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An optimal representation to Random Maximum *k* Satisfiability on the Hopfield Neural Network for High order logic($k \leq 3$)

Hamza Abubakar* [1,2]
[1]*School of Mathematical Sciences, Universiti Sains Malaysia*
[2]*Dept. of Mathematics, Isa Kaita College of Education, Katsina, Nigeria*
**Corresponding author: zeeham4u2c@yahoo.com*

Abstract

This paper proposes a new logical rule by incorporating Random Maximum *k* Satsifiability in the Hopfield neural network as a single model. The purpose is to combine the optimization capacity of the Hopfield neural network with non-systematic behaviour of the Random maximum *k* Satisfiability for classification problem. The energy function of a Hopfield neural network has been considered as a programming language for dynamics minimization mechanism. Several optimization and search problems associated with machine learning (ML), decision science (DS) and artificial intelligence (AI) have been expressed on the Hopfield neural network(HNN) optimally by modelling the problem into variables to minimize the objective function corresponding to Lyapunov energy function of the model. The computer simulation has been developed based on RANMAX*k*SAT logical rule in exploring the feasibility of the Hopfield neural network as a neuro-symbolic integration model for optimal classification problems. The perfromanmce of the proposed hybrid model has been compared with the existing models published in the literature in term of Global minimum ratio (zM), Fitness energy landscapes (FEL), Root Means square error (RMSE), Mean absolute errors and computation time (CPU). Hence, based on the experimental simulation results, it revealed that the RANMAX*k*SAT can optimally and effectively be represented in the Hopfield neural network (HNN) with 85.1 % classification accuracy.

Keywords: Hopfield Neural Network; Logic Mining; Random Maximum Satisfiability;Reverse analysis

1. Introduction

Artificial neural networks (ANNs) are a types of artificial intelligence (AI) and machine learning (ML) algorithms that arose from developments in cognitive and computational science research originated from the brain modelling structure. It is a computational mathematical model which aims to emulate from the computational and functionalities of the biological neural network(Anderson 2014). ANN has been clas-sified as intelligence approach to machine learning designed to address nonlinear statistical modelling problems and provide the most widely used tools for creating a predictive model for dichotomous find-ings in various field including disease detection or classification problem which is a modern alternative to logistic regression (Salgueiro*et al.* 2013). In Artificial Intelligence (AI), computational mathematics and optimization, where the target is to find the best representation of set of variables to satisfy a set of constraints(Jain and Kumar 2018). ANN offer a variety of advantages, such as the capacity to track complex non-linear pattern, the capacity to diagnose all the possible correlations among the predictor variables, and the availability of various training algorithms. There are varieties of real-world applica-tion found in ANN including Fuzzy logic modelling (Rushdi *et al.* 2018), data science and data mining (Idrees *et al.* 2020), forecasting problem(Amiri *et al.* 2018), prediction problem (Majid *et al.* 2021), identification and pattern recognition (Gao *et al.* 2018), classification (Makinde 2019), Electrohydrodynamic (EHD) flow(Sabir 2018), diagonosis problem (Almulla 2021), representation (Abubakar *et al.* 2020a), recognition problem(Al-Hmouz 2020) which are hard to solve using a traditional methods.

This study focused on Hopfield Neural Network (HNN) which is an important type of Artificial neural network (ANN) that simulates human networks associative memory invented by John J. Hopfield in 1982 (Skansi 2018). The structure of HNN consists of a single layer with one or more recurrent or fully interconnected neural networks. The network is known for its use in auto-association and optimization problem(Demircigil *et al.* 2017). It has a broad range of artificial intelligence application such as machine learning, associative memory, pattern recognition, VLSI and parallel processing of optical equipment. Technically, the HNN modelling approach requires the handling of a dynamic system in which the energy function or the Lyapunov function will describe the behaviour of the network and the problems to be addressed as minimization problem(Bharitkar *et al.* 2000).

The Boolean satisfiability problem in logic and computational science is called propositional satisfiability problems usually written in conjunctive normal form (CNF) which can be abbreviated as SATISFIABILITY, SAT, B-SAT or kSAT is the problem of determining if there exists an assignment that can satisfies a given Boolean formula. Boolean satisfiability, B-SAT or SAT is one of the most fascinating artificial intelligence tools in mathematical abstractions useful for reasoning and planning purposes. The classical propositional logic is one of the main approaches in solving planning problem in artificial intelligence and machine learning(Rintanen 2012). The Maximum$k$ Satisfiability problem is the optimization variant of kSAT, which consists of seeking an assignment that maximizes the number of clauses satisfied (Abubakar *et al.* 2020a). Maximum Random$k$ Satisfiability (RANMAXkSAT) is the optimization variant of Maximum$k$ Satisfiability problem, which consists of seeking an assignment that maximizes the random number of clauses satisfied(Abubakar *et al.* 2020b).

The major breakthrough in the machine learning and artificial intelligence community is the Neuro-symbolic integration which combines an artificial neural network with symbolic logic as a single model that can perform any type of symbolic operations. Neural-symbolic computation tries to generate two basic cognitive skills that are the ability to learn from the previous experience, and the ability to think from what has been learned(Donadello *et al.* 2017). For several years the hybridization of learning and reasoning through the neural-symbolic computation has been an area of research in the fields of machine learning (ML) and artificial intelligence (AI) communities. This is because of its capacity to analyze and evaluate complex non-linear patterns, including statistical, mathematical or engineering models(Khan *et al.* 2016; Asadi *et al.* 2014). The objective of Neural-symbolic computation approach is to reconcile artificial intelligence under a principled basis with the prevailing symbolic and connectionist paradigm such as classification and pattern recognition problem. In neural-symbolic computation approach, knowledge is conveyed in symbolic form, while a neural network computes learning and reasoning(Townsend *et al.* 2019). Therefore, the fundamental features of neural-symbolic computation make it easier to combine robust learning and effective inference in neural networks with the interpretability and general ability of symbolic knowledge extraction and logical system rationale.

The idea of hybridizing a logic program on the Hopfield neural network as a single network was first proposed by(Abdullah 1992). The purpose was to minimize the logical inconsistency of Hopfield neural network after the connection strengths have been defined from the logic program. The network converged to neural states corresponding to the optimal representation. The preserved neuron structures were measured using the HNN energy function known as Lyapunov function. Various studies have been conducted by incorporating to Wan Abdullah work to accommodate different variants of symbolic logic and Satisfiability problem. Horn logic has been integrated into the Hopfield neural network in (Sathasivam 2010).The effective method of relaxation to produce the optimum final neuron states was introduced into HNN by(Sarasivam 2010). The First-order logic learning was successfully incorporated into the artificial neural networks in(Guillame-Bert *et al.* 2010).The stochastic resonance approach for logic programming in the Hopfield neural network was proposed(Duan *et al.* 2020) . This stochastic approach reduced the neuronal oscillations of the Hopfield Neural Network during the recovery phase. Some studies have been conducted on the feasibility of Radial basis function neural to be incorporated with symbolic logic as a single network model including the work in (Hamadneh *et al.* 2012;Alzaeemi *et al.* 2020). The Maximum random *k* Satisfiability problem (MAXRANkSAT) is a vital generalization of Satisfiability problem. An algorithm for the Maximum Satisfiability problem has been proposed in (Hansen and Jaumard, 1990). Approximation method to maximum satisfiability has been proposed (Yannakakis, 1992). Maximum Satisfiability Problem has been integrated with the data mining and constrained clustering(Berg and Järvisalo 2017). Exact clustering via integer programming and maximum satisfiability was presented in(Miyauchi *et al.* 2018).The development of random satisfiability logic programming in the Hopfield network (HNN-RAN*k*SAT) has been proposed in (Abubakar *et al.* 2020b) . In a similar study conducted by (Sathasivam *et al.* 2020), Maximum Random kSatisfiability has been embedded in the Hopfield neural network. However, the current study upgraded the work of(Abubakar *et al.* 2020a) to include high order logic$k \leq 3$.In this paper, we will utilize the optimization capacity of the Hopfield neural network in finding the optimal representation of the RANMAX*k*SAT logical representation. The contributions of our work include:

1. To propose a new logical rule RANMAX*k*SAT by upgrading to $k \leq 3$(Abubakar *et al.* 2020a).
2. To implement the new logical rule in the Hopfield neural network (HNN-RANMAX*k*SAT).
3. To explore the performance of the proposed model based classification of medical data set for real-life application.

The main focused of this paper is to explore the feasibility of Hopfield neural network optimization capacity on the proposed RANMAX*k*SAT logic rule. The remaining parts of this paper is organized as follows: Section 2, is the Research methods which include; the proposed logical rule Random Maximum*k*satisfiability and the mapping of the proposed logical rule RANMAX*k*SAT in HNN as a single model based on Wan Abdullahi method.Section 3 presented the performance evaluation metrics. In section 4, the results and discussions have been presented and this papar concluded in section 5.

2. Materials and Methods

2.1 The Proposed Random Maximum *k* Satisfiability Logic

The satisfiability problem is considered as one of the most well-studied problems in Mathematical logic and computational theory due to its practical applications in combinatorial optimization. RANMAX*k*SAT belong to the families of non-systematic Boolean formula that consists of a maximum number of random literals per clause to be negated with the probability of ½. According to(Yolcu and Póczos, 2019). The general formulation for RANMAX*k*SAT will be restricted to $k \leq 3$ as follows.

$$F_{MAXRANkSAT} = \bigwedge_{i=0}^{n} F_{MAXkSAT} \bigwedge_{i=0}^{m} F_{RANkSAT} \tag{1}$$

where $F_{RANkSAT}$ and $F_{MAXkSAT}$ described in Equation 2) and 3) respectively as follows;

$$F_{\ RANkSAT} = \bigwedge_{i=0}^{w} C_i^{(3)} \bigwedge_{i=0}^{n} C_i^{(2)} \bigwedge_{i=0}^{m} C_i^{(1)} \tag{2}$$

$$F_{MAXkSAT} = \bigwedge_{i=0}^{r} \psi_i^{(3)} \bigwedge_{i=0}^{n} \lambda_i^{(3)} \bigwedge_{i=0}^{m} \varphi_i^{(3)} \tag{3}$$

where $\forall\ r, m,\ n\ \in N, r\ >\ 0, n\ >\ 0$ and $m\ >\ 0$. The clauses in $F_{RANkSAT}$ and $F_{MAXkSAT}$ defined as follows;

$$C_i^{(k)} = \begin{cases} (T_i \vee I_i \vee L_i)\ , & if\ k=3 \\ (T_i \vee I_i) \quad , & if\ \ k=2 \\ L_i \quad , & if\ \ k=1 \end{cases} \tag{4}$$

$$T_i^{(3)} = \begin{cases} (T_1 \vee T_2 \vee T_3)\ \wedge (\neg T_1 \vee T_2 \vee T_3) \wedge (T_1 \vee \neg T_2 \vee T_3)\ \wedge \\ (T_1 \vee T_2 \vee \neg T_3) \wedge (\neg T_1 \vee \neg T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee \neg T_3) \\ \wedge (T_1 \vee \neg T_2 \vee \neg T_3)\ \wedge (\neg T_1 \vee T_2 \vee \neg T_3) \wedge (\neg T_1 \vee \neg T_2 \vee \neg T_3) \end{cases} \tag{5}$$

$$I_i^{(2)} = (I_1 \vee I_2) \wedge \ (\neg I_1 \vee I_2) \ \wedge (I_1 \vee \neg I_2) \ \wedge (\neg I_1 \vee \neg I_2) \tag{6}$$

$$L_i^{(2)} \ = \ (L_1 \vee L_2) \tag{7}$$

where $T_i \in [T_i, \neg T_i]$, $L_i \in [L_i, \neg L_i]$, $I_i \in [I_i, \neg I_i]$ are representing the literals and their negation in RANMAXkSAT logical clauses respectively. Specifically, $C_i^{(1)}$denoted the first-order logic, $C_i^{(2)}$ denoted the second-order and third-order logical clause is denoted by $C_i^{(3)}$. $A_i^{(3)}$ and$B_i^{(2)}$designated as the second-order clause in $F_{MAXkSAT}$. In this work, F_α used to represent a Boolean formula in CNF where logical clauses are chosen uniformly, independently and without any replacement from $2^\alpha \begin{pmatrix} m+n+r \\ \kappa \end{pmatrix}$ non-trivial clause of length α. I_i exists in the $C_i^{(k)}$, if the $C_i^{(k)}$ contains eitherI_i or its negation $(\neg I_i)$ and the mapping of $g\left(F_\alpha\right) \to [-1, \ \ 1]$ defined as a logical interpretation of Boolean formula. Any Boolean formula for the satisfiability representation can be expressed as 1 for TRUE or -1 for otherwise. Theoretically from Equation (1), $F_{MAXRANkSAT}$ for $k \leq 3$ can be mathematically presented as follows;

$$F_{RANMAX-3SAT} = \begin{cases} \overbrace{\begin{matrix}(T_1 \vee T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee T_3) \wedge (T_1 \vee \neg T_2 \vee T_3) \wedge \\ (T_1 \vee T_2 \vee \neg T_3) \wedge (\neg T_1 \vee \neg T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee \neg T_3) \\ \wedge (T_1 \vee \neg T_2 \vee \neg T_3) \wedge (\neg T_1 \vee \neg T_2 \vee \neg T_3) \wedge (A_1 \vee A_2 \vee A_3) \wedge \end{matrix}}^{MAX-3SAT} \\ \overbrace{(I_1 \vee I_2 \vee \neg I_3) \wedge (L_1 \vee \neg L_2) \wedge \neg J}^{RANDAM-3SAT} \end{cases} \tag{8}$$

According to Equation 8, $F_{MAXRANkSAT}$ comprises of Equataion (9) to (12) as follows

$$C_1^{(3)} = (I_1 \vee I_2 \vee \neg I_3) \tag{9}$$

$$C_2^{(2)} = (L_1 \vee \neg L_2) \tag{10}$$

$$C_1^{(1)} = \neg J_1 \tag{11}$$

$$\begin{aligned} \lambda_i^{(3)} \ = \ & (T_1 \vee T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee T_3) \ \wedge (T_1 \vee \neg T_2 \vee T_3) \wedge \\ & (T_1 \vee T_2 \vee \neg T_3) \ \ \wedge (\neg T_1 \vee \neg T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee \neg T_3) \\ & \wedge (T_1 \vee \neg T_2 \vee \neg T_3) \ \wedge (\neg T_1 \vee \neg T_2 \vee \neg T_3) \ \ \wedge (A_1 \vee A_2 \vee A_3) \end{aligned} \tag{12}$$

Therefore, the result of Equation 8 is reduced to $F_{RANMAXkSAT} = -1$ (not satisfiable) if Equation (13) holds as follows.

$$(T_1, T_2, T_3, A_1, A_2, A_3, I_1, I_2 I_3, J_1) \ = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1) \tag{13}$$

where $(1, -1, 1, 1, 1, 1, -1, 1, 1, 1)$ are one of the representations that will render $F_{RANMAXkSAT} = -1$ (unsatisfiable). Hence, Equation (8) is considered as one of the constrained optimizations and search problems that can be found in maximization problem. It is observed by(Achlioptas *et al.* 2003; Abubakar *et al.* 2021)that RANMAXkSAT is not fully satisfiable, it is therefore considered as a constrained optimization that can be carried out on the Hopfield neural network model for optimal representation.

2.2 Hopfield neural network model structure

The architecture of HNN model consists of interconnected neurons and a powerful feature of content addressable memory that are crucial in solving various optimization and combinatorial tasks. The system consists of structured N neurons, each of which is represented by an *Ising* variable. The neurons in discrete HNN are utilized in bipolar representation whereby $S_i \in \{1, -1\}$, which strictly considers values of 1 and -1 (Sathasivam, 2010). The fundamental overview for neuron state activation in HNN is shown in Equation 14.

$$S_i = \begin{cases} 1 \quad , & if \ \sum_j T_{ij} S_j > \omega \\ -1 \ , & Otherwise \end{cases} \tag{14}$$

where T_{ij} is the synaptic weight from unit j to i. S_j is the state of neuron j and ω is the predefined threshold value. The connection in Hopfield neural net contains no connections with itself as follows

$$T^{(3)}_{ijk} = T^{(3)}_{kij} = T^{(3)}_{kji} \tag{15}$$

$$T^{(2)}_{ji} = T^{(2)}_{ij} \tag{16}$$

$$T^{(1)}_{i} = T^{(1)}_{j} \tag{17}$$

$$T_{jj} = T_{ii} = 0 \tag{18}$$

In resulting, HNN holds symmetrical features in terms of architecture. HNN model has similar intricate details to the *Ising* model of magnetism (Sathasivam, 2010). As the neuron state is termed in bipolar $S_i \in \{1, -1\}$ representation, the spin points follow in the direction of a magnetic field. This causes each neuron to flip until the equilibrium is reached. Thus, it follows the dynamics $S_i \rightarrow sgn\left[h_i(t)\right]$ where h_i is the local field of the connection of the neurons. The sum of the field induced by each neuron is given as follows.

$$h_i = \sum_{k}^{N} \sum_{j}^{N} T_{ijk} S_j S_k + \sum_{j}^{N} T_{ij} S_j + T_i \tag{19}$$

The task of the local field is to evaluate the final state of neurons and generate all the possible RANSAT induced logic that was obtained from the final state of neurons. One of the most prominent features of the HNN network is the fact that it always converges to stable states (Hopfield, 2007) and (Sathasivam, 2010). The Lyapunov energy function (LEF) utilized in HNN for RANkSAT logic programming is presented as follows,

$$S_i\left(t+1\right) = \begin{cases} 1 \ , & h_i = \sum_k^N \sum_j^N W_{ijk} S_j S_k + \sum_j^N W_{ij} S_j + W_i \geq 0 \\ -1 \ , & h_i = \sum_k^N \sum_j^N W_{ijk} S_j S_k + \sum_j^N W_{ij} S_j + W_i < 0 \end{cases} \tag{20}$$

The energy function of the HNN model is especially critical, it will decide the interoperability of the network. The value obtained from the equation will be verified as global or otherwise. The network would generate the right response when the induced neurons state reached global minimum energy. There are minimal works to integrate HNN with RANkSAT as a single computational network.

2.3 Mapping of RANMAX-*k*SAT in the Hopfield neural networks

The computational architectures of HNN have the synaptic connection between HNN network learns patterns thneurons that are symmetrically represented as $W^{(3)}_{ijk} = W^{(3)}_{kij} = W^{(3)}_{kji}$, $W^{(2)}_{ji} = W^{(2)}_{ij}$ and $W^{(1)}_{i} = W^{(1)}_{j}$. The at are *N*-dimensional vectors from the space $S_i \in [-1, 1]$ conform to the dynamics $S_i \rightarrow sign(h_i)$. The dimensionality of the pattern space prensented into HNN is reflected in the number of nodes, such that the net will have *N* nodes $\mathrm{S}_i(t) = x_1, x_2 ... x_n$.

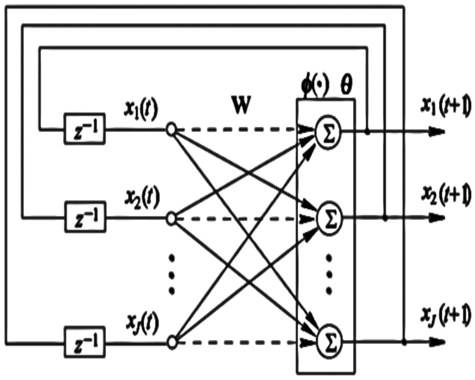

Fig. 1. Architecture of Hopfield neural network.

Figure 1 is the training architecture of Hopfield neural network algorithm. Where W_{ij} is the synaptic weight vector from starting from j neuron to i neuron. We defined S_j as the state of the neuron j in HNN and ς is the predefined value. The value of $\xi = 0$ has been specified in(Abubakar *et al.* 2020a; Abubakar *et al.* 2020b; Abubakar *et al.* 2020c; Sathasivam *et al.* 2020) to certify that the network's energy decreases to zero. The synaptic weight connection in the discrete HNN contains no connection with itself, zero self-connectivity i.e $W^{(3)}_{iii} = W^{(3)}_{jjj} = W^{(3)}_{kkk} = 0$ and $W_{jj} = W_{ii} = W_{kk} = 0$. The HNN model has similar intricate details to the *Ising* model of magnetism as described in(Neelakanta and De Groff 2018). The neurons status is the expression on in a bipolar form, the spin points implement the magnetic field trajectory. This will compel each neuron to flip until the equilibrium state is maintained as follows,

$$S_i \;\rightarrow\; sgn\left[h_i(t)\right] \tag{21}$$

where h_i is the local field that connects all neurons in HNN. The sum of the field is induced by each neuron state as follows,

$$h_i = \sum_{k}^{N} \sum_{j}^{N} W_{ijk} S_j S_k + \sum_{j}^{N} W_{ij} S_j + W_i \tag{22}$$

The task of the local field is to evaluate the final state of neurons and generate all the possible RANMAX3SAT induced logic that was obtained from the final state of neurons. One of the most prominent features of the HNN network is the fact that it always converges. The generalized fitness function $E_{F_{RANMAXkSAT}}$ that controls the combinations of neurons in HNN and $F_{RANMAXkSAT}$ is presented as follows.

$$E_{F_{RANMAXkSAT}} = \sum_{i=1}^{NN} \prod_{j=1}^{V} T_{ijk} \tag{23}$$

where V and NN are the number variables and the number of neurons generated in $F_{RANkSAT}$ respectively. We defined the inconsistency of $F_{RANMAXkSAT}$ representation as follows.

$$T_{ij} = \begin{cases} \frac{1}{2}\left(1 - S_\rho\right) \ , \ if \, \neg\rho \\ \frac{1}{2}\left(1 + S_\rho\right) \ , \ otherwise \end{cases} \tag{24}$$

The value $E_{F_{RANMAXkSAT}}$ is proportional to the value of "inconsistencies" of the logical clauses. The rule for updating the neural state is,

$$S_i\left(t+1\right) = \begin{cases} \;\;1 \ , & h_i \geq 0 \\ -1 \ , & h_i < 0 \end{cases} \tag{25}$$

The following equation represents the Lyapunov energy function of HNN model.

$$H=-\frac{1}{3}\sum_{i=1,}^{N}\sum_{j=1,}^{N}\sum_{k=1,}^{N}W_{ijk}S_iS_jS_k-\frac{1}{2}\sum_{i=1,}^{N}\sum_{j=1,}^{N}W_{ij}S_iS_j-\sum_{i=1}^{N}W_iS_i \tag{26}$$

Equation (26) has been applied to classify whether a solution is a global or local minimum energy. The HNN will generate the optimal assignment when the induced neurons state achieved global minimum energy. To best knowledge of the author, there is no work that utilized the optimization capacity of Lyapunov energy function of the Hopfield neural network(HNN) for optimal representation to RANMAXkSAT when the value of $k \leq 3$. Consequently, the quality of the final neuronal state can be maintained according to Equation (27) as utilized in(Abubakar *et al.* 2020a; Abubakar *et al.* 2020b; Abubakar *et al.* 2020c; Sathasivam *et al.*, 2020) as follows.

$$\left|H_{F_{RANMAXkSAT}}-H_{F_{RANMAXkSAT}}^{\min}\right|\leq\xi \tag{27}$$

where ξ is the pre-determined tolerance value.The value $\xi = 0.001$ was taken in(Abubakar *et al.* 2020a; Abubakar *et al.* 2020b; Abubakar *et al.* 2020c; Sathasivam *et al.* 2020).If the $F_{RANMAXkSAT}$ logical representation embedded in HNN does not satisfy criteria state in Equation (20), then the final state the neurons has been trapped in the wrong pattern. If the fitness requirement is not realized, the pattern will continue recursively, else the system will be stopped and presented. The fitness specification of HNN is accomplished if the network output coincides to a certain steady-state, which means that no further searching occurs in the fitness (objective) function.

2.4 Synaptics Weight Computation

The Random Maximum*k* Satsifiability (RANMAX*k*SAT) can be use as a logical rule in the Hopfield neural network for optimization problem. Depending on logical inconsistencies, Wan Abdullah methods (Wan Abdullah, 1992) has become one of the pioneer approach in synaptic weight extraction. Fitness function(objective function) that corresponds to RANMAX*k*SAT clauses is the minimization of logical inconsistencies.

$$\min_{i\in[1,\,-1],F_{MAXRANKSAT}=1}\neg F_{MAXRANkSAT} \tag{28}$$

As the number of "wrong" assignment decreases, the number of satisfied $F_{RANMAXSAT}$ clauses will increase. Wan Abdullah is one of the earliest methods of learning to derive synaptic weight based on logical inconsistencies(Sathasivam 2012). This can be achieved by storing atom truth values and generating a minimized cost function while satisfying random maximum logical clauses. Finding inconsistencies of Equation (8) can be represented in form of its negation. The cost function is defined as follows;

$$\begin{aligned}E_{F_{RANMAXkSAT}} &= \tfrac{1}{2}(1-W_{T_1})\tfrac{1}{2}(1-W_{T_2})\tfrac{1}{2}(1-W_{T_3})+\tfrac{1}{2}(1+W_{T_1})\tfrac{1}{2}(1-W_{T_2})\tfrac{1}{2}(1-W_{T_3})\\ &+\tfrac{1}{2}(1-W_{T_1})\tfrac{1}{2}(1+W_{T_2})\tfrac{1}{2}(1-W_{T_3})\ +\tfrac{1}{2}(1-W_{T_1})\tfrac{1}{2}(1-W_{T_2})\tfrac{1}{2}(1+W_{T_3})\\ &+\tfrac{1}{2}(1+W_{T_1})\tfrac{1}{2}(1+W_{T_2})\tfrac{1}{2}(1-W_{T_3})\ +\tfrac{1}{2}(1+W_{T_1})\tfrac{1}{2}(1-W_{T_2})\tfrac{1}{2}(1+W_{T_3})\\ &+\tfrac{1}{2}(1-W_{T_1})\tfrac{1}{2}(1+W_{T_2})\tfrac{1}{2}(1+W_{T_3})\ +\tfrac{1}{2}(1+W_{T_1})\tfrac{1}{2}(1+W_{T_2})\tfrac{1}{2}(1+W_{T_3})\\ &+\tfrac{1}{2}(1-W_{A_1})\tfrac{1}{2}(1-W_{A_2})\tfrac{1}{2}(1-W_{A_3})\ +\tfrac{1}{2}(1-W_{I_1})\tfrac{1}{2}(1-W_{I_2})\tfrac{1}{2}(1+W_{I_3})\\ &+\tfrac{1}{2}(1-W_{L_1})\tfrac{1}{2}(1+W_{L_2})+\tfrac{1}{2}(1+W_J)\end{aligned} \tag{29}$$

The acceptable synaptic weight of HNN-RANMAXkSAT can be achieved by equating the cost function $E_{F_{RANMAXkSAT}}$ in Equation (29) with energy function in Equation (19). In this experiment, the training process yields the optimum value of the cost function, which determines the system's accurate synaptic weights.The optimized global minimum energy estimation necessitates appropriate interpretations and synaptic weight adjustments. Since consistent interpretation cannot be found leading to

$E_{F_{RANMAX-3SAT}} = 0$ the concentration of the model will be decided to move by finding the corresponding value of $F_{RANMAX-3SAT}$. Applying the cost function (29) to energy function in (19), the respective synapses of $E_{F_{RANMAX-3SAT}}$ can be calculated and the result is presented in Table 1.

Table 1. HNN-RANMAXkSAT Synaptic Weights

Synaptic Weight	C_1	C_2	C_3	C_4	$F_{RANMAXkSAT}$
$W_{T_1}W_{T_2}W_{T_3}$	$\frac{1}{8}$	0	0	0	$\frac{1}{8}$
$W_{T_1}W_{T_2}$	$-\frac{1}{8}$	0	0	0	$-\frac{1}{8}$
$W_{T_2}W_{T_3}$	$-\frac{1}{8}$	0	0	0	$-\frac{1}{8}$
$W_{T_1}W_{T_3}$	$-\frac{1}{8}$	0	0	0	$-\frac{1}{8}$
W_{T_1}	$\frac{1}{8}$	0	0	0	$\frac{1}{8}$
W_{T_2}	$\frac{1}{8}$	0	0	0	$\frac{1}{8}$
W_{T_3}	$\frac{1}{8}$	0	0	0	$\frac{1}{8}$
$W_{I_1}W_{I_2}W_{I_3}$	0	$-\frac{1}{8}$	0	0	$-\frac{1}{8}$
$W_{I_1}W_{I_2}$	0	$\frac{1}{8}$	0	0	$\frac{1}{8}$
$W_{I_2}W_{I_3}$	0	$\frac{1}{8}$	0	0	$\frac{1}{8}$
$W_{I_1}W_{I_3}$	0	$-\frac{1}{8}$	0	0	$-\frac{1}{8}$
W_{I_1}	0	$-\frac{1}{8}$	0	0	$-\frac{1}{8}$
W_{I_2}	0	$\frac{1}{8}$	0	0	$\frac{1}{8}$
W_{I_3}	0	$\frac{1}{8}$	0	0	$\frac{1}{8}$
$W_{L_1}W_{L_2}$	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$
W_{L_1}	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$
W_{L_2}	0	0	$-\frac{1}{4}$	0	$-\frac{1}{4}$
W_J	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$

The synaptic weight displayed in table 1, has been computed using on Wan Abdullah method which will be stored in a HNN associative memory called Content Addressable Memories (CAM) and later restored the corrected pattern.

2.5 Experimental Setup

The objective is to incorporate a HNN in searching for optimal RANMAX*k*SAT logic representation. A real data sets, medical fertility data set (MFDS) were used in the in generating RANMAX*k*SAT logical clause. HNN-RANMAXkSAT simulations were conducted on Windows 8.1, Intel Core i3, 1.7 GHz 4 GB RAM processors with Dev C++ release version 5.11. Initially, the neuron has been randomized based on the objectives of this study and represent RANMAX*k*SAT to HNN. The algorithms in Figure 2 displays the execution of the HNN model within the network system. Table 2 indicates the appropriate control parameters utilized during each HNN model implementation.

3. Performance Evaluation

The RANMAXkSAT with large number of neurons as the input may achieve good results based on the training data used in this study; however, this could lead to a bad generalization (Alzaeemi and Sathasivam, 2020). In our experiments, we assess the performance of proposed logical rule model on a different number of neurons $10 \leq NN \leq 90$. In our experiments, different evaluate metrics have been adopted based on the logical rules. The measurements are evaluated based on the model accuracy, and errors accumulation that reflects the network complexity based on the number of neurons using the following formula.

$$z_m = \frac{1}{b\rho}\sum_{i}^{n} N_{F_{RANMAXkSAT}} \tag{30}$$

Table 2. List of Parameters

Parameter	Name
f_t	Number of clauses
f_c	Number of satisfied clause
n	Number of iteration
ρ	Number of trials
b	Number of neuron combinations
$N_{F_{RAND_{k-SAT}}}$	Number of Z*m*
$N_{P_{test}}$	Number of testing data
$P_{induced}^{Correct}$	Correct induced logic

$$MAE = \sum_{i=1}^{n} \frac{1}{n} \left| f_t - f_c \right| \tag{31}$$

$$RMSE = \sum_{i=1}^{n} \sqrt{\frac{1}{n}(f_t - f_c)^2} \tag{32}$$

where f_t and f_c are the the output value and target output value respectively, and n is a number of the iterations.

$$SBC = n\ \ln(MSE) + p\ \ln(n) \tag{33}$$

where p is the defined as number of the literals. The mean square error (MSE) define as:

$$MSE = \frac{\sum_{i=1}^{n} \left(o_i - f_c\right)^2}{n} \tag{34}$$

$$Accuracy\,(Q) = \frac{P_{induced}^{Correct}}{N_{P_{test}}} \times 100\% \tag{35}$$

The performance of the Hopfield neural network for Random maximimu kSatisfiability logical rule is preanted in Table 3.

3.0.1 Landscape Evaluation

If the RANMAX*k*SAT logical clauses mapped to HNN as one of the stored patterns ξ^v as an initial state, the neuron state may rotate. The final state of the neuron must be similar to the original state to make the HNN act as an associative memory. Time's similarity function is defined in (Kauffman and Weinberger, 1989) as follows,

$$m^v(t) = \frac{1}{N} \sum_{i=1}^{n} \xi_i^v s_i^v \tag{36}$$

The fitness function is computed by taking the average values according to the given RANMAX*k*SAT pattern.

$$f = \frac{1}{t_0.p} \sum_{t=1}^{t_o} \sum_{v=1}^{v} m^v(t) \tag{37}$$

In this case, t0 is twice the number of neurons (2*N*); and p is based on neuronal status. The fitness energy landscape value depends on the storage capacity of the model. Since the Hopfield network (HNN) is concerned about the energy model's flatness; consideration must be extended to the fitness energy environment. The quality of the fitness energy system is based on the concept of Kauffman in(Kauffman and Weinberger, 1989).

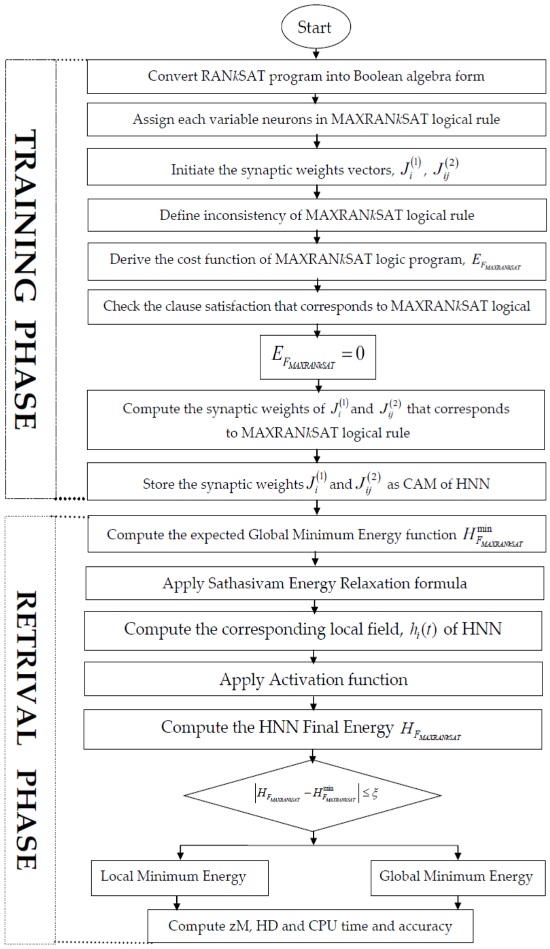

Fig. 2. Implementation of RAN3-SATRA for various models.

In this paper, a medical data set has been occupied in RAN*k*SATRA for analysis namely the medical fertility dataset (MFDS). The MFDS data was collected from the UCI machine learning repository website(Gil *et al.* 2012). It contains information about each data set for different purposes. The original MFDS has 100 instances, each with nine attributes and two classes. Normal fertility rate and abnormal fertility rate are the two classes. However, feature selection algorithms are employed to MFDS in order to extract more relevant features from the dataset. It reached the conclusion that some features have a lower influence on the entire quality of the data, and that non-serious data can sometimes act as noise in the data, highlighting the size of the features(Dua and Graff 2019). Some data points may have less impact on predicting fertility rates, both experimentally and medically. As a result of the foregoing, the final experimental dataset contains eight rather than nine attributes. The goal of this experiment is to conduct a thorough comparison of accuracy between them. The proposed RANMAX*k*SAT, MAX*k*SAT and RANkSAT using the same data set. The details of MFDS is shown in Table 3. The output will classify whether particular person is fertile or not.

4. Result and Discussion

The trend of the global minimum ratio (zM) of HNN model performance in searching for optima representation to RANMAX*k*SAT, MAX*k*SAT and RANkSAT logical clauses for MFDS classification has been presented in figure 3. The efficiency of the searching capacity of HNN can be observed by testing the consistency of the model energy. If the network global minimum energy is closer to one, that means, during the recovery phase, nearly all neurons achieved the required final state (100 percent satisfiable).The proposed logical rule model, HNN-RANMAX*k*SAT in comparison with existing models have entrenched with the real-life data sets. The investigation of a model's performance is separated into

Table 3. List of Attributes for MFDS (Gil *et al.* 2012)

logic	Details of each attributes
T_1	Frequency of alcohol consumption
T_2	Age at the time of Analysis
T_3	Childish diseases
I_1	Accident or serious trauma
I_2	Surgical Intervention
I_3	High fevers in the last
L_1	Smoking habit
L_1	Smoking habit
L_2	Diagnosis

two parts. The first significant part is to examine the quality of solution generated by different searching techniques by employing suitable training errors. Secondly is to analyze the robustness and efficiency of the proposed model based on implementation time and accuracy during the training and retrieval process. Therefore, this research main contribution is to explore the competency of HNN-RANMAXSAT in comparison with the existing models for the classification of medical fertility data set (MFDS).

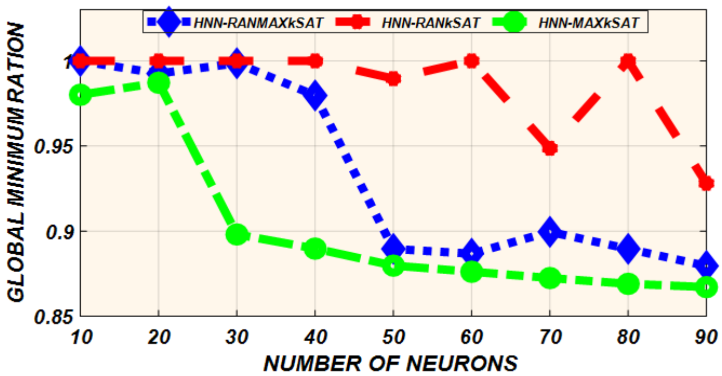

Fig. 3. zM for HNN Performance.

Table 4. Medical fertility data set (MFDS)

Logical rule	MAE	RMSE	Accuracy
RANMAX*k*SAT	0.3888	0.802	82.5 (%)
RAN*k*SAT	0.3611	0.030	79.6(%)
MAX*k*SAT	0.25	0.6944	68.1(%)

The trends of all model performances have been displayed in term of the Global minimum ratio(Z_m) for classification of medical fertility data set(MFDS)in figure 4. A model is considered a better model when it Z_m is close to 1. It can be seen from figure 4 that Z_m are closer to 1 even by manipulating a different number of neurons from $10 \leq NN \leq 90$ for classification of MFDS. The trend reveals the capability of the proposed model to attain global minimum energy by having Z_m closer to 1. Based on the findings of Z_m for all models achieved indistinguishable results.

Figure 5 to 7 displayed the Fitness landscape, MAE and RMSE during the training and retrieval process of all model understudy for classification of MFDS. It can deduce that all models exhibit lower Fitness landscape and minimal MAE and RMSE errors compared to the assigned threshold time. The general trend of MAE for HNN-RANMAX*k*SAT classification behaviour was reported to increase rapidly

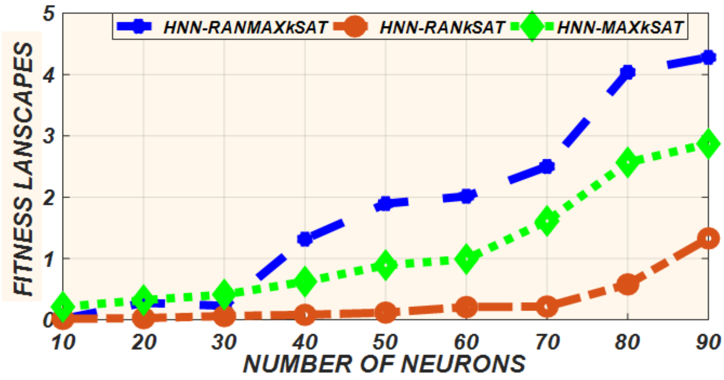

Fig. 4. Fitness landscape for HNN performance.

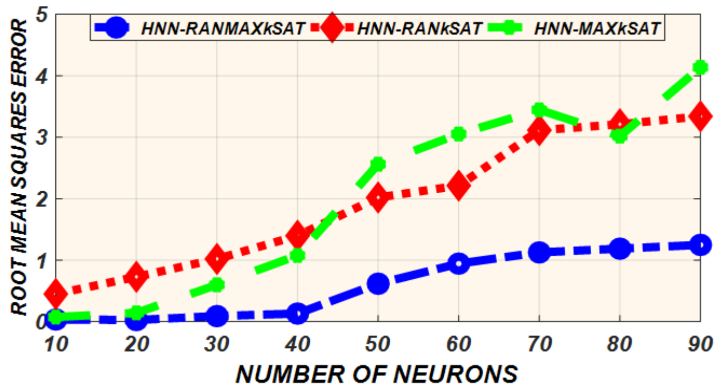

Fig. 5. RMSE for HNN performance.

with neurons complexity but still managed to achieve $E_{F_{RANkSAT}} = 0$, lower MAE accumulation than HNN-MAX*k*SAT and HNN-RAN*k*SAT. The FEL was supported by MAE and RMSE displayed in figure 6 and 7 for MFDS classification problem. HNN-RAN*k*SAT classification behaviour was reported to increase rapidly with neurons complexity. The proposed HNN-RANMAX*k*SAT was able to achieve the best global solution with lower FL, RMSE and MAE and accumulation than the existing one in classifying the MFDS. Figure 8 displayed the behaviours of models under study in terms of implementation time during the simulation cycle. The proposed HNN-RANMAX*k*SAT was faster than the existing models in classifying MFDS. Computational time is predefined as the expanse of time needed for the network to complete the overall computational process. Table 4 outlined the accuracy of all models for MFDS classifications problem. It can observe observed that, HNN-RANMAX*k*SAT and HNN-RAN*k*SAT enumerate the closer of accuracy with 82.5% and 79.6% respectively while HNN-MAXkSAT achieved an accuracy of 68.1%. This revealed the optimal capability of both models in attaining optimized induced logic for MFDS classification. The accuracy for both methods was promising, particularly HNN-RANMAXSAT, due to optimized induced logic generated at the end of the executions. By executing the simulation of HNN-RANMAX*k*SAT for data set classification, we can see the induced logic attained according to Equation (38) as follows.

$$P_{induced}^{Correct} = \begin{cases} \overbrace{\begin{array}{l}(T_1 \vee T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee T_3) \wedge (T_1 \vee \neg T_2 \vee T_3) \wedge \\ (T_1 \vee T_2 \vee \neg T_3) \wedge (\neg T_1 \vee \neg T_2 \vee T_3) \wedge (\neg T_1 \vee T_2 \vee \neg T_3) \\ \wedge (T_1 \vee \neg T_2 \vee \neg T_3) \wedge (\neg T_1 \vee \neg T_2 \vee \neg T_3) \wedge (A_1 \vee A_2 \vee A_3) \wedge \end{array}}^{MAX-3SAT} \\ \overbrace{(I_1 \vee I_2 \vee \neg I_3) \wedge (L_1 \vee \neg L_2) \wedge \neg J}^{RANDAM-3SAT} \end{cases} \tag{38}$$

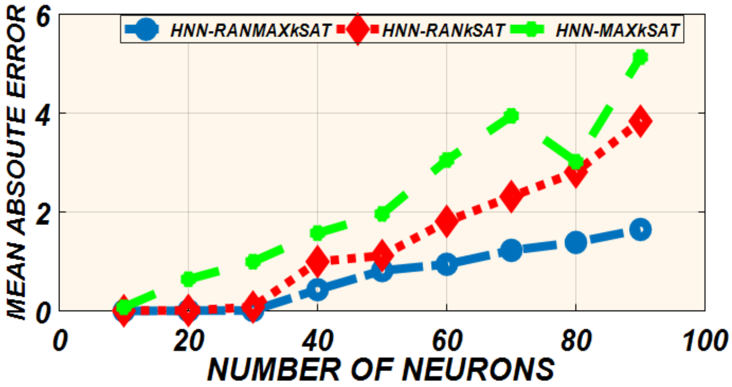

Fig. 6. MAE for HNN performance.

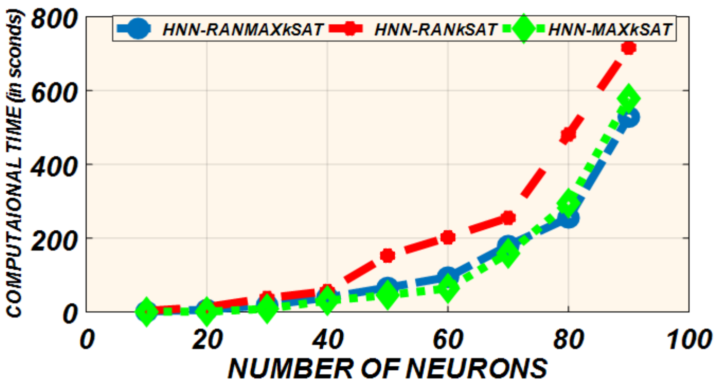

Fig. 7. CPU TIME for HNN Performance.

The attributes T_1, I_3, and J_2 in Equation (38) are trivial enough not to be analyzed. However, attributes like L_1 and J_1 will result in out-of-tune convergence. Considerably from table 3, the generated induced logic by HNN-RANkSAT in the testing stage accomplished an accuracy of 82.5%. This finding set forth HNN-RANMAXkSAT ability to acquire an optimized induced logic that could best represent the MFCDS data set classification. Contrary to that, HNN-RANkSAT, and HNN-MAXkSAT methods which achieved lower accuracy compared to HNN-RANMAXkSAT. This is due to the fact of the RANMAXkSAT mechanism has improve the clause satisfaction process which could lead to a better training stage in resulting in a construction of an optimized induced logic. From Equation (26), we can distinguish whether a Persons is fertile or not such attributes like A_1 and I_1 could exhibit fair fertility status. Other than that, the induced logic could reveal insignificant and trivial attributes, like I_3 and J_2 to sort out between fertile or not.

Based on the findings in table 4, we can infer that RANMAXkSAT is critical in our proposed model for producing better-induced logic that represents the precision of our changed network. The reason why RANMAXkSAT will fulfill such a factor is that the optimization operator in RANMAXkSAT introduces a random searching space and variance of the solution, which leads to a strong training point. The main drawbacks of RANMAXkSAT are the lack of satisfiability of generated induced logic which causes the tendency of overfitting. To overcome such an aspect, the alteration of the data sets is crucial. Rearrangement and permutation of the attributes, with a randomized selection of attributes, should be implemented. The no-free lunch theorem stated that there are no absolute or specific algorithms that can be utilized to solve every problem. However, in this research, we discovered that RANMAXkSAT works exceptionally well for MFDS in term of FEL, MAE, RMSE and accuracy in classifications.

5. Conclusion

Artificial neural network (ANN) possesses a comprehensive structure of training and testing stages, that made it one of the most efficient tools in patterns and knowledge extraction in solving real-life applications. This include classification, forecasting, risk analysis, detection and quantitative analysis. Therefore, we presume that this research contributes to amplifying the efforts to represent RANMAX*k*SAT in HNN for optimazation purpose. In this paper, a hybrid framework has been proposed which incorporated Hopfield neural network (HNN) in performing RANMAX*k*SATRA to assist logical rule in governing the behaviour of the data set. The algorithm has been compared by analytical tests on MAX*k*SAT and RAN*k*SAT using different numbers of neurons to confidently confirm the performance of the proposed algorithm. The results confirmed that the RANMAX*k*SAT outperformed the existing logical rule techniques on the preponderance of datasets substantially. The HNN-RANMAX*k*SATRA effectively to classify MFDS with a diverse number of features and training samples. The simulation results it has been proven that the RANMAX*k*SAT logical rule complied effectively with the Hopfield neural network for optimal representation in term of Fitness energy lanscapes (FEL), mean absolute error (MAE) and Root mean square error (RMSE) with able to classify 85.1 % of MFDS of the test samples better than MAX*k*SAT and RAN*k*SAT. Our future work, research is planned to be prolonged in two main lines. First, the proposed HNN-RANSATRA could be investigated for other data mining tasks like time series prediction and classifications problems.

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