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Einstein like (ε)-para Sasakian manifolds

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"*Dedicated to Professor H. Hilmi HACISALİHOĞLU on his seventieth birthday*".

ABSTRACT

Einstein like (ε)-para Sasakian manifolds are introduced. For an (ε)-para Sasakian manifold to be Einstein like, a necessary and sufficient condition in terms of its curvature tensor is obtained. The scalar curvature of an Einstein like (ε)-para Sasakian manifold is obtained and it is shown that the scalar curvature in this case must satisfy certain differential equation. A necessary and sufficient condition for an (ε)-almost paracontact metric hypersurface of an indefinite locally Riemannian product manifold to be (ε)-para Sasakian is obtained and it is proved that the (ε)-para Sasakian hypersurface of an indefinite locally Riemannian product manifold of almost constant curvature is always Einstein like.

Keywords: Einstein like (ε)-para Sasakian manifold; indefinite locally Riemannian product manifold.

INTRODUCTION

Satō in 1976 introduced an almost paracontact structure on a differentiable manifold, which is an analogue of the almost contact structure (Sasaki, 1960; Blair, 2002) and is closely related to almost product structure (in contrast to almost contact structure, which is related to almost complex structure). An almost contact manifold is always odd-dimensional but an almost paracontact manifold could be even-dimensional as well. Takahashi in 1969 studied almost contact manifolds equipped with associated pseudo-Riemannian metrics. The indefinite almost contact metric manifolds and indefinite Sasakian manifolds are also known as (ε)-almost contact metric manifolds and (ε)-Sasakian manifolds, respectively (Bejancu & Duggal, 1993; Duggal, 1990). Also, in 1989 Matsumoto, 1989 replaced the structure vector field ξ by $-\xi$ in an almost paracontact manifold and associated a Lorentzian metric with the resulting structure and called it a Lorentzian almost paracontact manifold. In a Lorentzian almost paracontact manifold given by Matsumoto, the semi-Riemannian metric has only index 1 and the structure vector field ξ is always timelike. Because of these circumstances, the authors in Tripathi *et al.* (2010) introduced (ε)-almost paracontact structures by associating a semi-Riemannian metric, not necessarily Lorentzian, with an almost paracontact structure, where the structure vector field ξ is spacelike or timelike according as $\varepsilon = 1$ or $\varepsilon = -1$.

In 1982 Sharma introduced and studied Einstein like para Sasakian manifolds. Motivated by his study, in this paper we introduce and study Einstein like (ε)-almost paracontact metric manifolds. The paper is organized as follows. Section 2 contains some preliminaries about (ε)-para Sasakian manifolds. In section 3, we give the definition of an Einstein like (ε)-almost paracontact metric manifold and give some basic properties. For an (ε)-para Sasakian manifold to be Einstein like, we also find a necessary and sufficient condition in terms of its curvature tensor. We also find the scalar curvature of an Einstein like (ε)-para Sasakian manifold and show that the scalar curvature in this case must satisfy certain differential equation. In section 4, we find a necessary and sufficient condition for an (ε)-almost paracontact metric hypersurface of an indefinite locally Riemannian product manifold to be (ε)-para Sasakian. Finally we prove that an (ε)-para Sasakian hypersurface of an indefinite locally Riemannian product manifold of almost constant curvature is always Einstein like.

2. PRELIMINARIES

Let M be an n-dimensional almost paracontact manifold (Satō, 1976) equipped with an almost paracontact structure (φ, ξ, η) consisting of a tensor field φ of type $(1,1)$, a vector field ξ and a 1-form η satisfying

$$\varphi^2 = I - \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \quad \eta \circ \varphi = 0.$$

By a semi-Riemannian metric (O'Neill, 1983) on a manifold M, we understand a non-degenerate symmetric tensor field g of type $(0,2)$. In particular, if its index is 1, it becomes a Lorentzian metric (Beem & Ehrlich, 1981). Throughout the paper we assume that $X, Y, Z, U, V, W \in \Gamma(TM)$, where $\Gamma(TM)$ is the Lie algebra of vector fields in M, unless specifically stated otherwise. Let g be a semi-Riemannian metric with $index(g) = \nu$ in an n-dimensional almost paracontact manifold M such that

$$g(\varphi X, \varphi Y) = g(X, Y) - \varepsilon\eta(X)\eta(Y), \tag{2.1}$$

where $\varepsilon = \pm 1$. Then M is called an (ε)-almost paracontact metric manifold equipped with an (ε)-almost paracontact metric structure $(\varphi, \xi, \eta, g, \varepsilon)$

(Tripathi *et al.,* 2010). In particular, if $index(g) = 1$, then an (ε)-almost paracontact metric manifold is a Lorentzian almost paracontact manifold. In particular, if the metric g is positive definite, then an (ε)-almost paracontact metric manifold is the usual almost paracontact metric manifold (Satō, 1976). The equation (2.1) is equivalent to

$$g(X,\ \varphi Y) = g(\varphi X,\ Y) \qquad \text{along with} \quad g(X,\ \xi) = \varepsilon\eta(X) \tag{2.2}$$

Note that $g(\xi,\ \xi) = \varepsilon$, that is, the structure vector field ξ is never lightlike. An (ε)-almost paracontact metric structure $(\varphi,\ \xi,\ \eta,\ g,\ \varepsilon)$ is called an $(\varepsilon)-$para Sasakian structure if

$$(\nabla_X\varphi)\ Y = -\ g(\varphi X,\ \varphi Y)\xi - \varepsilon\eta(Y)\,\varphi^2 X, \tag{2.3}$$

where ∇ is the Levi-Civita connection with respect to g. A manifold endowed with an (ε)-para Sasakian structure is called an (ε)-para Sasakian manifold. In an (ε)-para Sasakian manifold we have

$$\nabla\xi = \varepsilon\varphi, \tag{2.4}$$

$$\Phi(X,\ Y) \equiv g(\varphi X,\ Y) = \varepsilon g(\nabla_X\xi,\ Y) = (\nabla_X\eta)\ Y. \tag{2.5}$$

Example 2.1. Let R^3 be the 3-dimensional real number space with a coordinate system (x, y, z). We define

$$\eta = dz, \quad \xi = \frac{\partial}{\partial z},$$
$$\varphi\left(\frac{\partial}{\partial x}\right) = \frac{\partial}{\partial y}, \quad \varphi\left(\frac{\partial}{\partial y}\right) = \frac{\partial}{\partial x}, \quad \varphi\left(\frac{\partial}{\partial z}\right) = 0,$$
$$g_1 = (dx)^2 + (dy)^2 - (dz)^2,$$
$$g_2 = -(dx)^2 - (dy)^2 + (dz)^2.$$

Then the set $(\varphi, \xi, \eta, g_1)$ is a timelike Lorentzian almost paracontact structure, while the set $(\varphi, \xi, \eta, g_2)$ is a spacelike (ε)-almost paracontact metric structure. We note that $index(g_1) = 1$ and $index(g_2) = 2$.

Example 2.2. Let R^3 be the 3-dimensional real number space with a coordinate system (x, y, z). We define

$$\eta = dx - ydz, \quad \xi = \frac{\partial}{\partial x},$$
$$\varphi\left(\frac{\partial}{\partial x}\right) = 0, \quad \varphi\left(\frac{\partial}{\partial y}\right) = -\frac{\partial}{\partial y}, \quad \varphi\left(\frac{\partial}{\partial z}\right) = -\frac{\partial}{\partial z} - y\frac{\partial}{\partial x},$$
$$g_1 = (dy)^2 + (dz)^2 - \eta \otimes \eta,$$
$$g_2 = (dx)^2 + (dy)^2 + (dz)^2 - y(dx \otimes dz + dz \otimes dx),$$
$$g_3 = (dx)^2 + (dy)^2 - (dz)^2 - y(dx \otimes dz + dz \otimes dx).$$

Then, the set (φ, ξ, η) is an almost paracontact structure in R^3. The set $(\varphi, \xi, \eta, g_1)$ is a timelike Lorentzian almost paracontact structure. Moreover, the trajectories of the timelike structure vector ξ are geodesics. The set $(\varphi, \xi, \eta, g_2)$ is a spacelike Lorentzian almost paracontact structure. The set $(\varphi, \xi, \eta, g_3)$ is a spacelike (ε)-almost paracontact metric structure with $index(g_3) = 2$.

Example 2.3. (Tripathi *et al.,* 2010) Let (M^n, J, G) be a semi-Riemannian almost product manifold, such that

$$J^2 = I, (JX, JY) = G(X, Y).$$

Consider the product manifold $M^n \times R$. A vector field on $M^n \times R$ can be represented by $(X, f(d/dt))$, where X is tangent to M, f is a smooth function on $M^n \times R$ and t is the coordinates of R. On $M^n \times R$ we define

$$\eta = dt, \quad \xi = \frac{d}{dt}, \quad \varphi\left(X, f\frac{d}{dt}\right) = JX,$$
$$g\left(\left(X, f\frac{d}{dt}\right), \left(Y, h\frac{d}{dt}\right)\right) = G(X, Y) + \varepsilon fh.$$

Then $(\varphi, \xi, \eta, g)$is an (ε)-almost paracontact metric structure on the product manifold $M^n \times R$.

Example 2.4. (Tripathi *et al.,* 2010) Let R^3 be the 3-dimensional real number space with a coordinate system (x, y, z). We define

$$\eta = dz, \quad \xi = \frac{\partial}{\partial z},$$
$$\varphi\left(\frac{\partial}{\partial x}\right) = \frac{\partial}{\partial x}, \quad \varphi\left(\frac{\partial}{\partial y}\right) = -\frac{\partial}{\partial y}, \quad \varphi\left(\frac{\partial}{\partial z}\right) = 0,$$
$$g = e^{2\varepsilon x^3}(dx)^2 + e^{-2\varepsilon x^3}(dy)^2 + \varepsilon(dz)^2,$$

Then (φ, ξ, η, g) is an (ε)-para Sasakian structure.

For more details refer to Tripathi *et al*. (2010).

3. EINSTEIN LIKE (ε)-PARA SASAKIAN MANIFOLDS

We begin with the following definition analogous to Einstein like para Sasakian manifolds (Sharma, 1982).

Definition 3.1 An (ε)-almost paracontact metric manifold is said to be Einstein like if its Ricci tensor S satisfies

$$S(X,\ Y) = a\,g(X,\ Y) + b\,g(\varphi X,\ Y) + c\,\eta(X)\,\eta(Y)\,, \tag{3.1}$$

for some real constants a, b and c.

Proposition 3.2 In an Einstein like (ε)-almost paracontact metric manifold, we have

$$S(\varphi X,\ Y) = ag(\varphi X,\ Y) + bg(\varphi X,\ \varphi Y), \tag{3.2}$$

$$S(X,\ \xi) = \varepsilon a\eta(X) + c\eta(X). \tag{3.3}$$

Moreover, if the manifold is (ε)-para Sasakian, then

$$\varepsilon a + c = 1 - n, \tag{3.4}$$

$$r = na + btrace(\varphi) + \varepsilon c, \tag{3.5}$$

where r is the scalar curvature.

Proof. The equations (3.2) and (3.3) are obvious. In an (ε)-para Sasakian manifold, it follows that $S(X,\ \xi) = -(n-1)\eta(X)$, which in view of (3.3) implies (3.4). Now, let $\{e_1, \ldots,\ e_n\}$ be a local orthonormal frame. Then from (3.1), we have

$$r = \sum_{i=1}^{n} \{\varepsilon_i ag(e_i,\ e_i) + \varepsilon_i bg(\varphi e_i,\ e_i) + \varepsilon_i cg(\xi,\ e_i)\, g(\xi,\ e_i)\},$$

which gives (3.5).

Remark 3.3 From (3.1), it follows that the Ricci operator Q satisfies

$$QX = aX + b\varphi X + \varepsilon c\,\eta(X)\,\xi. \tag{3.6}$$

Differentiating (3.6), we find

$$(\nabla_Y Q)\,X = b(\nabla_Y \varphi)\,X + \varepsilon c(\nabla_Y \eta)\,(X)\,\xi + \varepsilon c\eta(X)\nabla_Y \xi.$$

Using (2.3), (2.5) and (2.4) in the above equation we get

$$\begin{aligned}(\nabla_Y Q)\,X &= -\,\varepsilon b\eta(X)\,Y + c\eta(X)\,\varphi Y \\ &\quad -(bg(X,\ Y) - 2\varepsilon b\eta(X)\,\eta(Y) - \varepsilon cg(\varphi X,\ Y))\,\xi.\end{aligned} \tag{3.7}$$

Now, using (3.7) we have

$$(divQ)\,X = \{\varepsilon(1-n)\,b + c\,trace(\varphi)\}\,\eta(X). \tag{3.8}$$

From (3.5) and (3.4) we get

$$r = b\,trace(\varphi) - \varepsilon(n-1)\,(c+n) \tag{3.9}$$

Using $Xr = 2(divQ)X$ and (3.9) in (3.8) we obtain that in an Einstein like (ε)-para Sasakian manifold, the scalar curvature r satisfies the following differential equation

$$b\,\xi r - 2cr = 2\varepsilon(1-n)\left(b^2 - c^2 - cn\right). \tag{3.10}$$

Proposition 3.4 In an Einstein like (ε)-para Sasakian manifold, if $trace(\varphi)$ is constant then

$$c\ \ trace(\varphi) = \varepsilon(n-1)b. \tag{3.11}$$

Proof. Using $Xr = 2(divQ)X$ in (3.8), we get

$$dr = 2(\varepsilon(1-n)\,b + c\,trace(\varphi))\,\eta. \tag{3.12}$$

Since $trace(\varphi)$ is constant, from (3.5), it follows that r is constant. Hence (3.12) gives (3.11).

From now on in this section the $trace(\varphi)$ will be assumed to be constant.

Theorem 3.5 An (ε)-para Sasakian manifold with constant $trace(\varphi)$ is Einstein like if and only if the $(0,2)$-tensor field $C_1^1(\varphi R)$ is a linear combination of g, Φ and $\eta\otimes\eta$ formed with constant coefficients.

Proof. In an (ε)-para Sasakian manifold the curvature tensor R satisfies [12]

$$
\begin{aligned}
R(X, Y)\varphi Z &= \varphi R(X, Y)Z + \varepsilon\Phi(Y, Z)X - \varepsilon\Phi(X, Z)Y - 2\varepsilon\Phi(Y, Z)\eta(X)\xi + 2\varepsilon\Phi(X, Z)\eta(Y)\xi \\
&\quad - \varepsilon g(Y, Z)\varphi X + \varepsilon g(X, Z)\varphi Y + 2\eta(Y)\eta(Z)\varphi X - 2\eta(X)\eta(Z)\varphi Y.
\end{aligned}
$$

Then we have

$$
S(Y, \varphi Z) = C_1^1(\varphi R)(Y, Z) + \varepsilon(n-2)\Phi(Y, Z) + (2\eta(Y)\eta(Z) - \varepsilon g(Y, Z))trace(\varphi). \quad (3.13)
$$

Since in an (ε)-para Sasakian manifold, it follows that of Tripathi *et al.* (2010) $S(X, \varphi Y) = S(\varphi X, Y)$, and also it can be verified that $C_1^1(\varphi R)(Y, Z) = C_1^1(\varphi R)(Z, Y)$; therefore the equation (3.13) is consistent. Now, if the manifold is Einstein like then from (3.2), (3.13) and (3.11), it follows that

$$
c\left(C_1^1(\varphi R)\right) = b(c + n - 1)g + c(a - \varepsilon(n-2))\Phi - \varepsilon(c + 2b(n-1))\eta \otimes \eta, \quad (3.14)
$$

which shows that $C_1^1(\varphi R)$ is a linear combination of g,Φ and $\eta \otimes \eta$ formed with constant coefficients. The converse is easy to follow.

Corollary 3.6 In an Einstein like (ε)-para Sasakian manifold with constant $trace(\varphi)$, the (0, 2) -tensor field $C_1^1(\varphi R)$ is parallel along the vector field ξ.

Proof. Since in an Einstein like (ε)-para Sasakian manifold $\nabla_\xi \Phi = 0$ and $\nabla_\xi \eta = 0$, therefore from (3.14) we conclude that $C_1^1(\varphi R)$ is parallel along the vector field ξ.

Theorem 3.7 In an Einstein like (ε)-para Sasakian manifold, we have

$$
L_\xi S = 2a\varepsilon\Phi + 2b\varepsilon(g - \varepsilon\eta \otimes \eta). \quad (3.15)
$$

Proof. In an (ε)-para Sasakian manifold, we obtain

$$
L_\xi \eta = \nabla_\xi \eta = 0, \quad L_\xi \Phi = 2\varepsilon(g - \eta \otimes \eta), \quad L_\xi g = 2\varepsilon\Phi. \quad (3.16)
$$

Now, taking Lie derivative of S in the direction of ξ in (3.1) and using (3.16), we obtain (3.15).

Theorem 3.8 In an Einstein like (ε)-para Sasakian manifold with constant $trace(\varphi)$, we have

$$
c\left(L_\xi\left(C_1^1(\varphi R)\right)\right) = 2\varepsilon b(c + n - 1)\Phi + 2\varepsilon c(a - \varepsilon(n-2))(g - \eta \otimes \eta). \quad (3.17)
$$

Proof. Taking Lie derivative of $C_1^1(\varphi R)$ in the direction of ξ in (3.14) and using (3.16), we get (3.17).

4 .(ε)-PARA SASAKIAN HYPERSURFACES

Let $\tilde{M}$ be a real $(n+1)$-dimensional manifold. Suppose $\tilde{M}$ is endowed with an almost product structure J and a semi-Riemannian metric $\tilde{g}$ satisfying

$$\tilde{g}(JX, JY) = \tilde{g}(X, Y), \tag{4.1}$$

for all vector fields X, Y in $\tilde{M}$. Then we say that $\tilde{M}$ is an indefinite almost product Riemannian manifold. Moreover, if on $\tilde{M}$ we have

$$(\tilde{\nabla}_X J)Y = 0, \tag{4.2}$$

for all $X,\ Y \in \Gamma(T\tilde{M})$, where $\tilde{\nabla}$ is the Levi-Civita connection with respect to $\tilde{g}$, we say that $\tilde{M}$ is an indefinite locally Riemannian product manifold.

Now, let M be an orientable non-degenerate hypersurface of $\tilde{M}$. Suppose that N is the normal unit vector field of M such that $\tilde{g}(N, N) = \varepsilon$, ξ belongs to M and

$$JN = \xi. \tag{4.3}$$

Let

$$JX = \varphi X + \eta(X)\, N. \tag{4.4}$$

Proposition 4.1 The set (φ, ξ, η, g) is an (ε)-almost paracontact metric structure, where g is the induced metric on M.

Proof. We have

$$X = J^2 X = \varphi^2 X + \eta(\varphi X)\, N + \eta(X)\, \xi,$$

where (4.4) and (4.3) are used. Equating tangential and normal parts we get $\varphi^2 = I - \eta \otimes \xi$ and $\eta \circ \varphi = 0$, respectively. We also have

$$N = J^2 N = J\xi = \varphi\xi + \eta(\xi)\, N,$$

where (4.4) and (4.3) are used. Equating tangential and normal parts we get $\varphi\xi = 0$ and $\eta(\xi) = 1$, respectively. Finally, we have $g(X, Y) = \tilde{g}(JX, JY)$, which in view of (4.4) gives (2.1).

The Gauss and Weingarten formulas are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + \varepsilon g(AX, Y)N, \tag{4.5}$$

$$\tilde{\nabla}_X N = -AX, \tag{4.6}$$

where ∇ is the Levi-Civita connection with respect to the semi-Riemannian metric g induced by $\tilde{g}$ on M and A is the shape operator of M .

Proposition 4.2 The (ε)-almost paracontact metric structure on M satisfies

$$(\nabla_X \varphi)\, Y = \eta(Y) AX + \varepsilon g(AX,\ Y)\xi, \tag{4.7}$$

$$(\nabla_X \eta)\, Y = -\varepsilon g(AX,\ \varphi Y), \tag{4.8}$$

$$\nabla_X \xi = -\varphi AX. \tag{4.9}$$

Proof. Using (4.4), (4.3), 4.5) and (4.6) in $(\tilde{\nabla}_X J)\, Y = 0$, we get

$$0 = (\nabla_X \varphi)\ Y - \eta(Y)\, AX - h(X,\ Y)\xi + ((\nabla_X \eta)\ Y)\, N + h(X,\ \varphi Y) N.$$

Equating tangential and normal parts we get (4.7) and (4.8), respectively. Eq. (4.8) implies (4.9).

Now we obtain the following theorem of characterization for (ε)-para Sasakian hypersurfaces.

Theorem 4.3 Let M be an orientable hypersurface of an indefinite locally Riemannian product manifold. Then M is an (ε)-para Sasakian manifold if and only if the shape operator is given by

$$A = -\varepsilon I + \varepsilon \eta \otimes \xi. \tag{4.10}$$

Proof. Let M be an (ε)-para Sasakian manifold. By using (2.4) and (4.9) we get

$$AX = -\varepsilon X + \varepsilon \eta(X)\, \xi + \eta(AX)\, \xi. \tag{4.11}$$

In particular, we have $A\xi = \eta(A\xi)\, \xi$. Thus, we have

$$\eta(AX) = \varepsilon g(\xi,\ AX) = \varepsilon g(A\xi,\ X) = \varepsilon g(\eta(A\xi)\, \xi,\ X) = \eta(A\xi)\, \eta(X). \tag{4.12}$$

Using this in (4.11) we get

$$A = -\varepsilon I + (\varepsilon + \eta(A\xi))\, \eta \otimes \xi. \tag{4.13}$$

Now, we use (4.13) in (4.7) to find

$$(\nabla_X \varphi)\ Y = -\varepsilon \eta(Y) X + 2\varepsilon \eta(X)\, \eta(Y)\xi + 2\eta(A\xi)\, \eta(X)\, \eta(Y)\xi - g(X,\ Y)\, \xi. \tag{4.14}$$

From (4.14) and (2.3) we get $\eta(A\xi) = 0$, which when used in (4.13) yields (4.10).

Conversely, using (4.10) in (4.7) we see that M is (ε)-para Sasakian manifold.

Now, assume that the indefinite almost product Riemannian manifold $\tilde{M}$ is of almost constant curvature (Yano, 1965) so that its curvature tensor $\tilde{R}$ is given by

$$\begin{aligned}\tilde{R}(X, Y, Z, W) &= k\{\tilde{g}(Y, Z)\,\tilde{g}(X, W) - \tilde{g}(X, Z)\,\tilde{g}(Y, W)\\ &+\tilde{g}(JY, Z)\tilde{g}(JX, W) - \tilde{g}(JX, Z)\tilde{g}(JY, W)\},\end{aligned} \tag{4.15}$$

for all vector fields X, Y, Z, W on $\tilde{M}$. If M is an (ε)-para Sasakian hypersurface, then in view of (4.10) and (4.15) the Gauss equation becomes

$$\begin{aligned}R(X, Y, Z, W) &= (k+\varepsilon)\,\{g(Y, Z)\,g(X, W) - g(X, Z)\,g(Y, W)\}\\ &+k\{g(\varphi Y, Z)g(\varphi X, W) - g(\varphi X, Z)g(\varphi Y, W)\}\\ &+g(X, Z)\,\eta(Y)\,\eta(W) - g(Y, Z)\,\eta(X)\,\eta(W)\\ &+g(Y, W)\,\eta(X)\,\eta(Z) - g(X, W)\,\eta(Y)\,\eta(Z).\end{aligned} \tag{4.16}$$

After calculating $R(X, Y)\xi$ from (4.16) and comparing the resulting expression with (Tripathi *et al.,* 2010)

$$R(X, Y)\,\xi = \eta(X)\,Y - \eta(Y)\,X, \tag{4.17}$$

we find that $k = 1 - 2\varepsilon$. With this value of k, from (4.16), we obtain

$$S = ((1-\varepsilon)\,(n-2))g + (1-2\varepsilon)\ trace(\varphi)\,\Phi + (\varepsilon\ - n)\eta\otimes\eta.$$

Thus we have proved the following:

Theorem 4.4 An (ε)-para Sasakian hypersurface of an indefinite locally Riemannian product manifold of almost constant curvature $(1-2\varepsilon)$ is Einstein like.

Remark 4.5 A hypersurface is called a *quasi-umbilical hypersurface* (Chen, 1973) if

$$h(X, Y) = \alpha\, g(X, Y) + \beta\, u(X)\, u(Y),$$

where α and β are some smooth functions and u is a 1-form. From (4.10) we see that the (ε)-para Sasakian hypersurface is quasi-umbilical.

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منطويات مثل – ساساكية شبيهة بمنطويات آينشتاين

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خلاصة

نقدم في هذا البحث منطويات مثل – ساساكية شبيهة بمنطويات آينشتاين. نحصل على شروط ضرورية وكافية ليكون المنطوي مثل – الساساكي منطوياً شبيهاً بمنطويات آينشتاين. وذلك بواسطة موتر التقوس. كما نحسب التقوس السلمي لهذه المنطويات ونبين أن التقوس، في هذه الحالة، لابد أن يحقق معادلة تفاضلية محددة. نحصل أيضاً على شروط ضرورية وكافية ليصبح الفوسطح القرب مثل – التلامس المتري لمنطوي جداء ريماني غير محدد، ليصبح هذا الفوسطح منطوياً مثل – ساساكي. ونثبت أن الفوسطح في هذه الحالة هو دائماً منطوي شبيه بمنطوي آينشتاين.