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A new approximate-analytical method to solve non-Fourier heat conduction problems

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Abstract

In this paper, the effect of laser as a heat source on a thin film was investigated. The non-Fourier heat conduction model of Cattaneo-Vernotte was used for thermal analysis of the problem. The thermal conductivity was assumed temperature-dependent, which resulted in a non-linear equation and by assuming the role of laser as a heat source, a non-homogeneous equation was obtained. The obtained equations were solved using approximate-analytical method of variational iteration method (VIM). It was concluded that the non-linear analysis is important in non-Fourier heat conduction problems. Significant differences were observed between the Fourier and non-Fourier solutions which stresses the importance of non-Fourier solutions in the similar problems.

Keywords: C-V model; laser heating; non-Fourier; thin film; variational iteration method.

1. Introduction

Laser has had increasing and important applications in the recent years. In the industry, laser is employed as a precise manufacturing tool and a concentrated heat source in applications such as cladding, cutting, surface hardening, welding, and machining. In medicine, laser has been utilized as a surgical tool and also for hyperthermia in cancer treatment. Its high temporal and spatial resolution, not involving or heating the unnecessary areas, and the minimal noise has made laser a very popular tool.

Since laser applies a high heat flux in a short period of time, the analysis of laser beam-induced heating is not possible using the classical Fourier's heat conduction law (Özişik & Tzou, 1994; Tang & Araki, 1999; Wang & Xu, 2002). The combination of Fourier's heat conduction law and the energy equation results in a parabolic equation. The parabolic equation leads to the non-physical conclusion, which implies an infinite speed of heat propagation. This paradox is not an issue in many common applications. However, the Fourier's law is not applicable to applications involving very high heat fluxes (Tung *et al.,* 2009), heat transfer at very low temperatures (Peshkov, 1944), and heat transfer at very small scales (Shirmohammadi, 2011). Therefore, improved models have to be employed in laser heating analysis.

The heat wave model, or the Cattaneo-Vernotte model, is an improved widely-used version of the Fourier's classical model (Catteneo, 1958; Vernotte, 1961). They defined a thermal relaxation time, whose macroscopic description is a time lag between the temperature gradient and the heat flux vector. This model has been recently utilized by many researchers. This model was used by Bargmann & Favata (2014) to analyze the laser heating in polycrystals. The non-Fourier heat transfer in combined analysis of heat conduction and thermal radiation in a differentially heated 2-D square cavity was studied by Sasmal & Mishra (2014). The non-Fourier heat conduction in a finite slab with insulated boundaries was numerically investigated by Rahbari *et al.* (2014). An analytical non-Fourier study was conducted by Zhao *et al*. (2014) on a solid sphere under arbitrary surface thermal disturbances. The non-Fourier heat conduction and thermal radiation problem in a concentric spherical shell was studied by Mishra & Sahai (2014). The thermal wave phenomenon in a thin film was analytically examined by Fong & Lam (2014).

In most studies on the non-Fourier heat conduction, the equations are linear due to the assumption of constant thermal properties, and nonlinear study of such problems is rare in the literature. The behavior of materials in nature is inherently nonlinear, so the nonlinear study of the mentioned problems is very important in some cases. In the past, in addition to the very limited and difficult analytical methods, numerical methods were used to solve nonlinear problems. These methods were also faced with convergence problem and had a high computational cost. New methods, known as semi-analytical (or approximate-analytical), have recently been proposed for solving nonlinear problems. Some of the most well-known semi-analytical methods include Adomian decomposition method (ADM) (Adomian, 1983); homotopy perturbation method (HPM) (He, 1998a; He, 1998b); homotopy analysis method (HAM) (Liao, 1992); differential transform method (DTM) (Zhou, 1986), and variational iteration method (VIM). There are some references to different problems that have been solved recently by means of semi-analytical methods (Dogan, 2013; Garg & Manohar, 2013; Bayat *et al*., 2015).

Variational iteration method (VIM), which is used in this study is one of the most flexible and powerful tools that was frequently used to solve a variety of linear and nonlinear equations. It was first developed by He (1999) and then many researchers emphasized on its appropriate convergence (Ramos, 2008; Tatari & Dehghan, 2007; Yang *et al*., 2010; Odibat, 2010; Salkuyeh, 2008; Saadati *et al*., 2009). Recently, many applications of this method have been reported. Elsayed (2013) studied about the thermal diffusion and the effect of thermo diffusion thixotropic fluid in the biological tissues using VIM. In the research done by Wu & Baleanu (2013), VIM was used for better description of the fluid flow in porous media and for this purpose a new variational iteration formula was developed. Saha Ray & Gupta (2014) solved the equation of Burgers-Huxley with the use of VIM and compared with exact solution and finally its simplicity and effectiveness was acknowledged. Finding a solution for the problem of Amperometric enzyme kinetics was the aim of a research done by Malvandi & Ganji (2013) using the VIM. By using VIM, an analytical solution for the problem of the suspended bridge was developed by Samaee *et al*. (2014). The problem of fractal heat conduction using VIM was solved by Liu *et al*. (2013).

The application of semi-analytical methods to solve the nonlinear problems of non-Fourier heat conduction has been reported just in few studies. Torabi *et al*. (2011) applied the homotopy perturbation method (HPM) to solve a non-linear convective-radiative non-Fourier conduction heat transfer equation with variable specific heat coefficient. Saedodin *et al*. (2011) used the variational iteration method (VIM) to solve the same problem. Differential transformation method (DTM) was applied to analyze non-linear convective–radiative hyperbolic lumped systems with simultaneous variation of temperature-dependent specific heat and surface emissivity by Torabi *et al*. (2013). In all of three mentioned references, the governing equations have been only dependent on time and in fact, ordinary differential equations (ODE) has been solved by semi-analytical methods and to the best knowledge of the authors, nonlinear partial differential equation (PDE) of non-Fourier heat conduction equation has not been solved yet by semi-analytical methods.

In this paper, the effect of heat transfer in a thin film under a laser heat source is investigated. For thermal analysis, the non-Fourier heat conduction model is used. Because of assuming the thermal conductivity variable with respect to the temperature, a nonlinear equation is obtained and solved using VIM. The application of a semi-analytical method such as VIM to solve nonlinear PDE of non-Fourier heat conduction equation is the novelty and originality of this study.

2. Physical modeling

Figure 1 indicates a simplified schematic of the problem. A film with the thickness of L and with the initial temperature of T_0 is heated by laser heat flux from the left side, whereas it is insulated from the right hand side.

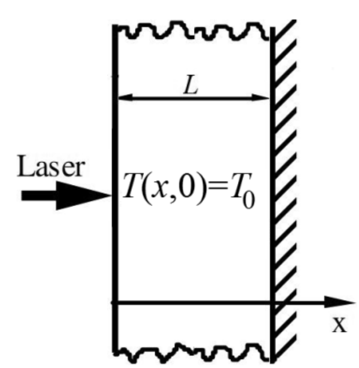

Fig. 1. Schematic of the problem

The energy equation assuming the presence of a heat source is as follows (Bergman & Incropera, 2011):

$$\rho c_p \frac{\partial T(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} - g = 0 \qquad (1)$$

Where ρ is the density of the body, c_p is the specific heat of the body, $T(x,t)$ is the temperature function and $q(x,t)$ is the function of heat flux, g is an internal heat source, and x and t are spatial and temporal variables, respectively. The constitutive equation governing the problem with respect to the heat wave model (Catteneo, 1958; Vernotte, 1961) can be written as follows:

$$\tau \frac{\partial q(x,t)}{\partial t} + q(x,t) + k\,\frac{\partial T(x,t)}{\partial x} = 0 \qquad (2)$$

Where τ is the thermal relaxation time and k is the thermal conductivity. A variation in thermal conductivity is considered as a linear function of temperature, as follows (Malekzadeh & Rahideh, 2007):

$$k = k_0\left[1 + \lambda(T(x,t) - T_0)\right] \qquad (3)$$

Where k_0 is the reference thermal conductivity and λ is the temperature coefficient of thermal conductivity. The Laser heat flux can be considered as a heat source (Blackwell, 1990; Zubair & Chaudhry, 1996). This heat source can be modelled as the following equation:

$$g(x,t) = I(t)(1-R)\mu \exp(-\mu x) \qquad (4)$$

The initial and boundary conditions are as follows:

$$T(x,0) = T_0, \quad q(x,0) = 0, \quad q(0,t) = 0, \quad q(L,t) = 0. \qquad (5)$$

Where $I(t)$ is the laser intensity, R is the surface reflectance and μ is the absorption coefficient. For making the obtained equations dimensionless, the following parameters are introduced:

$$t = \frac{c_0^2 t}{2\alpha_0}, \qquad x = \frac{c_0 x}{2\alpha_0}, \qquad \psi_0 = \frac{I_0\alpha_0(1-R)\mu}{\rho c_p c_0^2 T_0},$$

$$\beta = \frac{2\alpha_0\mu}{c_0}, \qquad \omega = \frac{\omega}{\omega_0}, \quad T(x,t) = \frac{T(x,t) - T_0}{T_0},$$

$$q(x,t) = \frac{\alpha_0 q}{T_0 k_0 c_0}, \quad \gamma = \lambda T_0, \quad \tau = \frac{\alpha_0}{c_0^2}, \quad \alpha_0 = \frac{k_0}{\rho c_p}. \qquad (6)$$

Thus the dimensionless form of Equations (1) and (2) are as follows:

$$\frac{\partial T(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} - 2\psi_0\phi(t)\exp(-\beta x) = 0 \qquad (7)$$

$$\frac{\partial q(x,t)}{\partial t} + 2q(x,t) + [1+\gamma T(x,t)]\frac{\partial T(x,t)}{\partial x} = 0 \qquad (8)$$

The initial and boundary conditions in the dimensionless form are also as follows:

$$T(x,0) = 0, \quad q(x,0) = 0, \quad q(0,t) = 0, \quad q(0.5,t) = 0. \qquad (9)$$

3. Basic idea of VIM

In this section, the basic concepts of VIM are presented. Consider the following nonlinear differential equation in its general form:

$$Lu(x,t) + Nu(x,t) = g(x,\mathrm{t}) \qquad (10)$$

Where L is a linear operator, N is a nonlinear operator and $g(x,t)$ is the non-homogeneous term, which in turn is an analytic continuous function. According to VIM, the correction functional can be defined as follows:

$$u_{n+1}(x,t) = \mathrm{u}_n(x,t) + \int_0^t \lambda(Lu_n(\mathrm{x},\tau) + Nu_n(\mathrm{x},\tau) - g(\mathrm{x},\tau))d\tau \qquad (11)$$

Where, λ is Lagrange multiplier that can be determined using various ways. The subscript of n represents the n^th^ approximate of the restricted variation, where $\delta u_n = 0$. One is free to choose the initial guess (u_0), but it can be obtained by solving the following equation:

$$Lu_0(x,t) = 0 \qquad (12)$$

Finally, the converged solution that is the exact solution of the problem can be obtained by the successive limit of approximations:

$$u_{n+1}(x,t) = u_n(x,t) \quad \text{or} \quad u(x,t) = \lim_{n\to\infty} u_n(x,t) \qquad (13)$$

4. VIM application

In this section, Equations (7) and (8) are solved based on VIM. To continue the solution process, heat source term in Equation (6) should be evaluated. One assumption for this term is taken from Torii & Yang (Torii & Yang, 2005), i.e. $\psi_0 = 1$ and $\phi(\mathrm{t}) = (1 + \sin(\omega t))$. To solve Equation (7), the correctional function is written as follows:

$$T_{n+1}(x,t) = T_n(x,t) + \int_0^t \lambda_T\left[\frac{\partial T_n(\mathrm{x},\tau)}{\partial \tau} + \frac{\partial q_n(\mathrm{x},\tau)}{\partial x} - [1+\sin(\omega t)]\exp(-\beta x)\right]d\tau \qquad (14)$$

The correctional function of the Equation 8 is of the following form:

$$q_{n+1}(x,t) = q_n(x,t) + \int_0^t \lambda_q\left[\frac{\partial q_n(x,\tau)}{\partial \tau} + 2q_n(x,\tau) + [1+\gamma T_n(x,\tau)]\frac{\partial T_n(x,\tau)}{\partial x}\right]d\tau \qquad (15)$$

To obtain the Lagrange multipliers appearing in the above equations, two methods are used. One method is the classic one, which is based on the calculus of variations and the other is a new method recently developed by Samaee *et al*. (2015).

4.1. Determining Lagrange multipliers by the classic method

The basis of the method is determination of the Lagrange multipliers by setting the variation of the desired function equal to zero. To determine λ in Equation 14 and λ_q in Equation 15, we should take variation from both sides of the equation and then set it equal to zero, i.e.:

$$\delta T_{n+1}(x,t)=\delta T_n(x,t) \\ +\delta\int_0^t \lambda_T\left[\frac{\partial T_n(x,\tau)}{\partial\tau}+\frac{\partial q_n(x,\tau)}{\partial x}-[1+\sin(\omega\ \ )]\exp(-\beta\ \ )\right]\tau \quad (16)$$

$$\delta q_{n+1}(x,t)=\delta q_n(x,t) \\ +\delta\int_0^t \lambda_q\left[\frac{\partial q_n(x,\tau)}{\partial\tau}+2q_n(x,\tau)+[1+\gamma T_n(x,\tau)]\frac{\partial T_n(x,\tau)}{\partial x}\right]d\tau \quad (17)$$

This procedure gives the following results for Lagrange multipliers:

$$\lambda_T=-1, \qquad \lambda_q=-e^{2(\tau-t)}. \quad (18)$$

4.2 Determining Lagrange multipliers by the new method

To determine Lagrange multipliers by the new method introduced by Samaee *et al*. (2015), we do as follows: we solve the linear part of equations by the Laplace transform method and set it equal to $(-1)^n$, where n is the highest order of the available derivative in the linear part of the equation. Boundary conditions appeared in the solution process should be set equal to zero. Finally, everywhere the variable t has appeared, it should have been replaced by $(-1)^n(\tau-t)$. By doing all the aforementioned steps, the Lagrange multipliers for the problem can be obtained as:

$$\lambda_T=-1+\frac{\{1+\omega(t-\tau)-\cos[\omega(t-\tau)]\}e^{-\beta x}}{\omega}, \ \lambda_q=-e^{2(\tau-t)}. \quad (19)$$

The only step remaining is determination of the initial functions. For this purpose, Equation 12 is used with the following initial conditions:

$$T_0(x,t)=0, \quad q_0(x,t)=0. \quad (20)$$

Therefore, VIM formula to determine the desired functions is obtained:

$$T_{n+1}(x,t)=T_n(x,t)+\int_0^t \lambda_T\left[\frac{\partial T_n(x,\tau)}{\partial\tau}+\frac{\partial q_n(x,\tau)}{\partial x}-[1+\sin(\omega t)]\exp(-\beta x)\right]d\tau \quad (21)$$

$$q_{n+1}(x,t)=q_n(x,t)+\int_0^t \lambda_q\left[\frac{\partial q_n(x,\tau)}{\partial\tau}+2q_n(x,\tau)+[1+\gamma T_n(x,\tau)]\frac{\partial T_n(x,\tau)}{\partial x}\right]d\tau \quad (22)$$

5. Results and discussion

To verify the accuracy of the solution obtained here, we have compared our results with the results of Lewandowska (2001) as an analytical study as well as by the results of Torii & Yang (2005) as a numerical study, as it is shown in Figure 2.

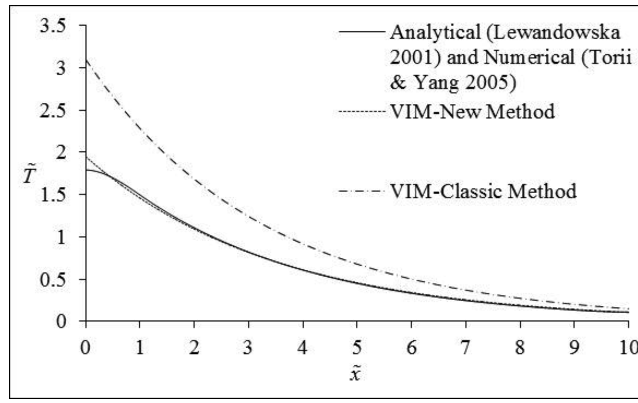

Fig. 2. The comparison between exact solution and VIM solutions for $\varphi(\tilde{t})=1$, $\beta=0.3$, $\gamma=0$ and $\tilde{t}=1$.

As it is clear from the figure, there are significant differences between the exact solution diagram and the diagram of the classic method of determination of the Lagrange multipliers. Especially, at the left boundary, i.e. $\tilde{x}=0$, there is an error of about 73 percent between them. However, with proceeding into the body, the error decreases and at the end reaches to 34.8%. The average error is also 51.18% that in the analytical studies is not acceptable. On the other hand, the diagram of the new method of determination of the Lagrange multiplier indicates much greater accuracy. The average error in this method is 2.57% that is much more acceptable than the classic one. However, at the left boundary the error is 8.9% and at the ending boundary it is about 4%. The reason for the large errors in boundaries, in fact, is the lack of imposing boundary conditions in the problem-solving process. We considered the linear operator in the dimensionless time direction and thus we lost the effect of boundary conditions in reaching the solution. This leads to the discrepancies at the boundaries to be more than the average error. Since the non-Fourier heat transfer problems are intensely dependent on time, the lack of applying boundary conditions would not cause significant error in solution of the problem.

An analytical researcher may believe that an error of 2.57 percent is even high for an analytical solution, but it should be noted that advantages of semi-analytical methods such as VIM compared to the conventional numerical methods make the use of these tools highly desirable for experimental scientists. Knowing the approximate solution of the problem helps these researchers to choose their equipment properly and reduce the cost and the number of their experiments.

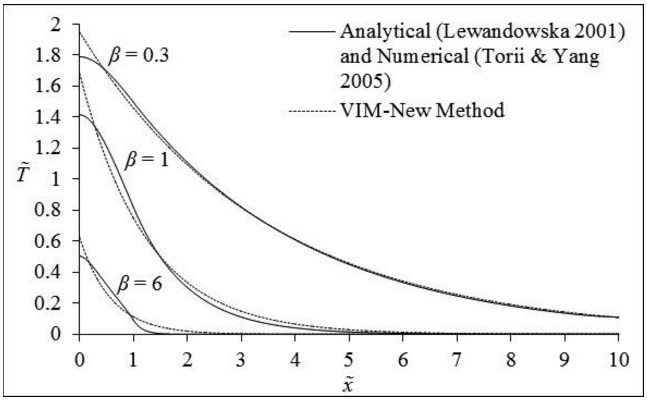

Fig. 3. Temperature distribution for different values of β. ($\varphi(\tilde{t})$=1, γ=0 and $\tilde{t}$=1)

Figure 3 shows the effect of variation of dimensionless absorption coefficient (β) on the temperature profile within the body. As it can be seen, with increasing β at a specific point within the body, the values of the dimensionless temperature decrease. This is natural, because according to Equation (7), with increasing β, the energy of the laser dissipates faster resulting in smaller (or negligible) increase in the film's temperature. On the other hand, with increasing $\tilde{x}$ for a constant value of β, the same behavior can be observed as it is also being expected from Equation (7).

In this figure, the comparison between the exact and VIM solutions are also shown. The accuracy of VIM in this figure is also considerable. As it can be seen, with increasing β, the error somewhat increases. The error values at the initial boundary and in the range of $0 \leq \tilde{x} \leq 2$ are shown in Table 1.

Table 1. Error values in different locations for different values of β (%)

Average error at $0 \leq \tilde{x} \leq 2$	Error at $\tilde{x}$=0	β
2.31	8.90	0.3
6.40	18.93	1
12.40	24.07	6

In Figure 4, the effect of dimensionless time variation on the temperature profiles at different locations on the body are shown. Over time, the thermal energy of the laser increases the temperature. Since with increase in the dimensionless location, the internal energy of the body decreases exponentially, the distant locations within the body would be affected negligibly over time by the laser heating. In Table 2, the error values for the VIM calculation and the exact solution are given.

Table 2. Error values in different locations for different values of $\tilde{t}$ (%)

Average error at $0 \leq \tilde{x} \leq 4$	Error at $\tilde{x}$=0	$\tilde{t}$
20.51	18.93	1
20.07	21.27	2
9.53	20.83	6

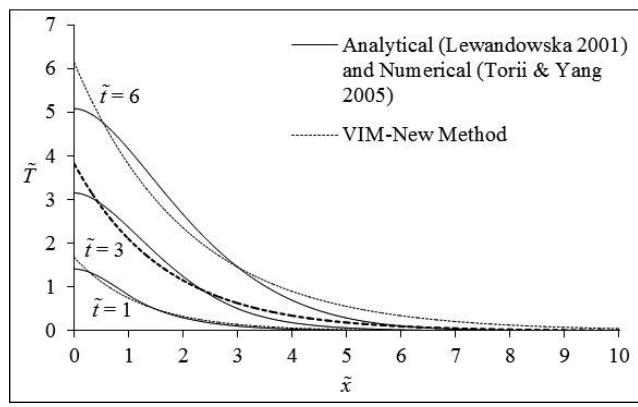

Fig. 4. The effect of different values of $\tilde{t}$ on temperature distribution for $\varphi(\tilde{t})$=1, γ=0 and β=1.

In Figure 5, the effect of variations of γ on the temperature profiles is investigated. At the beginning points of the body, increasing γ causes an increase in the body's temperature. Going deeper into the film, the difference between the curves decreases so that at one point, where $\tilde{x} \approx 0.23$, diagrams intersect with each other and then from this point onwards the increase in γ leads to a decrease in body's temperature. This is due to the effect of γ on the temperature's gradient. In fact, the greater the value of γ, the greater the temperature's gradient would be and this increase in the temperature's gradient leads to further decrease in temperature. Figure 5 also shows the important effect of applying the change in the thermal conductivity with temperature. It fact, in sensitive applications where small changes in temperature are important, nonlinear analysis of problem shows its significance well.

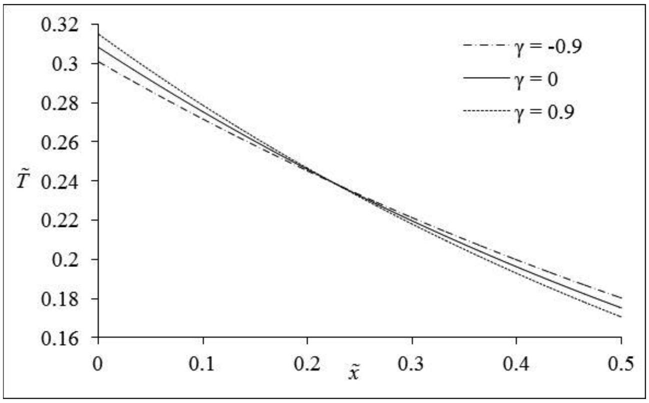

Fig. 5. Temperature distribution for different values of γ. (β=10, $\tilde{\omega}$ =10 and $\tilde{t}$=1)

At the end, it is worthy to have a comparison between the Fourier and the non-Fourier solutions. In Figure 6, this comparison is shown for two values of γ. The figure provides interesting results. Significant differences between the Fourier and non-Fourier solutions can be seen so that the slope of variations for the non-Fourier solution is more than the Fourier one. At the beginning points of the body, the non-Fourier solution has larger values and then as it has steeper slope than the Fourier one, finally at a point, where $\tilde{x} \approx 0.24$, these two solutions equal each other and then the Fourier solution has larger values than the non-Fourier one. This result is predictable, because in the non-Fourier solution, the internal resistance of the material is taken into consideration against the heat propagation.

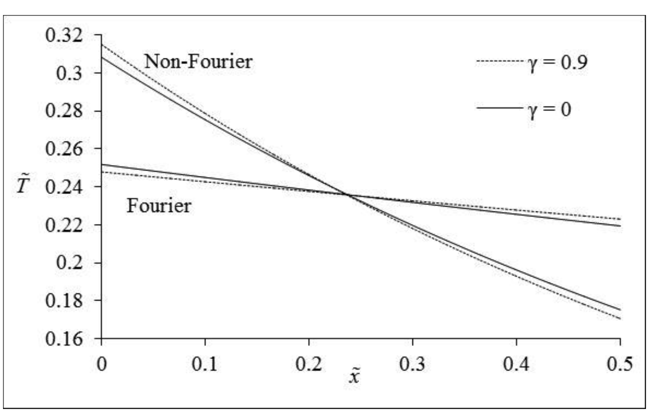

Fig. 6. The comparison between Fourier and non-Fourier solutions for different values of γ. (β=10, $\tilde{\omega}$=10 and $\tilde{t}$=1)

Another notable point in Figure 6 is different reactions of the Fourier and non-Fourier solutions against variations of the thermal conductivity with the change in temperature. In the non-Fourier solution, as it was already mentioned, the larger γ leads to a higher temperature gradient within the body and at the beginning parts of the body, the solution related to them are of higher temperature. While, in the Fourier solution, it is inverse so that the larger γ leads to lower temperature gradient within the body and at the beginning points of the body its corresponding temperature is less than the γ=0 case.

6. Conclusion

In this paper, the laser heating on a metal film was studied. For this purpose, the thermal wave model of Cattaneo-Vernotte was used. The heat conductivity was assumed to be temperature-dependent and based on this assumption the nonlinear equation was obtained. To solve the equations VIM was used and Lagrange multipliers were calculated in two ways. Using the obtained Lagrange multipliers, the problem was solved and the summary of results is as follows:

1. The accuracy of new method of determining Lagrange multipliers is considerably larger than classic one.
2. Although, the new method of determination of the Lagrange multipliers has somewhat smaller error, it can be a good and efficient choice for the researchers, who are seeking to determine a relatively good solution of the problem as quick as possible.
3. The fact that the variation of conductivity with temperature created a significant difference in the temperature profiles makes the nonlinear analysis of the problem important.
4. In the problem of laser heating, there is significant difference between the Fourier and the non-Fourier solutions that must be taken into consideration in the analysis of similar problems.

Nomenclature

c_0	reference speed of thermal wave (m s^{-1})
c_p	specific heat (J kg^{-1} K^{-1})
g	heat source (W m^{-3})
I	laser intensity (W m^{-2})
k	thermal conductivity (W m^{-1} K^{-1})
k_0	reference thermal conductivity (W m^{-1} K^{-1})
L	characteristic length (m) or linear operator
N	non-linear operator
q	heat flux (W m^{-2})
$\tilde{q}$	dimensionless heat flux
R	surface reflectance
T	temperature (K)
T_0	reference temperature (K)
$\tilde{T}$	dimensionless temperature
t	time (s)
$\tilde{t}$	dimensionless time
x	space direction (m)
$\tilde{x}$	dimensionless space direction

Greek symbols

α_0	reference thermal diffusivity (m^2 s)
β	dimensionless absorption coefficient
	dimensionless coefficient for taking into account of temperature-dependent conductivity
	lagrange multipliers or the slope of conductivity function (1/K)
μ	absorption coefficient
ρ	density (kg m^{-3})
	relaxation time (s)
	dimensionless rate of energy absorbed by object
ψ_0	constant coefficient for dimensionless heat source
ω	frequency of heat source (1/s)

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طريقة تقريبية تحليلية جديدة لحل مسائل لتوصيل الحرارة لا تتبع قانون فورييه (non-Fourier)

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خلاصة

في هذا البحث، تم فحص تأثير الليزر كمصدر للحرارة على شريحة رقيقة. تم استخدام نموذج توصيل الحرارة لا يتبع قانون فورييه من كاتانيو فيرنوت (Cattaneo-Vernotte) للتحليل الحراري للمشكلة. تم افتراض التوصيل الحراري اعتماداً على درجة الحرارة، مما أدى إلى الحصول على معادلة غير خطية وبافتراض دور الليزر كمصدر للحرارة، تم الحصول على معادلة غير متجانسة. تم حل المعادلات التي تم الحصول عليها باستخدام طريقة تقريبية تحليلية لأسلوب التكرار التغييري (VIM). توصلنا إلى أن التحليل اللا خطي مهم في مشاكل توصيل الحرارة التي لا تتبع قانون فورييه. وقد لوحظ وجود فروق ذات دلالة إحصائية بين الحلول باستخدام قانون فورييه والحلول بدون قانون فورييه والتي تؤكد على أهمية إيجاد حلول بدون قانون فورييه في المشاكل المماثلة.