Kuwait J. Sci. **44 (4)** pp 83-90, 2017

A simple way of improving the Bar–Lev, Bobovitch and Boukai Randomized response model

Tanveer A. Tarray[1],*, Housila P. Singh[2]

[1]*Dept. of Computer Science and Engineering, Islamic University of Science and Technology, Awantipora, Pulwama, Kashmir, India -192122*
[2]*School of Studies in Statistics, Vikram University, Ujjain, M.P., India-456010*
**Corresponding author: tanveerstat@gmail.com*

Abstract

Eichhorn & Hayre (1983) considered a randomized response procedure suitable for estimating the mean response, when the sensitive variable under investigation is quantitative in nature. They have obtained an estimate for the mean of the quantitative response variable under investigation and studied its properties. Bar–Lev *et al*. (2004) have suggested an alternative procedure, which use a design parameter (controlled by the experimenter) that generalizes Eichhorn & Hayre's (1983) results. They have also proved that the estimator proposed by them has uniformly smaller variance as compared to that of Eichhorn & Hayre (1983) in certain condition. In this paper we have suggested a simple procedure of improving the Eichhorn & Hayre (1983) and Bar–Lev *et al*. (2004) models along with its properties. It has been shown that the proposed procedure is uniformly better than Bar–Lev *et al*. (2004) procedure. The proposed procedure is also uniformly better than Eichhorn and Hayre's (1983) procedure under the same condition in which the Bar–Lev *et al*.'s (2004) procedure is more efficient than Eichhorn & Hayre's (1983) procedure. Numerical illustration is given in support of the present study.

Keywords: Estimation of proportion; randomized response sampling; respondents protection; sensitive quantitative variable.

AMS Subject Classification: 62D05.

1. Introduction

The problem of estimating the population mean of a sensitive quantitative variable is well recognized in survey sampling. Randomized response techniques (RRT) have been extensively used for personal interview surveys, ever since the pioneering work of Warner (1965). A rich growth of literature can be found in Fox & Tracy (1986), Chaudhuri & Mukerje (1988), Singh (2003) and among others. For recent references readers are referred to Gjestvang & Singh (2006, 2009), Bar–Lev *et al*. (2004), Singh & Mathur (2004, 2005), Odumade & Singh (2008, 2009), Hussain (2012), Singh & Tarray (2013, 2016) and Tarray and Singh (2015, 2016, 2017). The present study rely on the models suggested by Eichhorn & Hayre (1983) and Bar–Lev *et al*. (2004) so the description of these models are respectively given in section 1.1 and 1.2.

1.1 Eichhorn & Hayre (1983) procedure:

Eichhorn & Hayre (1983) suggested a multiplicative model to collect information on sensitive quantitative variables like income, tax evasion and amount of drug used. By their procedure, the interviewees are asked about their value of sensitive response variable. In return, they are allowed to respond with a coded (or scrambled) value composed of their true value for the variable of interest, multiplied by some random number. The interviewer does not know which random number was used by each of the interviewees for coding their responses, but fully knows the underling distribution which generated the coding number.

Let X be a random variabledenoting the quantitative response variable of interest and let S be a random variable denoting the random number used in the coding mechanism. Suppose that X (≥ 0) is independent of S and let Y = SX the coded response returns to the interviewee to the sensitive question, see Bar–Lev *et al*. (2004, p. 256). It is assumed that the distribution of the scrambling variable S is known. In other words,

$$\mu_x = E(X), \mu_s = E(S),\ \sigma_x^2 = V(X), \gamma^2 = V(S)$$

where μ_s and γ^2 are known and μ_x and σ_x^2 are unknown. We also denote $C_x = \sigma_x / \mu_x$ and $C_s = \gamma / \mu_s$ for the coefficient of variation of X and of S, respectively. The mean and variance of Y= XS are respectively given by

$$E(Y) = \mu_x \mu_s \tag{1}$$

and

$$V(Y) = \sigma_x^2 \mu_s^2 + \mu_x^2 (1 + C_x^2)\gamma^2 . \tag{2}$$

Eichhorn & Hayre (1983) based on a random sample

$(Y_1, Y_2, \ldots, Y_n)$ of coded (scrambled) responses suggested an unbiased estimator of the mean μ_x of the sensitive variable X as

$$\hat{\mu}_x = \overline{Y} / \mu_s . \quad (3)$$

where $\overline{Y} = \sum_{i=1}^{n} Y_i / n$, is the sample mean of the n coded responses. The variance of the estimator $\hat{\mu}_x$ is given by

$$V(\hat{\mu}_x) = \frac{\mu_x^2}{n}\left[C_x^2 + C_s^2(1 + C_x^2)\right] \quad (4)$$

which is larger than that resulting from a simple random sample with direct interviews; namely σ_x^2/n.

1.2 Bar–Lev, Bobovitch & Boukai's (2004) procedure

Exploiting both, the randomizing mechanism used in Warner's (1965) original randomized response model and the quantitative coding scheme in Eichhorn & Hayre (1983), Bar–Lev *et al*. (2004) have suggested a procedure whose description is given below.

In Bar–Lev, Bobovitch & Boukai (BBB, 2004) model, the distribution of the responses is given by

$$Y_i = \begin{cases} X_i S \text{ with probability } (1-P) \\ X_i \text{ with probability } P \end{cases} . \quad (5)$$

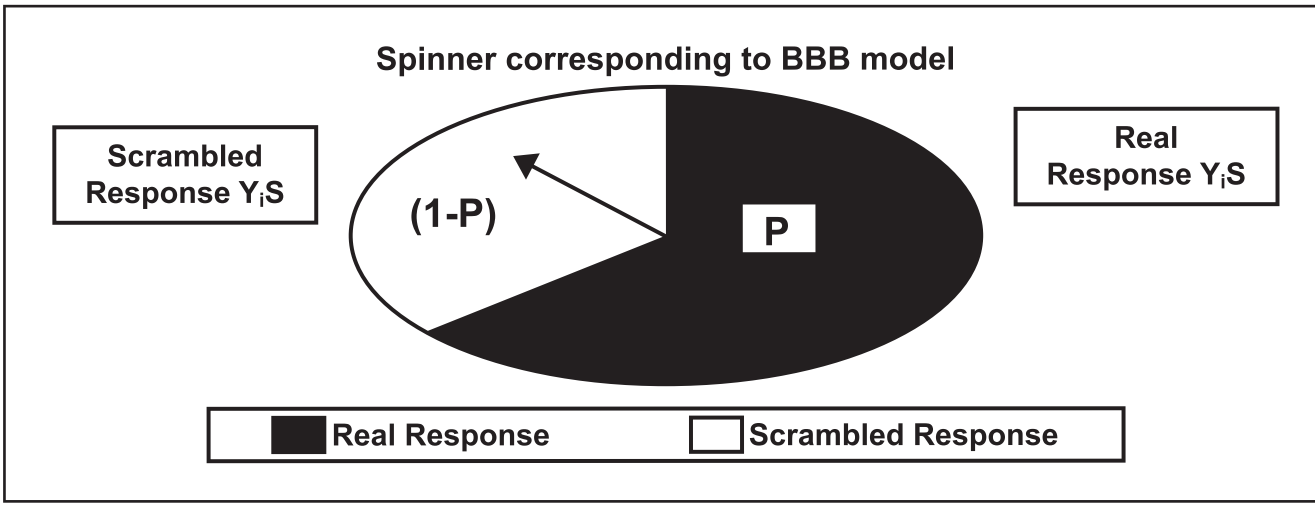

Fig. 1. Bar–Lev, Bobovitch & Boukai (2004; BBB) randomized response device

In other words, each respondent isselected in a simple random and replacement sample is requested to rotate a spinner unobserved by the interviewer, and if the spinner stops in the shaded area, then the respondent is requested to report the real response on the sensitive variable, say X_i; and if the spinner stops in the non-shaded area, then the respondent is required to report the scrambled response, say $X_i S$, where S is the scrambled variable. Let P be the radial non- shaded area of the spinner as shown in Figure1

An unbiased estimator of the population mean μ_x due to Bar–Lev *et al*. (2004) is given by:

$$\hat{\mu}_{x(BBB)} = \frac{1}{n\{(1-P)\mu_s + P\}} \sum_{i=1}^{n} Y_i \quad (6)$$

with variance under SRSWR sampling given by

$$V[\hat{\mu}_{x(BBB)}] = \frac{\mu_x^2}{n}[C_x^2 + (1 + C_x^2)C_P^2], \quad (7)$$

where

$$C_P^2 = \frac{\{(1-P)\mu_s^2(1+C_s^2) + P\}}{((1-P)\mu_s + P)^2} - 1 . \quad (8)$$

In this paper, we have made an effort in generalizing the results of Eichhorn & Hayre's (1983) and Bar–Lev *et al*.'s (2004) and provide an alternative estimator to the mean response of the sensitive variable which has uniformly smaller variance as compared to that of Eichhorn & Hayre's (1983) and Bar–Lev *et al*.'s (2004). The proposed estimator is also uniformly better than Eichhorn & Hayre's (1983) estimator under the same condition in which the Bar-Lev *et al*.'s (2004) estimator is more efficient than Eichhorn & Hayre's (1983) estimator. Numerical illustrations are also given in support of the present study.

2. The proposed procedure

In this section we suggest a quantitative randomized response procedure which generalizes that of Eichhorn & Hayre's (1983) and Bar-Lev *et al*.'s (2004) results. The description of the proposed procedure is given below:

Let a and b be any two known positive real numbers (Gjestvang&Singh, 2006) or the functions of the known parameters such as mean (μ_s) and variance (γ^2) of the scrambling variable S and we define $S^* = \frac{(aS + b\mu_s)}{(a+b)}$ such that $E(S^*) = \mu_s$. In the envisaged model, the distribution of the responses is given by

$$Y_i^* = \begin{cases} X_i S^* & \text{with probability } (1-P) \\ X_i & \text{with probability } P \end{cases}. \tag{9}$$

In other words, each respondent in a simple random and with replacement sample is requested to rotate a spinner unobserved by the interviewer, and if the spinner stops in the shaded area, then the respondent is request to report the real response on the sensitive variable, say X_i; and if the spinner stops in the non- shaded area, then the respondent is required to report the scrambled response, say $X_i S^*$. Let P be the radial non -shaded area of the spinner as shown in Figure 2

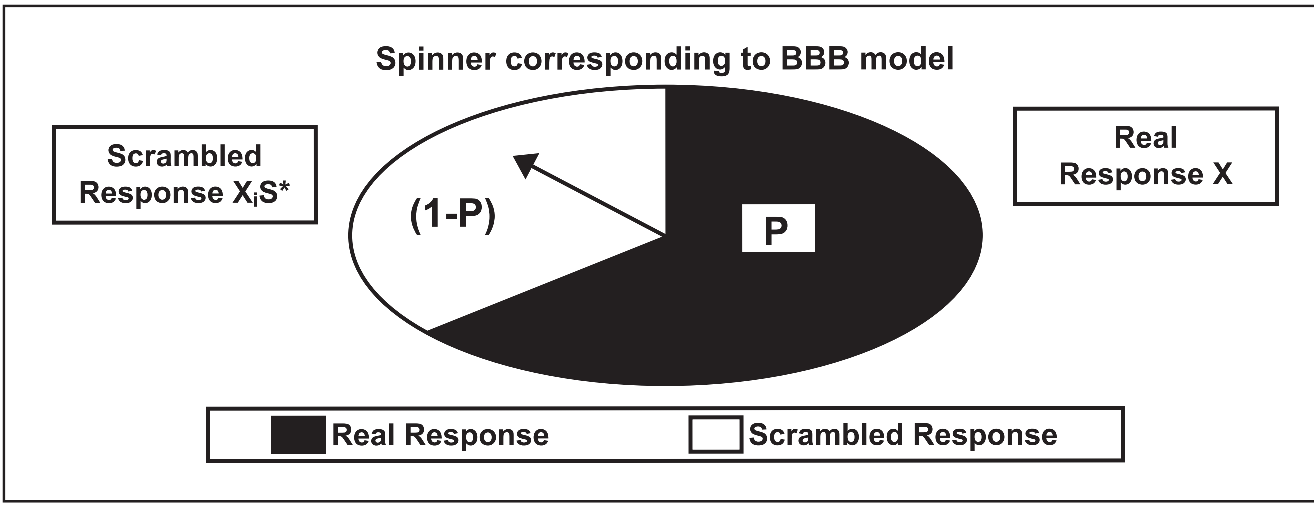

Fig. 2. Proposed randomized response device

It is interesting to mention that for (a, b) = (1,0) and P=0 , the proposed procedure reduces to that of Eichhorn & Hayre (1983) while for (a,b) = (1,0) it reduces to that of Bar-Lev *et al*. (2004). Thus the proposed procedure generalizes the work of Eichhorn & Hayre (1983) and Bar-Lev *et al*. (2004).

It can be easily seen that the expectation of Y^* is

$$\begin{aligned} E(Y^*) &= (1-P)E(XS^*) + PE(X) \\ &= (1-P)E(X)E(S^*) + PE(X) \\ &= (1-P)\mu_x\mu_s + P\mu_x \\ &= \mu_x[P + \mu_s(1-P)] \end{aligned} \tag{10}$$

and

$$\begin{aligned} V(Y^*) &= (1-P)E(X^2S^{*2}) + PE(X^2) - \{E(Y^*)\}^2 \\ &= (1-P)E(X^2)E(S^{*2}) + PE(X^2) - \{\mu_x[P+\mu_s(1-P)]\}^2 \\ &= (1-P)\mu_x^2(1+C_x^2)\mu_s^2(1+\eta^2C_s^2) + P\mu_x^2(1+C_x^2) - \{\mu_x[P+\mu_s(1-P)]\}^2 \\ &= \mu_x^2(1+C_x^2)[P+(1-P)\mu_s^2(1+\eta^2C_s^2)] - \{\mu_x[P+\mu_s(1-P)]\}^2, \end{aligned} \tag{11}$$

where $\eta = \frac{a}{(a+b)}$.

Hence, the proposed estimate for μ_x, based on random sample of the randomly coded responses $Y_1^*, Y_2^*, \ldots, Y_n^*$ is

$$\hat{\mu}_x^* = \frac{\overline{Y}^*}{[P + \mu_s(1-P)]} \tag{12}$$

Clearly, by (12), $\hat{\mu}_x^*$ is an unbiased estimate for μ_x. The variance of $\hat{\mu}_x^*$ is given by

$$V(\hat{\mu}_x^*) = \frac{\mu_x^2}{n}\left[C_x^2 + (1 + C_x^2)C_P^{*2}\right] \tag{13}$$

where $C_P^{*2} = \dfrac{[P + (1-P)\mu_s^2(1 + \eta^2 C_s^2)]}{(P + \mu_s(1-P))^2} - 1$. (14)

3. Efficiency comparison

From Equation (7) and (13) we have

$$V(\hat{\mu}_{x(BBB)}) - V(\hat{\mu}_x^*) = \frac{\mu_x^2(1 + C_x^2)}{n}\left[\frac{\{P + \mu_s^2(1 + C_s^2)(1-P)\}}{\{P + \mu_s(1-P)\}^2} - \frac{\{P + (1-P)\mu_s^2(1 + \eta^2 C_s^2)\}}{\{P + \mu_s(1-P)\}^2}\right]$$

$$= \frac{\mu_x^2 \mu_s^2 C_s^2 (1 + C_x^2)(1-P)(1-\eta^2)}{n\{P + \mu_s(1-P)\}^2}$$

which is always positive if

$1 - \eta^2 > 0$

i.e. if $\eta^2 < 1$

i.e. if $\eta < 1$ (15)

i.e. if $\dfrac{a}{(a+b)} < 1$

The condition $\dfrac{a}{(a+b)} < 1$ in Equation (15) is always true. Therefore the suggested estimator $\hat{\mu}_x^*$ is always better than Bar-Lev *et al*.'s (2004) estimator $\hat{\mu}_{x(BBB)}$. Hence the suggested model is more efficient than that of Bar-Lev *et al*. (2004). Thus we established the following theorem.

Theorem 3.1-The proposed model is uniformly better than that of Bar-Lev *et al*. (2004) i.e.

$$V(\hat{\mu}_x^*) < V(\hat{\mu}_{x(BBB)}) \tag{16}$$

Bar-Lev *et al*. (2004) have proved that if the scrambling distribution of S satisfies the condition

$$0 < \mu_s < \frac{2E(S^2)}{\{1 + E(S^2)\}}, \tag{17}$$

then

$$\frac{\sigma_x^2}{n} < V(\hat{\mu}_{x(BBB)}) < V(\hat{\mu}_x), \quad \forall P \in (0,1). \tag{18}$$

Thus we state the following theorem.

Theorem 3.2-If the scrambling distribution of S satisfies the condition (17), then from (16) and (18) we have following inequality:

$$\frac{\sigma_x^2}{n} < V(\hat{\mu}_x^*) < V(\hat{\mu}_{x(BBB)}) < V(\hat{\mu}_x), \quad \forall P \in (0,1). \tag{19}$$

It follows from Theorem -3.2 that if the scrambling distribution of S satisfies the condition (17), then from (19) it follows that the proposed estimator $\hat{\mu}_x^*$ is uniformly efficient than Eichhorn & Hayre's (1983) estimator $\hat{\mu}_x$ and Bar-Lev *et al*. (2004) estimator $\hat{\mu}_{x(BBB)}$.

4. Numerical illustration

To have the tangible idea about the performance of the proposed estimator $\hat{\mu}_x^*$ over Eichhorn & Hayre's (1983) estimator $\hat{\mu}_x$ and Bar-Lev *et al*.'s (2004) estimator , we have computed the percent relative efficiencies of the proposed estimator $\hat{\mu}_x^*$ with respect to and $\hat{\mu}_{x(EH)}$ by $\hat{\mu}_{x(BBB)}$ using the formulae:

$$\text{(i)}\quad PRE(\hat{\mu}_x, \hat{\mu}_x^*) = \frac{\left[C_x^2 + C_s^2(1+C_x^2)\right]}{[C_x^2 + C_P^{*2}(1+C_x^2)]} \times 100, \tag{20}$$

and

$$\text{(ii)}\quad PRE(\hat{\mu}_{x(BBB)}, \hat{\mu}_x^*) = \frac{\left[C_x^2 + C_P^2(1+C_x^2)\right]}{[C_x^2 + C_P^{*2}(1+C_x^2)]} \times 100, \tag{21}$$

for different values of Cx = 0.10,0.25,0.50,0.75; Cs = 1.5 (0.5) 3.0; P = 0.1(0.1)0.80, μs = 20(20)80 and h . Findings are displayed in Tables 1 and 2; where

$$C_P^2 = \left\{\frac{[P + \mu_s^2(1-P)(1+C_s^2)]}{[P+\mu_s(1-P)]^2} - 1\right\},$$

and

$$C_P^{*2} = \left\{\frac{[P + \mu_s^2(1-P)(1+\eta^2 C_s^2)]}{[P+\mu_s(1-P)]^2} - 1\right\}.$$

It is observed from Tables 1 and 2 that the percent relative efficiency are greater than 100 which follows that the proposed estimator $\hat{\mu}_x^*$ is more efficient than Eichhorn & Hayre (1983) estimator $\hat{\mu}_x$ and Bar-Lev *et al*. (2004) estimator $\hat{\mu}_{x(BBB)}$ with larger gain in efficiency. Thus our recommendation is to prefer the proposed estimator $\hat{\mu}_x^*$ over Eichhorn & Hayre (1983) estimator $\hat{\mu}_x$ and Bar-Lev *et al*. (2004) estimator $\hat{\mu}_{x(BBB)}$. Also we have shown the affect of a vale 'a' upon PRE $(\hat{\mu}_x, \hat{\mu}_x^*)$ and PRE $(\hat{\mu}_{x(BBB)}, \hat{\mu}_x^*)$, when other quantities are fixed in Figures 3 and 4 respectively.

Table 1. The PRE $(\hat{\mu}_x, \hat{\mu}_x^*)$

P	μ_s	C_x	C_s	a	b	η	PRE
	20	0.10	1.50	1	5	0.17	225.62
	40	0.25	2.00	5	10	0.33	392.13
	60	0.50	2.50	10	15	0.40	518.99
0.10	80	0.75	3.00	15	20	0.43	551.14
	20	0.10	1.50	20	25	0.44	224.30
	40	0.25	2.00	25	30	0.45	377.98
	60	0.50	2.50	30	35	0.46	468.06
0.20	80	0.75	3.00	35	40	0.47	476.64
	20	0.10	1.50	40	45	0.47	223.66
	40	0.25	2.00	45	50	0.47	369.90
	60	0.50	2.50	50	55	0.48	435.54
0.30	80	0.75	3.00	55	60	0.48	426.17
	20	0.10	1.50	60	65	0.48	222.96
	40	0.25	2.00	65	70	0.48	361.10
	60	0.50	2.50	70	75	0.48	402.87
0.40	80	0.75	3.00	75	80	0.48	378.38
	20	0.10	1.50	80	85	0.48	222.08
	40	0.25	2.00	85	90	0.49	350.23
	60	0.50	2.50	90	95	0.49	366.57
0.50	80	0.75	3.00	95	100	0.49	329.27
	20	0.10	1.50	100	105	0.49	220.89
	40	0.25	2.00	105	110	0.49	335.92
	60	0.50	2.50	110	115	0.49	324.61
0.60	80	0.75	3.00	115	120	0.49	277.22
	20	0.10	1.50	120	125	0.49	219.19
	40	0.25	2.00	125	130	0.49	315.92
	60	0.50	2.50	130	135	0.49	274.80
0.70	80	0.75	3.00	135	140	0.49	221.24
	20	0.10	1.50	140	145	0.49	216.57
	40	0.25	2.00	145	150	0.49	285.88
	60	0.50	2.50	150	155	0.49	214.39
0.80	80	0.75	3.00	155	160	0.49	160.51

Table 2. The PRE $(\hat{\mu}_{x(BBB)}, \hat{\mu}_x^*)$

P	μ_s	C_x	C_s	C_s	a	b	η	PRE
	20	0.10	1.50	1.50	1	5	0.17	257.72
	40	0.25	2.00	2.00	5	10	0.33	442.84
	60	0.50	2.50	2.50	10	15	0.40	581.41
0.10	80	0.75	3.00	3.00	15	20	0.43	614.76
	20	0.10	1.50	1.50	20	25	0.44	295.12
	40	0.25	2.00	2.00	25	30	0.45	487.20
	60	0.50	2.50	2.50	30	35	0.46	594.15
0.20	80	0.75	3.00	3.00	35	40	0.47	599.99
	20	0.10	1.50	1.50	40	45	0.47	342.57
	40	0.25	2.00	2.00	45	50	0.47	551.49
	60	0.50	2.50	2.50	50	55	0.48	635.48
0.30	80	0.75	3.00	3.00	55	60	0.48	614.39
	20	0.10	1.50	1.50	60	65	0.48	403.06
	40	0.25	2.00	2.00	65	70	0.48	633.60
	60	0.50	2.50	2.50	70	75	0.48	688.28
0.40	80	0.75	3.00	3.00	75	80	0.48	636.80
	20	0.10	1.50	1.50	80	85	0.48	482.52
	40	0.25	2.00	2.00	85	90	0.49	740.20
	60	0.50	2.50	2.50	90	95	0.49	751.85
0.50	80	0.75	3.00	3.00	95	100	0.49	663.79
	20	0.10	1.50	1.50	100	105	0.49	591.25
	40	0.25	2.00	2.00	105	110	0.49	883.39
	60	0.50	2.50	2.50	110	115	0.49	828.00
0.60	80	0.75	3.00	3.00	115	120	0.49	694.49
	20	0.10	1.50	1.50	120	125	0.49	748.14
	40	0.25	2.00	2.00	125	130	0.49	1085.28
	60	0.50	2.50	2.50	130	135	0.49	919.98
0.70	80	0.75	3.00	3.00	135	140	0.49	728.78
	20	0.10	1.50	1.50	140	145	0.49	989.89
	40	0.25	2.00	2.00	145	150	0.49	1389.69
	60	0.50	2.50	2.50	150	155	0.49	1032.58
0.80	80	0.75	3.00	3.00	155	160	0.49	766.78

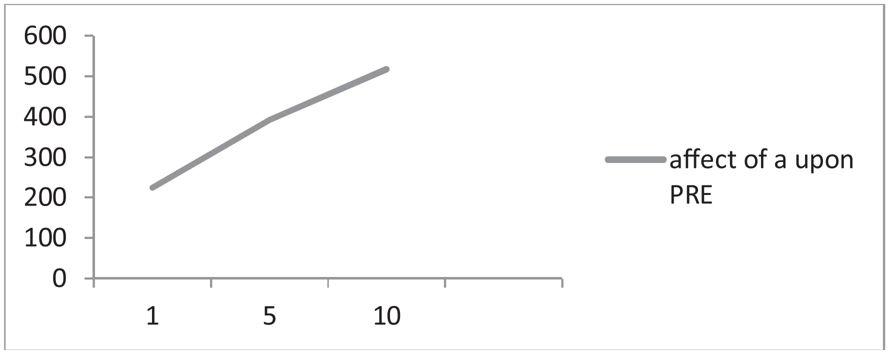

Fig. 3. Effects of 'a' upon the PRE ($\hat{\mu}_x, \hat{\mu}_x^*$), when other quantities are fixed:

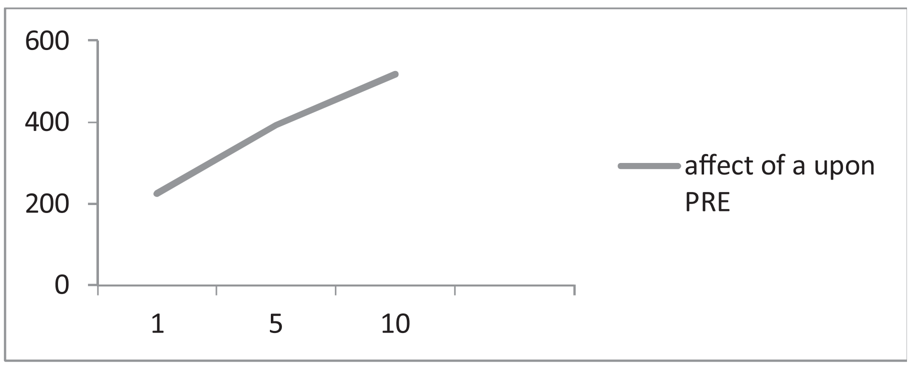

Fig. 4. Effects of 'a' upon the PRE ($\hat{\mu}_{x(BBB)}, \hat{\mu}_x^*$), when other quantities are fixed:

5. Conclusion

This paper illustrates enrichment over the Eichhorn & Hayre (1983) and Bar-Lev *et al*. (2004) models randomized response models. The proposed model is found to be more efficient both theoretically as well as numerically than the randomized response models studied by Eichhorn & Hayre (1983) and Bar-Lev *et al*. (2004). Thus the proposed randomized response procedure is therefore recommended for its use in practice as an alternative to Eichhorn & Hayre (1983) and Bar-Lev *et al*. (2004) models.

References

Chaudhuri, A. & Mukerjee, R. (1988). Randomized response: Theory and techniques. Marcel- Dekker, New York, USA.

Bar –Lev,SKE., Bobovitch, E. & Boukai, B. (2004). A note on randomized responsemodels for quantitative data. Metrika, **60**:225-250.

Eichhorn, B.H. & Hayre, L.S. (1983). Scrambled randomized response methods for obtaining sensitive quantitative dada. Journal of Statistics and planning Inference, **7**:307 -316.

Fox, J.A. & Tracy, P.E. (1986). Randomized Response: A method of Sensitive Surveys. Newbury Park, CA: SEGE Publications.

Gjestvang, C.R. & Singh, S. (2006). A new randomized response model. Journal of Royal Statistical Society, **68**: 523-530.

Gjestvang, C.R. & Singh, S. (2009). An improved randomized response model: Estimation of mean, Journal of Applied Statistics, **36**(12): 1361-1367.

Hussain, Z. (2012). Improvement of Gupta and Thornton scrambling model through double use of randomization device. International Journal of Academic ResearchBusiness and Society and Science, **2**(6),91-97.

Odumade, O. & Singh, S. (2008). Generalized forced quantitative randomized response model: A unified approach. Journal of Indian Society of Agricultural and Statistics, **62**(3): 244-252.

Odumade, O. & Singh, S. (2009). Improved Bar-Lev, Bobovitch and Boukai randomized response models. Communication in Statistics andTheory and . Methods, **38**(3):473-502.

Singh, H.P. & Mathur, N. (2004). Estimation of population mean with known coefficient of variation under optional response model using scrambled response technique.Statistics and Transactions, **6** (7):1079-1093.

Singh, H.P. & Mathur, N. (2005). Estimation of population mean when coefficient ofvariation is known using scrambled response technique. Journal of Statistics and Planning Inference, **131**:135-144.

Singh, H.P. & Tarray, T.A.(2013). A modified survey technique for estimating the proportion and sensitivity in a dichotomous finite population.International Journal of Advanced Sciences and Technology Research, **3**(6):459–472.

Singh, H.P. & Tarray, T.A.(2016). An improved Bar – Lev, Bobovitch and Boukai randomized response model using moments ratios of scrambling variable. Hacettepe Journal of Mathematics and Statistics, **45**(2):593-608.

Singh, S. (2003). Advanced sampling theory with applications. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Tarray, T.A. & Singh, H.P. (2015). Some improved additive randomized response models utilizing higher order moments ratios of scrambling variable. Model Assisted Statistics and Applications, **10**:361-383.

Tarray, T.A. & Singh, H.P. (2016). An adroit randomized response new additive scrambling model. Gazi University Journal of Sciences, **29**(1):159-165.

Tarray, T.A. & Singh, H.P. (2017). A Survey Technique for Estimating the Proportion and Sensitivity in a Stratified Dichotomous Finite Population. Statistics and Applications, **15**(1,2):173-191.

Warner, S.L. (1965). Randomized response: A survey technique for eliminating evasive answer bias. Journal of American Statistical Association, **60**:63-69.

Submitted : 21/07/2015

Revised : 29/11/2016

Accepted : 01/04/2017

طريقة مبسطة لتحسين نموذج بارليف، بوبوفيتش وبوكاي للعائد العشوائي Bar–Lev، Bobovitch and Boukai

تنوير تاراي[1]، *، هوسيلا سينغ[2]

[1] قسم علوم الحاسوب والهندسة، الجامعة الإسلامية للعلوم والتكنولوجيا، أوانتيبورا، بولواما، كشمير، الهند - 192122

[2] كلية الدراسات في الإحصاء، جامعة فيكرام، أوجين، الهند- 456010

*tanveerstat@gmail.com

خلاصة

تناول ايكهورن وهايري (1983) Eichhorn & Hayre طريقة العائد العشوائي مناسبة لتقدير متوسط العائد في حالة كون المتغير الحساس تحت الدراسة ذا طبيعة كمية. قاموا بالحصول على تقدير لمتوسط العائد العشوائي الكمي تحت الدراسة وقاموا بدراسة خواص التقدير. اقترح (2004) .Bar–Lev et al طريقة بديلة تستخدم معلمة تصميم (محكومة من خلال صاحب التجربة) تؤدي إلى تعميم نتائج (1983) Eichhorn & Hayre's. قاموا أيضاً بإثبات أن تباين التقدير المقترح من قبلهم دائماً أصغر من تباين تقدير (1983) Eichhorn & Hayre's في حالات محددة. نقترح في هذا البحث طريقة سهلة لتحسين نماذج كل من Eichhorn & Hayre's (1983) و (2004) .Bar–Lev et al والخواص المصاحبة. قمنا بتوضيح أن الطريقة المقترحة دائماً أفضل من طريقة (2004) .Bar–Lev et al. وكذلك فإن الطريقة المقترحة أفضل من (1983) Eichhorn & Hayre's في نفس الظروف التي يكون فيها طريقة (2004) .Bar–Lev et al أعلى كفاءة من طريقة (1983) Eichhorn & Hayre's . نقدم أيضاً دراسات عددية توضيحية لتدعيم الدراسة الحالية.