Kuwait J. Sci. **44 (1)** pp. 29-35, 2017

The relation between parameter curves and lines of curvature on canal surfaces

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Abstract

A canal surface is the envelope of a moving sphere with varying radius, defined by the trajectory $C(t)$ (spine curve) of its center and a radius function $r(t)$. In this paper, we investigate the parameter curves which are also lines of curvature on the canal surface. Last of all, for special spine curves we obtain the radius functions of canal surfaces.

Keywords: Canal surface; generalized tube; line of curvature; parameter curve; spine curve.

1. Introduction

A canal surface is defined as the envelope of a family of one parameter spheres and is useful for representing long thin objects, e.g., pipes, poles, ropes, 3D fonts or intestines of body. Canal surfaces are also frequently used in solid and surface modelling for CAD/CAM. Representative examples are natural quadrics, tori, tubular surfaces and Dupin cyclides.

A curve on a surface, which has the property whose tangent at each of its points p coincides with a principal direction at p is called a line of curvature, in other words, for a line of curvature

$$S(T) = k_n T,$$

where S is the shape operator of the surface and T is the tangent vector field and k_n is the normal curvature along the curve on the surface, respectively.

Developable surfaces are ruled surfaces with zero Gaussian curvature. They are also characterized by the property that each ruling is a line of curvature except at umbilic points or singular points. Canal surfaces correspond to cylinders that are also developable ruled surfaces (Izumiya *et al.,* 2007). Karadag *et al.* (2014) presented a method to be developable of a ruled surface in Minkowski 3-space. Lately, Dogan (2015) studied the relationship among characteristic curves on developable ruled surfaces.

Maekawa *et al.* (1998) researched necessary and sufficient conditions for the regularity of pipe (tube) surfaces. Then Xu *et al*. (2006) studied these conditions for canal surfaces and examined principle geometric properties of canal surfaces like computing the area and Gaussian curvature. Recently, Dogan & Yayli (2011; 2012) and Dogan (2012) investigated tubular surfaces with Bishop and Darboux frames instead of Frenet frame and gave the characterizations for special curves on them, such as geodesic and asymptotic curves, or lines of curvature.

Gross (1994) gave the concept of generalized tubes (briefly GT) and classified them in two types as ZGT and CGT. Here, ZGT refers to the spine curve (the axis) that has torsion-free and CGT refers to tube that has circular cross sections. He investigated the properties of GT and showed that the parameter curves of a generalized tube are also lines of curvature if and only if the spine curve is planar. In this study, we examine $s-$ and $\theta-$ parameter curves that are also lines of curvature on the canal surface.

Garcia *et al.* (2006) showed that immersed canal surfaces have at most two isolated periodic line of curvature by means of a connection of the differential equations for these curvature lines and real Riccati equations. Also, they showed that the maximal principal curvature lines are circles with the maximal principal curvature

$$k_2(s) = \frac{1}{r(s)}.$$

This paper is organized as follows. In section 2, we introduce a canal surface and give basic notions about it.

Then we observe the parameter curves that are also lines of curvature on generalized tubes and study the same property on canal surfaces in section 3. Furthermore, we obtain the radius function of the canal surface whose $s-$ parameter curves are also lines of curvature. In section 4, we conclude this paper.

2. Preliminaries

First of all, we give some coefficients of the first and second fundamental forms of a canal surface. Subsequently, we mention generalized tubes and prove the theorem characterizing the parameter curves are also lines of curvature on a surface.

Definition 1. A canal surface is defined as the envelope of a family of one parameter spheres. When $r'(t) < \left\| C'(t) \right\|$, the canal surface is regular and is parameterized as follows.

$$K(t,\theta) = C(t) - r(t)r'(t)\frac{C'(t)}{\left\|C'(t)\right\|^2}$$
$$\mp r(t)\frac{\sqrt{\left\|C'(t)\right\|^2 - r'(t)^2}}{\left\|C'(t)\right\|}(\cos\theta N(t) + \sin\theta B(t)),$$

where $C(t)$ is the spine curve, N and B are the principal normal and binormal of $C(t)$, respectively. If the spine curve $C(t)$ has arclength parameterization ($\left\|C'(t)\right\| = 1$), the canal surface is reparameterized as

$$K(s,\theta) = C(s) - r(s)r'(s)T(s)$$
$$\mp r(s)\sqrt{1 - r'(s)^2}\,(\cos\theta N(s) + \sin\theta B(s)),$$

where $\{T, N, B\}$ is the Frenet frame of the spine curve (Gray, 1998). Some coefficients of the first and second fundamental forms of the canal surface are as follows (Xu *et al*., 2006).

$$E = (\kappa g\cos\theta + h' - 1)^2 + (g\tau + h\kappa\sin\theta)^2$$
$$+ (g' - h\kappa\cos\theta)^2$$
$$F = g(g\tau + h\kappa\sin\theta), (2.1)$$
$$G = g^2,$$
$$\left\|K_s \times K_\theta\right\|^2 = g^2((\kappa g\cos\theta + h' - 1)^2$$
$$+ (g' - h\kappa\cos\theta)^2), (2.2)$$
$$f = \frac{(g' - h\kappa\cos\theta)\kappa g\sin\theta - \tau g(\kappa g\cos\theta + h' - 1)}{\sqrt{(\kappa g\cos\theta + h' - 1)^2 + (g' - h\kappa\cos\theta)^2}},$$

where $g = r(s)\sqrt{1 - r'(s)^2} \neq 0$, $h = r(s)r'(s) \neq 0$, κ and τ are the curvature and the torsion of $C(t)$, respectively. If the radius function $r(s) = r$ is a constant, then the canal surface is called a tube or a pipe surface. It is written as

$$K(s,\theta) = C(s) + r(\cos\theta N(s) + \sin\theta B(s)).$$

Definition 2.The parameterization of generalized tube around the spine curve $\Gamma(s)$ is

$$X(s,\theta) = \Gamma(s) + u(\theta)(\cos\theta N(s) + \sin\theta B(s)),$$

where $0 \leq \theta < 2\pi$ and u is twice differentiable, $u(\theta) > 0$, $u(0) = u(2\pi)$ (Gross, 1994). Now, we give some coefficients of the first and second fundamental forms of a generalized tube. The normal vector field N of a generalized tube can be computed as the cross product of tangent vectors of $\theta-$ parameter curve and $s-$ parameter curve, or vice versa.

$$X_\theta = (1 - \kappa u\cos\theta)T - u\tau\sin\theta N + u\tau\cos\theta B,$$
$$X_s = (u'\cos\theta - u\sin\theta)N + (u'\sin\theta + u\cos\theta)B,$$
$$N = X_\theta \times X_s = uu'\tau T$$
$$+ (1 - \kappa u\cos\theta)\begin{bmatrix}(u'\sin\theta + u\cos\theta)N \\ + (u\sin\theta - u'\cos\theta)B\end{bmatrix}.$$

Then we have

$$F = u^2\tau,$$
$$f = \frac{1}{\left\|N\right\|}[\tau\kappa uu'(u\sin\theta - u'\cos\theta)$$
$$- (1 - \kappa u\cos\theta)(u^2 + (u')^2].$$

Definition 3. Let M be a surface and let the curve $\alpha : I \subset \mathbb{R} \to M$. Then

$$(fE - eF)(u')^2 + (gE - eG)u'v'$$
$$+ (gF - fG)(v')^2 = 0$$

is called as the differential equation of lines of curvature on M (do Carmo, 1976).

Theorem 1. A necessary and sufficient condition for the parameter curves of a surface to be lines of curvature in a neighborhood of a nonumbilical point is that $F = f = 0$, where F and f are the respective the first and second fundamental coefficients (do Carmo, 1976).

Proof. Weingarten equations are given by

$$-S(x_u)=U_u=\frac{fF-eG}{EG-F^2}x_u+\frac{eF-fE}{EG-F^2}x_v,$$
$$-S(x_v)=U_v=\frac{gF-fG}{EG-F^2}x_u+\frac{fF-gE}{EG-F^2}x_v,$$

where U is the unit normal vector, E, F, G and e, f, g are the coefficients of the first and second fundamental forms of the surface, respectively.

($\Rightarrow$) Assume that the parameter curves in a neighborhood of a nonumbilical point of the surface are also lines of curvature. From the Definition (3) and the Weingarten equations we get

$$S(x_u)=-\frac{fF-eG}{EG-F^2}x_u,$$
$$S(x_v)=-\frac{fF-gE}{EG-F^2}x_v.$$

In other words,

$$eF-fE=0,$$
$$gF-fG=0.$$

From this, we have $F=f=0$.

($\Leftarrow$) Let $F=f=0$ in a neighborhood of a nonumbilical point of a surface. By the Weingarten equations it follows that

$$S(x_u)=\frac{e}{E}x_u,$$
$$S(x_v)=\frac{g}{G}x_v.$$

According to the definition of the line of curvature, $u-$ and $v-$ parameter curves become lines of curvature.

Theorem 2. The parameter curves of a generalized tube are lines of curvature if and only if the axis Γ is torsion-free, i.e., $\tau=0$ (Gross, 1994).

Proof. Let the spine curve Γ be a plane curve, that is, $\tau=0$. Then, $F=f=0$. Conversely, let the parameter curves be also lines of curvature. From Theorem (1) $F=f=0$. Since $F=u^2\tau=0$ and $u>0$, we have $\tau=0$, i.e., Γ is a plane curve.

We give an important lemma as regards the regularity of a canal surface.

Lemma 1. When

$\kappa(s_0)g(s_0)\cos\theta_0+h^{'}(s_0)-1=0,$

$g^{'}(s_0)-h(s_0)\kappa(s_0)\cos\theta_0=0$, where

$s_0\in[0,l]$ and $\theta_0\in[0,2\pi)$ (Xu *et al.*, 2006).

Proof. Since $h=rr^{'}\neq 0$ and $g=r\sqrt{1-r^{'2}}\neq 0$, we obtain

$$h(s_0)(h^{'}(s_0)-1)=-g(s_0)g^{'}(s_0).$$

If $\kappa(s_0)g(s_0)\cos\theta_0+h^{'}(s_0)-1=0$, then

$$h(s_0)(\kappa(s_0)g(s_0)\cos\theta_0+h^{'}(s_0)-1)=0$$
$$h(s_0)\kappa(s_0)g(s_0)\cos\theta_0+h(s_0)(h^{'}(s_0)-1)=0$$
$$h(s_0)\kappa(s_0)g(s_0)\cos\theta_0-g(s_0)g^{'}(s_0)=0$$
$$g(s_0)(h(s_0)\kappa(s_0)\cos\theta_0-g^{'}(s_0))=0.$$

Since $g(s_0)\neq 0$, $h(s_0)\kappa(s_0)\cos\theta_0-g^{'}(s_0)=0$. This completes the proof.

Thus from Equation (2.2) and Lemma (1) it follows that $K_s\times K_\theta=0$. Then the canal surface is singular at the points $p=K(s_0,\theta_0)$, that is, when $\kappa g\cos\theta+h^{'}-1\neq 0$, the canal surface is regular.

Remark 1. Garcia *et al.* (2006) showed that the maximal principal curvature lines ($\theta-$parameter curves) are circles with the maximal principal curvature

$$k_2(s)=\frac{1}{r(s)},$$

for immersed canal surfaces.

3. Some Characterizations for Lines of Curvature on Canal Surfaces

In this section, we investigate the parameter curves which are also lines of curvature and give some characterizations on canal surfaces around special spine curves.

Theorem 3. Let $K(s,\theta)$ be a regular canal surface and let F and f be its the first and second fundamental coefficients, respectively. Then we have

$$F=0 \Leftrightarrow f=0.$$

Proof. Assume that $F=0$. Then by Equation (2.1) we have $g(g\tau+h\kappa\sin\theta)=0$. Because $g\neq 0$, we get $g\tau=-h\kappa\sin\theta$. If we substitute the last equality in the expression of f, we obtain

$$f=\frac{(gg^{'}+h(h^{'}-1))\kappa\sin\theta}{\sqrt{(\kappa g\cos\theta+h^{'}-1)^2+(g^{'}-h\kappa\cos\theta)^2}}.$$

Since $gg^{'}+h(h^{'}-1)=0$, it follows that $f=0$. On the contrary, assume that $f=0$. In this case,

$$(g^{'}-h\kappa\cos\theta)\kappa g\sin\theta$$
$$-\tau g(\kappa g\cos\theta+h^{'}-1)=0.$$

If we arrange this equality, due to the fact that $gg' + h(h' - 1) = 0$, it concludes

$$(\kappa g\cos\theta + h' - 1)(g\tau + h\kappa\sin\theta) = 0.$$

Since $\kappa g\cos\theta + h' - 1 \neq 0$, we get $g\tau + h\kappa\sin\theta = 0$. Thus from Equation (2.1) $F = 0$.

Corollary 1. The parameter curves are also lines of curvature on a regular canal surface if and only if $g\tau + h\kappa\sin\theta = 0$.

From this time, we will investigate the equation $g\tau + h\kappa\sin\theta = 0$ which solves the problem that $s-$parameter curves of the canal surface are also lines of curvature. We will have a look at two different cases for this equation.

The Case 1. If $s-$parameter curves $\theta_0 = 0$ and $\theta_0 = \pi$ which are also lines of curvature are replaced in $g\tau + h\kappa\sin\theta = 0$, it follows $g\tau = 0$. Since $g \neq 0$, we have $\tau = 0$, i.e., the spine curve $C(s)$ is planar.

The Case 2. We go over $s-$parameter curves which are also lines of curvature except for $\theta_0 = 0, \pi$. If we substitute $g = r\sqrt{1-r'^2}$ and $h = rr'$ in $g\tau + h\kappa\sin\theta = 0$, we obtain

$$\tau r\sqrt{1-r'^2} = -\kappa r r' \sin\theta$$

$$\tau\sqrt{1-r'^2} = -\kappa r' \sin\theta. \qquad (3.1)$$

If we take square of both sides in Equation (3.1) and then arrange it, we get the radius function of the canal surface as follows.

$$\left(\tau^2 + \kappa^2\sin^2\theta\right)r'^2 = \tau^2$$

$$r' = \frac{|\tau|}{\sqrt{\tau^2 + \kappa^2\sin^2\theta}}$$

$$r(s) = \int \frac{|\tau(s)|}{\sqrt{\tau^2(s) + \kappa^2(s)\sin^2\theta}}\,ds + c. \qquad (3.2)$$

Corollary 2. (1) Let the spine curve $C(s)$ be a general helix.

Then $s-$parameter curves of the canal surface are also lines of curvature if and only if the canal surface is generated by a moving sphere with the linear radius function

$$r(s) = as + c,\ c > 0.$$

(2) Let the spine curve $C(s)$ be a circular helix. For $s-$parameter curves which are also lines of curvature, the canal surface is generated by a moving sphere with the linear radius function

$$r(s) = \frac{b}{\sqrt{b^2 + a^2\sin^2\theta}}s + c;\ c, \tau > 0.$$

Proof. (1) By Equation (3.2) we have

$$r(s) = \int \frac{|\tau(s)|}{\sqrt{\tau^2(s) + \kappa^2(s)\sin^2\theta}}\,ds + c.$$

Since the spine curve $C(s)$ is a general helix, the ratio of its curvatures $\frac{\tau}{\kappa} = \tan\phi$ is a constant. Because θ is also a constant, we obtain the radius function $r(s)$ as below.

$$r(s) = \int \sqrt{\frac{1}{1 + \frac{\kappa^2(s)}{\tau^2(s)}\sin^2\theta}}\,ds$$

$$r(s) = \frac{s}{\sqrt{1 + \cot^2\phi\sin^2\theta}} + c,\ c > 0.$$

Then the radius function have the linear equation like $r(s) = as + c$, where

$$a = \frac{1}{\sqrt{1 + \cot^2\phi\sin^2\theta}}.$$

(2) If the spine curve $C(s)$ is a circular helix, it can be parameterized as

$$C(s) = \left(a\cos\frac{s}{d}, a\sin\frac{s}{d}, b\frac{s}{d}\right),$$

where $a = \frac{\kappa}{\kappa^2 + \tau^2}$, $b = \frac{\tau}{\kappa^2 + \tau^2}$, $d^2 = a^2 + b^2$. Because of the fact that the curvatures $\kappa = \frac{a}{d^2}$ and $\tau = \frac{b}{d^2}$, from Equation (3.2) it concludes

$$r(s) = \frac{b}{\sqrt{b^2 + a^2\sin^2\theta}}s + c;\ c, \tau > 0.$$

The following example can be given for Corollary 2.

Example 1. For the circular helix $C(s) = (\cos\frac{s}{\sqrt{2}}, \sin\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}})$, $a = b = 1$ and $d = \sqrt{2}$. Then $\kappa = \tau = \frac{1}{2}$. From Equation (3.2) the linear radius function of the canal surface is obtained as $r(s) = \frac{1}{\sqrt{1 + \sin^2\theta}}s + c$, where c and θ are constants.

Salkowski (1909) studied a family of space curves with constant curvature and non-constant torsion (Salkowski curves). Then Monterde (2009) characterized them as the space curves whose normal vectors make a constant angle with a fixed line. According to this, the curvature and the torsion of a Salkowski curve can be given as $\kappa(s)\equiv 1$ and $\tau(s)=\tan(\arcsin(ms))$, where $m=\dfrac{1}{\tan\phi}$ and ϕ is the angle between the principal normal of the curve and the fixed line.

Corollary 3. Let the spine curve $C(s)$ be a Salkowski curve. Then $s-$parameter curves ($\theta = const.$) on the canal surface are also lines of curvature if and only if the canal surface is generated by a moving sphere with the radius function

$$r(s)=\frac{1}{\cos^2\theta}\sqrt{\cos^2\theta s^2+\sin^2\theta\tan^2\phi}+c.$$

Proof. Suppose that the spine curve $C(s)$ is a Salkowski curve. Then $\kappa(s)\equiv 1$ and $\tau(s)=\tan(\arcsin(ms))$. By using Equation (3.2) we obtain the radius function as

$$r(s)=\int\frac{\tan(\arcsin(ms))}{\sqrt{\tan^2(\arcsin(ms))+\sin^2\theta}}ds.$$

If we make the changing of variable $x=\arcsin(ms)$, it concludes

$$=\frac{1}{m}\int\frac{\sin x\cos x}{\sqrt{\sin^2 x+\sin^2\theta\cos^2 x}}dx.$$

For the changing of variable $\sin x=t$ we get

$$=\frac{1}{m}\int\frac{tdt}{\sqrt{\cos^2\theta t^2+\sin^2\theta}}.$$

Eventually,

$$\int\frac{\tan(\arcsin(ms))}{\sqrt{\tan^2(\arcsin(ms))+\sin^2\theta}}ds$$
$$=\frac{1}{m\cos^2\theta}\sqrt{\cos^2\theta m^2 s^2+\sin^2\theta}+c.$$

Then the radius function of the canal surface which is generated by the Salkowski curve is

$$r(s)=\frac{1}{\cos^2\theta}\sqrt{\cos^2\theta s^2+\sin^2\theta\tan^2\phi}+c.$$

Corollary 4. Let $s-$parameter curves be also lines of curvature on the regular canal surface $K(s,\theta)$. If $r(s)$ is an increasing function, then we have

$$-\kappa(s)<\tau(s)<\kappa(s),$$

where κ and τ are the curvature and torsion of the spine curve $C(s)$, respectively.

Proof. Assume that $s-$parameter curves are also lines of curvature. Then from Corollary (1) we have $g\tau+h\kappa\sin\theta=0$. For the regular canal surface, $r'<1$. If we leave alone $\sin\theta$ in Equation (3.1), we get

$$\sin\theta=-\frac{\tau}{\kappa}\frac{\sqrt{1-r'^2}}{r'}.$$

Because $r(s)$ is an increasing function, $0<r'(s)<1$. So, it concludes $\dfrac{\sqrt{1-r'(s)^2}}{r'(s)}>1$. If we take absolute value of the above equation and use the last inequality, we reach $\left|\dfrac{\tau}{\kappa}(s)\right|<1$. Therefore, we obtain $-\kappa(s)<\tau(s)<\kappa(s)$.

An example for $s-$parameter curves that are also lines of curvature on the canal surface can be given as follows.

Example 2. Vessiot (1919) showed that one family of lines of curvature on the canal surface is

$$\frac{d\theta}{ds}=-\tau(s)-\kappa(s)\cot\alpha(s)\sin\theta,$$
$$\cos\alpha(s)=-r'(s). \tag{3.3}$$

From the above equation it follows $\cot\alpha(s)=\dfrac{r'(s)}{\sqrt{1-r'(s)^2}}$. Furthermore, if $g(s)=r(s)\sqrt{1-r'(s)^2}$ and $h(s)=r(s)r'(s)$ are substituted in $g\tau+h\kappa\sin\theta=0$, we obtain

$$\tau(s)\sqrt{1-r'(s)^2}+r'(s)\kappa(s)\sin\theta=0. \tag{3.4}$$

By Equation (3.3) and Equation (3.4) we get

$$\frac{d\theta}{ds}=-\frac{\tau(s)\sqrt{1-r'(s)^2}+r'(s)\kappa(s)\sin\theta}{\sqrt{1-r'(s)^2}}=0$$
$$\theta=const.$$

At last, we view that one family of lines of curvature given in Equation (3.3) coincides with one family of $s-$parameter curves ($\theta = const.$) on the canal surface for our main equation $g\tau+h\kappa\sin\theta=0$.

4. Conclusions

In this paper, we observed the parameter curves which are also lines of curvature on generalized tubes and then we researched this property for canal surfaces. Afterwards, by taking special spine curves we obtained the radius function of a moving sphere which generates the canal surface and showed that one family of lines of curvature concurs with one family of $s-$parameter curves on canal surfaces.

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Submitted **:** 01/07/2015
Revised **:** 19/10/2015
Accepted **:** 01/11/2015

العلاقة بين منحنيات الوسيط وخطوط التقوس على سطوح قنوية

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خلاصة

السطح القنوي هو غلاف كرة متحركة لها نصف قطر متغير، يجرى تعريفه بواسطة المسار $C(t)$ (منحنى العماد) لمركزه و نصف قطره دالة $r(t)$. نقوم في هذا البحث بدراسة منحنيات الوسيط التي هي في نفس الوقت خطوط التقوس للسطح القنوي . أخيراً نحصل على دوال نصف القطر لسطوح قنوية لها منحنيات عماد خاصة.