Kuwait J. Sci. **40** (2) pp. 17-30, 2013

Solution of nonlinear q-schrödinger equation by two dimensional q-differential transform method

MRIDULA GARG AND LATA CHANCHLANI

Department of Mathematics, University of Rajasthan, Jaipur, Rajasthan, India

ABSTRACT

In the present paper, we first derive q-analogue of nonlinear Schrödinger equation from its discrete version and then solve it by two-dimensional q-differential transform method. The solution is obtained in the form of a series and in the case $q \to 1$, reduces to the exact solution of a nonlinear Schrödinger equation studied by Borhanifa and Abazari. We also draw some graphs of solution for different values of the parameter q using the software Mathematica.

Keywords: Two-dimensional q-differential transform method; nonlinear q-Schrödinger equation.

INTRODUCTION

The study of q-analysis is an old subject, which dates back to the end of the 19^{th} century. A detail account of it can be seen in the books by Slater (1966), Exton (1983), Gasper & Rahman (1990) and a thesis (Ernst, 2000). It has found many applications in such areas as the theory of partitions, combinatorics, exactly solvable models in statistical mechanics, computer algebra etc (Andrews, 1986). In recent years, mathematicians have considered q-difference equations for their links with other branches of mathematics such as quantum algebras and q-combinatorics.

The differential transform method is a semi numerical analytic method for solving differential equations. The concept of differential transform in one-dimension was first introduced by Zhou (1986), who solved linear and nonlinear initial value problems in electrical circuit analysis. The q-extension of one-dimensional differential transform method was introduced by Jing & Fan (1995) for solving the ordinary q-differential equations. Recently, Shahed & Gaber (2011) have introduced two-dimensional q-differential transform method for solving linear and nonlinear partial q-differential equations.

The Schrödinger equation is the fundamental equation of physics for describing quantum mechanical behavior. It is also often called the Schrödinger wave equation, and is a partial differential equation that describes how the wave function of a physical system evolves over time. The Schrödinger equation and its variants is one of the basic equations studied in the field of partial differential equations and has applications to geometry, to spectral and scattering theory and to integrable systems. Nonlinear problems are of interest to engineers, physicists and mathematicians because most physical systems are inherently nonlinear in nature. Various nonlinear dynamical systems and modified nonlinear Schrödinger equation (MNLSE) have been discussed in the papers (Konar & Sengupta, 1994; Srivastava & Konar, 2009; Jana & Konar, 2006). For complex systems, the linear multi-particle Schrödinger equation is often replaced by a nonlinear single-particle Schrödinger equation as in the density functional theory of solid state physics. The nonlinear Schrödinger equation (NLSE) is one of the most universal models that describe many physical nonlinear systems. In the book by Biswas & Konar (2006) it has been outlined and the NLSE for Kerr law nonlinearity from basic principles has been derived. A MNLSE is given by

$$i\frac{\partial}{\partial t}\psi(X,t) = -\frac{1}{2}\nabla^2\psi + \tau(X)\psi + \beta_d|\psi|^2\psi,\ \mathrm{X} \in R^d, \psi(X,0) = \psi_0(X), t \geq 0 \quad (0.1)$$

where $\tau(X)$ is the trapping potential, and β_d is a real constant. Equation (0.1) is a non-linear partial differential equation (PDE) of parabolic type that is not integrable, in general. The Equation (0.1) in the special case $\tau(X) = 0$ has been solved by Ablowitz & Clarkson (1999) by the method of inverse scattering transform (IST). Recently, Borhanifa & Abazari (2011) have solved the following NLSE

$$i\frac{\partial}{\partial t}\psi(x,t) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi - |\psi|^2\psi, x \in R, t \geq 0, \psi(x,0) = e^{ix},\ x \in R, \quad (0.2)$$

using two-dimensional differential transform method. Various nonlinear Schrödinger equations have also been solved by, Adomian decomposition method (Sadighi & Ganji, 2008), variational iteration method (Wazwaz, 2008) and other methods as well. In paper (Green & Biswas, 2010), 1-soliton solution of the nonlinear Schrödinger's equation that governs the propagation of solitons through optical fibers has been obtained.

DEFINITIONS

For $q \in C, 0 < |q| < 1$, q-analogue of a natural number n is given by (Ernst, 2000)

$$[n]_q = \sum_{k=1}^{n} q^{k-1} \text{ and } [0]_q = 0, \quad (0.3)$$

and q-factorial (Ernst, 2000) is defined by

$$[n]_q! = \prod_{k=1}^{n}[k]_q \text{ and } [0]_q! = 1. \tag{0.4}$$

The q-shifted factorial is defined as

$$(a;q)_n = \left\{ \begin{array}{ll} 1, & \text{if } n = 0 \\ \prod_{m=0}^{n-1}(1 - aq^m), & \text{if } n \in N \end{array} \right\}, \tag{0.5}$$

or equivalently

$$(a;q)_n = \frac{(a;q)_\infty}{(aq^n;q)_\infty}, \quad \text{where} \quad (a;q)_\infty = \prod_{m=0}^{\infty}(1 - aq^m). \tag{0.6}$$

$$\text{Also} \quad (a;q)_n = \sum_{k=0}^{n}(-1)^k \binom{n}{k}_q q^{\binom{n}{k}} a^k, \tag{0.7}$$

$$\text{and its inverted form} \quad a^n = \sum_{k=0}^{n} a^k \binom{n}{k}_q (a^{-1};q)_k. \tag{0.8}$$

q-analogue of power function

$$(a-b)^{(n)} = a^n \left({}^{b}/_{a}\,; q \right)_n = a^n \prod_{j=0}^{\infty} \left[\frac{1 - \left({}^{b}/_{a} \right) q^j}{1 - \left({}^{b}/_{a} \right) q^{j+n}} \right] = a^n \frac{\left({}^{b}/_{a}\,; q \right)_\infty}{\left(q^n\, {}^{b}/_{a}\,; q \right)_\infty}, (a \neq 0). \tag{0.9}$$

q-Exponential function

For $0 < |q| < 1$and $|x| < (1-q)^{-1}$the q-exponential function $E_q(x)$ is given by (Ernst, 2000)

$$E_q(x) = \sum_{k=0}^{\infty} \frac{x^k}{[k]_q!}. \tag{0.10}$$

The q-analogues of the trigonometric functions (Ernst, 2000) are defined by

$$Sin_q(x) = \frac{1}{2i}\left[E_q(ix) - E_q(-ix)\right] = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{[2k+1]_q!}, \tag{0.11}$$

$$Cos_q(x) = \frac{1}{2}\left[E_q(ix) + E_q(-ix)\right] = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{[2k]_q!}. \tag{0.12}$$

h-Derivative

The h-derivative of a function $f(x)$ as given in (Kac & Cheung, 2002) is defined by

$$(D_h f)(x) = \frac{f(x+h) - f(x)}{h}, \tag{0.13}$$

where $\lim_{h\to 0} D_h f(x) = \frac{df(x)}{dx}$, if $f(x)$ is differentiable.

q-Partial derivatives (Rajkovic′ *et al.*, 2003)

If $f(x, y)$ is a function of two variables, then its q-partial derivative with respect to the variable x is given by

$$D_{q,x} f(x,y) = \frac{f(x,y) - f(qx,y)}{(1-q)x}, (x \neq 0) \text{ and } D_{q,x} f(x,y)\big|_{x=0} = \lim_{x\to 0} D_{q,x} f(x,y). \tag{0.14}$$

1. FORMATION OF NONLINEAR q-SCHRÖDINGER EQUATION

A discrete version of the NLSE is used to describe the dynamics of pulses in optical waveguide arrays and photorefractive crystals (Porter, 2009). In this section, we form a nonlinear q-Schrödinger equation from its discrete version. A discrete version of nonlinear Schrödinger equation (DNLSE) considered in (Hernadez & Levi, 2003) is given by

$$i\dot{Q}_n = \frac{1}{h^2}\left[Q_n - \frac{1}{2}\left(1 - \varepsilon|Q_n|^2\right)(Q_{n+1} + Q_{n-1})\right], \tag{1.1}$$

Taking $\varepsilon = -1$, (1.1) becomes

$$i\dot{Q}_n = -\frac{1}{2h^2}(Q_{n+1} - 2Q_n + Q_{n-1}) - \frac{1}{2h^2}|Q_n|^2(Q_{n+1} + Q_{n-1}), \tag{1.2}$$

Now, to express (1.2) in terms of h-derivative defined in (0.13) of ψ function , we proceed as follows

$$Q_n = h\psi(nh,t), Q_{n-1} = h\psi((n-1)h,t), Q_{n+1} = h\psi((n+1)h,t) \tag{1.3}$$

$$\begin{aligned} D_h^2\psi((n-1)h,t) &= \frac{1}{h}\left[\frac{\psi((n+1)h,t)-\psi(nh,t)}{h} - \frac{\psi(nh,t)-\psi((n-1)h,t)}{h}\right] \\ &= \frac{1}{h^3}[Q_{n+1} - 2Q_n + Q_{n-1}] \end{aligned} \tag{1.4}$$

Now, using (1.3) and (1.4) in (1.2) it can be written as

$$iD_t\psi(nh,t) = -\frac{1}{2}D_h^2\psi((n-1)h,t) - \frac{1}{2}\left|\psi^2(nh,t)\right|[\psi((n+1)h,t) + \psi((n-1)h,t)]. \tag{1.5}$$

To find a q-analogue of NLSE from its discrete analogue given by (1.5), we make the following substitutions in (1.5)

$$x = nh,\ q^{-1}x = (n-1)h,\ qx = (n+1)h \tag{1.6}$$

and get the following equation

$$iD_{q,t}\psi(x,t) = -\frac{1}{2}D_{q,x}^2\psi\left(q^{-1}x,t\right) - \frac{1}{2}|\psi(x,t)|^2\left[\psi(qx,t) + \psi\left(q^{-1}x,t\right)\right],\ x \in R,\ t \geq 0, \tag{1.7}$$

which is a q-analogue of the NLSE.

In this paper we solve the nonlinear q-Schrödinger equation (1.7) using two-dimensional q-differential transform method. The equation (1.7) in the case $q \to 1$ gives the NLSE (0.2).

2. TWO-DIMENSIONAL *q*-DIFFERENTIAL TRANSFORM (Shahed & Gaber, 2011)

Two-dimensional q-differential transform $F_q(k,h)$ of the function $f(x,y)$ at $(x,y) = (a,b)$ is defined as follows

$$F_q(k,h) = \frac{1}{[k]_q![h]_q!}\left[D_{q,x,y}^{k+h}f(x,y)\right]_{(a,b)}. \tag{2.1}$$

The inverse two-dimensional q-differential transform of $F_q(k,h)$ is given by:

$$f(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} F_q(k,h)(x-a)^{(k)}(y-b)^{(h)}. \tag{2.2}$$

Here $f(x,y)$ is called the original function and $F_q(k,h)$ the transformed function.

Some basic properties of the two-dimensional q-differential transform are as given below (Shahed & Gaber, 2011, Theorems 3, 4, 5 and 8):

Let $F_q(k,h)$, $U_q(k,h)$ and $W_q(k,h)$ be q-differential transforms of the functions $f(x,y)$, $u(x,y)$ and $w(x,y)$ respectively, then the following results hold

(a) If $f(x,y) = u(x,y) \pm w(x,y)$, then $F_q(k,h) = U_q(k,h) \pm W_q(k,h)$.

(b) If $f(x,y) = au(x,y)$, a is constant, then $F_q(k,h) = aU_q(k,h)$.

(c) If $f(x,y) = (x-a)^{(n)}(y-b)^{(m)}, n, m \in N$, then $U_q(k,h) = \delta(k-n)\delta(h-m)$.

where δ is defined as $\delta(k) = \begin{cases} 1, & \text{when } k=0 \\ 0, & \text{otherwise} \end{cases}$.

(d) If $f(x,y) = D_{q,x}u(x,y)$, then $F_q(k,h) = [k+1]_q U_q(k+1,h)$.

(e) If $f(x,y) = D^{r+s}_{q,x,y}u(x,y)$, then $F_q(k,h) = \dfrac{\Gamma_q(k+r+1)\Gamma_q(h+s+1)}{\Gamma_q(k+1)\Gamma_q(h+1)} U_q(k+r,h+s)$.

Two-dimensional q-differential transform of product of two functions (Garg & Chanchlani, 2012)

If $f(x,y) = u(x,y)v(x,y)$, then its two-dimensional q-differential transform $F_q(k,h)$ at $(x,y) = (a,b)$ is given by

$$F_q(k,h) = \sum_{i=0}^{k}\sum_{j=0}^{h} \frac{q^{-i(k-i)-j(h-j)}}{[k-i]_q![h-j]_q!} \sum_{r=0}^{i}\sum_{s=0}^{k-i}\sum_{l=0}^{j}\sum_{m=0}^{h-j} A_q(r,i,s,k-i)A_q(l,j,m,h-j)$$
$$[r+s]_q![l+m]_q! a^{r+s-k+i} b^{l+m-h+j} U_q(r+s,l+m)V_q(i,j), \tag{2.3}$$

where $U_q(k,h)$ and $V_q(k,h)$ are two-dimensional q-differential transforms of functions $u(x,y)$ and $v(x,y)$ respectively and

$$A_q(k,n,j,m) = (q-1)^k q^{-j(m-j)+jk+\binom{k}{2}} \frac{[k]_q!}{[k+j-m]_q!} \binom{n}{k}_q \binom{m}{j}_q. \tag{2.4}$$

Also, the two-dimensional q-differential transform $F_q(k,h)$ of $f(x,y) = u(x,y)v(x,y)$ at $(x,y) = (0,0)$ is given by

$$F_q(k,h) = \sum_{i=0}^{k}\sum_{j=0}^{h} U_q(k-i,h-j)V_q(i,j). \tag{2.5}$$

3. APPLICATION OF TWO DIMENSIONAL *q*-DIFFERENTIAL TRANSFORM METHOD TO SOLVE NONLINEAR *q*-SCHRÖDINGER EQUATION

We consider the following nonlinear q-Schrödinger equation

$$iD_{q,t}\psi(x,t) = -\frac{1}{2}D_{q,x}^2\psi\left(q^{-1}x,t\right) - \frac{1}{2}\left|\psi^2(x,t)\right|\left[\psi(qx,t)+\psi\left(q^{-1}x,t\right)\right],\ x\in R, t\geq 0, \tag{3.1}$$

with the initial condition

$$\psi(x,0^+) = E_q(ix), \quad x\in R, \tag{3.2}$$

To solve the above problem, we write $\psi(x,t) = u(x,t) + iv(x,t)$ so that it transforms into

$$\begin{aligned} iD_{q,t}[u(x,t)+iv(x,t)] &= -\frac{1}{2}D_{q,x}^2\left[u\left(q^{-1}x,t\right)+iv\left(q^{-1}x,t\right)\right] \\ &-\frac{1}{2}\left[u^2(x,t)+v^2(x,t)\right]\left[\left[u(qx,t)+iv(qx,t)\right]+\left[u\left(q^{-1}x,t\right)+iv\left(q^{-1}x,t\right)\right]\right] \end{aligned} \tag{3.3}$$

$$\text{and}\quad u(x,0^+)+iv(x,0^+) = Cos_q(x) + iSin_q(x). \tag{3.4}$$

Equating real and imaginary parts in (3.3) and initial condition (3.4), we get following system of q-partial differential equations

$$\begin{cases} D_{q,t}u(x,t) = -\frac{1}{2}D_{q,x}^2 v\left(q^{-1}x,t\right) - \frac{1}{2}\left[u^2(x,t)+v^2(x,t)\right]\left[v(qx,t)+v\left(q^{-1}x,t\right)\right], \\ D_{q,t}v(x,t) = \frac{1}{2}D_{q,x}^2 u\left(q^{-1}x,t\right) + \frac{1}{2}\left[u^2(x,t)+v^2(x,t)\right]\left[u(qx,t)+u\left(q^{-1}x,t\right)\right], \end{cases} \tag{3.5}$$

with initial conditions
$$\begin{cases} u(x,0^+) = Cos_q(x), \\ v(x,0^+) = Sin_q(x). \end{cases}$$

On applying the two-dimensional q-differential transform (2.1) with $(a,b)=(0,0)$ to (3.5) and (3.6), using properties (a), (b), (c), (e) and result (2.5), we get

$$\begin{cases} U_q(k,h+1) = \dfrac{1}{[h+1]_q}[-\dfrac{1}{2}[k+1]_q[k+2]_q V_q(k+2,h)q^{-k-2} \\ \qquad -\dfrac{1}{2}\displaystyle\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} V_q(k-r-l,p)[q^{k-r-l}+q^{-k+r+l}] \\ \qquad \times [U_q(r,h-s-p)U_q(l,s)+V_q(r,h-s-p)V_q(l,s)]], \\ V_q(k,h+1) = \dfrac{1}{[h+1]_q}[\dfrac{1}{2}[k+1]_q[k+2]_q U_q(k+2,h)q^{-k-2} \\ \qquad +\dfrac{1}{2}\displaystyle\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U_q(r,h-s-p)[q^{r}+q^{-r}] \\ \qquad \times [U_q(k-r-l,p)U_q(l,s)+V_q(k-r-l,p)V_q(l,s)]], \end{cases} \tag{3.7}$$

and

$$U_q(k,h) = \begin{Bmatrix} 0, & k \text{ is odd} \\ \dfrac{(-1)^{\frac{k}{2}}}{[k]_q!}, & k \text{ is even} \end{Bmatrix}, \qquad \mathrm{V}_q(k,h) = \begin{Bmatrix} \dfrac{(-1)^{\frac{k-1}{2}}}{[k]_q!}, & k \text{ is odd} \\ 0, & k \text{ is even} \end{Bmatrix}, \tag{3.8}$$

where $U_q(k,h)$ and $V_q(k,h)$ are two dimensional q-differential transforms of $u(x,y)$ and $v(x,y)$ respectively.

We now obtain the values of $U_q(k,h)$ and $V_q(k,h)$, using (3.7) and (3.8) with the help of software Mathematica. As the expressions are large, we present only some of the values in the following Table 1 and Table 2.

Table 1. Transformed function $U_q(k,h)$

$U_q(k,h)$ $h \rightarrow k \downarrow$	0	1	2	3
0	1	0	$\frac{-1+q^2+4q^4-2q^5-3q^6}{4q^6(1+q)}$	0
1	0	$-\frac{-1+q^2+q^4}{2q^3}$	0	$\frac{(-1+q^2+2q^4+2q^6+6q^7-q^8-8q^9+q^{10}-6q^{11}-6q^{12}-4q^{13}+17q^{14}-9q^{15}-2q^{16}+8q^{17}+q^{19})}{8q^{15}(1+q)(1+q+q^2}$
2	$-\frac{1}{(1+q)}$	0	$\frac{(1-q^2-2q^4-4q^6+3q^7+q^9+4q^{10}-3q^{11}+q^{13}+q^{14})}{4q^{10}(1+q)^2}$	0
3	0	$\frac{(-1+q^2+q^4+q^6-q^7+q^8-q^9)}{2q^5(1+q)(1+q+q^2)}$	0	$\frac{(1-q^2-2q^4-2q^6-2q^8-8q^9+4q^{10}+7q^{11}-4q^{12}+14q^{13}+5q^{14}+17q^{15}-6q^{17}-34q^{18}+q^{19}-25q^{20}+51q^{21}+11q^{22}-4q^{23}-28q^{24}-7q^{25}+16q^{26}-10q^{27}+9q^{28}-5q^{29}+q^{30})}{8q^{21}(1+q)^2(1+q+q^2)^{2''}}$

Table 2. Transformed function $V_q(k,h)$

$V_q(k,h)$ $k\downarrow$ $h\rightarrow$	0	1	2	3
0	0	$1-\frac{1}{2q^2}$	0	$\frac{(1-q^2-2q^4-10q^6+3q^7+8q^8+3q^9+20q^{10}-23q^{11}-10q^{12}+9q^{13}+q^{14})}{8q^{12}(1+q)(1+q+q^2)}$
1	1	0	$\frac{(-1+q^2+2q^4+4q^5-3q^6-5q^7+7q^8-5q^9-q^{10})}{4q^8(1+q)}$	0
2	0	$\frac{-(-1+q^2+2q^4-2q^5+q^6}{2q^4(1+q)}$	0	$\frac{(-1+q^2+2q^4+2q^6+11q^8-4q^9-10q^{10}-5q^{11}-10q^{12}+q^{13}-14q^{14}+63q^{15}-34q^{16}-16q^{17}+4q^{18}+7q^{19}+2q^{20}-10q^{21}+5q^{22}+5q^{23}+q^{24}+q^{25}}{8q^{18}(1+q)^2(1+q+q^2)}$
3	$\frac{-1}{(1+q)(1+q+q^2)}$	0	$\frac{(-1+q^2+2q^4+q^6+6q^7-3q^8-8q^9+q^{10}-q^{11}+16q^{12}-16q^{13}+3q^{14}-11q^{15}+10q^{16}-q^{17}-q^{18}+q^{19})}{4q^{12}(1+q)^2(1+q+q^2)}$	0

Substituting the values of $U_q(k,h)$ and $V_q(k,h)$ from Tables 1 and 2, in (2.2) with $a = b = 0$, we shall get the expressions for $u(x,t)$ and $v(x,t)$, which in turn gives us $\psi(x,t) = u(x,t) + iv(x,t)$.

$u(x,t)$

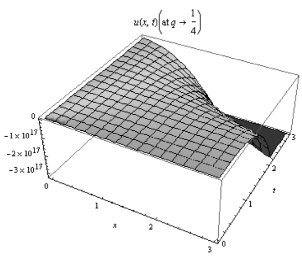

Fig. 1

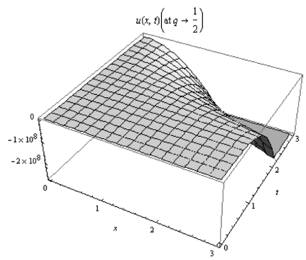

Fig.2

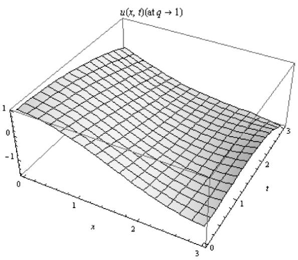

Fig.3

$v(x,t)$

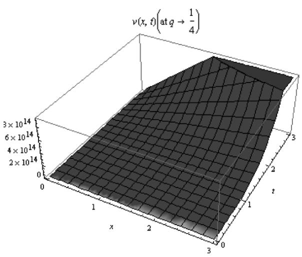

Fig.4

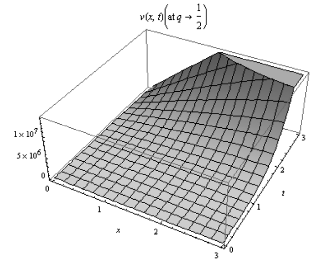

Fig.5

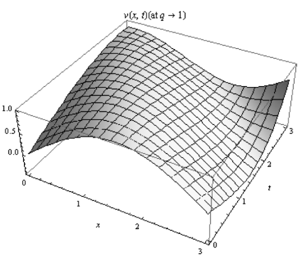

Fig.6

Figures 1 to 6, provide surface plots for approximate values of $u(x,t)$ and $v(x,t)$ for different values of q as $\frac{1}{4}, \frac{1}{2}$ and 1.

Further, in the limiting case $q \to 1$, we get

$$
\begin{aligned}
u(x,t) &= 1 - \left(\frac{x^2}{2} + \frac{tx}{2} + \frac{t^2}{8}\right) + \left(\frac{x^4}{24} + \frac{tx^3}{12} + \frac{t^2x^2}{16} + \frac{t^3x}{48} + \frac{t^4}{384}\right) - \dots\dots\dots\dots \\
&= Cos\left(x + \frac{t}{2}\right),
\end{aligned} \tag{3.9}
$$

$$
\begin{aligned}
v(x,t) &= \left(x + \frac{t}{2}\right) - \left(\frac{x^3}{6} + \frac{tx^2}{4} + \frac{t^2x}{8} + \frac{t^3}{48}\right) + \dots\dots\dots \\
&= Sin\left(x + \frac{t}{2}\right),
\end{aligned} \tag{3.10}
$$

$$\text{giving } \psi(x,t) = Cos\left(x+\frac{t}{2}\right) + iSin\left(x+\frac{t}{2}\right) = e^{i\left(x+\frac{t}{2}\right)} \tag{3.11}$$

which is same as given by Borhanifar & Abazari (2011).

CONCLUSION

In this paper, we have first developed a q-analogue of NLSE from its discrete version and then solved it by two-dimensional q-differential transform method. The solution is obtained in the form of a series and we have drawn some graphs of the solution for different values of the parameter q using the software Mathematica. It is noted that, in the case $q \to 1$, it reduces to the NLSE studied recently by Borhanifa & Abazari (2011).

REFERENCES

Ablowitz, M.J. & Clarkson, P.A. 1999. Solitons and symmetries. J. Eng. Math. **36**(1-2): 1-9.

Andrews, G. E. 1986. q-Series: Their development and application in analysis, combinatories, physics, and computer algebra. Conference Board of the Mathematical Sciences Regional Conference Series in Mathematics 66, American Mathematical Society, Providence, RI.

Biswas, A. & Konar, S. 2006. Introduction to non-Kerr law optical solitons. CRC Press, Boca Raton, FL. USA.

Borhanifa, A. & Abazari, R. 2011. Exact solutions for non-linear Schrödinger equations by differential transformation method. Journal of Applied Mathematics and Computing **35**(1): 37--51.

Ernst, T. 2000. The history of q-calculus and a new method (Licentiate Thesis). U.U.D.M. report: http://math.uu.se/thomas/Lics.pdf

Exton, H. 1983. q-Hypergeometric functions and applications. Ellis Horwood, Chichester.

Garg, M. & Chanchlani, L. 2012. On two-dimensional q-differential transform method. Afrika Matematika. doi:10.1007/s13370-013-0133-y.

Gasper, G. & Rahman, M. 1990. Basic hypergeometric series. Cambridge University. Press, Cambridge.

Green, P. & Biswas, A. 2010. Bright and dark solitons with time-dependent coefficients in a non-Kerr law media. Communications in Nonlinear Science and Numerical Simulation **15**(12): 3865-3873.

Hernández Heredero, R. & Levi, D. 2003. The discrete nonlinear schrödinger equation and its lie symmetry reductions. Journal of Nonlinear Mathematical Physics **10**(2): 77-94.

Jana, S. & Konar, S. 2006. Propagation of a mixture of modes of a laser beam in a medium with saturable nonlinearity. Journal of Electromagnetic Waves and Applications **20**(1): 65-77.

Jing, S.C. & Fan, H.Y. 1995. q-Taylor's formula with its q-remainder. Communications in Theoretical Physics **23**(1): 117-120.

Kac V. & Cheung, P. 2002. Quantum calculus. Universitext, Springer, New York.

Konar, S. & Sengupta, A. 1994. Propagation of an elliptic Gaussian laser beam in a medium with saturable nonlinearity. JOSA B **11**(9): 1644-1646.

Porter, M. A. 2009. Experimental results related to dnls equations, in discrete nonlinear schrödinger equation: Mathematical Analysis, Numerical Computations, and Physics Perspectives (P. G. Kevrekidis, Ed.), Springer Tracts in Modern Physics, Heidelberg, Germany.

Rajkovic′, P.M., Stankovic′, M.S. & Marinkovic′, S.D. 2003. On q-iterative methods for solving equations and systems. Novi Sad Journal of Mathematics **33**(2): 127-137.

Sadighi, A. & Ganji, D.D. 2008. Analytic treatment of linear and nonlinear Schrödinger equations: a study with homotopy-perturbation and Adomian decomposition methods. Physics Letters. A **372**(4): 465-469.

Shahed, M. & Gaber, M. 2011. Two-dimensional q-differential transformation and its application. Applied Mathematics and Computation **217**(22): 9165-9172.

Slater, L.J. 1966. Generalized hypergeometric functions. Cambridge University Press, Cambridge.

Srivastava, S. & Konar, S. 2009. Two-component coupled photovoltaic soliton pair in two-photon photorefractive materials under open circuit conditions, Optics & Laser Technology **41**(4): 419-423**.**

Wazwaz, A. 2008. A study on linear and nonlinear Schrödinger equations by the variational iteration method. Chaos Solitons Fractals **37**(4): 1136-1142.

Zhou, J.K. 1986. Differential transformation and its applications for electrical circuits. Huazhong University Press, Wuhan, China.

Submitted : *22/4/2012*
Revised : *20/9/2012*
Accepted : *4/10/2012*

حل لمعادلة شرودينغر غير الخطية بطريقة محول q التفاضلي ثنائي البعدية

[*]مريدولا جارج، [**]لاتا شانشلاني
قسم الرياضيات – جامعة راجستان – جيابور – الهند

خلاصة

نقوم في هذا البحث بإستخراج شبية لمعادلة شرودينغر غير الخطية وذلك من نسختها المتقطعة. ثم نقوم بعد ذلك بحل هذه المعادلة الشبيهة بإستخدام طريقة محول q التفاضلي ثنائي البعدية. ويظهر الحل على شكل سلسلة، ويختزل في هذه الحالة إلى الحل المضبوط لمعادلة شرودينغرغير الخطية التي درسها بورحنيفة وأبزاري. كما نرسم أيضاً بعض البيانات الحل وذلك لقيم مختلفة للوسيط q وذلك بإستخدام برمجيات ما ثماتيكا.