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On some tests for exponentiality based on the mean residual life function

Suja M. Aboukhamseen, Emad-Eldin A. A. Aly

Department of Statistics and Operations Research, Faculty of Science, Kuwait University

Corresponding Author: Email: saboukhamseen@yahoo.com

Abstract

We build on the work of Aly (1983) and Jammalamadaka & Taufer (2006) to develop new tests for exponentiality based on the mean residual life function. We obtain the asymptotic null distributions of the proposed tests and give approximations for their limiting critical values. We also give tables of their finite sample Monte Carlo critical values. We report the results of several Monte Carlo studies conducted to compare the proposed tests with a number of their competitors in terms of power.

Keywords: Brownian bridge; limit theorems; Monte Carlo simulations.

1. Introduction

Assume that the non-negative random variable, X, has the continuous distribution function $F(\cdot)$. The corresponding mean residual life (MRL) function at time t is defined as

$$m(t) = E\left(X - t \middle| X > t\right) = \frac{\int_t^{\infty} \overline{F}(x)dx}{\overline{F}(t)},$$

where $\overline{F}(\cdot) = 1 - F(\cdot)$. Shanbhag (1970) proved that

$$m(t) = \theta, \forall t > 0 \tag{1}$$

if and only if $F(\cdot)$ is exponential with mean θ.

Let $X_1, X_2, \cdots, X_{n+1}$ be a random sample from $F(\cdot)$. We consider the problem of testing the null hypothsis

$$H_\circ : F(x) = 1 - e^{-x/\theta}, \forall x \geq 0, \tag{2}$$

where $\theta > 0$ is unknown against the alternative that

$$H_A : F(x) \neq 1 - e^{-x/\theta}, \text{ for some } x \geq 0.$$

Let $X_{(0)} = 0$ and $X_{(1)} \leq \cdots \leq X_{(n+1)}$ be the order statistics of the given sample. Define the normalized spacings as

$$Y_i = (n-i+2)(X_{(i)} - X_{(i-1)}), i = 1,2,\cdots,n+1.$$

The sample MRL after $\overline{X}_{(k)}$ is given, for $k = 1,2,\cdots,n$, by

$$\overline{X}_{>k} = \frac{1}{n-k+1}\sum_{i=k+1}^{n+1} Y_i.$$

Jammalamadaka &Taufer (2006) exploited the charaterization (1) to propose a new family of omnibus tests for exponentiality. Their family of test statistics is defined as

$$T_n(\gamma) = \max_{1 \leq k \leq n-[n^{\gamma}]} \frac{\left|\overline{X} - \overline{X}_{>k}\right|}{\overline{X}}, \tag{3}$$

where $\overline{X}$ is the sample mean and $\gamma \in (0,1)$ is fixed. Note that the faimly of test statistics $T_n(\gamma)$ of (3) is scale parameter free.

Jammalamadaka &Taufer (2006) proved that for each fixed $\gamma \in (0,1)$, under the null hypothesis of exponentiality,

$$n^{\gamma/2} T_n(\gamma) \xrightarrow{D} \sup_{0 \leq u \leq 1} \left|W(u)\right| := T, \tag{4}$$

where $W(\cdot)$ is a Wiener process. It is well known that the distribution function of the random variable T of (4) is given by

$$P\{T \leq t\} = \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \exp\left\{-\frac{\pi^2(2k+1)^2}{8t^2}\right\}, t \geq 0,$$

(see, for example, Aly *et al.* (1994)). Consequently the limiting critical values of $n^{\gamma/2} T_n(\gamma)$ are 1.96, 2.241 and 2.807 for $\alpha = 0.1, 0.05$ and 0.01, respectively. Jammalamadaka &Taufer (2006) provided a Table of the finite sample critical values of $n^{\gamma/2} T_n(\gamma)$ for $\alpha = 0.05$. Their Table shows that these critical values are far from the corresponding limiting value of 2.241 even for $n = 200$.

The exponential distribution plays an important role in reliability, survival analysis and data analysis. The problem of testing for exponentiality continues to receive considerable attention. Hollander & Proschan (1972) proposed and studied tests for exponentiality against the class of new is better than used (NBU) distributions. Koul (1978) and Bhattacharjee & Sen (1995) proposed and studied tests against the class of new is better than used in expectation (NBUE) distributions. Bergman & Klefsjö

(1989) and Bandyopadhyay & Basu (1990) proposed and studied tests against the class of decreasing mean residual life (DMRL) distributions. Deshpande (1983) and Aly (1989) proposed and studied tests against the class of increasing failure rate average (IFRA) distributions. Aly (1990 a) considered the problem of testing against the class of increasing failure rate (IFR) distributions. Aly (1992) considered the problem of testing against the class of harmonic NBUE (HNBUE) distributions. Additional results and references on testing for exponentiality are given in Aly &Lu (1988); Aly (1990 b); Baringhaus & Henze (1992); Ebrahimi *et al*. (1992); Henze (1993); Ahmad & Alwasel (1999); Grzegorzewski & Wieczorkowski (1999); Baringhaus & Henze (2000); Taufer (2000); Alwasel (2001); Klar (2001); Henze & Meintanis (2002) and Jammalamadaka & Taufer (2003).

In this article we build on the works of Aly (1983) and Jammalamadaka & Taufer (2006). We propose and study some new tests for exponentiality based on the charaterization (1). We report the results of a Monte Carlo power study which shows that the proposed tests enjoy favourable power values compared to a number of competitors.

2. The proposed tests

The proposed tests are based on the scale parameter free testing process

$$T_n(u;\gamma) = (n+1)^{\frac{1}{2}} \left\{ \frac{\frac{1}{(n+1)} \sum_{i=n-[(n+1)u]+2}^{n+1} Y_i^{\gamma}}{\left(\frac{1}{n+1} \sum_{i=1}^{n+1} Y_i \right)^{\gamma}} - \frac{[(n+1)u]}{n+1} \Gamma(1+\gamma) \right\}, \tag{5}$$

where $\gamma > 0$ is fixed. Note that tests based on $T_n(\cdot;1)$ are related to the test proposed by Jammalamadaka & Taufer (2006) through the result that the process appearing on the right hand side of (3) can be expressed in terms of $T_n(\cdot;1)$ as follows

$$\frac{\overline{X} - \overline{X}_{>k}}{\overline{X}} = -\frac{(n+1)^{\frac{1}{2}} T_n(\frac{n-k+1}{n+1};1)}{n-k+1}. \tag{6}$$

2.1 Tests based on $T_n(\cdot;1)$

In the special case when $\gamma = 1$, we propose the following test statistics for testing for exponentiality.

$$T_{n,1} = \max_{1 \le k \le n+1} \left| T_n(\frac{k}{n+1};1) \right|, \tag{7}$$

$$T_{n,2} = \frac{1}{n+1} \sum_{k=1}^{n+1} T_n^2(\frac{k}{n+1};1), \tag{8}$$

$$T_{n,3} = (n+1) \sum_{k=1}^{n+1} \frac{T_n^2(\frac{k}{n+1};1)}{k(n-k+1)} \tag{9}$$

and

$$T_{n,4} = A(\log n) \max_{1 \le k \le n+1} \frac{n+1}{\left(k(n-k+1)\right)^{\frac{1}{2}}} \left| T_n(\frac{k}{n+1};1) \right| - D(\log n), \tag{10}$$

where

$$A(x) = \left(2 \log x\right)^{\frac{1}{2}}$$

and

$$D(x) = 2 \log x + \frac{1}{2} \operatorname{loglog} x - \frac{1}{2} \log \pi.$$

The limit in distribution of the test statistics $T_{n,i}$, $i = 1, \cdots, 4$ are given in the following Lemma.

Lemma A. Under $H_\circ$ of (2) and as $n \to \infty$ we have

$$T_{n,1} \xrightarrow{\mathrm{D}} \sup_{0 \le u \le 1} \left| B(u) \right| := T_1, \tag{11}$$

$$T_{n,2} \xrightarrow{\mathrm{D}} \int_0^1 B^2(u) du := T_2, \tag{12}$$

$$T_{n,3} \xrightarrow{\mathrm{D}} \int_0^1 \frac{B^2(u)}{u(1-u)} du := T_3 \tag{13}$$

and

$$P\{T_{n,4} \le x\} \to \exp\left(-2e^{-x}\right), \tag{14}$$

where $B(\cdot)$ is a Brownian bridge.

The critical values of T_1, T_2 and T_3 are respectively given in Tables 4, 1 and 5 of Section 8 of Chapter 3 of Shorack &Wellner (1986).

2.2 Tests based on $T_n(\cdot;\gamma)$

For $\gamma > 0$ we propose the following test statistics for testing for exponentiality.

$$T_{n,5}(\gamma) = \max_{1\le k\le n+1}\left|T_n(\frac{k}{n+1};\gamma)\right| \tag{15}$$

and

$$T_{n,6}(\gamma) = \left|\frac{1}{n+1}\sum_{k=1}^{n+1}T_n(\frac{k}{n+1};\gamma)\right|. \tag{16}$$

The limit in distribution of the test statistics $T_{n,5}(\gamma)$ and $T_{n,6}(\gamma)$ are given in the following Lemma.

Lemma B. Under $H_\circ$ of (2) and as $n \to \infty$ we have

$$T_{n,5}(\gamma) \xrightarrow{D} \sup_{0<u<1}\left|K(u;\gamma)\right| := T_5(\gamma) \tag{17}$$

and

$$T_{n,6}(\gamma) \xrightarrow{D} \left|T_6(\gamma)\right|, \tag{18}$$

where

$$T_6(\gamma) = \int_0^1 K(u;\gamma)du \tag{19}$$

and $K(\cdot;\gamma)$ is a mean zero Gaussian process with covariance function

$$E\{K(u;\gamma)K(v;\gamma)\} = \{\Gamma(2\gamma+1) - \Gamma^2(1+\gamma)\}\min(u,v) - \gamma^2\Gamma^2(1+\gamma)uv. \tag{20}$$

The critical values of $T_5(\gamma)$ will be obtained by simulating the Gaussian process $K(\cdot;\gamma)$. It is easy to see that $T_6(\gamma)$ is a mean zero Normal random variable with variance

$$\sigma^2(\gamma) = \frac{1}{3}\{\Gamma(2\gamma+1) - \Gamma^2(1+\gamma)\} - \frac{1}{4}\gamma^2\Gamma^2(1+\gamma).$$

For example,

$$\sigma^2(2) = \frac{8}{3} \text{ and } \sigma^2(\frac{1}{2}) = \frac{64-19\pi}{192}.$$

3. Proofs

By (5) and the substitution

$$Z_i = Y_{n-i+2}, i = 1, 2, \cdots, n+1$$

we obtain

$$T_n(\cdot;\gamma) = T_n^*(\cdot;\gamma), \tag{21}$$

where

$$T_n^*(u;\gamma) = (n+1)^{\frac{1}{2}} \left\{ \frac{\frac{1}{(n+1)} \sum_{i=1}^{[(n+1)u]} Z_i^\gamma}{\left(\frac{1}{n+1} \sum_{i=1}^{n+1} Z_i \right)^\gamma} - \frac{[(n+1)u]}{n+1} \Gamma(1+\gamma) \right\}. \tag{22}$$

By (21) $T_n(\cdot;\gamma)$ and $T_n^*(\cdot;\gamma)$ have the same asymptotic theory. For ease of presentation, we will prove the asymptotic results in terms of $T_n^*(\cdot;\gamma)$. Note that under $H_\circ$ of (2), $Z_1, Z_2, \ldots, Z_{n+1}$ are *iid* Exponential random variables with mean θ.

Proof of Lemma A: Note that

$$T_n^*(u;1) = \frac{S_n(u)}{\left(\frac{1}{n+1} \sum_{i=1}^{n+1} Z_i \right)}, \tag{23}$$

where

$$S_n(u) = (n+1)^{-\frac{1}{2}} \left\{ \sum_{i=1}^{[(n+1)u]} Z_i - \frac{[(n+1)u]}{n+1} \sum_{i=1}^{n+1} Z_i \right\} \tag{24}$$

is the well known cusum process (see, for example, Chapter 2 of Csörgő & Horváth (1997)). By (21), (23), Slutsky theorem and the well developed asymptotic theory of $S_n(\cdot)$ we get (11)-(14).

Proof of Lemma B: By (22) and the results of Aly (1983 &1988) we obtain, under $H_\circ$, for each $\gamma > 0$ and as $n \to \infty$;

$$T_n^*(u;\gamma) \xrightarrow{D} K(u;\gamma). \tag{25}$$

By the Continuous Mapping Theorem we obtain (17) and (18).

Remark 1 All of the proposed test statistics can be used to test

$$H_\circ : F(x) = 1 - e^{-(x-\theta_1)/\theta_2}, \ \theta_1 \in R, \theta_2 > 0, x \geq \theta_1,$$

against the alternative

$$H_A : F(x) \neq 1 - e^{-(x-\theta_1)/\theta_2}.$$

This follows from the well known result that, under $H_\circ$; $X_1 - X_{(1)}, X_2 - X_{(1)}, \ldots, X_n - X_{(1)}$ are *iid* random variables with exponential distribution with mean θ_2.

4. Monte Carlo simulations

4.1 Finite sample critical values for $T_{n,i}, i = 1,2,3,4$

We conducted a Monte Carlo Simulation to obtain the finite sample critical values of $T_{n,i}, i = 1,2,3,4$. In this study we generated 10,000 random samples of size $n = 10(10)50(50)200$ from the exponential distribution with mean one. It is worth noting that all of the test statistics presented in this paper are scale free. Therefore, the mean of the exponential distribution from which the samples are simulated has no bearing on the outcome. For each sample we computed each of the test statistics $T_{n,i}, i = 1,2,3,4$. For each test statistic we ordered the resulting 10,000 values and obtained the corresponding $(1-\alpha)$ percentile for $\alpha = 0.1, 0.05$ and 0.01. The resulting finite sample critical values for $\alpha = 0.05$ together with the corresponding limiting critical values are given in Table 1.

4.2 Limiting and finite sample critical values for $T_{n,5}(\gamma)$ and $T_{n,6}(\gamma)$

For $M = 2000$ we generate $N = 2000$ realizations

$$(Z_1^{(l)}(\gamma), \cdots, Z_M^{(l)}(\gamma)) : MVN(0, \Lambda), l = 1,2,\cdots,2000,$$

where for $i \leq j$,

$$\Lambda_{i,j} = \{\Gamma(2\gamma+1) - \Gamma^2(1+\gamma)\}\frac{i}{M+1} - \gamma^2\Gamma^2(1+\gamma)\frac{ij}{(M+1)^2}.$$

For $l = 1,2,\cdots,2000$, compute $\max_{1\leq i\leq M}\left|Z_i^{(l)}(\gamma)\right|$ and the corresponding $(1-\alpha)$ percentile for $\alpha = 0.1, 0.05$ and 0.01. This gives the limiting critical values for $T_{n,5}(\gamma)$ which are given in Table 1 for $\gamma = 0.5$ and 2. Of the various values for γ investigated, $\gamma = 0.5$ and $\gamma = 2$, are chosen as representative of the case when $\gamma < 1$ and $\gamma > 1$ respectively.

Table 1. Limiting critical values for $T_{n,5}(0.5)$ and $T_{n,5}(2)$

α	0.1	0.05	0.01
$T_{n,5}(0:5)$	0.58	0.66	0.79
$T_{n,5}(2)$	5.96	6.68	7.79

We conducted another Monte Carlo Simulation to obtain the finite sample critical values of $T_{n,5}(0.5), T_{n,5}(2), T_{n,6}(0.5)$ and $T_{n,6}(2)$. In this study we generated 10,000 random samples of size $n = 10(10)50(50)200$ from the exponential distribution with mean one. For each sample we computed each of the test statistics $T_{n,5}(0.5), T_{n,5}(2), T_{n,6}(0.5)$ and $T_{n,6}(2)$. For each test statistic, we ordered the resulting 10,000 values and obtained the corresponding $(1-\alpha)$ percentile for $\alpha = 0.1, 0.05$ and 0.01. In Table 2 we give the resulting finite sample critical values for $\alpha = 0.05$.

Table 2. Finite sample and limiting critical values for $T_{n,1} - T_{n,6}$

n	$\mathbf{T_{n,1}}$	$\mathbf{T_{n,2}}$	$\mathbf{T_{n,3}}$	$\mathbf{T_{n,4}}$	$\mathbf{T_{n,5}(0.5)}$	$\mathbf{T_{n,6}(0.5)}$	$\mathbf{T_{n,5}(2)}$	$\mathbf{T_{n,6}(2)}$
10	1.12	0.43	2.25	2.13	0.56	0.29	4.65	2.19
20	1.20	0.44	2.41	2.51	0.60	0.29	5.33	2.47
30	1.23	0.45	2.47	2.64	0.60	0.29	5.68	2.70
40	1.27	0.45	2.48	2.73	0.62	0.29	5.80	2.76
50	1.26	0.46	2.49	2.78	0.62	0.29	5.90	2.86
100	1.30	0.46	2.48	2.91	0.63	0.29	6.23	3.01
150	1.31	0.45	2.45	3.00	0.64	0.29	6.36	3.01
200	1.32	0.46	2.52	2.98	0.64	0.29	6.37	3.00
500	1.33	0.46	2.49	3.04	0.65	0.29	6.54	3.15
Asymptotic	1.36	0.46	2.50	3.66	0.66	0.29	6.68	3.20

4.3 Monte Carlo power comparisons

In this study we compared the empirical powers of the following tests:

1. $T_{n,1} - T_{n,4}, T_{n,5}(0.5), T_{n,5}(2), T_{n,6}(0.5)$ and $T_{n,6}(2)$ of Section 2.
2. The classical Kolmogorov-Smirnov test (KS_n).
3. The statistic L_n of Baringhaus &Henze (2000).
4. The statistic $T_n(\gamma)$ of Jammalamadaka & Taufer (2006) with $\gamma = 0.8$ and $\gamma = 0.9$.

5. The Cramer-Von-Mises test for Exponentiality,

$$C_n = \frac{1}{12n} + \sum_{i=1}^{n} \left\{ \frac{2i-1}{2n} - \left(1 - e^{-\frac{X_{(i)}}{\overline{X}}} \right) \right\}^2 .$$

6. The Anderson-Darling test for Exponentiality,

$$A_n = \left(1 + \frac{0.3}{n} \right) \left\{ -\frac{1}{n} \sum_{i=1}^{n} \left((2i-1) \ln \left(p_i (1 - p_{n-i+1}) \right) - n \right\},$$

where

$$p_i = 1 - e^{-\frac{X_{(i)}}{\overline{X}}}, i = 1, 2, \cdots, n.$$

We obtained the finite sample critical values of C_n and A_n by Monte Carlo simulation using 10,000 random samples of size $n = 20(30)80$ from the exponential distribution with mean one. In Table 3 we give the resulting finite sample critical values for $\alpha = 0.05$.

Table 3. Finite sample critical values of C_n and A_n.

n	20	50	80
C_n	0.22	0.22	0.22
A_n	1.31	1.32	1.31

We employed the following alternative distributions:

1. Weibull (θ); $f(x) = \theta x^{\theta-1} \exp\{-x^{\theta}\}, x > 0$ with $\theta = 0.8$ and 1.2.
2. Power (θ); $f(x) = \theta^{-1} x^{(1-\theta)/\theta}, 0 < x < 1$ with $\theta = 0.8$.
3. Lomax (θ); $f(x) = (1 + \theta x)^{-(1+\theta)/\theta}, x \geq 0$ with $\theta = 0.5$.
4. Dhillon (θ); $f(x) = \theta x^{\theta-1} \exp\{x^{\theta} + 1 - e^{x^{\theta}}\}, x > 0$ with $\theta = 0.5$
5. Log-logistic (θ); $f(x) = \dfrac{\theta x^{\theta-1}}{(1 + x^{\theta})^2}, x \geq 0$ with $\theta = 3$.
6. Compound Rayleigh (θ); $f(x) = \dfrac{2\theta x}{(1 + x^2)^{\theta+1}}, x \geq 0$ with $\theta = 1$.

For failure rate (FR) and mean residual life (MRL) classifications of the above distributions we refer to Table 2 of Jammalamadaka &Taufer (2006). We note here that the above selection includes distributions with increasing FR (IFR), decreasing FR (DFR), increasing then decreasing FR (IDFR) and decreasing then increasing FR (DIFR).

We generated 10,000 samples of size 20,50 and 80 from each of the alternative distributions. For each test statistic, each alternative distribution and each sample size we computed the fraction of times the test was significant at $\alpha = 0.05$. Some of the resulting empirical powers are given in Tables 4-6.

Table 4. Empirical powers at α=0.05 and n=20.

	W(1.2)	**W(0.8)**	**Log(3)**	**Lomax(0.5)**	**Dh(0.5)**	**P(0.8)**	**CRayl(1)**
KS_n	25	40	98	69	72	35	59
$T_n(0.8)$	20	42	99	72	68	19	17
$T_n(0.9)$	29	35	95	51	77	37	39
$T_{n,1}$	25	43	99	76	71	29	49
$T_{n,2}$	30	47	99	80	75	31	49
$T_{n,3}$	31	47	99	81	75	38	75
L_n	31	36	99	73	65	29	52
$T_{n,5}(0.5)$	20	24	96	54	52	17	40
$T_{n,6}(0.5)$	6	90	73	93	97	25	13
$T_{n,5}(2)$	2	26	97	65	42	29	34
$T_{n,6}(2)$	0	21	95	61	29	8	17
C_n	30	46	99	75	78	47	69
A_n	28	54	99	76	91	73	74

Table 5. Empirical powers at α=0.05 and n=50.

	W(1.2)	W(0.8)	Log(3)	Lomax(0.5)	Dh(0.5)	P(0.8)	CRayl(1)
KS_n	24	37	99	68	71	33	59
$T_n(0.8)$	19	40	99	69	64	20	14
$T_n(0.9)$	28	33	96	48	76	35	39
$T_{n,1}$	23	41	99	76	69	28	48
$T_{n,2}$	26	47	99	79	73	31	48
$T_{n,3}$	28	48	99	80	73	38	74
L_n	28	34	99	72	61	29	52
$T_{n,5}(0.5)$	18	24	96	53	47	20	38
$T_{n,6}(0.5)$	55	90	75	92	96	21	12
$T_{n,5}(2)$	3	23	97	65	39	27	28
$T_{n,6}(2)$	0	17	96	60	26	6	17
C_n	28	43	99	74	76	47	68
A_n	26	52	99	74	91	73	74

Table 6. Empirical powers at α=0.05 and n=80.

	W(1.2)	W(0.8)	Log(3)	Lomax(0.5)	Dh(0.5)	P(0.8)	CRayl(1)
KS_n	35	48	100	87	90	55	77
$T_n(0.8)$	30	53	100	90	79	27	20
$T_n(0.9)$	39	50	100	76	91	46	44
$T_{n,1}$	39	56	100	94	91	63	70
$T_{n,2}$	46	62	100	94	91	63	70
$T_{n,3}$	48	63	100	94	93	78	92
L_n	41	48	100	91	84	49	67
$T_{n,5}(0.5)$	29	37	100	78	73	40	59
$T_{n,6}(0.5)$	27	94	69	95	99	100	100
$T_{n,5}(2)$	7	35	100	85	60	45	46
$T_{n,6}(2)$	0	27	99	80	45	92	25
C_n	43	57	100	92	92	75	86
A_n	43	66	100	92	99	94	92

The power results of Tables 4-6 show that, in terms of power, the test statistics $T_{n,2}$ and $T_{n,3}$ enjoy favourable power values compared to the considered competitors.

4. Remarks

We comment here on the consistency of the proposed tests. By Theorem 7.4.2 of Csörgö (1983),

$$(n+1)^{-\frac{1}{2}} T_n^*(u;1) \xrightarrow{a.s.} \left\{ \frac{H_F^{-1}(u)}{H_F^{-1}(1)} - u \right\}$$

uniformly in $u \in (0,1)$, where

$$H_F^{-1}(u) = \int_0^u (1-y)dF^{-1}(y)$$

is the total time on test transform of F. Consequently, tests based on $T_n^*(\cdot;1)$ are consistent against all alternative distributions F for which

$$\frac{H_F^{-1}(u)}{H_F^{-1}(1)} \neq u$$

for some $u \in (0,1)$. Note that for the exponentials distribution

$$\frac{H_F^{-1}(u)}{H_F^{-1}(1)} = u$$

for all $u \in (0,1)$.

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عن بعض الاختبارات للتوزيع الأُسّي المعتمدة على دالة متوسط الحياة المتبقية

سجى منصور أبو خمسين* ، عماد الدين علي أحمد علي
قسم الإحصاء وبحوث العمليات – جامعة الكويت – الكويت
*المولف : saboukhamseen@yahoo.com

خلاصة

قمنا بتطوير اختبارات جديدة للتوزيع الأُسّي المعتمدة على دالة متوسط الحياة المتبقية. حصلنا على التوزيع التقاربي للاختبارات المقترحة وقدمنا تقريب لقيمهم الحرجة التقاربية. قدمنا أيضاً جداول لقيمهم الحرجة باستخدام طرق مونت كارلو للمحاكاة. استخدمنا طرق مونت كارلو للمحاكاة لعمل مقارنة لقوة الاختبارات المقترحة مع عدد من الاختبارات المنافسة.