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New exact solutions for generalized Gardner equation

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Abstract

In this research, we seek exact solutions of generalized Gardner equation. We make use of extended trial equation method to attain exact solutions of generalized Gardner equation. Firstly, we find some exact solutions such as soliton solutions, rational, Jacobi elliptic and hyperbolic function solutions of generalized Gardner equation by the help of extended trial equation method. Then, for some parameters, we draw two and three dimensional graphics of imaginary and real values of some exact solutions that we acquired by using this method.

Keywords: Extended trial equation method; generalized Gardner equation; hyperbolic function solutions; Jacobi elliptic solutions; soliton solutions; rational solutions.

1. Introduction

The research of exact solutions of nonlinear evolution equations (NLEEs) serves a highly important purpose in the observation of certain physical circumstances. The diversity of solutions of NLEEs occupies an important position in a lot of sciences such as biology, optical fibers, hydrodynamics, meteorology, chemical kinematics and electromagnetic theory.

In recent years, many authors have introduced several methods to look for exact solutions of NLEEs such as $\exp\left(-\phi(\xi)\right)$ -expansion method (Khan & Akbar, 2013a; Akter *et al.*, 2014), the modified simple equation method (Khan & Akbar, 2013b; Khan *et al.*, 2013; Khan & Akbar, 2014a; Khan & Akbar, 2014b), exp-function method (Khan & Akbar, 2014a), the enhanced (G'/G)-expansion method (Khan *et al.*, 2014; Islam *et al.*, 2013), generalized Kudryashov method (Tuluce Demiray *et al.*, 2014a; Bulut *et al.*, 2014a; Tuluce Demiray *et al.*, 2014b) and so on. In this study, ETEM (Pandir *et al.*, 2012; Pandir *et al.*, 2013; Bulut, 2013; Bulut *et al.*, 2014b; Tuluce Demiray & Bulut, 2015) will be used to find exact solutions of generalized Gardner equation.

Firstly, Gardner equation was presented nearly half a century ago (Miura, 1968; Miura *et al.*, 1968). The Gardner equation is an expansion of KdV equation. It introduces greatly the identical features as the classical KdV, but expands its order of availability to a larger interval of the parameters of the interior wave motion for a certain surrounding (Hamdi *et al.*, 2011). The event of shallow water wave is formed by Gardner equation, which is constitutively the KdV equation with dual power nonlinearity (Wazwaz, 2010). Experimental investigations have shown that Gardner equation models deep ocean waves rather than shallow water waves that is used by KdV equation (Daoui & Triki, 2014). This equation is very greatly handled in different fields such as plasma physics, quantum field theory, fluid dynamics, solid state physics and so on (Jawad, 2012).

Many scientists have handled different methods to seek exact solutions of generalized Gardner equation such as simple and direct method, generalized wave transformation, G'/G-expansion method, tan-cot function method, solitary wave ansatz method, Lie symmetries, general mapping deformation method, exp-function method combined with F-expansion method, a new Bernoulli sub-ODE method, tanh method, U-expansion method, auxiliary equation method, bilinear method, extended homoclinic test approach, sub-ODE method, restrictive Taylor approximation, Weierstrass and Jacobi elliptic functions, tanh-function method, the improved sub-ODE method, geometric singular perturbation method, etc. (Hamdi *et al.*, 2011; Wazwaz, 2010; Daoui & Triki, 2014; Jawad, 2012; Kumar *et al.*, 2013; Vaneeva *et al.*, 2015; Hong & Lu, 2012; Li & Ma, 2011; Zheng, 2011; Wazwaz, 2007; Usman *et al.*, 2013; Yang, 2012; Abdou, 2010; Lu & Liu, 2010; Rageh *et al.*, 2014; Zayed & Abdelaziz, 2011, Vassilev *et al.*, 2011; Zayed & Abdelaziz, 2010; Alejo, 2012; Zhang *et al.*, 2008; Li & Wang, 2007; Tang *et al.* 2008).

We investigate the following generalized Gardner equation (Zheng, 2011; Yang, 2012; Lu & Liu, 2010; Zhang *et al.*, 2008; Li & Wang, 2007),

$$u_t+\left(p+qu^n+ru^{2n}\right)u_x+u_{xxx}=0,\ n\geq 0, \tag{1}$$

where $u\,(x,t)$ is a function of two independent variables x and t that indicate the space variable in the direction of wave propagation and time, respectively. $u\,(x,t)$ denotes the amplitude of the relevant wave mode, the terms uu_x and u^2u_x demonstrate nonlinear wave steepening and u_{xxx} shows dispersive wave effects. The coefficients p,q and r are constants, which are explained by the steady oceanic background density and flow stratification through the linear eigenmode of interior waves (Hamdi *et al.*, 2011). If $n=1, q\neq 0, r\neq 0$, Equation (1) returns to the KdV-mKdV equation. If $n=1, q\neq 0, r=0$, Equation (1) transforms the KdV equation. If $n=1, q=0, r\neq 0$, Equation (1) converts to the mKdV equation (Zheng, 2011).

Our purpose in this work is to obtain exact solutions of generalized Gardner equation. In Section 2, we present basic structure of extended trial equation method (Pandir *et al.*, 2012; Pandir *et al.*, 2013; Bulut, 2013; Bulut *et al.*, 2014b; Tuluce Demiray & Bulut, 2015). In Sec. 3, as an example, we find exact solutions of generalized Gardner equation via extended trial equation method and then we draw two and three dimensional graphics of imaginary and real values of some exact solutions that we obtained by using this method.

2. Basic structure of the extended trial equation method

Step1. For a nonlinear partial differential equation (NLPDE)

$$P\left(u,u_t,u_x,u_{xx},\ldots\right)=0, \tag{2}$$

get the wave transformation

$$u\left(x_1,x_2,\ldots,x_N,t\right)=u\left(\eta\right),\eta=k\left(\sum_{j=1}^{N}x_j-\lambda t\right), \tag{3}$$

where $k\neq 0$ and $\lambda\neq 0$. Substituting Equation (3) into Equation (2) converts a nonlinear ordinary differential equation (NLODE),

$$N\left(u,u',u'',\ldots\right)=0. \tag{4}$$

Step2. Take transformation and trial equation as following:

$$u=\sum_{i=0}^{\delta}\tau_i\Gamma^i, \tag{5}$$

where

$$\left(\Gamma'\right)^2=\Lambda\left(\Gamma\right)=\frac{\phi\left(\Gamma\right)}{\psi\left(\Gamma\right)}=\frac{\xi_\theta\Gamma^\theta+\ldots+\xi_1\Gamma+\xi_0}{\zeta_\in\Gamma^\in+\ldots+\zeta_1\Gamma+\zeta_0}. \tag{6}$$

Taking into consideration Equations (5) and (6), we can get

$$\left(u'\right)^2=\frac{\phi\left(\Gamma\right)}{\psi\left(\Gamma\right)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)^2, \tag{7}$$

$$\begin{aligned}u''=&\frac{\phi'\left(\Gamma\right)\psi\left(\Gamma\right)-\phi\left(\Gamma\right)\psi'\left(\Gamma\right)}{2\psi^2\left(\Gamma\right)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)\\&+\frac{\phi\left(\Gamma\right)}{\psi\left(\Gamma\right)}\left(\sum_{i=0}^{\delta}i\left(i-1\right)\tau_i\Gamma^{i-2}\right),\end{aligned} \tag{8}$$

where $\phi\left(\Gamma\right)$ and $\psi\left(\Gamma\right)$ are polynomials. Substituting these terms into Equation (4) enables an equation of polynomial $\Omega\left(\Gamma\right)$ of Γ:

$$\Omega\left(\Gamma\right)=\sigma_s\Gamma^s+\ldots+\sigma_1\Gamma+\sigma_0=0. \tag{9}$$

With respect to balance rule, we can define a formula of $\theta,\in$ and δ. We can get some values of $\theta,\in$ and δ.

Step 3. Let the coefficients of $\Omega\left(\Gamma\right)$ all be zero will satisfy an algebraic equation system:

$$\sigma_i=0,\quad i=0,\ldots s. \tag{10}$$

Solving this equation system (10), we will identify the values of $\xi_0,\ldots,\xi_\theta;\ \ \zeta_0,\ldots,\zeta_\in$ and $\tau_0,\ldots,\tau_\delta$.

Step 4. Simplify Equation (6) to elementary integral form,

$$\pm\left(\eta-\eta_0\right)=\int\frac{d\Gamma}{\sqrt{\Lambda\left(\Gamma\right)}}=\int\sqrt{\frac{\psi\left(\Gamma\right)}{\phi\left(\Gamma\right)}}d\Gamma. \tag{11}$$

Applying a complete discrimination system for polynomial to classify the roots of $\phi\left(\Gamma\right)$, we solve the infinite integral (11) and categorize the exact solutions of Equation (2) via Mathematica (Pandir *et al.*, 2012).

3. Implementation of the extended trial equation method to generalized gardner equation

In this section, we seek the exact solutions of generalized Gardner equation (Zheng, 2011; Yang, 2012; Lu & Liu, 2010; Zhang *et al.*, 2008; Li & Wang, 2007) by using extended trial equation method.

In an attempt to obtain travelling wave solutions of the Equation (1), we get the transformation by using the wave variables

$$u(x,t)=u(\eta),\ \eta=k(x-\lambda t),\tag{12}$$

where k and λ are arbitrary constants.

Substituting Equation (13) into Equation (1),

$$u_t=-k\lambda u',\ u_x=ku',\ u_{xxx}=k^3u''',\tag{13}$$

we find

$$k(p-\lambda)u+qk\frac{u^{n+1}}{n+1}+rk\frac{u^{2n+1}}{2n+1}+k^3u''=0.\tag{14}$$

Taking into consideration the transformation

$$u(\eta)=v^{\frac{1}{n}}(\eta),\tag{15}$$

Equation (14) reduces to the following equation

$$\begin{aligned}&(p-\lambda)v^2+\frac{q}{n+1}v^3+\frac{r}{2n+1}v^4\\&+\frac{k^2(1-n)}{n^2}(v')^2+\frac{k^2}{n}vv''=0.\end{aligned}\tag{16}$$

Substituting Equations (5) and (8) into Equation (16) and using the balance principle in Equation (16), we obtain

$$\theta=2\delta+\in+2.\tag{17}$$

In an attempt to find exact solutions of Equation (1), if we choose $\in=0,\delta=1$ and $\theta=4$ in Equation (17), then

$$\begin{aligned}(v')^2&=\frac{\tau_1^2\left(\xi_0+\Gamma\xi_1+\Gamma^2\xi_2+\Gamma^3\xi_3+\Gamma^4\xi_4\right)}{\zeta_0},\\v''&=\frac{\tau_1\left(\xi_1+2\Gamma\xi_2+3\Gamma^2\xi_3+4\Gamma^3\xi_4\right)}{2\zeta_0},\end{aligned}\tag{18}$$

where $\xi_4\neq 0,\zeta_0\neq 0$. Respectively, solving equation system (10) provides

$$\begin{aligned}\xi_0&=\frac{[-2X+Y]^2\left[12X^2-5Y^2+4XY\right]}{256(2+n)^4r^4\xi_4^3\tau_1^4}\\&+\frac{[-2X+Y]^2\left[16(2+n)^2r^2\xi_2\xi_4\tau_1^2\right]}{256(2+n)^4r^4\xi_4^3\tau_1^4},\\\xi_1&=\frac{1}{8\xi_4^2}\left[-\xi_3^3+4\xi_2\xi_3\xi_4-\frac{4X^3\xi_3^3}{Y^3}\right]\\&+\frac{1}{8\xi_4^2}\left[\frac{X\xi_3\left(3\xi_3^2-8\xi_2\xi_4\right)}{Y}\right],\\\xi_2&=\xi_2,\xi_3=\xi_3,\xi_4=\xi_4,\end{aligned}\tag{19}$$

$$\begin{aligned}\zeta_0&=-\frac{k^2(1+n)(1+2n)\xi_4}{n^2r\tau_1^2},\\\lambda&=p-\frac{3q^2(1+2n)}{2r(1+n)(2+n)^2}\\&+\frac{r\left(3\xi_3^2-8\xi_2\xi_4\right)\tau_1^2}{8\left(1+3n+2n^2\right)\xi_4^2},\\\tau_0&=\frac{1}{4}\left[-\frac{2q(1+2n)}{(2+n)r}+\frac{\xi_3\tau_1}{\xi_4}\right],\tau_1=\tau_1,\end{aligned}$$

where $X=q(1+2n)\xi_4$ and $Y=r(2+n)\xi_3\tau_1$.

Substituting these results into Equations (6) and (11), we get

$$\pm(\eta-\eta_0)=A\int\frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_4}+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_3}{\xi_4}\Gamma^3+\Gamma^4}},\tag{20}$$

where $A=\sqrt{-\frac{k^2(1+n)(1+2n)\xi_4}{n^2r\tau_1^2}}$.

Integrating Equation (20), we obtain the solutions of Equation (1) as following:

$$\pm(\eta-\eta_0)=-\frac{A}{\Gamma-\alpha_1},\tag{21}$$

$$\pm(\eta-\eta_0)=\frac{2A}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\quad \alpha_2>\alpha_1,\tag{22}$$

$$\pm(\eta-\eta_0)=\frac{A}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|,\quad \alpha_1>\alpha_2,\tag{23}$$

$$\begin{aligned}&\pm(\eta-\eta_0)=\frac{2A}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\times\\&\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\\&\alpha_1>\alpha_2>\alpha_3,\end{aligned}\tag{24}$$

$$\begin{aligned}&\pm(\eta-\eta_0)=\frac{2A}{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\\&\alpha_1>\alpha_2>\alpha_3>\alpha_4,\end{aligned}\tag{25}$$

where

$$
\begin{aligned}
&F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},\\
&\varphi=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\\
&l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}.
\end{aligned}
\tag{26}
$$

Also, $\alpha_1,\alpha_2,\alpha_3$ and α_4 are the roots of the following equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0. \tag{27}$$

Substituting the solutions (21-25) into (5) and (15), by using Equation (12), the solutions of Equation (1) are obtained rational function solutions,

$$u_1(x,t)=\left(\pm\frac{A_1}{k(x-\lambda t)}\right)^{\frac{1}{n}}, \tag{28}$$

$$u_2(x,t)=\left(\frac{4A^2(\alpha_2-\alpha_1)\tau_1}{4A^2-\left[(\alpha_1-\alpha_2)(k(x-\lambda t))\right]^2}\right)^{\frac{1}{n}}, \tag{29}$$

travelling wave solution

$$
\begin{aligned}
u_3(x,t)=&\left(\frac{(\alpha_2-\alpha_1)\tau_1}{2}\right)^{\frac{1}{n}}\times\\
&\left\{1\pm\coth\left[\frac{(\alpha_1-\alpha_2)}{A}(k(x-\lambda t))\right]\right\}^{\frac{1}{n}},
\end{aligned}
\tag{30}
$$

soliton solution

$$u_4(x,t)=\frac{A_2}{\left(D+\cosh\left[B(k(x-\lambda t))\right]\right)^{\frac{1}{n}}}, \tag{31}$$

and Jacobi elliptic function solution

$$u_5(x,t)=\frac{A_3}{\left(M+N\,sn^2(\varphi,l)\right)^{\frac{1}{n}}}, \tag{32}$$

where

$$
\begin{aligned}
&A_1=\tau_1 A,\ A_2=\left(\frac{2\tau_1(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{\alpha_3-\alpha_2}\right)^{\frac{1}{n}},\\
&B=\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A},D=\frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2},\\
&A_3=\left(2\tau_1(\alpha_1-\alpha_3)(\alpha_4-\alpha_2)\right)^{\frac{1}{n}},\\
&M=\alpha_4-\alpha_2,\ N=\alpha_1-\alpha_4,\\
&l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)},\\
&\varphi=\frac{\pm\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(k(x-\lambda t)).
\end{aligned}
$$

Herein, A_2 denote the amplitude of the soliton, and B indicate the inverse width of the solitons. Thus, we can say that the solitons exist for $\tau_1<0$.

Remark 1. If the modulus $l\to 1$, then by using Equation (12), the solution (32) can be converted to the hyperbolic function solutions

$$u_6(x,t)=\frac{A_3}{\left(M+N\tanh^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(k(x-\lambda t))\right]\right)^{\frac{1}{n}}}, \tag{33}$$

where $\alpha_3=\alpha_4$.

Remark 2. If the modulus $l\to 0$, then by using Equation (12), the solution (32) can be turned to the periodic wave solutions

$$u_7(x,t)=\frac{A_3}{\left(M+N\sin^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(k(x-\lambda t))\right]\right)^{\frac{1}{n}}}, \tag{34}$$

where $\alpha_2=\alpha_3$.

In Figure 1, we plot two and three dimensional graphics of Equation (30), which demonstrate the vitality of solutions with suitable parametric choices. Moreover, in Figure 2, we draw two and three dimensional graphics of Equation (31), which indicate the dynamics of solutions with appropriate parametric selections. Finally, in Figure 3, we plot two and three dimensional graphics of Equation (32), which denote the vitality of solutions with certain parametric options.

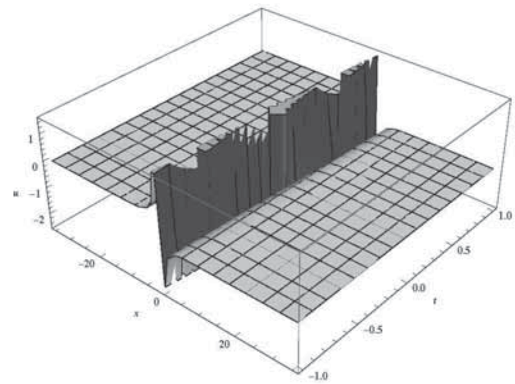

Fig. 1-(a)

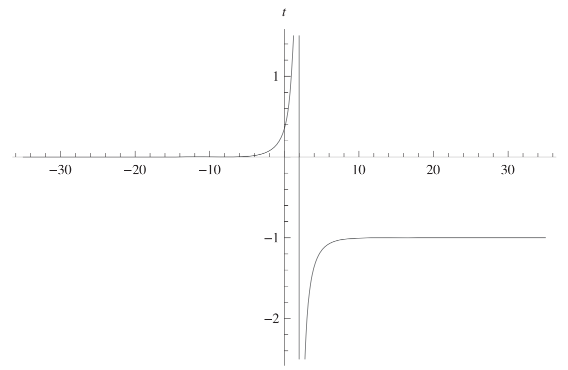

Fig. 1-(b)

Fig.1. Graph of Equation (30) is demonstrated at $k=1, \lambda=2, r=2, n=1, \tau_1=1, \xi_4=-3, \alpha_1=3, \alpha_2=2, -35<x<35, -1<t<1$ and the second graph indicates Equation (30) for $-35<x<35,\ t=1$.

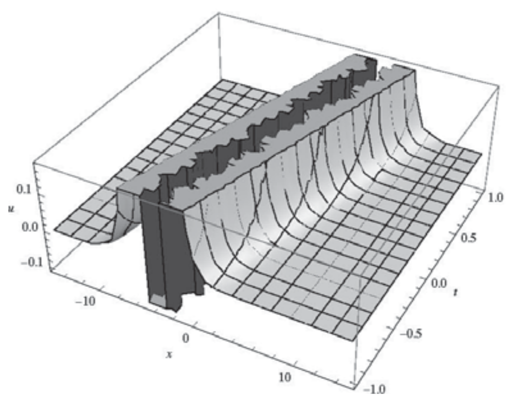

Fig. 2-(a)

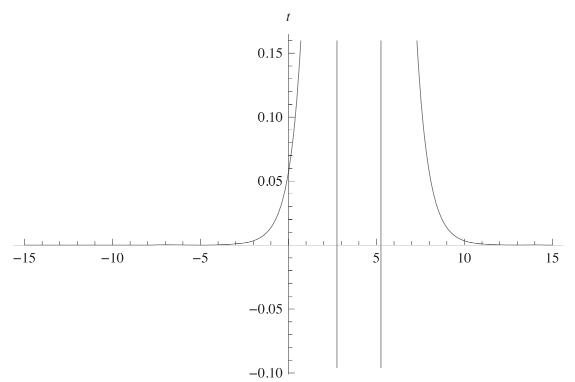

Fig. 2-(b)

Fig.2. Graph of Equation (31) is demonstrated at $k=3, \lambda=4, r=3, n=1, \tau_1=2, \xi_4=-2, \alpha_1=1, \alpha_2=2, \alpha_3=3, -15<x<15, -1<t<1$ and the second graph indicates Equation (31) for $-15<x<15,\ t=1$.

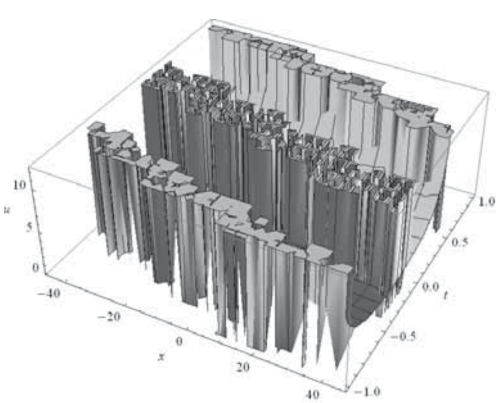

Fig. 3-(a)

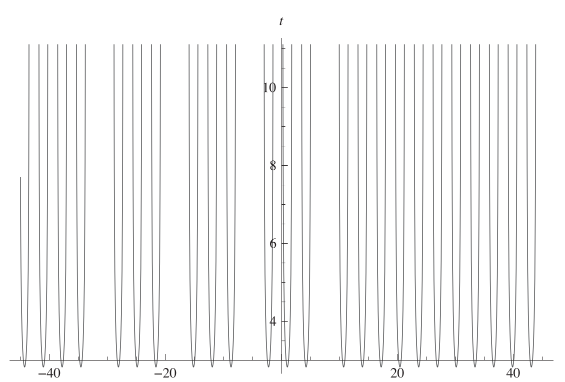

Fig. 3-(b)

Fig.3. Graph of Equation (32) is demonstrated at $k=4, \lambda=1, r=5, n=2, \tau_1=-2, \xi_4=-3, \alpha_1=1, \alpha_2=2, \alpha_3=3, \alpha_4=4, -45<x<45, -1<t<1$ and the second graph indicates Equation (32) for $-45<x<45,\ t=1$.

Remark 3. The exact solutions of Equation (1) were found by using extended trial equation method, have been calculated by means of Mathematica Release 9. If we compare with the exact solutions of generalized Gardner equation reported by the other authors, we have acquired the similar solution with the solution Equation (3.15) in (Lu & Liu, 2010) in this study as the solution Equation (29). Also, we have procured the similar solution with the solution Equation (52) in (Daoui & Triki, 2014), the solution Equation (3.12) in (Lu & Liu, 2010) and the solution Equation (31) in (Zhang *et al.*, 2008) in this study as the solution Equation (30). Furthermore, we have attained the similar solution with the solution Equation (16) in (Yang, 2012) in this study as the solution Equation (31). To our knowledge, other solutions of Equation (1) that we obtained here, are new and are not trackable in the former literature.

4. Conclusion

In this work, we find exact solutions of generalized Gardner equation by using ETEM. Besides, we obtain exact solutions including Jacobi elliptic function, hyperbolic function and periodic wave solution of generalized Gardner equation via ETEM. These functions yield a system of differential equations. Particularly, Jacobi elliptic function solutions are of substantial applications of periodic meromorphic functions. There are a lot of examples of these functions in the applied sciences such as fluid dynamics, optical fibers, electromagnetic theory, special relativity and heat transfer in several fields of physics. These solutions give to us numerous various aspects of the solutions of NLEEs. Moreover, we plot two-dimensional and three-dimensional graphics of these exact solutions.

From these results, the advantage of this method is that ETEM supplies powerful mathematical tools for obtaining the analytical solutions of GGE and this method is highly influential in terms of looking for new solutions such as soliton solutions, rational, Jacobi elliptic and hyperbolic function solutions. It is quite obvious that not only extended trial equation method plays important position in researching NLEEs but also it is substantially forceful in providing analytical solutions of NLEEs. We believe that this method can also be performed to other NLEEs which arising in the theory of solitons.

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حلول مضبوطة جديدة لمعادلة غاردنر المعممة

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خلاصة

نبحث في هذه الورقة عن حلول مضبوطة لمعادلة غاردنر المعممة. نستفيد في مسعانا من استخدام طريقة المعادلة المحاولة الممتدة للوصول إلى حلول مضبوطة لمعادلة غاردنر المعممة. نقوم أولاً بإيجاد حلول مضبوطة مثل حلول سوليتون وكذلك حلول الدوال الزائدية، الناقصية، النسبية و حلول جاكوبي، لمعادلة غاردنر المعممة وذلك بمساعدة طريقة المحاولة الممتدة. نقوم بعد ذلك باستخدام بعض الوسيطات لرسم بيانات ثنائية و ثلاثية الأبعاد لقيم حقيقية و تخيلية لبعض الحلول المضبوطة التي حصلنا عليها باستخدام هذه الطريقة.