On some variations of dominating identification in graphs

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Abstract

In this study, we introduce the locating-dominating value and the location-domination polynomial of graphs and location-domination polynomials of some families of graphs were identified. Locating-dominating set of graph G is defined as the dominating set which locates all the vertices of G. And, location-domination number G is the minimum cardinality of a locating-dominating set in G.

Keywords: Lexicographic product; locating-dominating set; location-domination number; location-domination polynomial; locating-dominating value

1. Introduction

Let G be a simple undirected graph. If every pair of vertices in G has a path between them, then G is said to be a *connected graph*, else, G is *disconnected*. *Neighborhood* of vertex v of G is set $N(v) = \{u \in V(G) \ : \ u \ is \ adjacent \ to \ v\}$. The number of elements in $N(v)$ is the *degree* of v, denoted by $d_G(v)$. A vertex v with $d_G(v) = 0$ is an *isolated vertex*. If every vertex of connected graph G has two neighbors, then G is called a *cycle*. The *girth* of a graph G is the number of edges in its shortest cycle. If two distinct vertices u and v of G have the property that $N(u) - \{v\} = N(v) - \{u\}$, then u and v are called *twin vertices* (or simply twins) in G. A set $T \subseteq V(G)$ is said to be a *twin-set* in G if every two elements of T are twin vertices of G. The *complement* of G, denoted by $\overline{G}$, has the same vertex set as G and x is adjacent to y in $\overline{G}$ if and only if x is not adjacent to y in G. For a graph G, if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$, then H is called a *subgraph* of G. A non empty subset H of a connected graph G is called an *induced subgraph* of G, if H contains all the edges $uv \in E(G)$ with $u, v \in V(H)$. The *component* is the maximal connected subgraph of G.

A set $D \subseteq V(G)$ is called *dominating* if for every vertex $u \in V(G) \setminus D$, $N(u) \bigcap D \neq \phi$. A set L_d of vertices of a graph G is called a *locating-dominating set* if, for every two distinct vertices $u, v \in V(G) - L_d$, $\phi \neq N(u) \bigcap L_d \neq N(v) \bigcap L_d \neq \phi$. The *location-domination number* of a graph G, denoted by $LD(G)$, is the minimum cardinality of a locating-dominating set of G. Caceres *et al.* (2013), showed that each locating-dominating set is both locating and dominating set. However, the converse is not true in general.

The initial application of locating-dominating sets was fault-diagnosis in maintenance of multiprocessor systems (Karpovsky, Chakrabarty & Levitin, 1998). The purpose of fault detection is to test the system and locate faulty processors. Locating-dominating sets have since been extended and applied. The decision problem for locating-dominating sets for directed graphs has been shown to be an NP-complete problem (Charon, Hudry & Lobstein, 2002).

Balbuena *et al.* (2015), studied the locating-dominating sets of graphs containing no triangle. They also gave upper bound on the smallest size of such sets in term of the order of the graphs. Blidia *et al.* (2007), have worked on the location-domination number of trees. The study revealed upper and lower bounds of a tree graph of order greater than or equal to 3, in terms of leaves and support vertices. And classify all the graphs satisfying the upper bound. Fazil *et al.* (2016) worked on the locationg-dominating sets in hyper graph. Mphako-Banda & Ncambalala, 2019; Alaeiyan *et al.,* 2014; Imran *et al.,* 2014; Bertrand *et al.,* 2004; Honkala *et al.,* 2004; Charon *et al.,* 2003; Slater, 2002; Slater, 1988; Colbourn *et al.,* 1987; Finbow & Hartnell, 1987; Fall & Slater, 1984 came up with helpful results.

2. Location-domination polynomials and Locating-dominating values

In this section, we introduce two new graph invariants, ie., the location-domination polynomial of a graph and the locating-dominating value of a vertex of a graph.

Initially recall some results on the location-domination number to proceed.

Proposition 2.1 *(Murtaza, 2020). Suppose that u, v are twins in a connected graph G and L_d is a locating-dominating set of G. Then either u or v is in L_d. Moreover, if $u \in L_d$ and $v \notin L_d$, then $(L_d - \{u\}) \cup \{v\}$ is a locating-dominating set of G.*

Remark 2.2 *(Murtaza, 2020). Let $T \subseteq V(G)$ be a twin-set of order $k \geq 2$. Then every locating-dominating set L_d of G contains at least $k-1$ vertices of T.*

Lemma 2.3 *(Balbuena, 2015). Let G be a graph of girth at least 5, and let C be a dominating set of G. Let $X = \{x \in V(G) \setminus C : |N(x) \bigcap C| = 1\}$, then C is a locating-dominating set of G if and only if there is an injective function $f : X \to C$ such that $f(x) \in C \bigcap N(x)$ for all $x \in X$.*

Akbari *et al.* (2010), introduced the concept of domination polynomial of G. This family is related to all the dominating sets of G. Birkhoff (1912), introduced the concept of chromatic polynomial of G, which is related to all the color classes of G. Heilmann & Lieb (1972) came up with the concept of matching polynomial which corresponds to all the edges of G. Gutman & Harary (1983) introduced the concept for the independent polynomial of G, which corresponds to all the independent sets of G. Salman *et al.* introduced the concept for the resolving polynomial of G which corresponds to all the resolving sets of G. Javaid *et al.* (2018) proposed the concept of fixing polynomial of the graph, defines the fixing sets of G. In this paper, we studied the location-domination polynomial of the graph which corresponds to all the locating-dominating sets of G.

For a graph G of order n and location-domination number $LD(G)$, the location-domination polynomial $LD(G, x)$ is a generating polynomial for the locating-dominating sequence $(l_{LD(G)}, l_{LD(G)+1}, \ldots, l_n)$ which helps in counting all the locating-dominating sets of cardinality j; $LD(G) \leq j \leq n$, for G. j-set is the subset of the vertices of the graph G of cardinality j. Let $\mathcal{L}(G, j)$ denote the family of all the locating-dominating sets of G which are j-sets. Let $l_j = |\mathcal{L}(G, j)|$.

Definition 2.4 $LD(G, x) = \sum\limits_{j=LD(G)}^{n} l_j x^j$, *is defined as location-domination polynomial of G. Where, $l_j = 0$ iff $j = LD(G) = 0$ or $j < LD(G)$.*

Some properties related to location-domination polynomial $LD(G, x)$ of graph G are listed below.

Properties 2.5 (1) $l_{|G|} = 1$ *and* $l_{|G|-1} = |G|$.

(2) $LD(G, x)$ *is monic.*

(3) *Since $l_i = 0$ for $j = LD(G) = 0$ or $j < LD(G)$, so $LD(G, x)$ has no constant term.*

(4) *Each value of the locating-dominating sequence $(l_{LD(G)}, l_{LD(G)+1}, \ldots, l_n)$ is non-zero.*

(5) *For any $a, á \in [0, \infty)$ such that $a < á$, $LD(G, a) < LD(G, á)$. It concludes that $LD(G, x)$ is strictly increasing function on $[0, \infty)$.*

(6) If G_1 is any subgraph of a graph G, then $deg(LD(G, x)) \geq deg(LD(G_1, x))$.

Proposition 2.6 *Let G be a graph with t components $G_1, G_2, ..., G_t$, then $LD(G, x) = \prod\limits_{i=1}^{t} LD(G_i, x)$.*

Proof. Since all the components are disjoint, the locating-dominating sets of each component are disjoint with other components. This implies that any locating-dominating set L_d of cardinality say r, where $LD(G) \leq r \leq |G|$, there must exist t disjoint locating-dominating sets one from each $G_i, 1 \leq i \leq t$, such that L_d is the union of these t locating-dominating sets. Thus, the coefficient of x^r in both polynomials $LD(G,x)$ and $\prod_{i=1}^{t} LD(G_i,x)$ is the same. Hence the polynomials are identical and completes the proof.

Proposition 2.7 *Let G be the graph of order $n \geq 2$ with $t \geq 2$ isolated vertices. If $LD(G,x)$ is the location-domination polynomial of G, then $t = n - l_{n-1}$.*

Proof. Suppose $A = \{x \in V(G) : d_G(x) = 0\}$ and $|A| = t$. Then for any vertex $u \in V(G) \setminus A$, the set $V(G) \setminus \{u\}$ is a locating-dominating set of G. Also, for any locating-dominating set B and any vertex $v \in A$, $B \setminus \{v\}$ is not a locating-dominating set of G. Accordingly, the result follows.

Proposition 2.8 *Let G be a graph of order $n \geq 2$ with $t \geq 2$ isolated vertices and G_1 be a graph induced by the set $V(G) \setminus A$, A is the set of isolated vertices (if $n = 2$, $t = 2$, then G_1 is not a graph). Then $LD(G_1,x) = \frac{LD(G,x)}{x^t}$.*

Proof. Let G_2 be a graph induced by A. Since G_1 is a subgraph induced by $V(G)\setminus A$, so $LD(G_1,x) = \sum_{i=LD(G_1)}^{m} l_i x^i$, $m = |V(G) \setminus A|$. Because A has at least two isolated vertices, G_2 must have two components. Thus, G_1 and G_2 have at least three components. Applying Proposition 2.6, we have $LD(G,x) = LD(G_1,x)x^t$, and then $LD(G_1,x) = \frac{LD(G,x)}{x^t}$.

Corollary 2.9 *Let G be a graph of order $n \geq 2$ with $t \geq 2$ isolated vertices and G_1 be its subgraph induced by the set $V(G) \setminus A$, A is the set of $t \geq 2$ isolated vertices. If $LD(G,x) = \sum_{i=LD(G)}^{n} l_i x^i$ and $LD(G_1,x) = \sum_{j=LD(G_1)}^{n-t} l_j x^j$ are location-domination polynomials of G and G_1, respectively. Then $t = deg(LD(G,x)) - deg(LD(G_1,x))$.*

Consider the set G^* which contains all locating-dominating sets of cardinality $LD(G)$, then $|G^*| = l_{LD(G)}$. The definition of locating-dominating value of each vertex of G as follows: for each vertex $v \in V(G)$, the *locating-dominating value*, $LDV(v)$, is the total number of the elements of G^* for which v belongs.

Proposition 2.10 *Let G be a graph, then*

(1) $\sum_{v \in V(G)} LDV(v) = |G^*| \cdot LD(G)$.

(2) *If u and v are twin vertices in G, then $LDV(u) = LDV(v)$.*

(3) *If G has $t \geq 2$ components $G_1, G_2, ..., G_t$, then $|G^*| = \prod_{i=1}^{t} |G_i^*|$. Further, for $v \in V(G)$, $LDV(v) = LDV(G_i(v)) \prod_{j=1 j\neq i}^{t} |G_j^*|$.*

3. Location-domination polynomials and Locating-dominating values of some families of graphs

This section considers location-domination polynomials of various graph families, such as the complete multipartite graphs, Petersen graph and lexicographic product of cycle graph of order $n_1 \geq 3$ with $n_2 \geq 2$ isolated vertices.

Let G be a complete multipartite graph and $P_1, P_2, P_3, ..., P_q$ be its q-partites. It is clear that each partite is a twin set. Also, for any $u \in P_{q_1}$ and any $v \in P_{q_2}$, $q_1 \neq q_2$, $N(u) \neq N(v)$. Thus, $LD(G) = \sum_{j=1}^{q} |P_j| - q$

We prove the following theorem for $|P_j| = n_j$, $1 \leq j \leq q$, which describes the location-domination polynomial of the complete multipartite graph.

Theorem 3.1 *Let $G = K_{n_1,n_2,n_3,\ldots,n_q}$ be the complete multipartite graph with $n_1 \geq n_2 \geq n_3 \geq \ldots. \geq n_q$, then for each $n_j \geq 2$, $LD(G,x) = x^{|G|} + \sum\limits_{i=1}^{q}[\sum\limits_{1=j_1<j_2<\ldots<j_i}^{q}(n_{j_1}n_{j_2}\ldots n_{j_i})x^{|G|-i}]$.*

Proof. Since each partite P_j with $|P_j| = n_j$, $1 \leq j \leq q$, is in fact a twin set, therefore every minimal locating-dominating set of $G = K_{n_1,n_2,n_3,\ldots,n_q}$ contains all but one vertex from each P_j. Applying Properties 2.5(1), we have $l_{|G|} = 1$, $l_{|G|-1} = |G|$, and for $l_{|G|-i}$, $2 \leq i \leq LD(G)$, we choose all vertices of G except i vertices with one vertex from each P_j. It is possible to do this in exactly $\sum\limits_{1=j_1<j_2<\ldots<j_i}^{q} n_{j_1}n_{j_2}\ldots n_{j_i}$. This proves the result.

From Theorem 3.1, there are $\prod\limits_{j=1}^{q} n_j$ locating-dominating sets of minimum cardinality, and each locating-dominating set must contain all but one vertex from each partite, so we have the following corollary:

Corollary 3.2 *Let $G = K_{n_1,n_2,n_3,\ldots,n_q}$ be the complete multipartite graph with $n_1 \geq n_2 \geq n_3 \geq \ldots. \geq n_q$ and each $n_j \geq 2$, then for any $v \in V(G)$, there is a partite $P_k, 1 \leq k \leq q$ such that $LDV(v) = \prod\limits_{j=1 j\neq k}^{q} n_j(n_k - 1)$.*

The Petersen graph $G = P(5,2)$ is the graph with the vertex set $V(G) = \{u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4, v_5\}$, and the edge set $E(G) = \{u_i u_{(i+1)mod5}, v_i v_{(i+1)mod5}, u_i v_{(i+j)mod5}, 1 \leq i \leq 5, j = i - 1\}$, with $u_0 = u_5$ and $v_0 = v_5$.

The following theorem points to the Petersen graph's location-domination number.

Theorem 3.3 *(Balbuena, 2015) Let $G = P(5,2)$ be the Petersen graph, then $LD(G) = 4$.*

Following proposition gives the location-domination polynomial of Petersen graph.

Proposition 3.4 *Let $G = P(5,2)$, then $LD(G,x) = x^{10}+10x^9+45x^8+120x^7+200x^6+192x^5+65x^4$.*

Proof. Since the girth of Petersen graph $G = P(5,2)$ is 5. A dominating set D is also a locating if it does not have two vertices that are each uniquely dominated by one vertex. Let V denotes the vertex set of Petersen graph, D dominating set and $E = V \setminus D$. Here are the cases for E.

1. When $|E| = 0, 1, 2, or\ 3$, then D is a dominating set because no vertex of E is isolated from the set D. By Lemma 2.3, D is also a locating set, so l_j= $\binom{10}{j}$, for $j = 7, 8, 9, 10$.

2. When $|E| = 4$, then there exist 10 possibilities to choose E with 1 vertex isolated from D, so there exist $\binom{10}{4} - 10 = 200$ dominating sets of size 6. All of them are also locating, and $l_6 = 200$.

3. When $|E| = 5$, then there exist $10 * 6$ possibilities to choose E with 1 vertex isolated from D, so there exist $\binom{10}{5} - 60 = 192$ dominating sets of size 5. All of them are also locating, and $l_5 = 192$.

4. When $|E| = 6$, then there exist $10 * 12$ possibilities to choose E with 1 vertex isolated from D, and 15 possibilities to choose E with 2 vertices isolated from D. So there exist $\binom{10}{6} - 120 - 15 = 75$ dominating sets of size 4. A dominating set D is not locating if there exists a vertex $v \in D$ with two adjacent vertices u and w such that $N[u] \bigcup N[w] \setminus \{v\} \subset V \setminus D$. In this case set E contains another 4 vertices different from u and w. Vertices of D induce a star $K_{1,3}$. There are 10 ways to pick this star, so there are 10 different sets of dominating characteristics not located in the Petersen graph. 65 of the remaining dominating sets are also locating and $l_4 = 65$.

Consequently, $LD(G,x) = x^{10} + 10x^9 + 45x^8 + 120x^7 + 200x^6 + 192x^5 + 65x^4$.

Definition 3.5 *Lexicographic product between the graphs G and $\acute{G}$, denoted by $G[\acute{G}]$, is the graph with vertex set $V(G) \times V(\acute{G}) = \{(u,v) \ : \ u \in V(G), \ v \in V(\acute{G})\}$ and (u_1,v_1) is adjacent to (u_2,v_2) if $u_1 = u_2$ and v_1 is adjacent to v_2, or u_1 is adjacent to u_2.*

Here is the result for a graph G which is the lexicographic product of a cycle graph with n_1 nodes and the complement of a complete graph with n_2 nodes.

Theorem 3.6 *Let G be the graph $C_{n_1}[\overline{K_{n_2}}]$ with $n_1 \geq 3$ and $n_2 \geq 2$, then*

$$LD(G) = \begin{cases} n_1(n_2-1) & \text{when } n_1 \neq 4, \\ n_1(n_2-1)+2 & \text{when } n_1 = 4. \end{cases}$$

Proof. Let $V(G) = \{(u_i,v_j); \ 1 \leq i \leq n_1, \ 1 \leq j \leq n_2\}$, where $u_i \in V(C_{n_1})$ and $v_j \in V(\overline{K_{n_2}})$. Then for each i, where $1 \leq i \leq n_1$, the set $T_i = \bigcup_{j=1}^{n_2} \{(u_i,v_j)\}$ is a twin set. Since there are n_1 twin sets and each twin set is of cardinality n_2. Thus, every locating-dominating set of the minimum cardinality contains at least $n_1(n_2-1)$ vertices, and hence $LD(G) \leq n_1(n_2-1)$.
Consider the set $L_d = \bigcup_{i=1}^{n_1} \bigcup_{j=1}^{n_2-1} \{(u_i,v_j)\}$. Then for each i, $1 \leq i \leq n_1$ and $j = n_2$,

$$N(u_i,v_j) \bigcap L_d = \bigcup_{k_1=1(k_1\neq i)}^{n_1} \bigcup_{k_2=1}^{n_2-1} \{(u_{k_1},v_{k_2})\}.$$

This implies that all the remaining vertices $(u_i,v_j) \notin L_d$, $1 \leq i \leq n_1$ and $j = n_2$, have different non-empty neighborhoods in L_d. Thus, the set L_d is a locating-dominating set of cardinality $n_1(n_2-1)$. Hence, $LD(G) = n_1(n_2-1)$ for $n_1 \geq 3$, $n_1 \neq 4$ and $n_2 \geq 2$.
Further, for $n_1 = 4$ and $n_2 \geq 2$, the set $L_d^* = \bigcup_{i=1}^{n_1} \bigcup_{j=1}^{n_2-1} \{(u_i,v_j)\}$ does not form a locating-dominating set because $N(u_i,v_{n_2}) = N(u_j,v_{n_2}), 1 \leq i \leq 2, j = i+2$. However, the set $L_d^* \bigcup_{i=1}^{2} \{(u_i,v_{n_2})\}$ is a locating-dominating set of the minimum cardinality. Thus, for $n_1 = 4$ and $n_2 \geq 2$, $LD(G) = n_1(n_2-1)+2$.

Theorem 3.7 *Let $G = C_{n_1}[\overline{K_{n_2}}]$, then for $n_1 \neq 4 \geq 3$, $n_2 \geq 2$,*

$$LD(G,x) = \sum_{j=0}^{n_1} \binom{n_1}{j} n_2^{n_1-j} x^{n_1(n_2-1)+j}.$$

And, for $n_1 = 4, \ n_2 \geq 2$,

$$LD(G,x) = x^{4n_2} + 4n_2 x^{4n_2-1} + 4n_2^2 x^{4n_2-2}.$$

Proof. Note that there are n_1 twin-sets of cardinality n_2 in G. From these n_1 twin-sets, we can choose k twin-sets from which we will choose all the n_2 vertices, and this can be done in $\binom{n_1}{k}$ different ways. From the remaining n_1-k twin-sets, we can choose n_2-1 vertices out of n_2 vertices, which can be done in $n_2^{n_1-k}$ ways. This implies that

$$LD(G,x) = \sum_{j=0}^{n_1} \binom{n_1}{j} n_2^{n_1-j} x^{n_1(n_2-1)+j}.$$

Now, for $n_1 = 4$ and $n_2 \geq 2$, $LD(G) = n_1(n_2-1)+2$, so $l_{4n_2} = 1$ and $l_{4n_2-1} = 4n_2$. In this case, we notice that $V(G) = T_1 \bigcup T_2$, where $T_1 = \bigcup_{j=1}^{n_2} \{(u_1,v_j),(u_3,v_j)\}$ and $T_2 = \bigcup_{j=1}^{n_2} \{(u_2,v_j),(u_4,v_j)\}$ are the twin sets. Since $|T_1| = |T_2| = 2n_2$, therefore $l_{4n_2-2} = 4n_2^2$, and hence, the result holds.

From Theorem 3.7, we have the following corollary:

Corollary 3.8 *Let* $G = C_{n_1}[\overline{K_{n_2}}]$*,* $n_1 \geq 3$*,* $n_2 \geq 2$*, and* T_1*,* T_2*,...,*T_t *are the twin sets of* G *such that* $V(G) = \bigcup_{i=1}^{t} T_i$*, then for any* $v \in T_k, 1 \leq k \leq t$*,* $LDV(v) = \prod_{i=1 i \neq k}^{t} |T_i|(|T_k| - 1)$.

4. Conclusion

The location-domination polynomial of a graph is a new concept within graph theory. This article introduces the location-domination polynomial of a graph G and discusses its properties. Additionally, location-dominating value has also been introduced. Our research also computed the location-domination numbers and polynomials of several families of graphs.

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Submitted: 27/03/2021
Revised: 04/10/2021
Accepted: 03/11/2021
DOI: 10.48129/kjs.13175