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Description of the energy levels and electric quadrupole transitions of the ^{92}Nb and ^{92}Mo nuclei using nuclear shell model

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Abstract

In the current study,^{92}Nb and ^{92}Mo isotopes have been determined for calculating energy levels and electric quadrupole transition probabilities. Two interactions that have been applied in this study are surface delta and modified surface delta interactions. The calculations have been achieved by using appropriate effective charges for proton and neutron as well as parameter length of harmonic potential. Computed results have been compared with the experimental values. After this comparison, energy and the transition probability values have a good agreement with the experimental values, also there are values of the total angular momentum and parity are determined and confirmed for some of the experimental energies, undetermined and unconfirmed experimentally. Theoretically, new values of quadrupole electric transition probabilities have been explored which have not been known in the experimental data.

Keywords: appropriate effective charges, harmonic potential, parities, Shell model, total angular moment.

1. Introduction

In early 1934, Elsasser author observed that there are "special numbers" of fermions (neutrons and protons) that Grant to the nuclei a distinctively stable configuration (sorlin, 2014). The author suggested that these numbers are associated with closed shells in a model of non-interacting nucleons occupying energy levels generated by a potential well. subsequently, Mayer, Haxel, Suess, and Jensen (Mayer,1949; Haxel et al., 1949) indicated that this nuclear potential could be constituted by a one-body Harmonic Oscillator(HO) an(L.L) term and a spin-orbit (SO) potential for creating the shell gaps of the harmonic oscillator at 8, 20, 40 and 50 so shell gaps at 28, 50, 82 and 126. (Elliott and Lane,1954). These magic numbers can be explained according to the nuclear shell model of the nucleus which conceives each nucleon in moving in single-particle orbits within some potential and regulates the energy levels in terms of quantum numbers (nlj) (Bhatt et al.,1992) This model became the standard tool for interpretation and calculation of many aspects of nuclear structure such as total angular momentum, parity, nuclear energies, and wave functions and some implications for nuclear many-body theory(Talmi,2005), an convenient double magic nucleus is considered as inert core which the nucleons in that the nucleons that give J = 0 total angular momentum in the core are not moving and the valence nucleons that are more than this are included in the calculations. Accordingly, these nucleons can't be included in valence nucleons out of the core. Assuming that the valence nucleons may be distributed in the shells just above the core, these shells are

considered as model space. Nucleons in the model space can be placed in all combinations in each orbit. Different combinations cause different energy levels of the nucleus (Akkoyun,2020). This work is aimed to calculate the energy levels and electric quadrupole transition probabilities for isotopes with Z =41,42. These isotopes include^{92}Nb and ^{92}Mo using two interactions.

2. Nuclear Shell Model formalism

The most essential matter in the nuclear structure at low energies is the residual interaction between nucleons in the valence shell. This residual interaction depends on the numbers of valence nucleons (protons and neutrons)as well as the available valence orbitals, this interaction is defined as the collision force between the nucleons that occur due to the perturbation of the Hamiltonian effect, which is equal to the sum of two basic parts and is given according to the following relationship (Brown, 2010; Frank et al., 2009; Jassim and Sahib, 2018)

$$\widehat{H} = H_D + \hat{V} \quad (1)$$

Where: H_D is given in the following mathematical form:

$$H_D = \sum_{i=1}^{A}(\frac{-\hbar}{2m}\nabla^2 + U_i) = \sum_{i=1}^{A}\varepsilon_i \quad (2)$$

Where $\hat{V}$ describes the residual interaction between the two particles and takes the following form:

$$\hat{V} = \sum_{i>k}^{A} V_{ik}(r_i - r_k) - \sum_{i=1}^{A} U_i\ (r_i) \quad (3)$$

Where H_D is defined as follows: H_D: It is the diagonal Hamiltonian that describes the independent nucleons movement among each other in the same intermediate field (Hamiltonian effect without perturbation) and which contains the single-particle energies of nucleons(Ghorui and Praharaj, 2013). The residual interactions can be small if a suitable field is chosen, thus it is neglected in the independent particle envelope model, in this model, the nucleons interact only indirectly through this field. In addition, the principle of exclusion must be achieved by Pauli, and the central potentials are usually approximated using square well potential, harmonic oscillating, or Wood-Saxon potentials(Bürger, 2007). Many studies described the residual interactions using shell model calculations such as; Majeed et al. who applied the active reactions jun45 and jj44b of $^{76\text{-}66}$Ni isotopes to calculate the excitations energies (Majeed et al., 2014), Herndl and Brown studied the properties of the ^{89}Tc,^{92}Rh, ^{94}pd, and ^{100}Sn isotopes by employing the active interaction (SLGT0) (Herndl and Brown, 1997). Finally, Majeed and Obaid studied the energy levels of the nuclei ^{134,136} Te and ^{134,136}Sn, using the active reactions jj56pna and jj56pn(Majeed and Obaid, 2016). For the mixed order case, a state's group similar to (J^{π}, T) values are taken, this means they can give the same values for (J^{π}, T)for the levels, then mixing order is built for these states and get the energies values by a diagonal process of the interaction matrix, after that, when getting the energies values, the states can be defined, where the state with the lowest energy value is the ground state and the states after which are the excited states, the basis on which we relied on in the case of the mixed arrangement is assuming that the (E) plane represents an energy level through the following mathematical formula(Brown and Richter, 2006; Caurier et al., 2005; Griffiths and Schroeter, 2018; Sorlin and Porquet, 2008).

$$E = E_K{}^{(0)} + E_K{}^{(1)} \quad (4)$$

Where, the contribution $E_K{}^{(0)}$ results from the Hamiltonian H_D that describes the independent-particle motion, while the contribution $E_K{}^{(1)}$ derives from the residual interaction $\hat{V}$.The element of the Hamiltonian matrix can be described according to the particles in the outer shell (outside the closed-shell) and the arrangement of the state j with the following equation(Otsuka, 2009; Otsuka et al., 2005).

$$H_{ij} = (\varepsilon_i + \varepsilon_j)\delta_{ij} + \langle j_i j_j | \mathrm{V} | j_i j_j \rangle \quad (5)$$

Where ε_i and ε_j is single-particle energies for orbit i, j respectively, these energies can be found from the vicinity of the closed-shell, with a mass number that exceeds one nucleon from the closed core, this can be as in the following equation (Brussard,and Glaudemans, 1977):

$$\varepsilon_i = BE(core+1) - BE(core) \quad (6)$$

While $\langle j_i j_j | \mathrm{V} | j_i j_j \rangle$ in equ. (5) stands for the matrix element of the residual two-body interaction by using surface delta interaction and modified surface delta interaction.

The surface delta interaction between valence nucleons can be defined according to the following formula (Brussard,and Glaudemans, 1977) :

$$v_{SDI}(r_1,r_2) = -4\pi A_T \delta(r_1 - r_2)\, \delta(r_1 - R_0) \quad (7)$$

where A_T is the strength parameter for isospin T=0 or1, while r_1 and r_2 are the position vectors of the interacting particles and R_0 is the nuclear radius. While another interaction is the modified surface delta interaction is given by(Brussard, and Glaudemans, 1977) :

$$V_{MSDI}(r_1,r_2) = \left[-4\pi A_T \delta(r_1 - r_2)\delta(r_1 - R_0) + B\big(\tau(1).\tau(2)\big) + C\right] \quad (8)$$

When $\langle \tau(1).\tau(2) \rangle = 2T(T+1) - 3$

and $\langle B\big(\tau(1).\tau(2)\big) + C \rangle = \begin{Bmatrix} -3B + C \; for \; T = 0 \\ B + C \; for \; T = 1 \end{Bmatrix}$

$\tau(1), \tau(2)$ are isospin for particles 1 and 2, while B and C are strength parameters.

Through equations (5 and 6), it is possible to calculate the energies values of nucleons in the pure arrangement and the energy matrix elements, Hamilton, in the mixed arrangement, but in the nuclei that contain two different nucleons (protons - neutrons) in different orbits, the energy matrix elements (Hamilton) is defined as in the following equation (DeShalit and Feshbach, 1974; Otsuka , 2009).

$$H_{ij} = (\varepsilon_i + \varepsilon_j)\delta_{ij} + \frac{1}{2}\left[\langle \langle j_i j_j | \mathrm{V} | j_i j_j \rangle \rangle_{J,0} + \langle \langle j_i j_j | \mathrm{V} | j_i j_j \rangle \rangle_{J,1}\right] \quad (9)$$

The main information reference for total angular momentum and parity is electromagnetic transitions because the electromagnetic interactions are more declared than nuclear forces inside nuclei. Gamma rays emission is associated with nuclear decay, transformations, and internal interactions, while the decay rate of the emitted gamma photon results from multipolar transitions from an initial state j_i to a final state j_f as shown in the equation below (Cáceres, 2008):

$$T\big(\sigma\lambda; J_i \to J_f\big) = \left[\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left(\frac{E\gamma}{\hbar c}\right)^{2\lambda+1} \times B\big(\sigma\lambda; J_i \to J_f\big)\right] \quad (10)$$

The symbols above equation can be explained as follow :(σ)the type of polarity, λ the polarity order, (j_i, j_f) the initial and the final states respectively,(Eγ) the emitted gamma photon energy in MeV units, while $B\ (\sigma\lambda;\ j_i \to j_f)$defined the reduced transition probability. The decay rate can be related to half-life according to the relationship:

$$T(\sigma\lambda; J_i \rightarrow J_f) = \frac{ln\,2}{T_{1/2}\ (1+\alpha)} \quad (11)$$

($T_{1/2}$) Represents the half-life at the initial state and (α) is the conversion factor of the emitted gamma radiation. From equations (10) and (11) the $B,(\sigma\lambda;\ j_i \rightarrow j_f)$ is computed by the following equation:

$$B(\sigma\lambda; J_i \rightarrow J_f) = \frac{\lambda[(2\lambda+1)!!]^2}{8\pi(\lambda+1)} \frac{\hbar\, ln\,2}{T_{1/2}\ (1+\alpha)} \left(\frac{\hbar c}{E\gamma}\right)^{2\lambda+1} \quad (12)$$

The measurement unit for the reduced electrical transmission probability is $e^2(fm)^{2\lambda}$.
The reduced transition probability is given by the following equation:

$$B(\sigma\lambda; J_i \rightarrow J_f) = \frac{1}{2J_i+1} \left| < J_f\ \alpha_f \right| \left| O^{\sigma\lambda} \right| \left| J_i\alpha_i > \right|^2 \quad (13)$$

As $O^{\sigma\lambda}$ is the multipolar electromagnetic operator of the configuration (λ).
The reduced transition probability for a quadrupole transition can be written based on equation (13) as follow:

$$B(E2; J_i \rightarrow J_f) = \frac{1}{2J_i+1} \left| < J_f\ \alpha_f \right| \left| O^{E2} \right| \left| J_i\alpha_i > \right|^2 \quad (14)$$

Several successful studies explained the mixing configuration of energy levels and that calculate the electric-quadrupole transition: Researchers Hasan and Obeed calculated the energy levels of a ^{30}P phosphorous nucleus for both the mixed and pure arrangements(Hasan and Obeed, 2009), Hasan studied the effect of mixed configuration on nuclei ^{93}Tc, ^{92}Mo, ^{91}Nb, ^{90}Zr (Hasan, 2009). All those researchers studied only the energy levels of the mixed configurations of the mentioned nuclei, while researcher Werner et al. studied the structure of the isotopes ^{92}zr, ^{94}Mo, and ^{96}Ru, to calculate the excitation energies and transitions of the reduce electric quadrupole (Werner et al., 2002). Finally, studying the isotopes rich in neutrons^{132}Sn for calculating reduced electrical transitions B (E2; $2^+ \rightarrow 0^+$) was studied by the researcher Aissaoui et al. (Aissaoui et al., 2009).

3. Numerical details

3.1.1 Energy levels :

The Niobium nucleus contains two nucleons (one proton and one neutron) within the model space ($1g_{9/2}$, $1g_{7/2}$)for proton and ($2d_{5/2}$,$1g_{7/2}$,$3s_{1/2}$,$2d_{3/2}$ and $1h_{11/2}$)for neutron. While the molybdenum nucleus has 42 protons and50 neutrons outside the closed core of a zirconium-90 nucleus. The valence nucleons(two protons) of this nucleus are located within the model space ($1g_{9/2}$,$1g_{7/2}$) From the aforementioned orbits of both above nuclei, the single particles' energies of proton and neutron have been calculated from equation (6)and tabulated in tables. (1and 2) While table.3, shows the necessary coefficients for the interaction strength for the surface delta and modified surface delta interactions for the niobium nucleus and molybdenum nucleus. These parameters are necessary to compute the matrix elements by using interactions(SDI and MSDI) on respectively, while the eigenvalues of the energy have been computed according to

the equations (5,9). These values are very essential for calculating the predicted energy level values for all allowable angular momentum and parity for both niobium and molybdenum nuclei.Figures(1and 2) displays comparing the theoretical energy levels with experimental values[Baglin,2012]for both nuclei in under a study.

3.1.2. Reduced electric quadrupole transition probabilities $(B(E2;\downarrow))$

The other properties of the nucleus are reduced electric quadrupole transition probabilities which it has been calculated from equation (14). These calculations relied on several necessary parameters are the size of the harmonic oscillator and the effective charges.Table.4 shows these parameters for both niobium and molybdenum nuclei using surface delta and modified surface delta interactions. These coefficients were selected to obtain the best fit for the theoretical values with the experimental values

4. Numerical results and discussion

One of the nuclear properties is the energy level. In addition, the states (total angular momentum and parity) can be known by comparing them with the experimental values (Baglin,2012), Figure (1)displays comparing the theoretical energy levels with experimental values with their total angular momentum and the parities for niobium nucleus. From that comparison shows follows:

1. The expected theoretical energy values in MeVunit {1.9641 , 2.0354 , 2.4300 , 2.5516 , and 3.5287 } have appeared at the surface delta interaction, while the values { 1.9672 , 2.0432 , 2.4546 , 2.5775 and 3.5483} have determine the modified surface delta interaction. These values have appeared very closely identical to the experimental energy values in MeVunit{1.9720, 2.1280, 2.4330, 2.6100, and 3.5300} respectively, for the same total angular momentum and parity.

2. Total angular momentum and parity have been $\{4^+, 6^-\}$ of surface delta interaction, while the values $\{4^+, 6^-\}$ have located at the modified surface delta interaction, these values have confirmed the experimental energy values in MeV unit { 2.5800, 2.2033}respectively.

3. It has been expected a confirmation of only the total angular momentum{7, 5 , 4, 6 , 4 , 5 , 1 and 3} of surface delta interaction, whilevalues{7, 5 , 4, 6 , 4 , 5 , 1 and 3} have specified the modified surface delta interaction ,these values have confirmed the experimental energy values in MeV unit {0.2857, 0.3574 , 0.4802 , 0.5012, 1.4728,1.5657,1.6503 and 2.4630 }respectively.

4. Parity only{+ ,+ , + , + , + , +, − , − and +} have been allocated at the surface delta interaction, while the parity{+,+,+ ,+ ,+ , +,− , − and + } have been designated at modified surface delta interaction these parities have confirmed the experimental energies{2.1470, 2.1620 ,2.2130, 2.2920 , 2.4030 , 2.5940 , 2.6560 , 2.7000 and 3.4890 } respectively.

5. It is expected that the total angular momentum and parity{10^-, 2^-, 8^-, 4^- , 0^+ , 1^+, 7^+ , 2^+, 5^+, 6^+ and 3^+ } have been allocated on surface delta interaction, while the total angular momentum and parity {10^-, 2^-, 8^-, 4^- , 0^+ , 7^+, 3^+ , 5^+, 2^+, 5^+, 9^- and 3^+ } have been determined at modified surface delta interaction, these values have determined the experimental energies in MeV unit { 2.3110 , 2.3350, 2.4980 , 2.5150 , 3.5500 , 3.9200 , 4.0320 ,4.1720 , 4.2850 , 4.3550 and 4.4500} respectively, through the present theoretical calculations.

6. The total angular momentum {1, 6, 4, 6, 7, 2, and 3} have been decided at surface delta interaction, while the total angular momentum {1, 6, 4, 6,7, 1 and 2} have been specified at modified surface delta interaction the above values have been designated the experimental energies in MeVunit { 1.8510, 2.0330, 2.3620, 2.5300, 2.6660, 4.0790, 4.1350 and 3.7530 } for the above total angular momentum respectively

7. It is noticed through theoretical calculations that there are values that have not been compared with the practical values because they are far from them. While the figure.2 illustrates comparing between the predicted energy levels with experimental values of the valence nucleons for molybdenum nucleus (Baglin, 2012)

8. The expected theoretical energy value in MeV unit (3.1672) for the surface delta reaction appeared identical with the experimental energy value (3.0913) for the same total angular momentum and parity.

9. The expected theoretical energy in MeV unit (0.9699) is on the surface delta interaction, while the values {1.0211, 3.2298} are defined at the modified surface delta interaction. These values have appeared somewhat compatible with available experimental energy values in MeV unit {1.5095, 3.0913} respectively, for the same total angular momentum and parity.

10. The total angular momentum and parity{ 4^+, 1^+, 3^+, 5^+ , 7^+ and 2^+} is for the surface delta interaction, while the values { 4^+, 1^+, 3^+, 5^+ , 7^+ and 2^+} have been located at the modified surface delta interaction ,these values have confirmed the experimental energy values in MeV unit {3.0636, 3.3686 and 4.9707} respectively.

11. It expected a confirmation only for the total angular momentum {8,6 and 6} these have been assigned for the surface delta interaction, while the values {8,6 and 6} for the modified surface delta interaction, these values have confirmed the experimental energy values in MeV unit {2.8386, 2.9226 and 5.0036} respectively. Parity (+) has specified for the experimental energy value (4.9790) MeV unit for interactive surface delta interaction and modified surface delta interaction respectively by predicted theoretical calculations.

12. The total angular momentum and parity (0^+) have been specified for the experimental energy (4.5444) MeV for interactive surface delta interaction and modified surface delta interaction respectively by expected theoretical calculations.

13. Through theoretical calculations, it has been noticed that there are values that have not been compared with the practical values because they are far from them. While tables(4 and 5) are shown comparing the theoretical reduced electric quadrupole transition probabilities($B(E2;\downarrow)$) with experimental values for ^{92}Nb and ^{92}Mo isotopes.

Table5 illustrates the comparison between the predicted theoretical and experimental values(Baglin, 2012). These comparisons showed the following For reduced electric quadrupole transition probabilities ($B(E2;\downarrow)$): ^{92}Nb nucleus; there is excellent compatibility on transition ($3_1^+ \longrightarrow 2_1^+$) in $e^2(fm)^4$ unit for both the delta surface and the modified surface delta interactions for the values (30.8268)(30.8268) respectively, these values have been compared with the experimental value (Baglin, 2012) (29.60 ± 37.00) in units of $e^2(fm)^4$. There are also several new values of the expected theoretical results found in our calculations that are not known experimentally yet. Table 6. displays the reduced electric quadrupole transition probabilities($B(E2;\downarrow)$, for ^{92}Mo nucleus, the explanations can be as follows; There has been very excellent compatibility for the transition ($2_1^+ \longrightarrow 0_1^+$) in $e^2(fm)^4$ unit for both the delta surface and the modified surface delta interactions for the values (207.5644) (207.1713) respectively, these results have been compared with the experimental value(207.22 ± 12.33) (Baglin,2012)in $e^2(fm)^4$ units, there are also several new values of the expected theoretical results found in our calculations that are not known experimentally yet

Table 1. Single particle energies according to the mixing configurations of orbits for ^{92}Nb nucleus

Configurations	**Energy Values(MeV)** **(** Baglin, 2013**)**
$2d_{5/2}$ (n) , $1g_{9/2}$ (p)	-11.8378
$2d_{5/2}$ (n) , $1g_{7/2}$ (p)	-9.8747
$1g_{7/2}$(n) , $1g_{9/2}$(p)	-9.9556
$1g_{7/2}$ (n), $1g_{7/2}$ (p)	-7.9925
$3s_{1/2}$(n) , $1g_{9/2}$ (p)	-10.6330
$3s_{1/2}$ (n), $1g_{7/2}$ (p)	-8.6699
$2d_{3/2}$ (n), $1g_{9/2}$ (p)	-9.7955
$2d_{3/2}$ (n) , $1g_{7/2}$ (p)	-7.8324
$1h_{11/2}$(n), $1g_{9/2}$ (p)	-9.6677
$1h_{11/2}$(n), $1g_{7/2}$ (p)	-7.7046

Table 2. Single-particle energies according to the mixing configurations of orbits for nucleus ^{92}Mo nucleus.

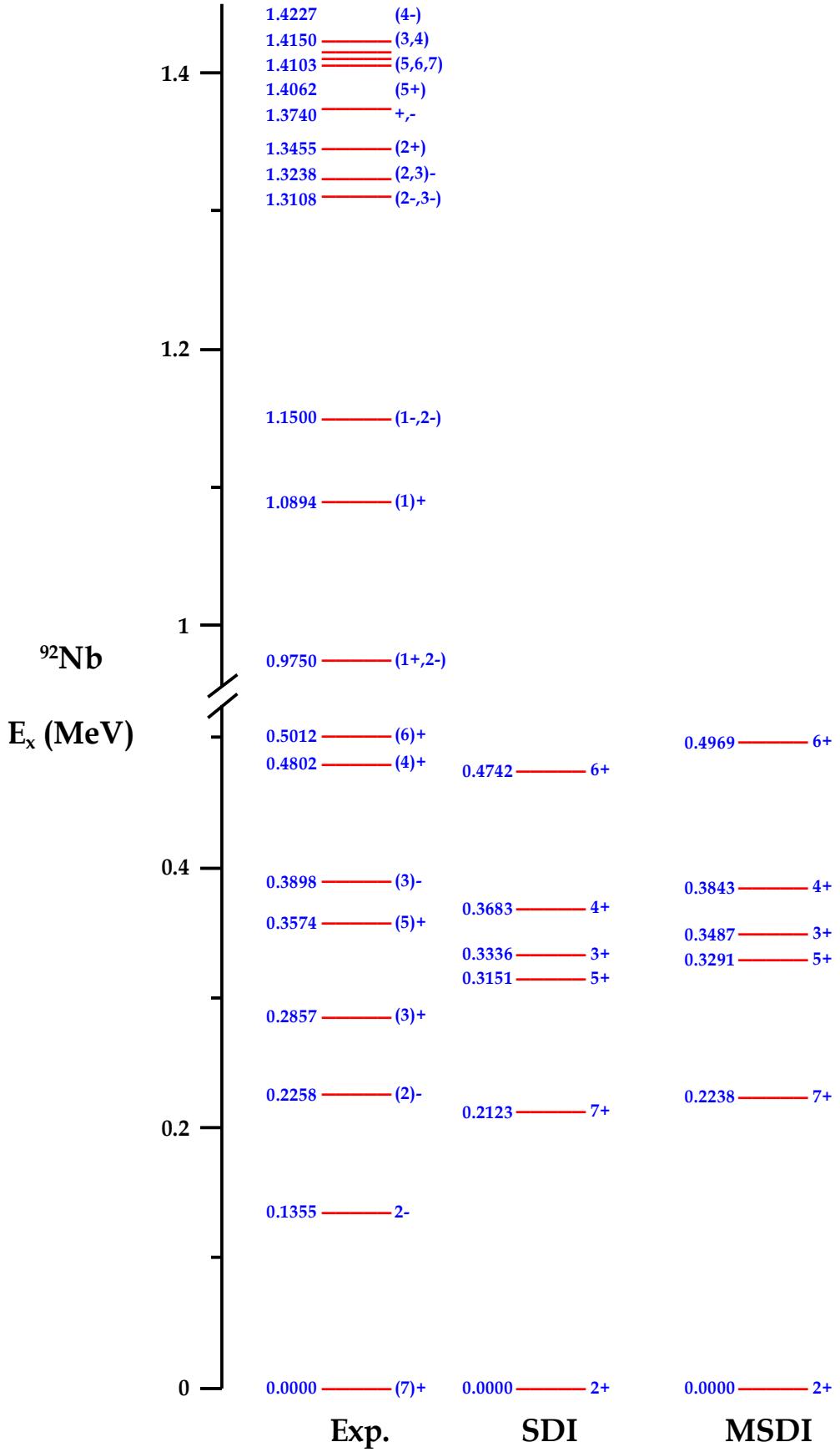
Configurations	**Energy Values(MeV)(** Baglin, 2013**)**
$1g_{9/2}$ (p), $1g_{9/2}$(p)	-9.2867
$1g_{9/2}$(p), $1g_{7/2}$ (p)	-7.3236
$1g_{7/2}$(p) , $1g_{7/2}$(p)	-5.3605

Table 3. Interaction strength parameters for the surface delta and modified surface delta interactions for ^{92}Nb and ^{92}Mo nuclei.

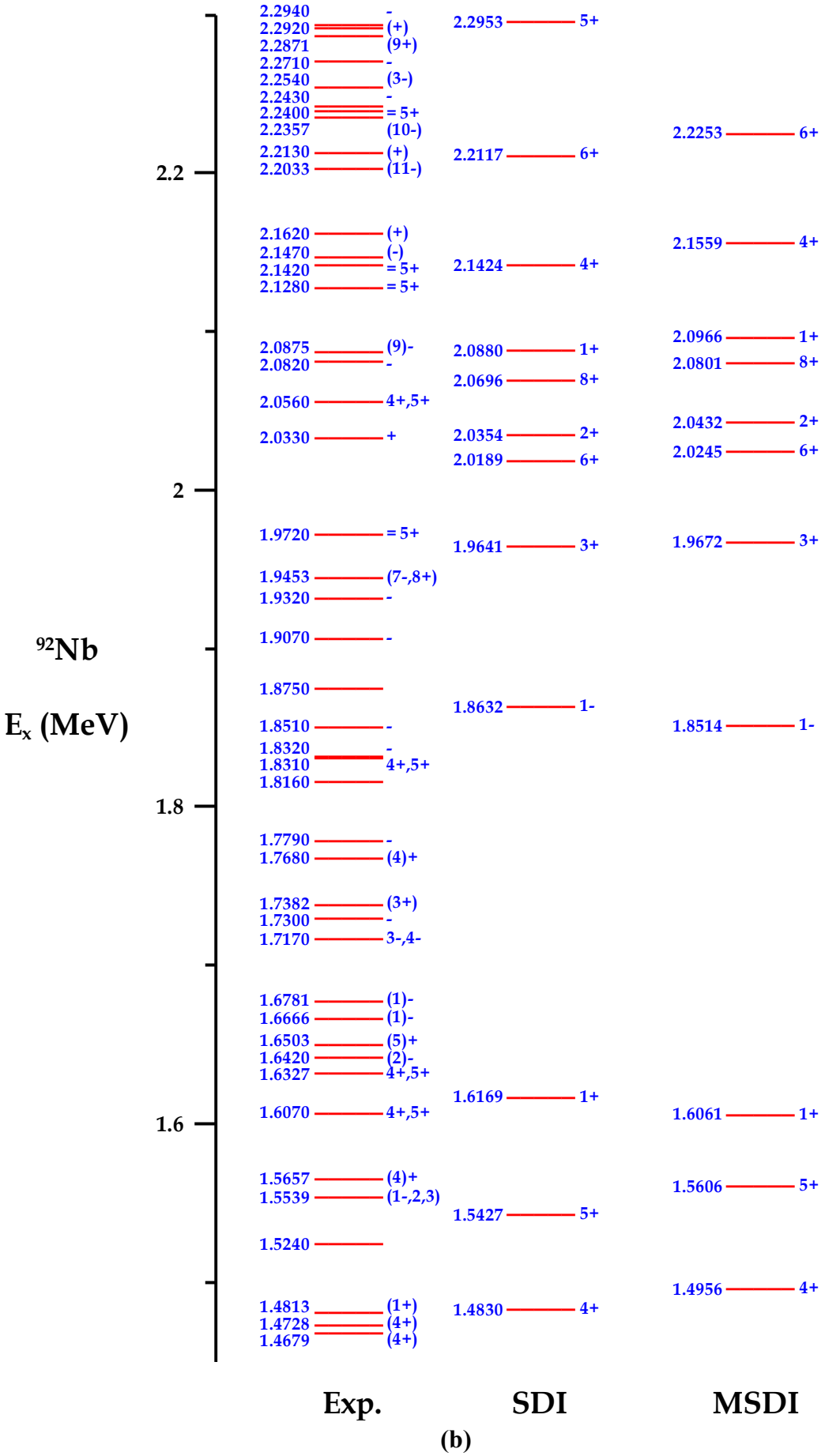
Parameters	**Values(MeV)**
$A_T(SDI)$	0.2079
A_T(MSDI)	0.2173
B	0.1089
C	0

Table 4. Parameters of Harmonic oscillator size inverse and effective charge values using surface delta and modified surface delta interactions for ^{92}Nb and ^{92}Mo nuclei

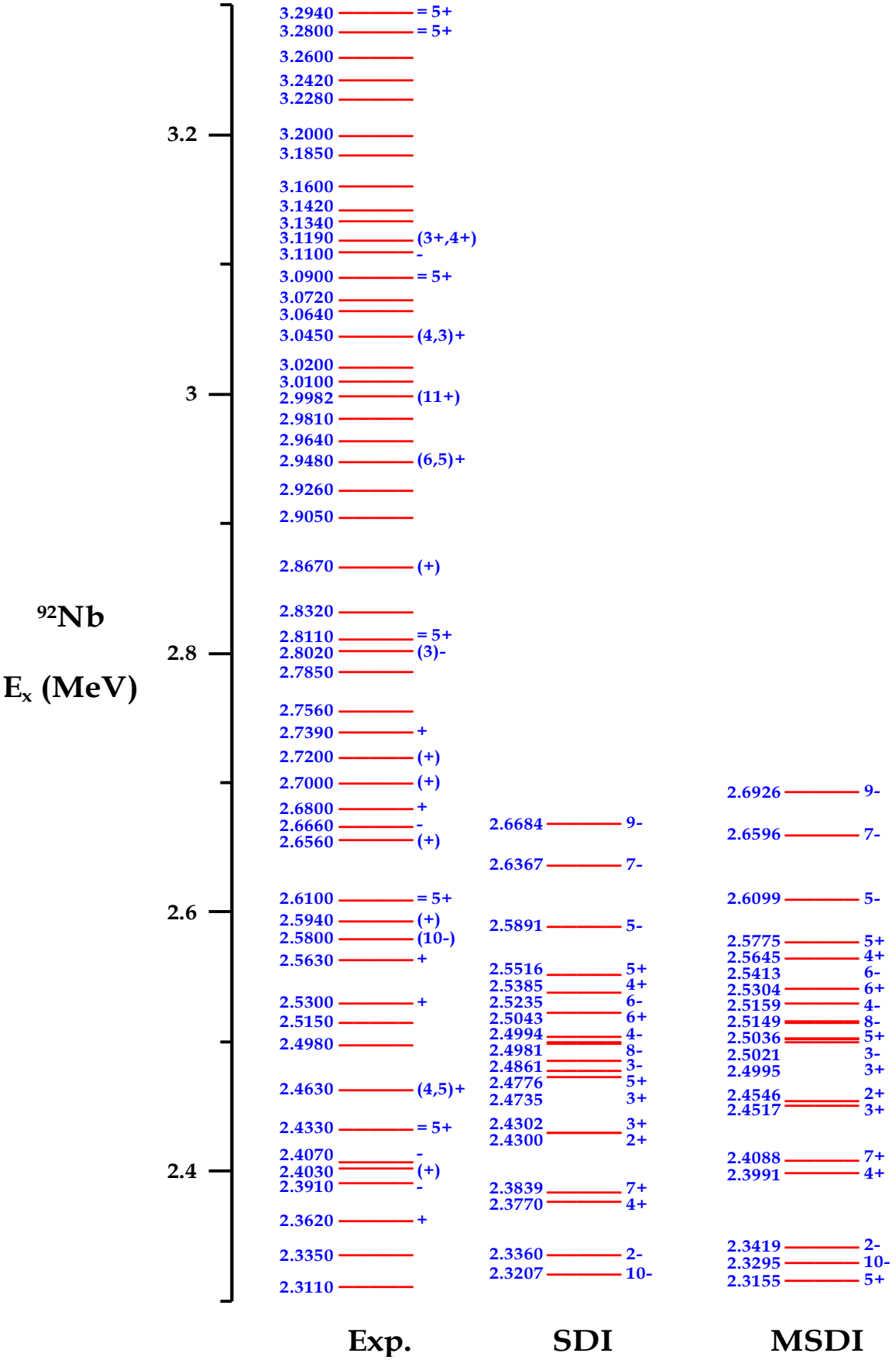
Parameters	**Values**
^{92}Nb nucleus	
α	0.458 $(fm)^{-1}$
e_{eff}	0.89e
^{92}Mo nucleus	
α	0.458$(fm)^{-1}$
e_{eff}	2.006e


92Nb
Ex (MeV)
1.4
1.2
1
0.4
0.2
0
1.4227 (4-)
1.4150 (3,4)
1.4103 (5,6,7)
1.4062 (5+)
1.3740 +,-
1.3455 (2+)
1.3238 (2,3)-
1.3108 (2-,3-)
1.1500 (1-,2-)
1.0894 (1)+
0.9750 (1+,2-)
0.5012 (6)+
0.4802 (4)+
0.3898 (3)-
0.3574 (5)+
0.2857 (3)+
0.2258 (2)-
0.1355 2-
0.0000 (7)+
0.4742 6+
0.3683 4+
0.3336 3+
0.3151 5+
0.2123 7+
0.0000 2+
0.4969 6+
0.3843 4+
0.3487 3+
0.3291 5+
0.2238 7+
0.0000 2+
Exp.
SDI
MSDI

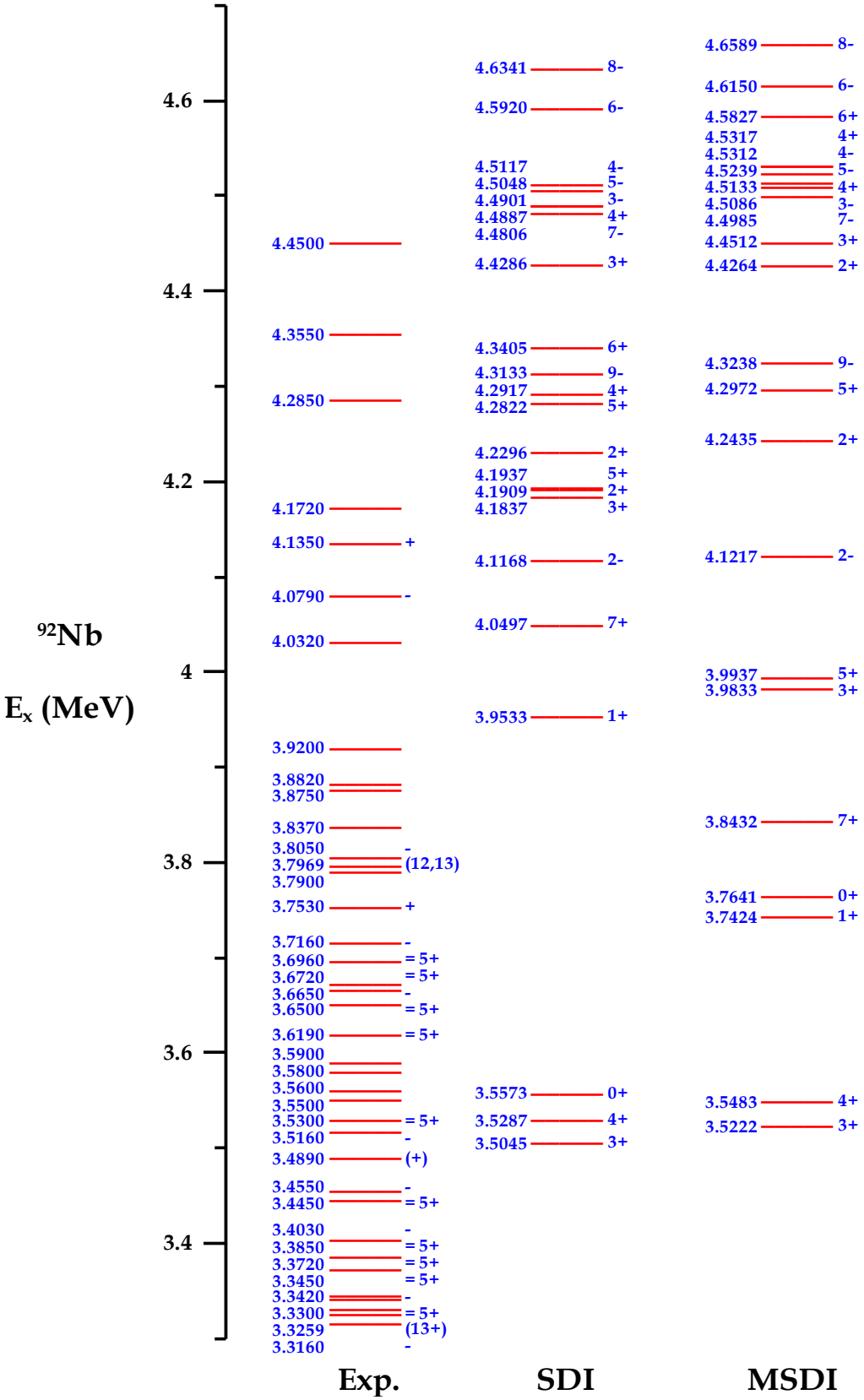
(a)


92Nb
Ex (MeV)
2.2
2
1.8
1.6
2.2940 -
2.2920 (+)
2.2871 (9+)
2.2710 -
2.2540 (3-)
2.2430 -
2.2400 = 5+
2.2357 (10-)
2.2130 (+)
2.2033 (11-)
2.1620 (+)
2.1470 (-)
2.1420 = 5+
2.1280 = 5+
2.0875 (9)-
2.0820 -
2.0560 4+,5+
2.0330 +
1.9720 = 5+
1.9453 (7-,8+)
1.9320 -
1.9070 -
1.8750
1.8510 -
1.8320 -
1.8310 4+,5+
1.8160
1.7790 -
1.7680 (4)+
1.7382 (3+)
1.7300 -
1.7170 3-,4-
1.6781 (1)-
1.6666 (1)-
1.6503 (5)+
1.6420 (2)-
1.6327 4+,5+
1.6070 4+,5+
1.5657 (4)+
1.5539 (1-,2,3)
1.5240
1.4813 (1+)
1.4728 (4+)
1.4679 (4+)
2.2953 5+
2.2117 6+
2.1424 4+
2.0880 1+
2.0696 8+
2.0354 2+
2.0189 6+
1.9641 3+
1.8632 1-
1.6169 1+
1.5427 5+
1.4830 4+
2.2253 6+
2.1559 4+
2.0966 1+
2.0801 8+
2.0432 2+
2.0245 6+
1.9672 3+
1.8514 1-
1.6061 1+
1.5606 5+
1.4956 4+
Exp.
SDI
MSDI

(b)

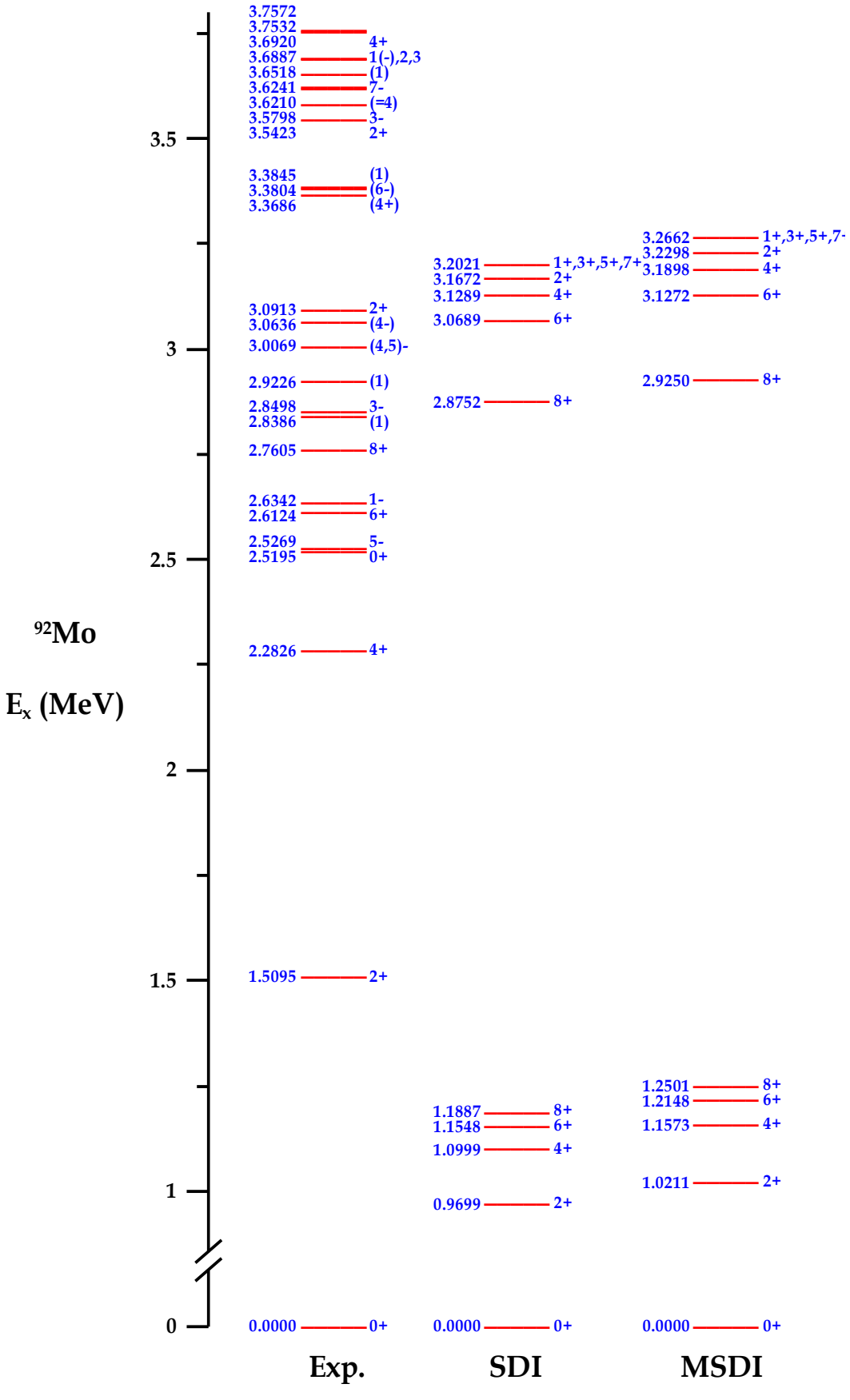

92Nb
Ex (MeV)
3.2
3
2.8
2.6
2.4
3.2940 = 5+
3.2800 = 5+
3.2600
3.2420
3.2280
3.2000
3.1850
3.1600
3.1420
3.1340
3.1190 (3+,4+)
3.1100 -
3.0900 = 5+
3.0720
3.0640
3.0450 (4,3)+
3.0200
3.0100
2.9982 (11+)
2.9810
2.9640
2.9480 (6,5)+
2.9260
2.9050
2.8670 (+)
2.8320
2.8110 = 5+
2.8020 (3)-
2.7850
2.7560
2.7390 +
2.7200 (+)
2.7000 (+)
2.6800 +
2.6660 -
2.6560 (+)
2.6100 = 5+
2.5940 (+)
2.5800 (10-)
2.5630 +
2.5300 +
2.5150
2.4980
2.4630 (4,5)+
2.4330 = 5+
2.4070 -
2.4030 (+)
2.3910 -
2.3620 +
2.3350
2.3110
Exp.
2.6684 9-
2.6367 7-
2.5891 5-
2.5516 5+
2.5385 4+
2.5235 6-
2.5043 6+
2.4994 4-
2.4981 8-
2.4861 3-
2.4776 5+
2.4735 3+
2.4302 3+
2.4300 2+
2.3839 7+
2.3770 4+
2.3360 2-
2.3207 10-
SDI
2.6926 9-
2.6596 7-
2.6099 5-
2.5775 5+
2.5645 4+
2.5413 6-
2.5304 6+
2.5159 4-
2.5149 8-
2.5036 5+
2.5021 3-
2.4995 3+
2.4546 2+
2.4517 3+
2.4088 7+
2.3991 4+
2.3419 2-
2.3295 10-
2.3155 5+
MSDI

(c)

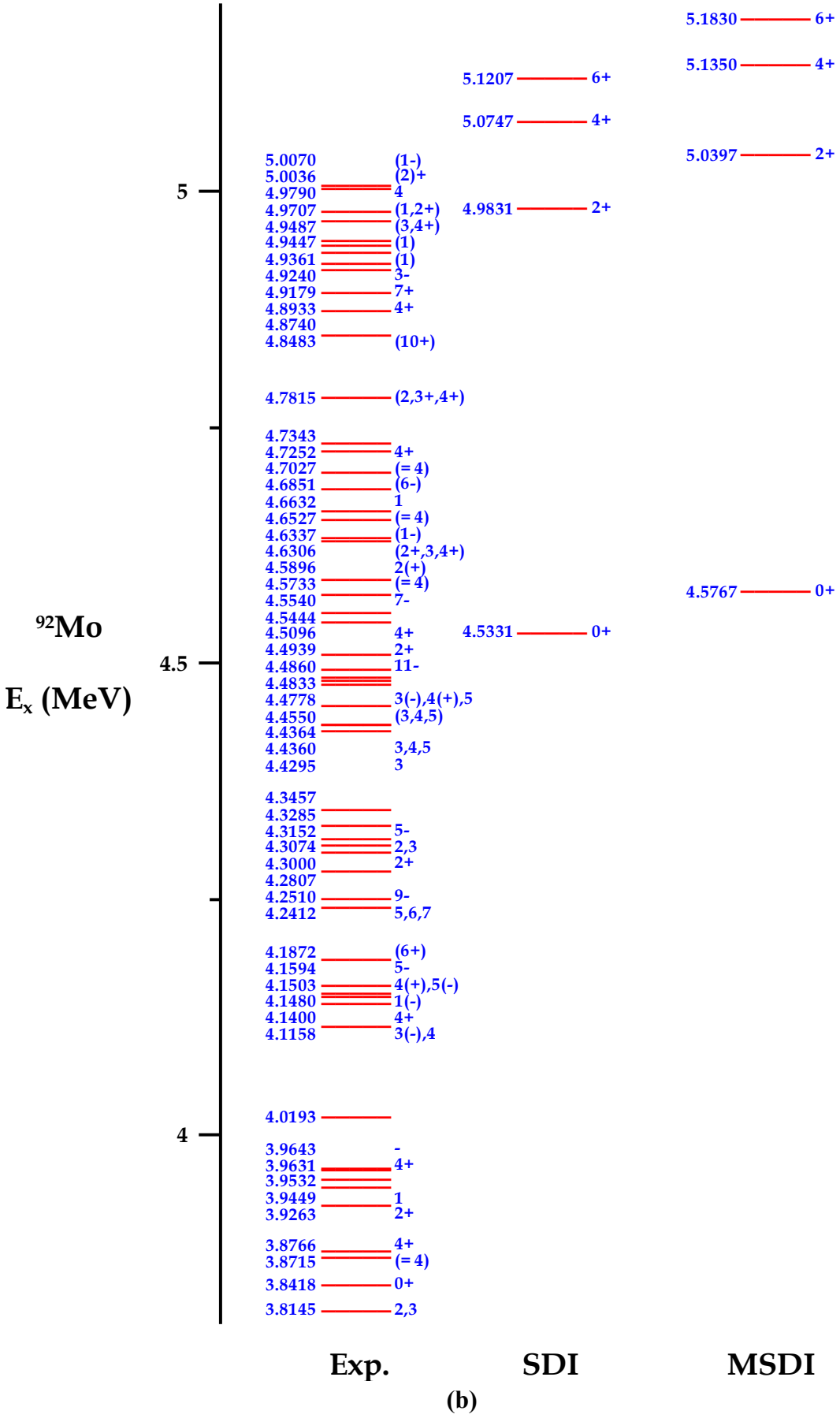


(d)

Fig. 1. a, b, c, d Comparison of the calculated energy levels with experiment scheme for ^{92}Nb isotope (Baglin, 2013).


92Mo
Ex (MeV)
3.5
3
2.5
2
1.5
1
0
3.7572
3.7532
3.6920 4+
3.6887 1(-),2,3
3.6518 (1)
3.6241 7-
3.6210 (=4)
3.5798 3-
3.5423 2+
3.3845 (1)
3.3804 (6-)
3.3686 (4+)
3.0913 2+
3.0636 (4-)
3.0069 (4,5)-
2.9226 (1)
2.8498 3-
2.8386 (1)
2.7605 8+
2.6342 1-
2.6124 6+
2.5269 5-
2.5195 0+
2.2826 4+
1.5095 2+
0.0000 0+
3.2021 1+,3+,5+,7+
3.1672 2+
3.1289 4+
3.0689 6+
2.8752 8+
1.1887 8+
1.1548 6+
1.0999 4+
0.9699 2+
0.0000 0+
3.2662 1+,3+,5+,7-
3.2298 2+
3.1898 4+
3.1272 6+
2.9250 8+
1.2501 8+
1.2148 6+
1.1573 4+
1.0211 2+
0.0000 0+
Exp.
SDI
MSDI

(a)



(b)

Fig.2. a, b Comparison of the calculated energy levels with experiment scheme for ^{92}Mo isotope (Baglin, 2013).

Table 5. Comparison of the theoretically reduced transition probability $B(E2)\downarrow$ with the experimental data values of ^{92}Nb nucleus (Baglin, 2012).

$J_i \longrightarrow J_f$	$B(E2)e^2\ (fm)^4$		
	Exp. Values	Theor. Values	
		SDI	MSDI
$3_1^+ \longrightarrow 2_1^+$	29.60 ± 37.00	30.8268	30.8268
$5_1^+ \longrightarrow 7_1^+$	50.08 ± 1.23	10.0829	10.0829
$4_1^+ \longrightarrow 5_1^+$	14.80 ± 32.07	85.0073	85.0073
$4_1^+ \longrightarrow 3_1^+$	---	52.3122	52.3122
$4_1^+ \longrightarrow 2_1^+$	---	41.1024	41.1024
$6_1^+ \longrightarrow 5_1^+$	---	73.9412	73.9412
$6_1^+ \longrightarrow 4_1^+$	---	18.7254	18.7254
$6_1^+ \longrightarrow 7_1^+$	---	63.3782	63.3782
$1_1^+ \longrightarrow 3_1^+$	≥ 0.03	107.8939	107.8939
$8_1^+ \longrightarrow 6_1^+$	---	0.428	0.428
$8_1^+ \longrightarrow 7_1^+$	---	2.7085	2.7085
$0_1^+ \longrightarrow 2_1^+$	---	205.9792	205.9792
$0_1^+ \longrightarrow 2_2^+$	---	0	0
$4_2^+ \longrightarrow 2_1^+$	---	0.1293	0.1365
$4_2^+ \longrightarrow 5_1^+$	---	0.0602	0.0611
$4_2^+ \longrightarrow 3_1^+$	---	0.0049	0.0044
$4_2^+ \longrightarrow 4_1^+$	---	0.1079	0.1193
$4_2^+ \longrightarrow 6_1^+$	---	0.1336	0.142
$5_2^+ \longrightarrow 5_1^+$	---	0.0226	0.0231
$5_2^+ \longrightarrow 3_1^+$	---	0.0265	0.0284
$5_2^+ \longrightarrow 7_1^+$	---	0.0255	0.0272
$5_2^+ \longrightarrow 4_1^+$	---	0.0056	0.0062
$5_2^+ \longrightarrow 6_1^+$	---	0.0203	0.0216
$5_2^+ \longrightarrow 4_2^+$	---	0.7	0.8041
$3_2^+ \longrightarrow 2_1^+$	---	0.3992	0.3927
$3_2^+ \longrightarrow 5_1^+$	---	0.9693	0.9524
$3_2^+ \longrightarrow 3_1^+$	---	1.1925	1.1595
$3_2^+ \longrightarrow 4_1^+$	---	0.0089	0.0087
$3_2^+ \longrightarrow 1_1^+$	---	0.4701	0.4587
$3_2^+ \longrightarrow 6_1^+$	---	0	0
$3_2^+ \longrightarrow 4_2^+$	---	0.1009	0.1177
$3_2^+ \longrightarrow 5_2^+$	---	0.0788	0.0866
$6_2^+ \longrightarrow 7_1^+$	---	2.1138	2.0855
$6_2^+ \longrightarrow 5_1^+$	---	0.6903	0.6713
$6_2^+ \longrightarrow 4_1^+$	---	0.7312	0.7181
$6_2^+ \longrightarrow 6_1^+$	---	0.2549	0.2622
$6_2^+ \longrightarrow 4_2^+$	---	4.5482	5.0562
$6_2^+ \longrightarrow 5_2^+$	---	1.2794	1.4022
$2_2^+ \longrightarrow 2_1^+$	---	4.4889	4.6104
$2_2^+ \longrightarrow 3_1^+$	---	3.6107	3.7434
$2_2^+ \longrightarrow 4_1^+$	---	0.7168	0.6741
$2_2^+ \longrightarrow 1_1^+$	---	4.4286	4.4249
$2_2^+ \longrightarrow 4_2^+$	---	0.0635	0.0723
$2_2^+ \longrightarrow 3_2^+$	---	0.4533	0.4973

Table 6. Comparison between the theoretically reduced transition probability $B(E2)\downarrow$ with experimental data values of ^{92}Mo nucleus (Baglin, 2012).

$J_i \longrightarrow J_f$	$B(E2)e^2\ (fm)^4$		
	Exp. values	*Theor. values*	
		SDI	*MSDI*
$2_1^+ \longrightarrow 0_1^+$	207.22 ± 12.33	207.5644	207.1713
$4_1^+ \longrightarrow 2_1^+$	< 592.08	244.9892	244.9608
$6_1^+ \longrightarrow 4_1^+$	80.42 ± 2.71	168.4857	168.4156
$8_1^+ \longrightarrow 6_1^+$	32.34 ± 0.54	66.8595	66.7977
$1_1^+ \longrightarrow 2_1^+$	---	92.8245	92.989
$3_1^+ \longrightarrow 2_1^+$	---	47.6355	47.6256
$3_1^+ \longrightarrow 4_1^+$	< 2.96	63.1928	63.1338
$5_1^+ \longrightarrow 4_1^+$	---	51.8388	51.7637
$5_1^+ \longrightarrow 6_1^+$	---	62.1207	62.0491
$7_1^+ \longrightarrow 6_1^+$	---	46.4696	46.4098
$7_1^+ \longrightarrow 8_1^+$	---	35.1091	35.0901
$8_2^+ \longrightarrow 6_1^+$	---	1.2365	1.1729
$8_2^+ \longrightarrow 8_1^+$	---	89.7675	88.3228
$6_2^+ \longrightarrow 4_1^+$	---	7.9805	7.8182
$6_2^+ \longrightarrow 6_1^+$	---	25.6819	25.5526
$6_2^+ \longrightarrow 8_1^+$	---	1.9289	1.8626
$6_2^+ \longrightarrow 8_2^+$	---	0.0032	0.0035
$4_2^+ \longrightarrow 2_1^+$	---	13.3695	13.1876
$4_2^+ \longrightarrow 4_1^+$	---	3.0098	3.0476
$4_2^+ \longrightarrow 6_1^+$	---	11.5669	11.3413
$4_2^+ \longrightarrow 6_2^+$	---	0.0166	0.0185
$2_2^+ \longrightarrow 0_1^+$	61.67 ± 7.40	0.3114	0.3328
$2_2^+ \longrightarrow 2_1^+$	106.08 ± 32.07	0.7887	0.8229
$2_2^+ \longrightarrow 4_1^+$	---	23.2511	22.901
$2_2^+ \longrightarrow 4_2^+$	---	0.0127	0.0145
$0_1^+ \longrightarrow 2_1^+$	---	1037.822	1035.857
$0_1^+ \longrightarrow 2_2^+$	---	1.5569	1.6639

5. Conclusions

In our calculations, expected energy levels, as well as the electric quadrupole transitions for ^{92}Nb and ^{92}Mo isotopes, have been expected by using surface delta and modified surface delta interactions. These predicted results have been compared with experimental values, this comparison has explained that there is a good agreement for most of the energy levels as well as the quadrupole transitions with the experimental data. This agreement value of theoretical energies has been very clear through using the modified surface delta interaction because more than one parameter has been used in the modified surface delta interaction. The total angular momentum and parity in our calculations have been located and confirmed by some

experimental values. Some parities values have determined the experimental data non- parities states. It has been found new theoretical values for electric quadrupole transitions unknown experimental values for both ^{92}Nb and ^{92}Mo isotopes. Through the current study, it can be concluded that the nuclear shell model using the modified delta surface interaction is successful for calculating some nuclear properties from described energy levels and states (total momentum and parity), in addition, it has explained the electric quadrupole transition probabilities within the model space which has been used for the^{92}Nb and ^{92}Mo nuclei and this success has been very clear during our calculations.

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