xi$ is $\pi/2$ (resp. $[0,\pi] - \{\pi/2\}$), i.e.

$$< N, \xi >= 0 \quad (\text{resp.} \quad < N, \xi >= c)$$

where $c$ is a non-zero constant.

Assume that the curve $\tilde{\gamma}(t)$ is parameterized by the arc-length with Frenet frame apparatus $\left(\tilde{T}, \tilde{N}, \tilde{B}, \tilde{\kappa}, \tilde{\tau}\right)$. Then,

$$\begin{cases} \tilde{T}(t) = \frac{d\gamma^i}{dt}\partial_{x^i} + \frac{dX^i}{dt}\partial_{u^i} \\ \quad = \frac{d\gamma^i}{dt}(\partial_{x^i})^h(\gamma(t)) \\ \quad + \left(\frac{dX^i}{dt} + \frac{d\gamma^i}{dt}X^k\Gamma^i_{jk}\right)\partial_{u^i}(\gamma(t)) \\ \quad = (E^h + (\nabla_E X)^v)(\gamma(t)) \end{cases} \quad (6)$$

where $E = \gamma'(t)$.

Let $\theta$ be the angle between $\tilde{T}$ and $\xi$. Using Equations (2) and (4), we get

$$g_1(\tilde{T}, \xi) = \cos\theta = \tfrac{1}{2}g(E, X). \quad (7)$$

Differentiating both side of the Equation (7) with respect to $s$, and using Equations (3),(4),(5) and (6), we have

$$\begin{aligned} \frac{d}{ds}g_1(\tilde{T},\xi) &= g_1(\nabla_{1\ \tilde{T}(t)}\tilde{T},\xi) + g_1(\tilde{T},\nabla_{1\ \tilde{T}(t)}\xi) \\ &= \tilde{\kappa}g_1(\tilde{N},\xi) + g_1(\tilde{T}(t),(2\nabla_E X \\ &+ R(u,\nabla_E X)u)^h + (R(E,u)u)^t) \\ &= \tilde{\kappa}g_1(\tilde{N},\xi) + 8g(E,\nabla_E X) - 8R(E,X,X,\nabla_E X) \\ &= -\theta'\sin\theta \end{aligned}$$

and

$$\begin{aligned} g_1(\tilde{N},\xi) = \tfrac{8}{\tilde{\kappa}}\Big(&R(E,X,X,\nabla_E X) \\ &-g(E,\nabla_E X)\Big) - \tfrac{\theta'\sin\theta}{\tilde{\kappa}} \end{aligned} \quad (8)$$

where $\xi(t) = 2X^h$ and $R$ is the curvature tensor of $\mathbb{M}$.

Let $(T, N)$ be a Frenet frame on $\gamma$. The unit vector $X$ can be expressed using Equation (7) by

$$X = \frac{2}{r}\cos\theta\ T + 2\beta N \quad (9)$$

where $r = g(E,E)$ and $\beta$ is $C^\infty$ function given from

$$\frac{4}{r^2}\cos^2\theta + 4\beta^2 = 1$$

by

$$\beta = \pm\frac{1}{r}\sqrt{(\tfrac{r}{2})^2 - \cos^2\theta}. \quad (10)$$

The differentiation of the Equation (9) with respect to $s$ gives

$$\begin{aligned} \nabla_E X &= 2\left(\tfrac{\cos\theta}{r}\right)' T + 2\kappa\cos\theta\ N + 2\beta' N - 2r\beta\kappa T \\ &= 2\left((\tfrac{\cos\theta}{r})' - r\beta\kappa\right)T + 2(\kappa\cos\theta + \beta')N. \end{aligned} \quad (11)$$

The vectors $X$ and $\nabla_E X$ are orthogonal (i.e $g(X, \nabla_E X) = 0$). Using Equations (11) and (7), the vector $E$ can be given by

$$E = 2\cos\theta\ X + \tfrac{2r}{\|\nabla_E X\|}\left((\tfrac{\cos\theta}{r})' - r\beta\kappa\right)\nabla_E X. \quad (12)$$

Substituting Equations (11), (10) and (12) into Equation (8) and taking an account that

$$\begin{aligned} R(E,X,X,\nabla_E X) &= 2r\left((\frac{\cos\theta}{r})' - r\beta\kappa\right)\frac{R(\nabla_E X, X, X, \nabla_E X)}{\|\nabla_E X\|} \\ &= 2r\left((\frac{\cos\theta}{r})' - r\beta\kappa\right)\mathbb{K}(s), \end{aligned}$$

where $\mathbb{K}(s)$ is the sectional curvature of $\mathbb{M}$, we get

$$\begin{aligned} g_1(\tilde{N},\xi) = 16r\tfrac{(1-\mathbb{K}(s))}{\tilde{\kappa}}\Big(&(\tfrac{\cos\theta}{r})' \\ &\pm r\kappa\sqrt{(\tfrac{r}{2})^2 - \cos^2\theta}\Big) - \tfrac{\theta'\sin\theta}{\tilde{\kappa}}. \end{aligned} \quad (13)$$

## 5. Main results

Proposition 4. In the unit tangent bundle of a unit sphere $\mathbb{S}^2$, all legendre and slant curves are $\tilde{N}$-legendre curves.

Proof. Let $\tilde{\gamma}(t) = (\gamma(t), X(t))$ be a legendre or slant curve in $T_1\mathbb{S}^2$ parameterized by the arc-length. In $\mathbb{S}^2$, the sectional curvature $\mathbb{K}$ is equal to 1, and using the Equation (13) we get

$$g_1(\tilde{N},\xi)=0.$$

From Definition (3), $\tilde{\gamma}$ is a $\tilde{N}$-legendre curve in $T_1\mathbb{S}^1$. Thus, the proof is completed.

Theorem 5. Let $\tilde{\gamma}$ be a non-slant curve in $T_1\mathbb{S}^2$, then $\tilde{\gamma}$ is $\tilde{N}$ -slant curve if the angle $\theta$ satisfies

$$\theta=\arccos c\int\tilde{\kappa}$$

where $c$ is a non-zero constant.

Proof. Let $\tilde{\gamma}(t)=(\gamma(t),X(t))$ be a non-slant curve in $T_1\mathbb{S}^2$ parameterized by the arc length. Taking account that $\mathbb{K}=1$, and using Definition (3) and Equation (13), we have

$$g_1(\tilde{N},\xi)=-\frac{\theta'\sin\theta}{\tilde{\kappa}}=c\quad\text{constant}$$

and

$$(\cos\theta)'=-c\tilde{\kappa}.$$

The resolution of a last ODE gives

$$\theta=\arccos c\int\tilde{\kappa}$$

and thus the proof is completed.

Proposition 6. Let $\tilde{\gamma}=(\gamma,X)$ be a slant curve in $T_1\mathbb{M}$ and $\gamma$ be a curve of velocity 2, where $\mathbb{M}$ is a non-unit sphere (i.e. $\mathbb{M}\neq\mathbb{S}^2$). If the torsion $\tilde{\tau}$ of the curve $\tilde{\gamma}$ is equal to $\mathbb{K}$, then $\tilde{\gamma}$ is $\tilde{N}$-legendre (resp. $\tilde{N}$-slant) curve if and only if $\gamma$ is a geodesic (resp. has a non-zero constant curvature $\kappa$).

Proof. Let $\tilde{\gamma}=(\gamma,X)$ be a slant curve in $T_1\mathbb{M}$ and $\gamma$ be a curve of velocity 2, where $\mathbb{M}$ is a non-unit sphere (i.e. $\mathbb{M}\neq\mathbb{S}^2$). If we suppose that $\tilde{\tau}=\mathbb{K}$ and using Equations (1) and (13), we get

$$\begin{aligned}g_1(\tilde{N},\xi)&=32\frac{(1-\tilde{\tau})}{\tilde{\kappa}}(\pm2\kappa)\\&=\overline{c}\kappa,\end{aligned}$$

where $\overline{c}$ is a non-zero constant. Thus, the curve $\tilde{\gamma}$ is $\tilde{N}$ -legendre (resp. $\tilde{N}$-slant) curve if and only if $\kappa=0$ (resp. nonzero constant). This completes the proof.

Example 7. Let $\mathbb{S}^2_p$ be a unit sphere of diameter $p$. All slant and $\tilde{N}$-slant curves $\tilde{\gamma}$ in $T_1\mathbb{S}^2_p$ of Frenet frame apparatus $\left(\tilde{T},\tilde{N},\tilde{B},\tilde{\kappa},1/p^2\right)$ are curves of projection curves $\gamma$, the circle in $\mathbb{S}^2_p$.

Theorem 8. Let $\tilde{\gamma}=(\gamma,X)$ be a slant curve in $T_1\mathbb{M}$ and $\gamma$ be a curve of velocity 2, where $\mathbb{M}$ is a non-unit sphere (i.e. $\mathbb{M}\neq\mathbb{S}^2$). The curve $\tilde{\gamma}$ is $\tilde{N}$-slant curve if and only if the ratio

$$\frac{(1-\mathbb{K})\kappa}{\tilde{\kappa}}$$

is a non-zero constant.

Proof. Let $\tilde{\gamma}(t)=(\gamma(t),X(t))$ be a slant curve in $T_1\mathbb{M}$ parameterized by the arc-length and suppose that the curve $\gamma$ has velocity 2. From Equation (13), we have

$$\begin{aligned}g_1(\tilde{N},\xi)&=\pm16r\frac{(1-\mathbb{K})}{\tilde{\kappa}}\kappa\\&=\pm(32(1-\mathbb{K}))\frac{\kappa}{\tilde{\kappa}}\\&=c\frac{(1-\mathbb{K})\kappa}{\tilde{\kappa}},\end{aligned}$$

where $c$ is a non-zero constant. Then, using Definition (3), we can say that $\tilde{\gamma}$ is a $\tilde{N}$-slant curve if and only if the ratio

$$\frac{(1-\mathbb{K})\kappa}{\tilde{\kappa}}$$

is a non-zero constant which completes the proof.

Example 9. Under the condition of the Thm. 8, in $T_1\mathbb{R}^2$ all slant curves $\tilde{\gamma}$ with a geodesic curve projection $\gamma$ are $\tilde{N}$-legendre curves. If $\gamma$ is not geodesic then $\tilde{\gamma}$ is $\tilde{N}$-slant curve if and only if the ratio

$$\frac{\kappa}{\tilde{\kappa}}$$

is a non-zero constant.

Theorem 10. Let $\tilde{\gamma}=(\gamma,X)$ be a non-slant curve in $T_1\mathbb{M}$ and $\gamma$ be a curve of velocity 2, where $\mathbb{M}$ is a non-unit sphere (i.e. $\mathbb{M}\neq\mathbb{S}^2$). If the angle $\theta$ is linear, then

1. The curve $\tilde{\gamma}$ is $\tilde{N}$-legendre curve if and only if

$$(\mathbb{K}-1)(a\pm\kappa)=a/16$$

for constant $a=\theta'/4$.

2. The curve is $\tilde{N}$-slant curve if and only if

$$\theta=\arcsin\left(\frac{c\tilde{\kappa}}{16(\mathbb{K}(s)-1)(a_1\pm4\kappa)-a_1}\right)$$

where $a_1=\theta'$ and $c$ are non-zero constants.

Proof. Let $\tilde{\gamma}(t)=(\gamma(t),X(t))$ be a non-slant curve in $T_1\mathbb{M}$, where $\mathbb{M}$ is a non-unit sphere, which means that

$\mathbb{K} \neq 1$. And let be $\gamma$ a curve with a velocity 2. If the angle $\theta$ is linear (i.e. $\theta = a_1 t + b$), then the Equation (13) turns into

$$g_1(\tilde{N}, \xi) = 16\frac{(1-\mathbb{K}(s))}{\tilde{\kappa}}\big( - a_1 \sin\theta \pm 4\kappa \sin\theta\big) - \frac{a_1 \sin\theta}{\tilde{\kappa}}. \quad (14)$$

The $\tilde{N}$-legendre condition of the curve $\tilde{\gamma}$ means that the Equation (14) vanishes. Hence

$$16\frac{(1-\mathbb{K}(s))}{\tilde{\kappa}}(-a_1 \sin\theta \pm 4\kappa \sin\theta) = \frac{a_1 \sin\theta}{\tilde{\kappa}}$$

and thus

$$(\mathbb{K}-1)(a \pm \kappa) = a/16$$

with $a = a_1/4$. Thus, item 1 in Theorem (10) is proved.

The $\tilde{N}$-slant condition of the curve $\tilde{\gamma}$ gives

$$g_1(\tilde{N}, \xi) = c = 16\frac{(1-\mathbb{K}(s))}{\tilde{\kappa}}(-a_1 \sin\theta \pm 4\kappa \sin\theta) - \frac{a_1 \sin\theta}{\tilde{\kappa}},$$

where $c$ is a non-zero constant, and thus we get

$$\sin\theta = \frac{c\tilde{\kappa}}{16(1-\mathbb{K}(s))(-a_1 \pm 4\kappa) - a_1}$$

$$\theta = \arcsin\left(\frac{c\tilde{\kappa}}{16(1-\mathbb{K}(s))(-a_1 \pm 4\kappa) - a_1}\right).$$

Thus, item 2 in Theorem (10) is proved.

## 6. Conclusion

One of the important hypersurface of the tangent bundle $TM$ is the unit tangent bundle $T_1M$ which is an almost contact manifold. Zhong *et al*. (2013), studied the slant curves, which are generalizations of the Legendrian curves in $T_1M$, as curves whose tangent vector make a constant angle with the Reeb vector field. In this paper, we studied N-Legendre and N-slant curves whose normal vector make a constant angle with the Reeb vector field. Also, some important characterizations about these curves are given in $\mathbb{S}^2$, $\mathbb{R}^2$, "a surface of sectional curvature of $\mathbb{K} \neq 1$" and "a linear angle case".

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# منحنيات ان-ليجندر N-Legendre و ان-سلانت N-Slant في وحدة حزمة مماس الأسطح

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## ملخص

لنفرض أن $(T_1\mathbb{M}, g_1)$ هي وحدة حزمة مماس لبعض الأسطح $(\mathbb{M}, g)$ مقدمة مع مقياس ساساكي (Sasaki) المُستحث. في البحث الحالي، نحدد نوعين من المنحنيات تسمى منحنيات ان-ليجندر (N-Legendre) و ان-سلانت (N-Slant) كمنحنيات لديها ناتج ضرب داخلي لمتجه طبيعي ومتجه ريب (Reeb) يعادل الثابت الصفري واللاصفري، على التوالي، وتم الحصول على العديد من الخصائص الهامة لهذه المنحنيات.