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An easy construction of generalized neighbor designs in minimal circular blocks

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Abstract

Neighbor designs (NDs) are used in the experiments where neighbor effects may arise. Neighbor designs neutralize these effects and are, therefore, considered to be robust against neighbor effects. Minimal neighbor designs are always most economical among the neighbor designs and are, therefore, preferred by the experimenters. Method of cyclic shifts provides these designs in circular blocks only for odd v (number of treatments). For v even, minimal circular generalized neighbor designs in which only $\frac{v}{2}$ unordered pairs of distinct treatments that do not appear as neighbors will be the better alternate to the minimal neighbor designs. In this article, such minimal generalized neighbor designs are constructed in circular blocks for v even.

Keywords: Direct effects; minimal circular blocks; neighbor designs; neighbor effects; robust to neighbor effects.

1. Introduction

Neighbor designs (NDs) are used in the experiments where neighbor effects may arise. Neighbor designs neutralize these effects and are, therefore, considered to be robust against neighbor effects. Minimal neighbor designs are always most economical among the neighbor designs and are, therefore, preferred by the experimenters. Tomar *et al.* (2005) suggested that competition among neighboring units becomes a source of bias and treatments comparisons are not much affected by neighbor effects with the use of neighbor balanced designs (NBDs). If each treatment appears exactly once with all other treatments as neighbor then design is minimal neighbor balanced. If every treatment occurs equal number of times, say λ' as neighbor of every other treatment, design is called neighbor balanced, here λ' is positive integer. If $\lambda' = 1$ then it is minimal NBD. If each treatment appears either (i) $\lambda'_1 = 1$ and $\lambda'_2 = 0$, or (ii) $\lambda'_1 = 1$ and $\lambda'_2 = 2$ times with other treatments as neighbor (left or right) then designs are called minimal generalized neighbor designs (GNDs).

Rees (1967) finds application of minimal NBDs in serology for virus research. In experiments of agriculture, horticulture, and forestry, neighbor effects arise due to plots' nature, plots' layout, pest infections from neighboring plots, etc. Minimal NBDs are available in literature to neutralize neighbor effects economically for v odd. For v even, our proposed minimal circular GNDs (MCGNDs) will reduce the bias due to neighbor effects economically and efficiently. Williams (1952) constructed NBDs in linear blocks. Rees (1967) used neighbor designs in virus research using circular blocks. Rees (1967) constructed neighbor designs also for $k \leq v$. Hwang (1973) constructed NBDs for some cases of v odd. Cheng (1983) constructed neighbor designs for block size (k) equal to v. NBDs reduce bias due to neighbor effects, see Azais (1987), Langton (1987), Azais *et al.* (1993) and Kunert (2000). Iqbal *et al.* (2009) constructed some series of NBDs through method of cyclic shifts. Akhtar *et al.* (2010) and Ahmed & Akhtar (2011) presented NBDs for $k = 5$ and 6 respectively. Shehzad *et al.* (2011a) constructed minimal circular blocks neighbor designs for some limited cases. Misra *et al.* (1991) relaxed the conditions of balance property up to some extent and constructed GNDs. GNDs will be minimal GN_2-designs if λ' may take only two values (i) $\lambda'_1 = 1$ and $\lambda'_2 = 0$, or (ii) $\lambda'_1 = 1$ and $\lambda'_2 = 2$. Nutan (2007) and Kedia & Misra (2008) constructed some classes of circular GNDs (CGNDs) in which some are GN_2-designs. Ahmed *et al.* (2009), Zafaryab *et al.* (2010), Shehzad *et al.* (2011b) and Iqbal *et al.* (2012) presented MCGNDs for some limited cases. In this article, MCGNDs are obtained for v even in equal and unequal block sizes. In our proposed designs, $\frac{v}{2}$ pairs of treatments do not appear as neighbors.

This article is organized as: Method of cyclic shifts is described in Section 2, along with the conditions for existence of MCGNDs. Efficiency measure is described in Section 3. MCGNDs are obtained for $m \equiv 0$ (mod 4) in Section 4 and for $m \equiv 3$ (mod 4) in Section 5. Discussion and conclusion are given in Section 6.

2. Method of cyclic shifts

Iqbal (1991) introduced method of cyclic shifts which is simplified here for minimal CNBDs and minimal CGNDs.

Let $S_j = [q_{j1}, q_{j2}, \cdots, q_{j(k-1)}]$ be i sets, $j = 1, 2, \cdots, i$, $1 \leq q_{ju} \leq v-1$ and $u = 1, 2, \cdots, k-1$.

- If each of $1, 2, \cdots, v-1$ appears exactly once in S^* then designs will be minimal CNBD.
- If each of $1, 2, \cdots, v-1$ appears either (a) 1 and 0, or (b) 1 and 2 in S^* then designs will be minimal CGND.

Here S^* contains:

(i) Each element of all sets S_j.

(ii) Sum (mod v) of all elements in each set S_j.

(iii) Complements of all elements in (i) and (ii), here complement of a being $v - a$.

In this article, we deal with the construction of MCGNDs in which $\frac{v}{2}$ unordered pairs will not appear as neighbors. To construct minimal CGNDs for $v = 2ik_1 + 2k_2 + 2k_3 + \cdots + 2k_h + 2$, using Rule I, proceed as.

- If sum of A is divisible by v then it will produce required MCGNDs, here A = $[1, 2, \cdots, m]$. For this, one or more elements can be replaced with their complements, here $2m = v - 2$.
- Divide resultant A in i classes of k_1 size , one class each of $k_2, k_3, \cdots$, and k_h sizes in such a way that sum of each class is divisible by v.
- Required sets of shifts will be obtained by deleting one element (any) from each class.

Example 2.1

S = [1, 2, 3] provides minimal CGND for $v = 10$ and $k = 4$.

Proof: S^* = [1, 2, 3, 6, 9, 8, 7, 4] which contains each of $1, 2, \cdots, 9$ once but 5 does not appear. Hence S = [1, 2, 3] provides minimal CGND in blocks of size 4. Blocks of the design are generated as follows from the given set(s) of shifts.

Each set requires v blocks. Assign to each block 0 to $v-1$ as first unit e lement. Add 1 (mod v) to first unit elements for second unit elements. Then add 2 (mod v) to second unit values and so on, see Table 1.

Example 2.2

Table 1. Blocks obtained from S = [1, 2, 3]

Blocks									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	0
3	4	5	6	7	8	9	0	1	2
6	7	8	9	0	1	2	3	4	5

S_1 = [3,4,7], S_2 = [1,5] produce following MCGND for $v = 16$, $k_1 = 4$ and $k_2 = 3$.

Proof: S^* = [3,4,7,14,1,5,6,13,12,9,2,15,11,10], Here each of $1, 2, \cdots, 15$ appears once except 8 which does not appear. Hence S_1 and S_2 generate MCGND for $v = 16$, $k_1 = 4$, $k_2 = 3$.

Table 2 and 3 jointly present MCGND for $v = 16$, $k_1 = 4$, $k_2 = 3$. In this design, unordered pairs (0,8), (1,9), (2,10), (3,11), (4,12), (5,13), (6,14), (7,15) do not appear as neighbors. Hence we save $[16(15) - 16(7)] = 53.33\%$ experimental units at the cost of losing $[\frac{8}{120} \times 100\%] = 6.67\%$ neighbor balance.

Table 2. Blocks obtained from $S_1 = [3, 4, 7]$

Blocks															
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2
7	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6
14	15	0	1	2	3	4	5	6	7	8	9	10	11	12	13

Table 3. Blocks obtained from $S_2 = [1, 5]$

Blocks															
17	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0
6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5

3. Efficiency of neighbor effects

The model repeatedly discussed for residual effects in literature is the traditional model given by,

$$y_{ijk} = \mu + \tau_{d(k,j)} + \gamma_{d(k-1,j)} + \pi_k + \xi_{ij} + \varepsilon_{ijk}, \tag{1}$$

The efficiency factor for neighbor effect is the harmonic mean of non-zero Eigen values of their respective information matrix, see James & Wilkinson (1971) and Pearce *et al.* (1974). Design with high value of E_n is considered efficient to estimate neighbor effects. Our proposed designs possess high values of E_n, therefore, these are suitable for this purpose.

Example 3.1

[2,3,4,5,7,13], [1,6,8,9,10], [11,12,14,15,16] produce MCGND for $v = 34$, $k_1 = 6$ and $k_2 = 5$ with $E_n = 0.90$.

4. Minimal CGNDs for $m \equiv 3 \pmod 4$

Here, MCGNDs are obtained for $m \equiv 3 \pmod 4$ with $m = \frac{v-2}{2}$.

Theorem 4.1

If $m \equiv 3 \pmod 4$, sets obtained from A = $[1, 2, \cdots, \frac{(3m-1)}{4}, \frac{(3m+7)}{4}, \frac{(3m+11)}{4}, \cdots, m, \frac{5(m+1)}{4}]$ will produce MCGNDs for:

- $v = 2ik + 2$.
- $v = 2ik_1 + 2u_1k_2 + 2$.
- $v = 2ik_1 + 2u_1k_2 + 2u_2k_3 + 2$ and so on.

Proof: Let

$$\begin{aligned}
S &= 1 + 2 + \cdots + \tfrac{(3m-1)}{4} + \tfrac{(3m+7)}{4} + \tfrac{(3m+11)}{4} + \cdots + m + \tfrac{5(m+1)}{4} \\
&= 1 + 2 + \cdots + \tfrac{(3m-1)}{4} + \tfrac{(3m+3)}{4} + \tfrac{(3m+7)}{4} + \tfrac{(3m+11)}{4} + \cdots + m + \tfrac{5(m+1)}{4} - \tfrac{(3m+3)}{4} \\
&= [1 + 2 + \cdots + m] + \tfrac{(2m+2)}{4} \\
&= \tfrac{m(m+1)}{2} + \tfrac{2(m+1)}{4} \\
&= \tfrac{2(m+1)(m+1)}{4}, \quad \text{Since} \quad v = 2(m+1) \\
&= \tfrac{v(m+1)}{4}.
\end{aligned}$$

$\frac{(m+1)}{4}$ will be integer for $m \equiv 3 \pmod 4$.
S (mod v) $\equiv 0$ if $\frac{(m+1)}{4}$ is integer. Hence proved.

4.1 MCGNDs in equal block sizes when $m \equiv 3 \pmod 4$

For the following cases, minimal CGNDs can be constructed in equal block sizes using i sets derived from theorem 4.1.

(i) $v = 2ik + 2$, $k = 5, 9, 13, \cdots$, $i = 3, 7, 11, \cdots$

(ii) $v = 2ik + 2$, $k = 7, 11, \cdots$, $i = 1, 5, 9, \cdots$

Example 4.1

Following sets generate minimal CGND for v = 32 and k = 5 with $E_n = 0.92$.
S_1 =[4,6,10,11], S_2 =[5,7,8,9], S_3 =[13,14,15,20]

4.2 MCGNDs in two different block sizes when $m \equiv 3 \pmod 4$, through i sets for k_1 and one for k_2

For the following cases, MCGNDs can be obtained in two different block sizes from i sets for k_1 and one set for k_2 derived from theorem 4.1.

(i) $v = 2k_1(i + 1)$, $k_1 = 4l = k_2 + 1$, i & l integers.

(ii) $v = 2k_1(i + 1)$, $k_1 = 4l + 2 = k_2 + 1$, i odd.

(iii) $v = 2k_1(i + 1)$, k_1 (odd) $= k_2 + 1 > 3$, $i \equiv 3 \pmod 4$.

(iv) $v = 2k_1(i + 1) - 2$, $k_1 \equiv 1 \pmod 4 = k_2 + 2$, $i \equiv 0 \pmod 4$.

(v) $v = 2k_1(i + 1) - 2$, $k_1 \equiv 3 \pmod 4 = k_2 + 2$, $i \equiv 2 \pmod 4$.

(vi) $v = 2ik_1 + 8$, $k_1 = 4l$, k_2 = 3, i & l integers and $m \equiv 3 \pmod 4$.

(vii) $v = 2ik_1 + 8$, $k_1 = 4l + 2$, k_2 = 3, i even.

(viii) $v = 2ik_1 + 8$, k_1 (odd) > 3, k_2 = 3, $i \equiv 0$ (mod 4).

(ix) $v = 2ik_1 + 10$, $k_1 \equiv 1$ (mod 4) , k_2 = 4, $i \equiv 3$ (mod 4).

(x) $v = 2ik_1 + 10$, $k_1 \equiv 3$ (mod 4), k_2 = 4, $i \equiv 1$ (mod 4).

(xi) $v = 2ik_1 + 12$, $k_1 = 4l + 2$, k_2 = 5, and i odd.

(xii) $v = 2ik_1 + 12$, $k_1 = 4l + 2$, k_1 (odd) > 5, $k_2 = 5$, $i \equiv 2$ (mod 4) and $m \equiv 3$ (mod 4).

Example 4.2

Following sets generate MCGND for v = 16, k_1 = 4, k_2 = 3 with $E_n = 0.85$.

$$S_1 = [2,3,10], \qquad S_2 = [5,4]$$

4.3 MCGNDs in two different block sizes when $m \equiv 3$ (mod 4) , through i sets for k_1 and two for k_2

For the following cases, MCGNDs can be obtained in two different block sizes using i sets for k_1, two for k_2 derived from theorem 4.1.

(i) $v = 2k_1(i + 2) - 2$, $k_1 = 5, 9, \cdots, k_1 = k_2 + 1, i = 3, 7, 11, \cdots$

(ii) $v = 2k_1(i + 2) - 2$, $k_1 = 7, 11, \cdots, k_1 = k_2 + 1, i = 1, 5, 9, \cdots$

(iii) $v = 2k_1(i + 2) - 6$, $k_1 = 5, 9, \cdots, k_1 = k_2 + 2, i = 1, 5, 9, \cdots$

(iv) $v = 2k_1(i + 2) - 6$, $k_1 = 7, 11, \cdots, k_1 = k_2 + 2, i = 3, 7, 11, \cdots$

(v) $v = 2k_1(i + 2) - 10$, $k_1 = 5, 9, \cdots, k_1 = k_2 + 3, i = 3, 7, 11, \cdots$

(vi) $v = 2k_1(i + 2) - 10$, $k_1 = 7, 11, \cdots, k_1 = k_2 + 3, i = 1, 5, 9, \cdots$

Example 4.3

Following sets generate minimal CGND for $v = 40$, $k_1 = 7$ and $k_2 = 6$ with $E_n = 0.93$.
S_1 = [3,4,5,6,7,14], S_2 = [10,11,12,13,25], S_3 = [8,16,17,18,19]

4.4 MCGNDs in three different block sizes when $m \equiv 3$ (mod 4), through i sets for k_1 and one set for k_2, one for k_3

For the following cases, MCGNDs can be obtained in three different block sizes from i sets for k_1, one set for k_2, one for k_3 derived from theorem 4.1.

(i) $v = 2k_1(i + 2) - 4$, $k_1 = 4l + 2 = k_2 + 1$, $k_3 = k_1 - 2$, i odd.

(ii) $v = 2k_1(i + 2) - 4$, $k_1 = k_2 + 1 > 3$, $k_3 = k_1 - 2$, $i \equiv 0$ (mod 4).

(iii) $v = 2k_1(i + 2) - 4$, $k_1 \equiv 1$ (mod 4)$= k_2 + 1$, $k_3 = k_1 - 3$, $i \equiv 1$ (mod 4).

(iv) $v = 2k_1(i + 2) - 6$, $k_1 \equiv 3$ (mod 4)$= k_2 + 1$, $k_3 = k_1 - 3$, $i \equiv 3$ (mod 4).

(v) $v = 2k_1(i + 2) - 8$, $k_1 = 4l$, $l > 1$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i, l integer and $m \equiv 3$ (mod 4).

(vi) $v = 2k_1(i + 2) - 8$, $k_1 = 4l + 2$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i even, l integer, $m \equiv 3$ (mod 4).

(vii) $v = 2k_1(i + 2) - 8$, $k_1 > 5$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, $i \equiv 2$ (mod 4).

(viii) $v = 2k_1(i + 1) + 6$, $k_1 \equiv 1$ (mod 4)$= k_2 + 1$, k_3 = 3, $i \equiv 0$ (mod 4).

(ix) $v = 2k_1(i+1)+6$, $k_1 \equiv 3$ (mod 4)$= k_2+1$, k_3 = 3, $i \equiv 2$ (mod 4).

(x) $v = 2k_1(i+1)+8$, $k_1 = 4l = k_2+1$, $l > 1$, $k_3 = 4$.

(xi) $v = 2k_1(i+1)+8$, $k_1 = 4l+2 = k_2+1$, k_3 = 4, i odd.

(xii) $v = 2k_1(i+1)+8$, $k_1 = k_2+1 > 5$, k_3 = 4, $i \equiv 3$ (mod 4).

(xiii) $v = 2k_1(i+1)+10$, $k_1 \equiv 1$ (mod 4)$= k_2+1$, k_3 = 5, $i \equiv 3$ (mod 4).

(xiv) $v = 2k_1(i+1)+10$, $k_1 \equiv 3$ (mod 4)$= k_2+1$, k_3 = 5, $i \equiv 0$ (mod 4).

(xv) $v = 2k_1(i+1)+4$, $k_1 = 4l+2$, $l > 1$, $k_2 = k_1-2$, k_3 = 3, i even, $m \equiv 3$ (mod 4).

(xvi) $v = 2k_1(i+1)+4$, $k_1 > 5$, $k_2 = k_1-2$, $k_3 = 3$, $i \equiv 1$ (mod 4).

(xvii) $v = 2k_1(i+1)+6$, $k_1 \equiv 1$ (mod 4)$= k_2+2$, k_3 = 4, $i \equiv 0$ (mod 4).

(xviii) $v = 2k_1(i+1)+6$, $k_1 \equiv 3$ (mod 4)$= k_2+2$, k_3 = 4, $i \equiv 2$ (mod 4).

(xix) $v = 2k_1(i+1)+8$, $k_1 = 4l$, $l > 2$, $k_2 = k_1-2$, $k_3 = 5$, i integer and m (mod 4) $\equiv 0$.

(xx) $v = 2k_1(i+1)+8$, $k_1 = 4l+2 = k_2+2$, k_3 = 5, i odd.

(xxi) $v = 2k_1(i+1)+8$, $k_1 > 5$, $k_2 = k_1-2$, k_3 = 5, $i \equiv 3$ (mod 4).

(xxii) $v = 2k_1(i+1)+2$, $k_1 \equiv 1$ (mod 4), $k_2 = k_1-3$, k_3 = 3, $i \equiv 2$ (mod 4).

(xxiii) $v = 2k_1(i+1)+2$, $k_1 \equiv 3$ (mod 4), $k_2 = k_1-3$, k_3 = 3, $i \equiv 0$ (mod 4).

(xxiv) $v = 2k_1(i+1)+4$, $k_1 = 4l+2 = k_2+3$, k_3 = 4, i even.

(xxv) $v = 2k_1(i+1)+4$, $k_1 > 5$, $k_2 = k_1-3$, k_3 = 4, $i \equiv 1$ (mod 4).

(xxvi) $v = 2k_1(i+1)+6$, $k_1 \equiv 1$ (mod 4)$= k_2+3$, k_3 = 5, $i \equiv 2$ (mod 4).

Example 4.4

Following sets generate minimal CGND for v = 32, k_1 = 6, k_2 = 5 and k_3 = 4 with $E_n = 0.92$.
S_1 = [9,11,13,14,15], S_2 = [6,7,8,10], S_3 = [4,5,20]

4.5 MCGNDs in three different block sizes when $m \equiv 3$ (mod 4), through i sets for k_1, one set for k_2, two for k_3

For the following cases, MCGNDs can be obtained in blocks of three different sizes through i sets for k_1, one set for k_2, two for k_3 derived from theorem 4.1.

(i) $v = 2k_1(i+3)-8$, $k_1 = 4l = k_2+1$, $k_3 = k_1-2$.

(ii) $v = 2k_1(i+3)-8$, $k_1 = 4l+2 = k_2+1$, $k_3 = k_1-2$, i odd.

(iii) $v = 2k_1(i+3)-8$, k_1 (odd) > 3, $k_2 = k_1-1$, $k_3 = k_1-2$, $i \equiv 1$ (mod 4).

(iv) $v = 2k_1(i+3)-12$, $k_1 = 4l+2 = k_2+1$, $k_3 = k_1-3$, i even.

(v) $v = 2k_1(i+3)-12$, k_1 (odd) > 3, $k_1 = k_2+1$, $k_3 = k_1-3$, $i \equiv 3$ (mod 4).

Example 4.5

Following sets generate minimal CGND for $v = 32$, $k_1 = 5$, $k_2 = 4$ and $k_3 = 3$ with $E_n = 0.85$.
S_1 = [3,5,9,14], S_2 = [7,8,11], S_3 = [10,20], S_4 = [13,15]

5. Minimal CGNDs for $m \equiv 0$ (mod 4)

Here, MCGNDs are generated for m (mod 4) $\equiv 0$, where $m = \frac{v-2}{2}$.

Theorem 5.1

If $m \equiv 0$ (mod 4), sets obtained from B = [1, 2, $\cdots$, m] will produce proposed MCGNDs for:

- $v = 2ik + 2$.
- $v = 2ik_1 + 2u_1k_2 + 2$.
- $v = 2ik_1 + 2u_1k_2 + 2u_2k_3 + 2$, and so on.

Proof: Let
$S = 1 + 2 + \cdots + m$
$= \frac{(m+1)m}{2}$
$= \frac{2(m+1)m}{4} = v.\frac{m}{4}$ Since $v = 2(m+1)$
$\frac{m}{4}$ will be integer for m $\equiv 0$ (mod 4).
S (mod v) $\equiv 0$ if $\frac{m}{4}$ is integer. Hence proved.

5.1 MCGNDs in equal block sizes when $m \equiv 0$ (mod 4)

For the following cases, minimal CGNDs can be constructed in equal block sizes using i sets derived from theorem 5.1

(i) $v = 2ik + 2$, $k = 4l$, i, l integer and $m \equiv 0$ (mod 4).

(ii) $v = 2ik + 2$, $k = 4l + 2$, i even, l integer and $m \equiv 0$ (mod 4).

(iii) $v = 2ik + 2$, k (odd) > 3, $i \equiv 0$ (mod 4) and $m \equiv 0$ (mod 4).

Example 5.1

Following sets generate minimal CGND for $v = 18$ and $k = 4$ with $E_n = 0.88$.
S_1 = [3, 4, 5], S_2 = [1, 2, 7]

5.2 MCGNDs in two different block sizes when $m \equiv 0$ (mod 4) through i sets for k_1 and one for k_2

For the following cases, minimal CGNDs can be constructed in two different block sizes from i sets for k_1 and one for k_2 derived from theorem 5.1.

(i) $v = 2k_1(i + 1)$, $k_1 \equiv 1$ (mod 4)$= k_2 + 1$, $i \equiv 0$ (mod 4).

(ii) $v = 2k_1(i + 1)$, $k_1 \equiv 3$ (mod 4)$= k_2 + 1$, $i \equiv 2$ (mod 4).

(iii) $v = 2k_1(i + 1) - 2$, $k_1 = 4l + 2 = k_2 + 2$, i even, l integer.

(iv) $v = 2k_1(i + 1) - 2$, $k_2 = k_1 - 2$, k_1 (odd) > 3, $i \equiv 1$ (mod 4).

(v) $v = 2ik_1 + 8$, $k_1 \equiv 1$ (mod 4), $k_2 = 3$, $i \equiv 1$ (mod 4).

(vi) $v = 2ik_1 + 8$, $k_1 \equiv 3$ (mod 4), $k_2 = 3$, $i \equiv 3$ (mod 4).

(vii) $v = 2ik_1 + 10$, $k_1 = 4l$, $l > 1$, $k_2 = 4$, i integer.

(viii) $v = 2ik_1 + 10$, $k_1 = 4l + 2$, $k_2 = 4$, i even.

(ix) $v = 2ik_1 + 10$, k_1 (odd) > 3, k_2 = 4, $i \equiv 0$ (mod 4).

(x) $v = 2ik_1 + 12$, $k_1 \equiv 1 \pmod 4$, $k_2 = 5$, $i \equiv 3 \pmod 4$.

(xi) $v = 2ik_1 + 12$, $k_1 \equiv 3 \pmod 4$, $k_2 = 5$, $i \equiv 1 \pmod 4$.

Example 5.2

Following sets generate MCGND for v = 18, k_1 = 5, k_2 = 3 with $E_n = 0.88$.

S_1 = [2,4,5,6], S_2 = [7,8]

5.3 MCGNDs in two different block sizes when $m \equiv 0 \pmod 4$, through i sets for k_1 and two for k_2

For the following cases, minimal CGNDs can be constructed in two different block sizes from i sets for k_1, two for k_2 derived from theorem 5.1.

(i) $v = 2k_1(i+2) - 2$, $k_1 = 4l + 2 = k_2 + 1$, i odd.

(ii) $v = 2k_1(i+2) - 6$, $k_1 = 4l = k_2 + 2$, i & $l > 1$.

(iii) $v = 2k_1(i+2) - 6$, $k_1 = 4l + 2 = k_2 + 2$, i even.

(iv) $v = 2k_1(i+2) - 6$, $k_2 = k_1 - 2$, k_1 (odd) > 3, $i \equiv 2 \pmod 4$.

(v) $v = 2k_1(i+2) - 10$, $k_2 = k_1 - 3$, $k_1 = 4l + 2$, i odd.

(vi) $v = 2k_1(i+2) - 10$, $k_2 = k_1 - 3$, k_1 (odd) > 3, $i \equiv 0 \pmod 4$.

Example 5.3

Following sets generate MCGND for $v = 34$, $k_1 = 6$ and $k_2 = 5$ with $E_n = 0.90$.
S_1 = [3,5,6,7,11], S_2 = [4,8,9,12], S_3 = [13,14,15,16]

5.4 MCGNDs in three different block sizes when $m \equiv 0 \pmod 4$, through i sets for k_1, one set for k_2 and one for k_3

For the following cases, MCGNDs can be obtained in blocks of three different sizes from i sets for k_1, one set for k_2, one for k_3 derived from theorem 5.1.

(i) $v = 2k_1(i+2) - 4$, $k_1 \equiv 1 \pmod 4 = k_2 + 1$, $k_3 = k_1 - 2$, $i \equiv 1 \pmod 4$.

(ii) $v = 2k_1(i+2) - 4$, $k_1 \equiv 3 \pmod 4 = k_2 + 1$, $k_3 = k_1 - 2$, $i \equiv 3 \pmod 4$.

(iii) $v = 2k_1(i+2) - 6$, $k_1 = 4l$, $l > 4$, $k_2 = k_1 - 1$ & $k_3 = k_1 - 3$, i, l integer.

(iv) $v = 2k_1(i+2) - 6$, $k_1 = k_2 + 1 = 4l + 2$, $k_3 = k_1 - 3$, i even.

(v) $v = 2k_1(i+2) - 6$, $k_1 > 5$, $k_2 = k_1 - 1$ & $k_3 = k_1 - 3$, $i \equiv 2 \pmod 4$.

(vi) $v = 2k_1(i+2) - 8$, $k_1 \pmod 4 \equiv 1$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, $i \equiv 3 \pmod 4$.

(vii) $v = 2k_1(i+2) - 8$, $k_1 \pmod 4 \equiv 3$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, $i \equiv 1 \pmod 4$.

(viii) $v = 2k_1(i+2) + 6$, $k_1 = 4l + 2 = k_2 + 1$, $l > 1$, $k_3 = 3$, i even.

(ix) $v = 2k_1(i+2) + 6$, $k_1 = k_2 + 1 > 5$, $k_3 = 3$, $i \equiv 1 \pmod 4$.

(x) $v = 2k_1(i+1) + 8$, $k_1 \equiv 1 \pmod 4 = k_2 + 1$, $k_3 = 4$, $i \equiv 3 \pmod 4$.

(xi) $v = 2k_1(i+1) + 8$, $k_1 \equiv 3 \pmod 4 = k_2 + 1$, $k_3 = 4$, $i \equiv 2 \pmod 4$.

(xii) $v = 2k_1(i+1) + 10$, $k_1 = 4l$, $l > 1$, $k_2 = k_1 - 1$, $k_3 = 5$, i integer.

(xiii) $v = 2k_1(i+1) + 10$, $k_1 = k_2 + 1 = 4l + 2$, $k_3 = 5$, i odd.

(xiv) $v = 2k_1(i+1) + 10$, $k_1 = k_2 + 1 > 5$, $k_3 = 5$, $i \equiv 3$ (mod 4).

(xv) $v = 2k_1(i+1) + 4$, $k_1 \equiv 1$ (mod 4) , $k_2 = k_1 - 2$, $k_3 = 3$, $i \equiv 2$ (mod 4).

(xvi) $v = 2k_1(i+1) + 4$, $k_1 \equiv 3$ (mod 4), $k_2 = k_1 - 2$, $k_3 = 3$, $i \equiv 2$ (mod 4).

(xvii) $v = 2k_1(i+1) + 6$, $k_1 = k_2 + 2 = 4l + 2$, $k_3 = 4$, i even.

(xviii) $v = 2k_1(i+1) + 6$, $k_1 > 7$, $k_2 = k_1 - 2$, $k_3 = 4$, $i \equiv 1$ (mod 4).

(xix) $v = 2k_1(i+1) + 8$, $k_1 \equiv 1$ (mod 4), $k_2 = k_1 - 2$, $k_3 = 5$, $i \equiv 0$ (mod 4).

(xx) $v = 2k_1(i+1) + 8$, $k_1 \equiv 3$ (mod 4) , $k_2 = k_1 - 2$, $k_3 = 5$, $i \equiv 2$ (mod 4).

(xxi) $v = 2k_1(i+1) + 2$, $k_1 = 4l$, $l > 1$, $k_2 = k_1 - 3$, $k_3 = 3$, i integer.

(xxii) $v = 2k_1(i+1) + 2$, $k_1 = 4l + 2$, $l > 1$, $k_2 = k_1 - 3$, $k_3 = 3$, i odd.

(xxiii) $v = 2k_1(i+1) + 2$, $k_1 > 5$, $k_2 = k_1 - 3$, $k_3 = 3$, $i \equiv 3$ (mod 4).

(xxiv) $v = 2k_1(i+1) + 4$, $k_1 \equiv 1$ (mod 4), $k_2 = k_1 - 3$, $k_3 = 4$, $i \equiv 2$ (mod 4).

(xxv) $v = 2k_1(i+1) + 4$, $k_1 \equiv 3$ (mod 4), $k_2 = k_1 - 3$, $k_3 = 4$, $i \equiv 0$ (mod 4).

(xxvi) $v = 2k_1(i+1) + 6$, $k_1 = 4l + 2 = k_2 + 3 > 5$, $k_3 = 5$, i even, l integer.

(xxvii) $v = 2k_1(i+1) + 6$, $k_1 > 5$, $k_2 = k_1 - 3$, $k_3 = 5$, $i \equiv 2$ (mod 4).

Example 5.4

Following sets generate minimal CGND for v = 26, k_1 = 5, k_2 = 4 and k_3 = 3 with $E_n = 0.87$.
S_1 = [3,5,7,9], S_2 = [4,10,11], S_3 = [8,12]

5.5 MCGNDs in three different block sizes when $m \equiv 0$ (mod 4), from i sets for k_1, one set for k_2 and two for k_3

For the following cases, MCGNDs can be obtained in blocks of three different sizes through i sets for k_1, one set for k_2 and two for k_3, derived from theorem 5.1.

(i) $v = 2k_1(i+3) - 8$, $k_1 \equiv 1$ (mod 4)$= k_2 + 1$, $k_3 = k_1 - 2$, $i \equiv 2$ (mod 4).

(ii) $v = 2k_1(i+3) - 8$, $k_1 \equiv 3$ (mod 4)$= k_2 + 1$, $k_3 = k_1 - 2$, $i \equiv 0$ (mod 4).

(iii) $v = 2k_1(i+3) - 12$, $k_1 \equiv 3$ (mod 4)$= k_2 + 1$, $k_3 = k_1 - 3$, $i \equiv 2$ (mod 4).

(iv) $v = 2k_1(i+3) - 14$, k_1 (odd) > 3, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, $i \equiv 1$ (mod 4).

Example 5.5

Following sets generate minimal CGND for $v = 42$, $k_1 = 5$, $k_2 = 4$ and $k_3 = 3$ with $E_n = 0.88$.
S_1 = [3,4,14,20], S_2 = [7,8,9,12], S_3 = [10,13,17], S_4 = [15,16], S_5 = [18,19]

6. Discussion and conclusion

Neighbor effects may arise in experiments of serology for virus research and in agriculture experiments, due to nature of plots, etc. In the presence of neighbor effects, misleading conclusions may be drawn in the variety competition experiments. Minimal NBDs are available in literature to neutralize these effects economically for v odd.

To overcome this problem for v even, complete solution is given in this article to construct proposed MCGNDs. For v even, our proposed designs have been proved efficient mathematically and logically to reduce the bias due to neighbor effects, therefore, practitioners are recommended to implement these designs in their experiments where neighbor effects may arise. With the collaboration of practitioners in future research, author(s) will apply these designs on their relevant experiments and then give numerical results for further recommendations.

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