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Heat flux effect in η_i-mode driven solitary and shock waves in electron-positron-ion plasma

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Abstract

The specific role of ion heat flux on the characteristics of the linear and nonlinear ion temperature gradient (ηi) driven mode in inhomogeneous electron-positron-ion plasma is presented. Inhomogeneity in density, temperature, and the magnetic field is considered. A modified linear dispersion relation is obtained, and its different limiting cases are when $\eta_i \gg 2/3$, ω^D(gradient in magnetic field) = 0 and β(density ratio of plasma species) = 1 are discussed. Furthermore, an expression for the anomalous transport coefficient of the present model is obtained. Nonlinear structure solutions in the form of solitons and shocks show that mode dynamics enhance in the presence of ion heat flux in electron-positron-ion plasma. The present study is essential in energy confinement devices such as tokamak because the heat flux observed experimentally in tokamak plasma is much higher than those described by collisions. Further, it could be helpful to understand the nonlinear electrostatic excitations in the interstellar medium.

Keywords: Anomalous transport; electron-positron-ion plasma; heat flux; ion temperature gradient mode; nonlinear structures.

1. Introduction

The study of electron-positron and electron-positron-ion (e-p-i) plasmas in the last few years, as reported, has made it a popular and interesting in plasma physics due to its applications and presence in astrophysical and laboratory environments [1–4]. The concentration of positrons (with the same mass and opposite charge) transforms the electronion plasma to e-p-i plasma, with modified dynamics reported by some authors [5–7]. In e-p-i plasma, several low frequencies waves can propagate in the presence of ions which otherwise do not exist in electron-positron plasma [8]. The e-p-i plasma study is essential and relevant to astrophysics, cosmology, and laboratory experiments [1]. The e-p-i plasma is reported in the astrophysical side such as in the early universe, magneto-spheres, solar flares, and in the core of our galaxy, in the intergalactic and outer galactic regions [9–11]. On the other hand, in the laboratory, its creation is found due to an interaction of high laser pulses with dense solid-state materials. The energetic electromagnetic waves can create such plasma [12, 13]. Different properties of e-p-i plasma are studied linearly as well as nonlinearly. Nonlinearity could generate various structures, such as solitons, shocks, peakons, cuspons, etc. [14, 15]. In the nonlinear regime, the shock waves are studied under the relativistic collision condition is e-p-i plasma [16]. Using the Pseudopotential method, ion-acoustic (IA) solitary waves are investigated in superthermal and magnetized e-p-i plasma [8, 17]. The solitary and shock wave profiles have been studied in e-p-i plasmas showing that stability range modifies with the positron number density and other plasma parameters [18, 19]. The research in e-p-i plasma is extended in different directions by authors such as by Popel *et al*. considering Boltzmannian electron, by Nejoh *et al.* to the ion-acoustic soliton in unmagnetized e-p-i plasma, and by Mushtaq *et al*. to the effectiveness of positron number density in two-dimensional magnetosonic waves.[2, 4, 20]. It has been observed that as thermal energy $k^B T$ gets greater in comparison to rest mass energy $m^0 c^2$ of electron or positron, then pair production and inhalation are more important in e-p-i plasma. These observations are noted near the Active Galactic Nuclei (AGN) black hole, where the ion temperature is 10^{13}K, and due to short cooling time where the electron temperature is in the order of 10^9K [21].

To a great extent researchers agree that ion heat transport driven turbulences in Magneto Hydrodynamics (MHD) quiescent tokamak H-mode plasmas are controlled mainly by the ion temperature gradient. The temperature gradient is of general importance in poloidal flows resulting from radial electric fields. Also, the high-temperature rise is considered the central issue in the confinement and transport of plasma species in confinement devices, e.g., in a tokamak. Experimentally it is reported that, in the presence of high neutral beam heating, the intensifying density, fluctuations are observed when T_{io}

becomes more significant than a specific value, typically, $T_{io} > 4$ keV [22]. One reason for such fluctuations is the drift wave instability, driven by the ion temperature gradient. Experimentally η_i which is defined as $\eta_i = d \ln T_{io}/d \ln n_{io}$, is an acceptable critical parameter in the pellet injection [23, 24]. The η_i nonlinear effect becomes visible due to the saturation of number density and energy confinement time. This mode was first observed by Rudakov *et al*. in the slab geometry [25]. Coppi investigated by taking the pressure effect in the background in this mode [26]. The ITG mode has a counterpart known as the electron temperature gradient (ETG) mode (η_e), which accounts for micro instabilities [27]. It is reported that both η_e and η_i mode instabilities are very strong compared to the collision-driven fluxes and electron gyroradius ρ_e effects. The ITG mode instability arises as a result of free energy stored in the form of temperature gradients. The toroidal geometry also studies the ITG mode, including magnetic field curvature and impurity effects [28]. In toroidal ITG mode $\eta_i > 1$. Jerman *et al.* took the Righi-Liduce heat flux in the energy balance equation for ion and observed the change in ITG mode instabilities [24]. In connection to localized structure formation, drift mode is one of the primary sources for L-H transition to the region of enhanced confinement in tokamak devices, wherever heat flux effects are noticeable. Moreover, ITG, the trapped electron, and the pressure gradient are the most promising sources of instabilities. This work investigates the influence of ion heat flux in ion temperature gradient driven solitons and shocks in an e-p-i plasma. Furthermore, we also derive the expression for anomalous ion-energy transport coefficient in the presence of positron concentration. This study helps to understand small and large amplitude electrostatics turbulence and cross field ion energy transport in electron-positron-ion plasma. This work extends the very recent investigations done by Zakir *et al*. in electron-ion plasma [30]. This article is divided into the following sections: in section 2, formalism related to ITG mode and the corresponding dispersion relation have been derived. In subsection 2.1 of the same area, we derive the anomalous ion energy transport, and in subsection 2.2 discusses some interesting limiting cases. Next, in subparts 3.1 and 3.2 of section 3, we discuss the solitary and shock wave solutions in an e-p-i plasma system. Section 4 gives a brief discussion on the numerical analysis, and in the last section, we present the concluding remarks.

2. Theoretical model and dispersion relation

We are assuming electron-positron-ion plasma having background magnetic field $B_0(x)\,\hat{z}$ (where $\hat{z}$ is the unit vector along the z-axis). Further, consider gradients in number density, temperature and in magnetic filed are in x-direction i.e., $d_x n_{i0}$ and $d_x T_{i0}$ and $d_x B_{i0}$ for ions (where n_{i0}, T_{i0}, B_{i0} are the unperturbed ion number density temperature and the strength of the external field). Comparatively masses of electrons and positrons are very small to that of ions. The dynamical role of ion is taken into account and Maxwellian distribution is applied to the assumed plasma species. Adding that the mode frequency is small with respect to the ion gyrofrequency $\omega_{ci} = (eB/m_i c)$, here e refers to the magnitude of the ion charge, m_i mass of ion and c designates the speed of light. The model is restricted to electrostatic fluctuations where $\nabla \times E = 0$ and the equilibrium quasi neutrality condition is defined as $n_{io}(x) + n_{po}(x) \simeq n_{eo}(x)$. For the dynamics of the considered mode, we use the ion momentum equation, given as:

$$m_i n_i(\partial_t + v_i.\nabla)\mathbf{v}_i = en_i\nabla\phi - \nabla P_i, \tag{1}$$

where $E_z = -\nabla\varphi$ represents electric filed and φ is the electric potential. The ion number density and the temperature are expanded about the unperturbed position in such a way that $n_i = n_{i0} + n_{i1}$, $T_i = T_{i0} + T_{i1}$ where $n_{i1} \ll n_{i0}$ and $T_{i0} \ll T_{i1}$ (terms having a subscript "0" represent the equilibrium and those designated with subscript "1" denote the perturbed). In the low frequency limits $\omega \ll \omega_{ci}$, the ion fluid velocity under drift approximation is written as

$$\mathbf{v}_i = \mathbf{v}_{EB} + \mathbf{v}_{Di} + \mathbf{v}_{pi} + \mathbf{v}_{\pi i} + v_{iz}\hat{\mathbf{z}}, \tag{2}$$

where $\mathbf{v}_{EB} = \frac{c}{B_0}(\hat{z}\times\nabla\phi)$ is the $\mathbf{E}\times\mathbf{B}$ drift , $\mathbf{v}_{Di} = \frac{c}{eB_0 n_i}(\hat{z}\times\nabla p_i)$, is the ion diamagnetic drift, $\mathbf{v}_{pi} = -\frac{c}{B_0\omega_{ci}}(\partial_t + \mathbf{v}_i.\nabla)\hat{z}\times\mathbf{v}_i$ is the ion polarization drift and $\mathbf{v}_{\pi i} = \frac{1}{en_i B_0}(\hat{z}\times\nabla.\pi_i)$ is the stress tensor drift. The different symbols ϕ, p, π_i, used in these equations referred to the electrostatic potential, pressure and stress tensor and v_{iz} is the ion fluid velocity along the z-axis. Here $p_i = n_i T_i$. The continuity equation for the ion species of the mode is given by

$$\partial_t n_i + \nabla.(n_i\mathbf{v}_i) = 0. \tag{3}$$

For temperature calculations, we use the energy equation that is given by

$$\frac{3}{2}\left(\partial_t + \mathbf{v}_i\cdot\nabla\right)T_i + T_i(\nabla\cdot\mathbf{v}_i)n_i = \frac{1}{n_i}\nabla.[(5cT_i/2eB_0)\hat{z}\times\nabla T_i], \tag{4}$$

$(5cT_i/2eB_0)\hat{z}\times\nabla T_i$ called the Righi-Leduce heat flux term for ions. Our conditions of quasineutrality is restricted to the first order as

$$n_{i_1} = n_{e_1} - n_{p_1}. \tag{5}$$

Equations (1), (3) and (4) along with Eq. (5) form a complete set which we will use for closing our system. In the following part of this section we derive the dispersion relation of our model. For this purpose neglecting the nonlinearities by considering small amplitude limits: assuming $n_i = n_{i0} + n_{i1}$, $T_i = T_{i0} + T_{i1}$ where $n_{i1} \ll n_{i0}$ and $T_{i0} \ll T_{i1}$. Thus we can write Eq. (3) with drift approximation as

$$D_t^i N + \tau(\mathbf{v}_{ni}.\nabla\Phi) - \frac{1}{2}\rho_i^2\tau^{-1}\partial_t\nabla^2(\Phi + T + N) + \partial_z v_{iz} = 0. \tag{6}$$

In Eq. (6) different terms introduced have the following representations: $D_t^i = (\partial_t + \mathbf{v}_E.\nabla)$, $\mathbf{v}_{ni} = (\frac{cT_{io}}{eB_0})\nabla\ln n_{io}\times\hat{z}$, $\tau = \frac{T_{eo}}{T_{io}}$, $\Phi = (\frac{e\phi}{T_{eo}})$, $T = \frac{T_{i1}}{T_{io}}$ and $N = \frac{n_{io1}}{n_{io}}$. Restricting our model to electrostatic limits and by plugging in the value of p_i in the ion momentum equation we get

$$D_t^i v_{iz} = -c_s^2\partial_z\Phi - \tau^{-1}c_s^2\partial_z(T+N) - v_{iz}\partial_z v_{iz}, \tag{7}$$

here $c_s = \rho_s\omega_{ci}$ and ρ_s is plasma species gyroradius. Using the drift approximation in Eq. (4) we obtain the nonlinear energy balance equation for the ion fluid in plasma as

$$D_t^i T = \frac{2}{3}D_t^i N + \tau(\eta_i - \frac{2}{3})\mathbf{v}_{ni}.\nabla\Phi + \frac{5}{3}\mathbf{v}_B.\nabla T. \tag{8}$$

Since we in this model assume that our plasma species follow Maxwellian distribution for which density expression is written as

$$n_s = n_{s0}\exp(\frac{e\phi}{T_s}), \tag{9}$$

where $s = e, p$ for electron and positron and $s = i$ for ion, the electron with charge $(-e)$ and positron with charge $(+e)$ in exponent of Eq. (9). In linear limits, a wave provides valuable information about the mode phase velocity. So to check this we consider small amplitude perturbations where the different perturbed quantities vary exponentially in Fourier space as $\exp(ik_y y + ik_z z - i\omega t)$ (where k_y, k_z is the perpendicular, parallel components of the wave vector and ω is the angular frequency). Equations (6)-(8) are written in linearized form by using Fourier transformation as

$$\omega N - \tau\omega_{ni}\Phi + \frac{1}{2}\rho_i^2\tau^{-1}\omega k^2\Phi + \frac{1}{2}\rho_i^2\tau^{-1}\omega k^2(N+T) - k_z v_{iz} = 0, \tag{10}$$

$$v_{iz} = \frac{c_s^2 k_z}{\omega}(\Phi + \tau^{-1}(N+T)), \tag{11}$$

and

$$T = \frac{2}{3}\frac{\omega N}{(\omega - \frac{5}{3}\omega_D)} - \tau\frac{(\eta_i - \frac{2}{3})}{(\omega - \frac{5}{3}\omega_D)}\omega_{ni}\Phi, \tag{12}$$

where $\omega_D = (\mathbf{k}.\mathbf{v}_B)$, $\mathbf{k}$ is a wave vector and $\mathbf{v}_B = (\frac{cT_{io}}{eB_0})\nabla\ln B_0\times\hat{z}$, which is the ion ∇B_0 drift. The normalized density expression can be written by using Eq. (9) in Eq. (5).

$$N = (\frac{n_{eo}}{n_{io}} - \frac{n_{po}}{n_{io}}\frac{T_{eo}}{T_{po}})\Phi = \beta\Phi, \tag{13}$$

here $\beta = \frac{n_{eo}}{n_{io}} - \frac{n_{po}}{n_{io}}\frac{T_{eo}}{T_{po}}$, where n_{eo}, n_{po} are the equilibrium number densities of electrons and positrons while T_{eo}, T_{po} are the equilibrium temperature of electrons and positrons. To get a linear dispersion relation we combine Eqs. (10)-(13) and after some manipulation we obtain a solution as:

$$\begin{aligned}&\omega^3\{\beta + \frac{1}{2}\rho_i^2\tau^{-1}k^2(1+\frac{5}{3}\beta)\} - [\{\tau + \frac{1}{2}\rho_i^2 k^2(\eta_i - \frac{2}{3})\}\omega_{ni}\\&+\frac{5}{3}\omega_D\{\frac{1}{2}\rho_i^2\tau^{-1}k^2(1+\beta)+\beta\}]\omega^2 - \{c_s^2k_z^2(1+\frac{5}{3}\tau^{-1}\beta)\\&-\frac{5}{3}\omega_D\tau\omega_{ni}\}\omega + c_s^2k_z^2\{(\eta_i - \frac{2}{3})\omega_{ni}\\&+\frac{5}{3}\omega_D(1+\tau^{-1}\beta)\} = 0\end{aligned} \tag{14}$$

2.1 Anomalous transport

As reported in previous studies that ITG mode is responsible for ion energy transport in the various toroidal device. Thus, this section focuses on how ion temperature gradient mode can cause anomalous heat flux across the confining magnetic field. The mechanism used here is as: we know that the temperature gradient, which is opposite to the density gradient over a broad region, makes the drift wave unstable, producing anomalous heat flux. To obtain a relation for this purpose we assume $\omega = \omega_r + i\omega_i$. First term in this expression represents the real and the second one denotes the imaginary parts of the frequencies. Thus expressing Eq. (12) as:

$$T_{i1} = \frac{2}{3}T_{i0}\left[\frac{\omega N - \tau(\eta_i - 3/2)\omega_{ni}}{(\omega - 5/3\omega_D)}\right]\Phi. \tag{15}$$

and by using $\omega = \omega_r + i\omega_i$, we can write the above equation as

$$T_{i1} = \frac{2}{3}T_{i0}\left[\frac{\omega_r + i\omega_i N - \tau(\eta_i - 3/2)\omega_{ni}}{(\omega_r + i\omega_i - 5/3\omega_D)}\right]\Phi. \tag{16}$$

Since ion motion is mostly dominated by $\mathbf{E}\times\mathbf{B}$ force, for which the heat flux is written as

$$\langle\Gamma x\rangle = v_E T_{i1}^* + v_E^* T_{i1}. \tag{17}$$

Further, Eq. (16) can be expressed as

$$T_{i1} = \mathcal{X} + i\mathcal{Y} \tag{18}$$

where $\mathcal{X} = \frac{\frac{2}{3}T_{i0}}{((\omega_r - \frac{5}{3}\omega_D)^2 + \omega_i^2)}[(\omega_r\beta - \tau(\eta_i - \frac{2}{3})\omega_{ni})\omega_i + i\omega_i\beta(\omega_r - \frac{5}{3}\omega_D) + \omega_i^2\beta]\Phi$ and $\mathcal{Y} = \frac{\frac{2}{3}T_{i0}}{((\omega_r - \frac{5}{3}\omega_D)^2 + \omega_i^2)}[\tau(\eta_i - \frac{2}{3})\omega_{ni}\omega i - \frac{5}{3}\omega_D\omega_i\beta]\Phi$. From Eq. (17) one can write

$$\langle\Gamma x\rangle = -\frac{2}{3}\frac{cT_{e0}}{eB_0}\omega_i k_y T_{i0}\left[\frac{\frac{10}{3}\omega_D\beta + 2\tau(1 - \frac{2}{3}\eta_i)\omega_{ni}}{(\omega_r - \frac{5}{3}\omega_D)^2 + \omega_i^2}\right]|\Phi|^2. \tag{19}$$

The ion excursion length is defined as $\partial_t\xi = \mathbf{v}_E$, where $\xi = \pi/k$. It gives the saturated wave amplitude in the limit $\Phi \leq \pi(\omega_r\omega_{ci})/k_\perp^2 v_{th}^2$. As the Fourier transform of electromagnetic drift is $\mathbf{v}_E = i\mathcal{Z}\Phi$ with $\mathcal{Z} = (cT_{e0}/eB)k_y$. Thus incorporating these definitions and using Fick's law we can write the ion thermal conductivity as

$$\chi_i = \frac{8\pi^2}{3}\frac{\omega_i\omega_{ci}}{T_{e0}k_\perp^2 k_y v_{th}^2}\left[\frac{\frac{10}{3}\omega_D\beta + 2\tau(1 - \frac{2}{3}\eta_i)\omega_{ni}}{(\omega_r - \frac{5}{3}\omega_D)^2 + \omega_i^2}L_T\right]\omega_r^2. \tag{20}$$

From the expression obtained above we see that thermal conductivity of the mode show the effect of positron number density, which in turns may influence the ion energy thermal transport.

2.2 Limiting Cases

Equation (14) is a third-order linear dispersion relation of the ITG driven mode in electron-positron-ion plasma. The above dispersion relation shows that the plasma waves modify in ion temperature gradient and heat flux. The dispersion relation is cubic in ω. Experimentally it has been found that this solution describes a propagating wave that can be destabilized because of the presence of an ion temperature gradient [31]. Furthermore, the additional term in the dispersion relation shows an apparent effect of the ion heat flux.

Case I. From Eq. (14) if we remove the Larmor radius effect and by letting $\eta_i \gg \frac{2}{3}$, then one obtain

$$\begin{aligned}&\omega^3\beta - (\tau\omega_{ni} + \frac{5}{3}\omega_D\beta)\omega^2\\ &-\{c_s^2k_z^2(1 + \frac{5}{3}\tau^{-1}\beta) - \frac{5}{3}\omega_D\tau\omega_{ni}\}\omega\\ &+c_s^2k_z^2\{\eta_i\omega_{ni} + \frac{5}{3}\omega_D(1 + \tau^{-1}\beta)\} = 0.\end{aligned} \tag{21}$$

Case II. Ignoring the ion heat flux such that considering homogeneous magnetic field ($\omega_D = 0$), we can get the dispersion relation for the e-p-i plasma without heat flux as

$$\beta\omega^3 - \tau\omega_{ni}\omega^2 - (1 + \frac{5}{3}\tau^{-1}\beta)c_s^2k_z^2\omega + c_s^2k_z^2\eta_i\omega_{ni} = 0. \tag{22}$$

This Eq. (22) is the same as obtained in Ref. [29].
Case III. Further by putting $\beta = 1$ the above relation reduces to the electron ion dispersion relation as

$$\omega^3 - \tau\omega_{ni}\omega^2 - c_s^2k_z^2(1 + \frac{5}{3}\tau^{-1})\omega + c_s^2k_z^2\eta_i\omega_{ni} = 0. \tag{23}$$

Which is the same dispersion relation obtained in Ref. [23] by Eq. (5.151). The dispersion relation given in different limiting cases shows that the driving term here is η_i and the instability is given by $\omega^3 = -c_s^2k_\parallel^2\eta_i\omega_{ni}$.

3. Nonlinear structures

3.1. Solitons

In this section of our work, we obtain a set of nonlinear equations, the combination of which give a nonlinear partial differential equation known as the Korteweg-de-Vries (KdV) equation and its solution, in turn, generates the ITG-mode driven solitary waves in the presence of ion heat flux in inhomogeneous e-p-i plasma. To get a solitary wave solution, the amplitude of the waves here is assumed large enough. For the derivation of solitary wave equation a co-moving frame ξ = $(y + \alpha z - ut)$, is introduced. Here u represents the nonlinear wave speed and α is a small angle along the z-axis in a dispersive medium. Equations (6), (7) and (8) transform in the new frame as:

$$\partial_\xi N - \frac{\tau v_{ni}}{u}\partial_\xi\Phi - \frac{1}{2}\rho_i^2\tau^{-1}\partial_\xi^3(\Phi + N + T) - \frac{\alpha}{u}\partial_\xi v_{iz} = 0, \tag{24}$$

$$v_{iz} = \frac{c_s^2\alpha}{u}[\{\Phi + \tau^{-1}(N + T)\} - \frac{1}{2}\frac{c_s^2\alpha^2}{u^2}\{\Phi + \tau^{-1}(N + T)\}^2], \tag{25}$$

$$T = \frac{1}{(1 - \frac{5}{3}\frac{v_B}{u})}\{\frac{2}{3}\beta - \frac{\tau}{u}(\eta_i - \frac{2}{3})v_{ni}\}\Phi = \mathcal{L}\Phi, \tag{26}$$

where $\mathcal{L} = \frac{1}{(1 - \frac{5}{3}\frac{v_B}{u})}\{\frac{2}{3}\beta - \frac{\tau}{u}(\eta_i - \frac{2}{3})v_{ni}\}$. Combination of equations (24)-(26) produce the following nonlinear partial differential equation as

$$\begin{aligned}&\{\beta - \frac{\tau v_{ni}}{u} - \frac{c_s^2\alpha^2}{u^2}\{1 + \tau^{-1}(\beta + \mathcal{L})\}\}\partial_\xi\Phi \\ &+ \frac{c_s^4\alpha^4}{u^4}\{1 + \tau^{-1}(\beta + \mathcal{L})\}^2\Phi\partial_\xi\Phi \\ &-\{\frac{1}{2}\rho_i^2\tau^{-1}(1 + \beta + \mathcal{L})\}\partial_\xi^3\Phi = 0,\end{aligned} \tag{27}$$

which in more simple form becomes

$$A_1\partial_\xi\Phi + A_2\Phi\partial_\xi\Phi + A_3\partial_\xi^3\Phi = 0, \tag{28}$$

with the values of A_1, A_2 and A_3 are given by

$$A_1 = \{\beta - \frac{\tau v_{ni}}{u} - \frac{c_s^2\alpha^2}{u^2} - \frac{c_s^2\alpha^2}{u^2}\tau^{-1}(\beta + \mathcal{L})\},$$

$$A_2 = \frac{c_s^4\alpha^4}{u^4}\{1 + \tau^{-1}(\beta + \mathcal{L})\}^2,$$

$$A_3 = -\frac{1}{2}\rho_i^2\tau^{-1}\{1+\beta+\mathcal{L}\},$$

or in more compact form

$$\partial_\xi \Phi + A\Phi\partial_\xi\Phi + B\partial_\xi^3\Phi = 0, \tag{29}$$

here $A = \frac{A_2}{A_1}$ and $B = \frac{A_3}{A_1} \gg 1$. A general solution of Eq. (29) has the form

$$\Phi = \phi_m \sec\mathrm{h}^2(\frac{\xi}{W}), \tag{30}$$

where $\phi_m = 3u/A$ and $W = \sqrt{4B/u}$. The amplitude and width parameters in the solution given by Eq. (30) clearly shows that solitary waves driven by ITG mode get modify in the presence of ion heat flux, electron-positron density and temperature ratios. From the potential relation obtained in this section, we are now in a position to get the expression for the electric field. This can be done by using Eq. (30) which gives

$$E = E_0 \sec\mathrm{h}^2(\frac{\xi}{W})\tanh(\frac{\xi}{W}), \tag{31}$$

here $E_0 = \frac{6u}{A}$. The corresponding surface charge density for the solitary wave is given by

$$\sigma = \sigma_0 \sec\mathrm{h}^2(\frac{\xi}{W})\tanh(\frac{\xi}{W}), \tag{32}$$

where $\sigma_0 = \frac{3u}{2\pi A}$, and the pressure of the solitary wave is

$$P = P_0 \sec\mathrm{h}^4(\frac{\xi}{W})\tanh^2(\frac{\xi}{W}), \tag{33}$$

in this equation $P_0 = 2\pi\sigma_0^2$. The point at which $\xi = \frac{1}{2}w[\ln(\frac{\sqrt{2}+1}{\sqrt{2}-1})]$ is the maximum pressure point of the solitary wave.

3.2. Shocks

This is another kind of nonlinear structure that appears in the large amplitude limit and exist in fluids as a result of dissipative effects. For modelling this we introduce neutrals effect in the system and neutral-ion collision term $\nu_n v_i$ (ν_n is the neutral collisional frequency), that in turns produce the collisional drift as $\mathbf{v}_c = c\nu_n/B_0\omega_{ci}\nabla\phi$ in the momentum equation. Solving our model equations in the new introduced frame we finally obtain the following expression

$$\partial_\xi \Phi + A\Phi\partial_\xi\Phi - C\partial_\xi^2\Phi = 0, \tag{34}$$

with $C = \frac{\rho_i^2\nu_n\alpha}{u}$. Equation (34) denotes Burger equation which has solution of the form

$$\Phi = \frac{2C}{A}\left[1-\tanh\xi\right], \tag{35}$$

where $C = \frac{A_3}{A_1}$. The profile of the shock wave strictly depends on the sign and magnitude of C/A ratio. Looking at the values of A, A_1 and A_3 one may observe the effect of ion heat flux on the shock structures.

4. Numerical Results

The obtained linear and nonlinear dispersion relations are solved numerically for the purpose to show the visible effects of new terms on the considered mode. For qualitative behavior of the numerical analysis we use the data given in (Qamar *et al.*, 2003; Davydova & Pankin 1998; Zakir *et al.*, 2016), some of these are: $m_i = 1.67\times10^{-24}g$, $n_e = 10^{14}cm^{-3}$, $B = 1.4\times10^4G$, $T_{eo} = 10^5eV$, $T_{io} = 0.1T_{eo}$, $n_{po} = 0.001n_{eo}$, $T_{Po} = 0.1T_{eo}$, $\eta_i = 2$, $c_s = 10^6cm/s$, ion gyrofrequency $\omega_{ci} = 10^4rad/s$, in new coordinates $u = 10^6cm/s$ and $\alpha = 0.1rad$. Based on various derived relations of our study we here discuss the linear and the nonlinear outcomes of our work.

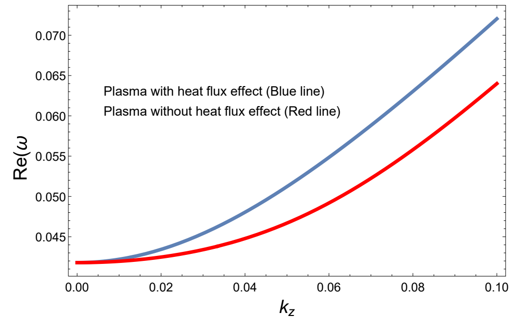

Fig.1. Shows real part of frequency of the mode (Eq. 15 and Eq.16) against k_z, using parameters as mentioned in text.

In the first figure, we numerically investigated the effect of ion heat flux on ITG mode in electron-positron-ion plasma. In this case, based on Eq. (14), we have shown that the phase velocity of the present mode in the absence of ion heat flux is maximum compared to the situation, where ion heat flux is considered. It is clear from this figure that ion heat flux is significant, means result in an increase in the phase velocity in linear mode.

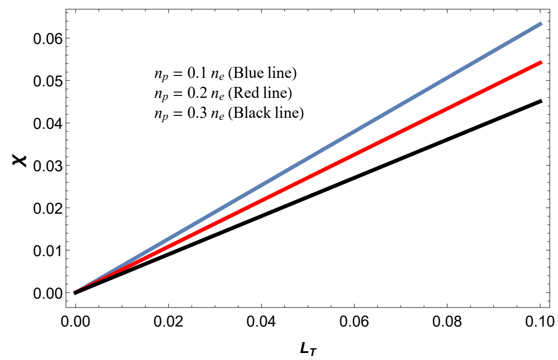

Fig.2. Shows the thermal conductivity based on Eq. (20) against LT, using parameters as mentioned in text.

Figure 2 shows the effect of positron to electron number density ratio on the thermal conductivity. This illustration of the analytical relation (Eq. (20)) depicts that ion energy thermal transport decreases on the increase of this ratio. The change becomes more clear for large value of $\partial_x \ln T_{i0}$. In figure 3 the effect of positron to electron temperature ratio (T_{po}/T_{eo}) is shown on the phase velocity of the mode. We see here that phase velocity of the mode modifies. Our numerical results also depict that in comparison to a large value of T_{po}/T_{eo}, the variation in phase velocity is high for a smaller value of T_{po}/T_{eo}.

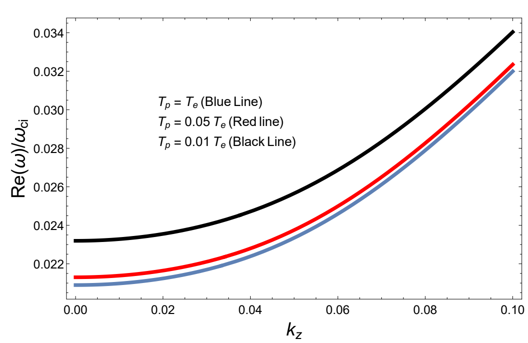

Fig.3. Shows real frequency plot of the mode (Eq. 15) against k_z. Fixed other parameters and changing $T_p = T_e$ (the blue line), $T_p = 0.05 T_e$ (the red line) while $T_p = 0.01 T_e$ (the black line).

In figure 4 we see the effect of positron to electron density ratio on the phase velocity of the ITG driven mode. In this case, we see an increase in the phase velocity of the mode. We can say that the slope of the mode, in this case, increases for larger k_z value.

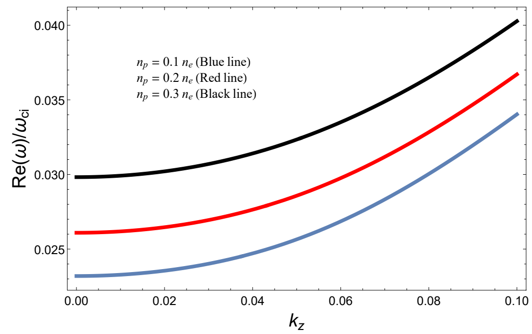

Fig.4. Showing phase velocity variation against k_z . Keeping other parameters fixed and changing $n_p = 0.1 n_e$ (the blue line), $n_p = 0.2 n_e$ (the red line) while $n_p = 0.3 n_e$ (the black line).

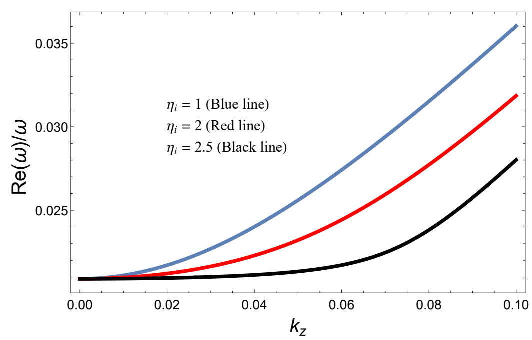

Fig.5. Shows normalized phase velocity against k_z . Fixed other parameters and changing $\eta i = 1$ (Blue), $\eta i = 2$ (Red) while $\eta i = 2.5$ (Black).

The mode parameter η_i effect on phase velocity is shown in figure 5. From our numerical analysis here it is clear that the phase velocity of the mode strongly depends on this parameter. Giving larger value to this parameter phase velocity of the ITG mode in (e-p-i) magnetized plasma reduces.

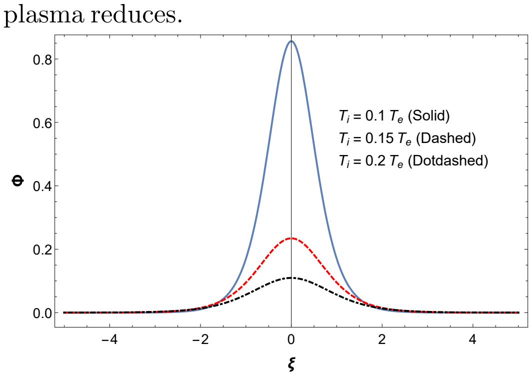

Fig.6. Result of ion to electron temperature ratio on the propagation of solitary wave in ion temperature gradient mode (ϕ vs ξ). Fixed other parameters and changing $Ti = 0.1Te$ (Solid line), $Ti = 0.15Te$ (Dotted line) while for $Ti = 0.2Te$ (Dotdashed line).

Next, we discuss numerical results of the ion temperature gradient driven mode nonlinear waves in e-p-i plasma. In figure 6 we have tried to see the influence of ion to electron temperature ratio on solitary waves. It is observed in this case that, a minimum value of this ratio (T_{io}/T_{eo}) results in high potential pulses and for the higher value, we see that these nonlinear structures are going to disappear. It might happen due to the reason that the ion-acoustic wave in fluid theory exist for small ratio (T_{io}/T_{eo}).

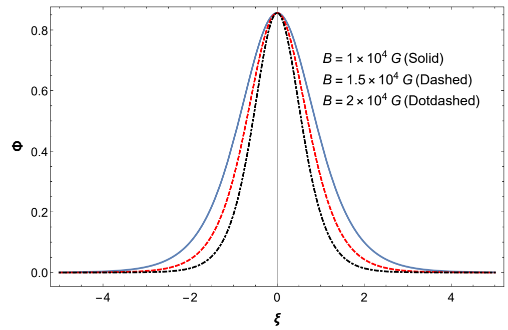

Fig.7. Magnetic field gradient effect of the mode on solitary waves (ϕ vs ξ). Fixed other parameters and changing $B = 1 \times 10^4 G$ (Solid line), $B = 1.5 \times 10^4 G$ (Dotted line) while for $B = 2 \times 10^4 G$ (Dotdashed line).

Figure 7 reflects the effect of the background magnetic field. It is clear that this parameter strongly contributes to the dispersive properties of nonlinearity. From this, we can also say that the magnetic field plays the main role in confining. Comparing these results with the previous study of Ref. [30], we observe an increase in the width and amplitude of the solitary waves in e-p-i plasma having heat flux effect. Adding here that these nonlinear structures spread out with a constant speed as noted. In figure 8 we show how the width and amplitude of the solitary structures change with the ratio of the positron to electron number density. Looking to the numerical results in this case we see that an abrupt change occurs at $n_p = 0.25 n_e$, in the solitary structures, where the hump type soliton inverts to dip type. A noticeable point is that both the amplitude and dispersive properties of the solitary wave modify but major change occur in the amplitude of the soliton.

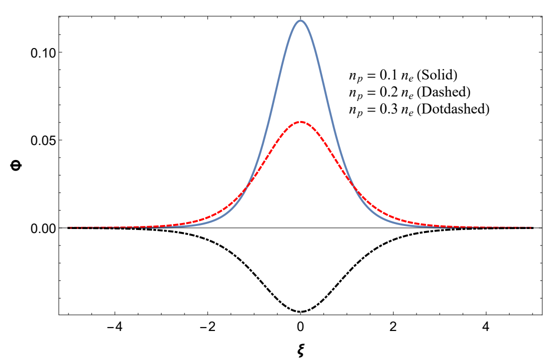

Fig.8. Positron density variation effect on solitary waves in ion temperature gradient mode (ϕ vs ξ). Fixed other parameters and changing $n_p = 0.1 n_e$ (Solid line), $n_p = 0.2 n_e$ (Dotted line) while for $n_p = 0.3 n_e$ (Dotdashed line).

Influence of T_{po}/T_{eo} on solitary structures is shown in figure 9. Here we see that temperature effect is very effective because for a very small change in temperature change in amplitude is drastic.

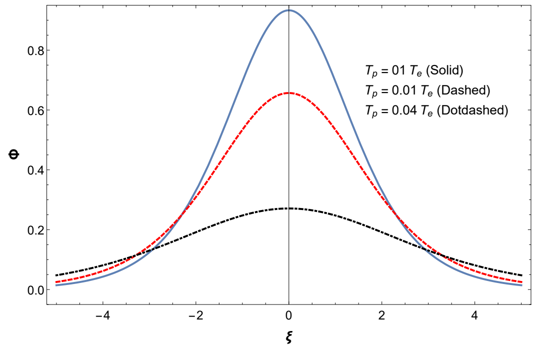

Fig.9. Shows positron to electron temperature ratio on propagation of solitary waves(ϕ vs ξ). Fixed other parameters and changing $Tp = Te$ (Solid line), $Tp = 0.05Te$ (Dotted line) while for $Tp = 0.01Te$ (Dotdashed line).

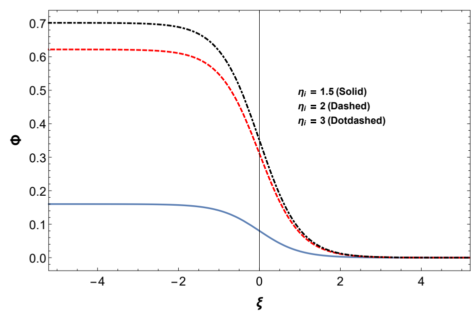

Fig.10. Shocks potential variation with respect to ion temperature gradient. Fixed other parameters and changing $\eta i = 1.5$ (Solid line), $\eta i = 2$ (Dashed line) while $\eta i = 3$ (Dotdashed line).

Based on Eq. (35) we here show the effect on ion temperature gradient coefficient on shock structure. In our numerical investigations, we found that ITG coefficient has an effective influence on the structure of shocks as shown in figure 10. It shows the importance of the present study. In this case, we observe that change in amplitude of the shocks is large in the beginning but small later on. The positron to electron temperature ratio on the shock amplitude is shown in figure 11. We see that a sharp fall is seen in the amplitude of the shock structure in this case. In figure 12 we have shown the effect of n_{po}/n_{eo}.The potential pulses are shown in this figure modify for different ratios of the positron to electron number density. We see that for step difference of 0.05, the change in potential pulse in the beginning is small, but it seems that for some higher value its contribution becomes viable as shown.

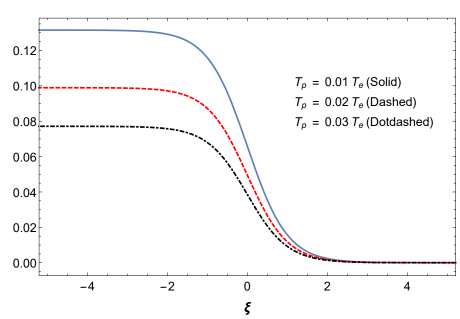

Fig.11. Shows shock potential variation with respect to the positron to electron temperature ratio on the propagation of shocks. Fixed other parameters and changing $Tp = 0.01Te$ (Solid line), $Tp = 0.02Te$ (Dashed line) while $Tp = 0.03Te$ (Dotdashed line).

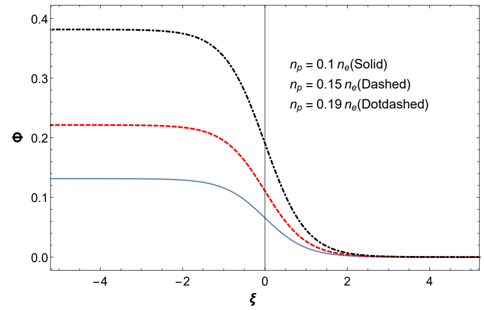

Fig.12. Result of positron to electron number density ratio on propagation of shock waves. Fixed other parameters and changing $n_p = 0.1 n_e$ (Solid line), $n_p = 0.15 n_e$ (Dotted line) while for $n_p = 0.19 n_e$ (Dotdashed line).

5. Conclusion

The present work investigated the linear/nonlinear ITG mode-driven solitary and shock waves in electron-positron-ion plasma. The influence of different ratios such as density and temperature of (electron/positron) are shown on the characteristics of linear/nonlinear mode in electron-positron-ion plasma generally and the consequences of ion heat flux on these modes particularly. In the linear regime, we concluded from our numerical results that propagation of the mode strongly depends on ion heat flux. The expression obtained for the thermal conductivity of the mode shows that ion energy thermal transport is modified in positron concentration and ion heat flux. This effect is noted in the large-amplitude limit, where the ion heat flux influences the solitary and shock wave solutions. We in this work observed that hump-type solitary waves turn into a dip type for an increase in the positron to electron density ratio, which is noted for numerical value $n_{po}/n_{eo} = 0.25$, as shown. Conclusively, we can write that positron presence is essential in plasma from this tipping point of the positron number density. We can say that inclusion of this component changes the plasma dynamics. Our findings may help understand the role of ion heat flux term driven by ion temperature exhibits similar behavior as predicted by theory and confirmed by gyrokinetic modelings such as in ASDEX upgrading H-mode devices and astrophysical plasma [34]. In conclusion, we stress that the present investigations are prime for understanding the salient features of fluctuations and crossfield energy transport in magnetically confined devices with positron concentration.

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