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A note on some new modifications of ridge estimators

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Abstract

Ridge estimator is an alternative to ordinary least square estimator, when there is multicollinearity problem. There are many proposed estimators in literature. In this paper, we propose some new estimators. A Monte Carlo experiment has been conducted for the comparison of the performances of the estimators. Mean squared error (MSE) is used as a performance criterion. The benefits of new estimators are illustrated using a real dataset. According to both simulation results and application, our new estimators have better performances in the sense of MSE in most of the situations.

Keywords: Monte Carlo simulation; MSE; multicollinearity; OLS; ridge estimator.

1. Introduction

Consider the following standard multiple linear regression model

$$y = X\beta + \varepsilon \quad (1)$$

where y is an $n\times 1$ vector of dependent variable, X is an $n\times p$ design matrix consisting explanatory variables as columns, where p is the number of explanatory variables, β is a $p\times 1$ vector of regression coefficients and ε is an $n\times 1$ error vector distributed normally with zero mean and variance σ^2. The ordinary least squares (OLS) estimator of the coefficient vector β is defined as follows:

$$\hat{\beta} = \left(X'X\right)^{-1} X'y. \quad (2)$$

The well-known problem of multicollinearity arises due to linear dependencies between the explanatory variables, which lead to large variance and large mean squared error (MSE). As a consequence, we observe ineffective inference and prediction in regression focus parameters. For instance, this situation occurs in econometric data.

In literature, there are various methods to solve multicollinearity. One of the most popular method is the ridge regression, firstly suggested by Hoerl & Kennard (1970), which is defined as

$$\hat{\beta}_R = \left(X'X + kI_p\right)^{-1} X'y \quad (3)$$

where $k > 0$.

The mean squared error (MSE) of the ridge estimator $\hat{\beta}_R$ is given as follows

$$MSE\left(\hat{\beta}_R\right) = \sigma^2 \sum_{j=1}^{p} \frac{\lambda_i}{\left(\lambda_j + k\right)^2} + k^2 \beta' \left(X'X + kI_p\right)^{-2} \beta \quad (4)$$

where λ_j's are the eigenvalues of the matrix $X'X$.

The first term of Equation (4), namely, the asymptotic variance function is monotonically decreasing and the second term, namely, the squared bias function is a monotonically increasing in parameter k. Thus, there is some k such that $MSE\left(\hat{\beta}_R\right)$ is less than $MSE\left(\hat{\beta}_{OLS}\right) = \sum_{j=1}^{p}\left(1/\lambda_j\right)$. However, $MSE\left(\hat{\beta}_R\right)$ depends on the parameters σ^2, β and k which are unknown in practice. Thus, k should be estimated from the data. Most of the papers on ridge regression discusses the methods of estimating the ridge parameter k.

In recent papers, the new suggested estimators have been compared to the one proposed by Hoerl & Kennard (1970) and each other. Many of the studies in this area suggest different estimators of ridge parameter. For detailed discussions we refer to the following studies; Hoerl *et al.* (1975), Lawless & Wang (1976), Saleh & Kibria (1993), Kibria (2003), Khalaf & Shukur (2005), Alkhamisi *et al.* (2006), Muniz & Kibria (2009), Mansson *et al.* (2010) and Muniz *et al.* (2012).

The purpose of this study is to investigate the estimation methods of ridge parameter in the literature and make a comparison between them by conducting a Monte Carlo simulation. We also suggest some new modifications of the estimator defined by Lawless & Wang (1976). We use MSE criterion to compare the performances of the estimators. The organization of this paper is as follows. In section 2, we present the methodology and propose some new estimators. In section 3, we provide the details of Monte Carlo simulation. Moreover, results and discussions aregiven in section 4. Finally, we analyze a real data example to illustrate the benefits of new estimators in section 5.

2. Methodology and proposed estimators

Firstly, we review the generalized ridge regression (Alkhamisi & Shukur, 2007). To do so, we write the general model (1) in canonical form. Assume that the matrix D includes the eigenvectors of $X'X$ as columns such that $D'X'XD=\Lambda=\text{diag}\left(\lambda_1,\lambda_2,\ldots,\lambda_p\right)$ where λ_j 's are the eigenvalues of the matrix $X'X$. Let us substitute $Z=XD$ and $\alpha=D'\beta$ in model (1), then the canonical version of (1) is given by the following equation

$$y=Z\alpha+\varepsilon. \tag{5}$$

Thus the generalized ridge estimator is given as follows

$$\hat{\alpha}_R=\left(Z'Z+K\right)^{-1}Z'\text{y} \tag{6}$$

where $K=\text{diag}\left(k_1,k_2,\ldots,k_p\right)$ such that $k_j>0$ for each $j=1,2,\ldots,p$. The OLS estimator of α can be computed as follows

$$\hat{\alpha}_{OLS}=\Lambda^{-1}Z'\text{y}. \tag{7}$$

The MSE of $\hat{\alpha}_R$ and $\hat{\alpha}_{OLS}$ are respectively obtained as

$$MSE\left(\hat{\alpha}_R\right)=\sum_{j=1}^{p}\frac{\sigma^2\lambda_i}{\left(\lambda_j+k_j\right)^2}+\sum_{j=1}^{p}\frac{k_j^2\alpha_j^2}{\left(\lambda_j+k_j\right)^2} \tag{8}$$

and

$$MSE\left(\hat{\alpha}_{OLS}\right)=\sigma^2\sum_{j=1}^{p}\frac{1}{\lambda_j}. \tag{9}$$

Hoerl & Kennard (1970) showed that the value of k_j minimizing (8) is

$$k_j=\frac{\sigma^2}{\alpha_j^2} \tag{10}$$

where σ^2 is the error variance and α_j is the i^{th} element of α. Since σ^2 and α_j^2 are not known, they suggested to use the common unbiased estimators $\hat{\sigma}^2$ and $\hat{\alpha}$ respectively and obtained $\hat{k}_j=\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}$ where $\hat{\sigma}^2=\left(y'y-\hat{\alpha}'Z'y\right)/\left(n-p\right)$.

Now, we review some proposed estimators in previous research works:

The estimator

$$k_{HK}=\frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2} \tag{11}$$

was suggested by Hoerl & Kennard (1970) where $\hat{\alpha}_{\max}$ is the maximum element of $\hat{\alpha}_i$. Likewise the estimator

$$k_{HKB}=\frac{p\hat{\sigma}^2}{\sum_{j=1}^{p}\hat{\alpha}_j^2} \tag{12}$$

which is the harmonic mean of $\hat{k}_j$ and suggested by Hoerl *et al.* (1975).

$$k_{LW}=\frac{p\hat{\sigma}^2}{\sum_{j=1}^{p}\lambda_j\hat{\alpha}_j^2} \tag{13}$$

which is the harmonic mean of $k_{LW(j)}=\frac{\hat{\sigma}^2}{\lambda_j\hat{\alpha}_j^2}$ and proposed by Lawless & Wang (1976) whereas

$$k_{AD}=\frac{2p\hat{\sigma}^2}{\sum_{j=1}^{p}\lambda_{\max}\hat{\alpha}_j^2} \tag{14}$$

as an estimator which is the harmonic mean of $k_{AD(j)}=\frac{2\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_j^2}$ proposed by Dorugade (2014). The estimators

$$k_{KM8}=\max\left(\frac{1}{\sqrt{\frac{\lambda_{\max}\hat{\sigma}^2}{\left(n-p\right)\hat{\sigma}^2+\lambda_{\max}\hat{\alpha}_j^2}}}\right) \tag{15}$$

and

$$k_{KM12}=\text{median}\left(\frac{1}{\sqrt{\frac{\lambda_{\max}\hat{\sigma}^2}{\left(n-p\right)\hat{\sigma}^2+\lambda_{\max}\hat{\alpha}_j^2}}}\right) \tag{16}$$

were defined by Muniz *et al.* (2012).

We define our new estimators which are modifications of $k_{LW(j)}=\frac{\hat{\sigma}^2}{\lambda_j\hat{\alpha}_j^2}$ proposed by Lawless & Wang (1976) for the generalized ridge regression. To achieve our desired estimators, we apply the square root transformation to this individual parameter and get $\sqrt{\frac{\hat{\sigma}^2}{\lambda_j\hat{\alpha}_j^2}}$ in a similar manner performed in Muniz & Kibria (2009) and

Mansson *et al.* (2010). After this transformation, we apply arithmetic mean, geometric mean and harmonic mean transformations and we also use maximum, minimum and median functions following Kibria (2003) and Muniz *et al.* (2012).

Thus, we get the following new estimators: Let $k_{Yj} = \sqrt{\frac{\hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2}}$,

$$k_{Y1} = \frac{1}{p}\sum_{j=1}^{p}\sqrt{\frac{\hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2}} \tag{17}$$

which is the arithmetic mean of k_{Yj} 's.

Similarly, the estimator

$$k_{Y2} = \left(\prod_{j=1}^{p}\sqrt{\frac{\hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2}}\right)^{1/p} \tag{18}$$

is the geometric mean of k_{Yj} 's whereas the estimator

$$k_{Y3} = \text{median}\left(k_{Yj}\right)_{j=1}^{p} \tag{19}$$

is the median of k_{Yj} 's.

Likewise the estimator we suggest

$$k_{Y4} = \max\left(k_{Yj}\right)_{j=1}^{p} \tag{20}$$

is the maximum of k_{Yj} 's and the estimator

$$k_{Y5} = \text{median}\left(1/k_{Yj}\right)_{j=1}^{p} \tag{21}$$

is the median of $1/k_{Yj}$'s. In a similar fashion, we propose the estimator

$$k_{Y6} = \max\left(1/k_{Yj}\right)_{j=1}^{p} \tag{22}$$

which is the median of $1/k_{Yj}$'s and the estimator

$$k_{Y7} = \frac{1}{p}\sum_{j=1}^{p}\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2}}} \tag{23}$$

the mean of $1/k_{Yj}$'s.

Finally, we present the estimators

$$k_{Y8} = \frac{p}{\sum_{j=1}^{p}\sqrt{\frac{\lambda_j \hat{\alpha}_j^2}{\hat{\sigma}^2}}} \tag{24}$$

and

$$k_{Y9} = \frac{p}{\sum_{j=1}^{p}\sqrt{\frac{\hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2}}} \tag{25}$$

which are and the harmonic means of k_{Yj} 's and $1/k_{Yj}$'s respectively.

All of these estimators are compared via a Monte Carlo simulation and details of the simulation are given in section 4.

3. The Monte Carlo simulation

In this section, we provide the design of the Monte Carlo simulation, which is conducted to compare the performances of the estimators. In order to conduct a meaningful simulation, we need to specify the effective properties of the estimators and the performance criteria. Effective factors in this simulation are the degree of correlation ρ among variables, the error variance σ^2, the number of explanatory variables p and the data size n. Also the mean squared error of the estimators has been chosen to be the performance criteria for the simulation. In order to get different degrees of multicollinearity and to generate the explanatory variables, we used the following generally used equation (Kibria (2003)):

$$x_{ij} = \left(1-\rho^2\right)^{1/2} z_{ij} + \rho z_{ip} \tag{26}$$

where $i = 1,2,\ldots,n, j = 1,2,\ldots,p$, ρ^2 is the correlation between the explanatory variables and z_{ij}'s are independent pseudo-random numbers following the standard normal distribution. The vector of dependent variable y is generated by

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} + \varepsilon_i \tag{27}$$

where $i = 1,2,\ldots,n$ satisfying $\beta'\beta = 1$ where β is the eigenvector corresponding to the largest eigenvalue of $X'X$ in order to get a minimized MSE due to Kibria (2003) and ε_i has zero mean and variance σ^2.

We consider three different degrees of correlation, namely, $\rho = 0.90, 0.95$ and 0.99. The sample size varies as $n = 50, 100$ and 200. The number of explanatory variables are chosen as $p = 4$ and 8. Finally, the error variance is chosen as $\sigma^2 = 1.0$ and 5.0. For the values of ρ, n, p and σ^2, the simulation is repeated 5000 times by producing the errors in Equation (27). For each replicate we compute $MSE\left(\hat{\alpha}_R\right)$ and $MSE\left(\hat{\alpha}\right)$ by using the following equation

$$MSE\left(\hat{\alpha}_r\right) = \frac{1}{5000}\sum_{r=1}^{5000}\left(\hat{\alpha}_r - \alpha\right)'\left(\hat{\alpha}_r - \alpha\right) \tag{28}$$

where $\hat{\alpha}_r$ is the estimator given in previous section at the r^{th} replication.

4. Results and discussions

The results of the simulation have been presented in this section. Performance of an estimator is quantified through the MSE criterion. The average mean squared error (AMSE) values of the estimators, according to ρ, n, p and σ^2 have been given in Tables 1-4. According to tables, all new proposed estimators have better performance than OLS estimator, namely, they have less AMSE than OLS estimator.

Increasing the sample size has a positive effect on the estimators, i.e, for large values of sample size, ASME values decrease as it is seen from Figure 1. It is obvious from tables that when the error variance σ^2 increases, AMSE values increase for all estimators. This result is represented for specific situations, namely, for $p=8, \rho=0.99, n=100$ in Figure 2. Moreover, an increase in the degree of correlation makes a negative effect as it is observed from Figure 3.

For the case $p=4$ and $\sigma^2=1.0$, the estimator k_{Y6} has the best performance among all of the estimators. However, k_{KM8} is superior to other when $\rho=0.99$ and small sample sizes. For the case $p=4$ and $\sigma^2=5.0$, k_{HKB} becomes the best estimator for lower degrees of correlation and k_{Y4} has the lowest AMSE values for high degrees of correlation.

Moreover, for the case $p=8$ and $\sigma^2=1.0$, k_{Y6} has the best estimator most of the time and k_{Y4} has the lowest AMSE when $\rho=0.99$. Similarly, when $p=8$ and $\sigma^2=5.0$, although k_{Y4} has the lowest AMSE most of the time, k_{HKB} is superior to other estimators for lower degree of correlation and large sample sizes.

Table 1. Average MSEs of the estimator when $p = 4$, $\sigma^2 = 1.0$

ρ	0.90			0.95			0.99		
n	50	100	200	50	100	200	50	100	200
Y1	0.3275	0.2120	0.1171	0.5061	0.3478	0.2122	0.8369	0.6956	0.5800
Y2	0.3301	0.2141	0.1119	0.5259	0.3662	0.2128	0.8789	0.7369	0.6196
Y3	0.3303	0.2127	0.1110	0.5361	0.3687	0.2113	0.9210	0.7695	0.6430
Y4	0.3476	0.2281	0.1360	0.5000	0.3505	0.2300	0.7955	0.6695	0.5593
Y5	0.3034	0.1985	0.1072	0.4811	0.3288	0.1977	0.9808	0.7538	0.5796
Y6	0.2333	0.1433	0.0883	0.3576	0.2278	0.1489	0.7011	0.5361	0.4110
Y7	0.2796	0.1778	0.0990	0.4451	0.2949	0.1799	0.8821	0.6845	0.5329
Y8	0.3551	0.2296	0.1154	0.5856	0.4081	0.2256	1.0020	0.8504	0.7113
Y9	0.3401	0.2173	0.1114	0.5876	0.3906	0.2145	1.5461	1.1214	0.7715
LW	0.3912	0.2472	0.1187	0.6935	0.4727	0.2408	1.3995	1.1668	0.8996
HK	0.3412	0.2147	0.1118	0.5908	0.3851	0.2132	2.0763	1.3688	0.7990
HKB	0.3048	0.1864	0.1034	0.5017	0.3218	0.1883	1.6540	1.0655	0.6414
AD	0.4151	0.2545	0.1194	0.8508	0.5302	0.2475	4.5628	2.9008	1.3476
KM8	0.3001	0.2008	0.1081	0.4408	0.3227	0.2009	0.6666	0.5320	0.4849
KM12	0.3399	0.2221	0.1140	0.5476	0.3873	0.2211	0.8626	0.7748	0.6789
OLS	0.4191	0.2553	0.1195	0.8651	0.5332	0.2478	4.7208	2.9418	1.3526

Table 2. Average MSEs of the estimator when $p = 4$, $\sigma^2 = 5.0$

ρ	0.90			0.95			0.99		
n	50	100	200	50	100	200	50	100	200
Y1	1.3244	0.9091	0.5156	1.8762	1.4019	0.9107	2.3209	2.1786	2.0446
Y2	1.4955	1.0095	0.5453	2.2153	1.6270	1.0050	2.8943	2.7179	2.5044
Y3	1.5195	1.0207	0.5445	2.2940	1.6716	1.0129	3.1440	2.9083	2.6499
Y4	1.0670	0.7590	0.4707	1.3751	1.0714	0.7632	1.5299	1.4527	1.4133
Y5	1.5530	1.0046	0.5354	2.5217	1.7215	1.0004	4.5115	3.8330	3.0642
Y6	1.2195	0.8069	0.4613	1.7448	1.2636	0.8176	2.1889	2.0900	1.9193
Y7	1.5094	0.9768	0.5230	2.4047	1.6591	0.9758	3.8951	3.4471	2.8753
Y8	1.6341	1.0801	0.5633	2.5611	1.8252	1.0682	3.7627	3.4242	3.0232
Y9	1.7290	1.0977	0.5582	3.0385	2.0087	1.0835	7.7538	5.8839	4.0391
LW	1.7626	1.1505	0.5812	2.9230	2.0334	1.1311	5.4392	4.5521	3.6558
HK	1.1732	0.7721	0.4508	2.0031	1.3138	0.7729	8.4268	5.3113	2.8338
HKB	0.9558	0.6269	0.3874	1.5987	1.0296	0.6291	6.8312	4.1164	2.1880
AD	2.0454	1.2652	0.5963	4.1946	2.6331	1.2348	22.6858	14.4506	6.7237
KM8	1.5875	1.0652	0.5605	2.2805	1.7238	1.0642	1.8245	2.1179	2.4347
KM12	1.7188	1.1216	0.5701	2.8197	2.0059	1.1139	3.6712	3.9506	3.5952
OLS	2.0956	1.2766	0.5975	4.3257	2.6660	1.2388	23.6041	14.7092	6.7628

Table 3. Average MSEs of the estimator when $p = 8$, $\sigma^2 = 1.0$

ρ	0.90			0.95			0.99		
n	50	100	200	50	100	200	50	100	200
Y1	0.5670	0.3797	0.2168	0.7396	0.5627	0.3627	1.0358	0.9590	0.8317
Y2	0.7215	0.4517	0.2330	0.9407	0.7007	0.4146	1.2997	1.2182	1.0199
Y3	0.7554	0.4567	0.2333	1.0144	0.7271	0.4197	1.4980	1.3626	1.1021
Y4	0.4583	0.3292	0.2274	0.5928	0.4615	0.3457	0.8610	0.7695	0.6990
Y5	0.8515	0.4882	0.2415	1.1811	0.8097	0.4477	2.1570	1.7801	1.3128
Y6	0.3968	0.2591	0.1600	0.5850	0.4062	0.2654	0.9830	0.8592	0.7221
Y7	0.7139	0.4313	0.2235	1.0146	0.7028	0.4038	1.8353	1.5452	1.1684
Y8	0.9032	0.5157	0.2499	1.1809	0.8445	0.4665	1.7150	1.5654	1.2471
Y9	1.1448	0.5443	0.2532	1.7840	0.9923	0.4941	4.0779	2.9249	1.8220
LW	1.2359	0.5864	0.2640	1.6434	1.0502	0.5240	2.4264	2.1084	1.5833
HK	0.9926	0.4804	0.2397	1.7876	0.8400	0.4433	5.2419	3.2713	1.6524
HKB	0.5774	0.3047	0.1778	1.0317	0.5035	0.2985	3.1291	1.8972	0.9943
AD	1.8030	0.6249	0.2684	3.8716	1.3128	0.5614	11.3430	7.2704	3.0909
KM8	0.6642	0.4639	0.2368	0.7984	0.6998	0.4254	0.8952	0.9351	0.9341
KM12	1.1122	0.5537	0.2561	1.5154	0.9858	0.5022	2.0181	1.9750	1.5903
OLS	1.8328	0.6271	0.2687	3.9512	1.3198	0.5623	11.5169	7.3363	3.1027

Table 4. Average MSEs of the estimator when $p = 8$, $\sigma^2 = 5.0$

ρ	0.90			0.95			0.99		
n	50	100	200	50	100	200	50	100	200
Y1	2.5268	1.7697	1.0046	3.0642	2.5095	1.6473	3.5730	3.4507	3.2286
Y2	3.3543	2.1772	1.1446	4.2378	3.2832	1.9962	5.2919	5.1758	4.4695
Y3	3.6498	2.2636	1.1619	4.7105	3.5243	2.0715	6.1660	6.0272	4.9788
Y4	1.3927	1.1165	0.7606	1.5550	1.4004	1.0912	1.6847	1.5909	1.5635
Y5	4.3526	2.4463	1.2071	6.1623	4.1120	2.2461	10.9866	9.2113	6.7023
Y6	2.5909	1.7378	0.9666	3.3631	2.6181	1.6582	3.9440	4.2138	3.7068
Y7	4.0102	2.3353	1.1719	5.5955	3.8621	2.1579	9.2201	8.1761	6.1248
Y8	4.0289	2.4214	1.2117	5.2722	3.8510	2.2028	7.3750	6.9591	5.5651
Y9	5.8094	2.7305	1.2669	9.2274	5.0156	2.4796	20.9435	15.4628	9.6460
LW	4.8429	2.6461	1.2689	6.5049	4.4026	2.3819	10.3513	8.8126	6.6613
HK	3.8459	1.6566	0.9060	7.6554	3.1001	1.5839	24.4082	15.7781	7.1248
HKB	2.2390	1.0145	0.5831	4.3854	1.8160	0.9686	14.4423	9.1194	4.2350
AD	8.9600	3.1143	1.3401	19.2805	6.5465	2.8023	56.6544	39.9325	17.6938
KM8	3.4722	2.4634	1.2369	4.0338	3.6972	2.2569	4.7571	4.6206	4.5134
KM12	5.6448	2.7720	1.2806	7.8728	4.9735	2.5133	10.9230	10.4046	8.4176
OLS	9.1638	3.1356	1.3434	19.7561	6.5992	2.8117	57.5846	40.3097	17.7640

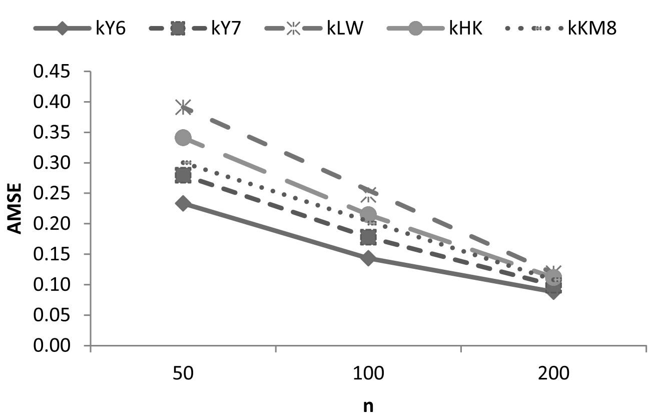

Fig. 1. AMSE values of some estimators for changing values of n when $p=4, \sigma^2=1.0, n=100$

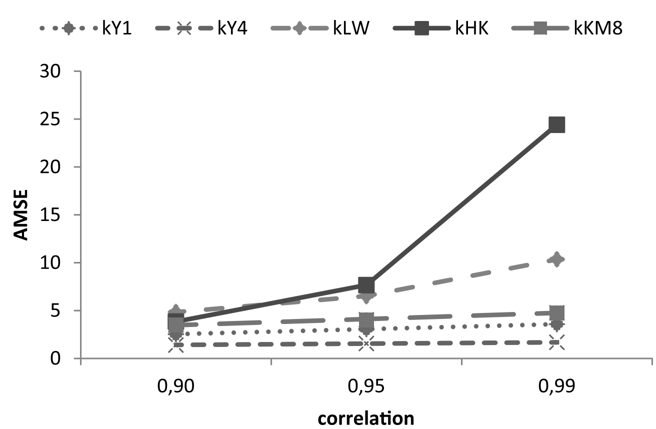

Fig. 3. AMSE values of some estimators for different degrees of correlation when $p=8, \sigma^2=5.0, n=50$

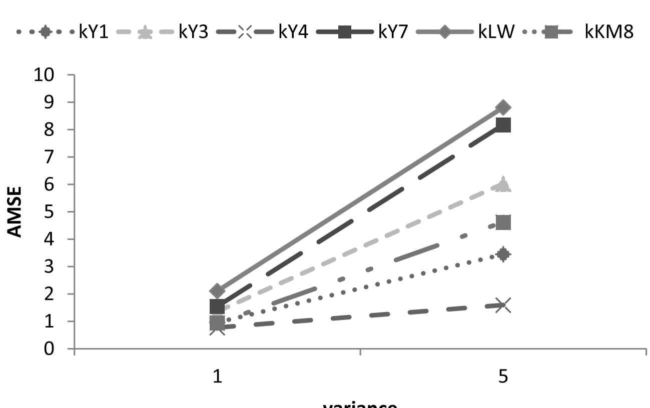

Fig. 2. AMSE values of some estimators for changing values of variance when $p=8, \rho=0.99, n=100$

5. Real data application

In order to show the performances of new estimators, we use a well-known real data set which was studied originally by Stamey *et al.* (1989). The data represents the relationship between the dependent variable y, the logarithm of prostate-specific antigen (lpsa) and the explanatory variables which are log (cancer volume) (lcavol), log (prostate weight) (lweight), age (age), the logarithm of benign prostatic hyperplasia amount (lbph), log (capsular penetration) (lcp), seminal vesicle invasion (svi), Gleason score (gleason), and percent of Gleason scores 4 or 5 (pgg45).

The eigenvalues of the matrix $X'X$ are obtained as 8.0931, 20.2390, 44.5238, 64.7986, 175.6339, 210.9038, 6.1907e+4 and 4.7908e+5. The condition number $\kappa = \max eigvalue / \min eigvalue$ of the data is approximately 5.9196e+04which show that there is a severe collinearity problem with this dataset. The estimated theoretical MSE values of the estimators considered in this study are reported in Table 5 by using equations (8) and (9) (Please see the Appendix for R codes.). According to Table 5, k_{Y9} and k_{KM8} have the least MSE values. Moreover, the estimators $k_{Y1}, k_{Y2}, k_{Y3}, k_{Y8}$ have less MSE values than that of OLS.

Table 5. Estimated theoretical MSE values of the estimators

Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
0.10133	0.10548	0.10552	0.13782	0.11760	0.48129	0.18193	0.10786
Y9	LW	HK	HKB	AD	KM8	KM12	OLS
0.10073	0.11005	0.12056	0.11731	0.11015	0.10089	0.10333	0.11015

6. Conclusion

In this study, we proposed new ridge estimators, which are modifications of the estimator k_{LW} defined by Lawless & Wang (1976) and studied the properties of new modified estimators for choosing ridge parameter, when there is multicollinearity between the explanatory variables. We compared the estimators proposed earlier to our new proposed estimators through a Monte Carlo simulation having 5000 replications for each combination. Average mean squared error (AMSE) has been chosen to be the evaluation criterion for the simulation. We created tables consisting of AMSE values according to different values of the sample size n, the degree of correlation ρ, the number of predictors p and the variance of error terms σ^2. We have provided some figures for selected situations. According to tables and figures, we may say that our new suggestions for ridge estimators are better than the others for most of the cases. Especially k_{Y4} and k_{Y6} have smaller ASME values in most of the situations. Moreover, we considered a real dataset to illustrate the performances of estimators in the sense of MSE and presented the benefit of using the new estimators. k_{Y9} and k_{KM8} have quite less MSE value than the others and OLS.

References

Alkhamisi, M.A., Khalaf, G. & Shukur, G. (2006). Some modifications for choosing ridge parameters. Communications in Statistics - Theory and Methods, **35**(11):2005-2020.

Alkhamisi, M.A. & Shukur, G. (2007). A Monte Carlo study of recent ridge parameters. Communications in Statistics - Simulation and Computation, **36**(3):535-547.

Dorugade, A.V. (2014). New ridge parameters for ridge regression. Journal of the Association of Arab Universities for Basic and Applied Sciences, **15**:94-99.

Hoerl, A.E. & Kennard, R.W. (1970). Ridge regression: Biased estimation for nonorthogonal problems. Technometrics, **12**(1):55-67.

Hoerl, A.E., Kennard, R.W. & Baldwin, K.F. (1975). Ridge regression: some simulations. Communications in Statistics - Theory and Methods, **4**(2):105-123.

Khalaf, G. & Shukur, G. (2005). Choosing ridge parameter for regression problems. Communications in Statistics - Theory and Methods, **34**(5):1177-1182.

Kibria, B.M.G. (2003). Performance of some new ridge regression estimators. Communications in Statistics-Simulation and Computation, **32**(2):419-435.

Lawless, J.F. & Wang, P. (1976). A simulation study of ridge and other regression estimators. Communications in Statistics - Theory and Methods, **5**(4):307-323.

Mansson, K., Shukur, G. & Kibria, B.G. (2010). On some ridge regression estimators: A Monte Carlo simulation study under different error variances. Journal of Statistics, **17**(1):1-22.

Muniz, G. & Kibria, B.G. (2009). On some ridge regression estimators: An empirical comparisons. Communications in Statistics - Simulation and Computation, **38**(3):621-630.

Muniz, G., Kibria, B.G., Mansson, K. & Shukur, G. (2012). On developing ridge regression parameters: a graphical investigation. Sort: Statistics and Operations Research Transactions, **36**(2):115-138.

Saleh, A.M. & Kibria, B.M.G. (1993). Performance of some new preliminary test ridge regression estimators and their properties. Communications in Statistics-Theory and Methods, **22**(10):2747-2764.

Stamey, T.A., Kabalin, J.N., McNeal, J.E., Johnstone, I.M., Freiha, F., *et al.* (1989). Prostate specific antigen in the diagnosis and treatment of adenocarcinoma of the prostate: II. radical prostatectomy treated patients, Journal of Urology, **141**(5):1076–1083.

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نبذة حول بعض التعديلات الجديدة في مقدرات ريدج (ridge)

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ملخص

مقدر ريدج (ridge) هو بديل لمقدر المربعات الصغرى الاعتيادية عند وجود مشكلة العلاقات الخطية المتعددة. يوجد العديد من المقدرات المقترحة المنشورة. ونقترح في هذا البحث بعض المقدرات الجديدة. تم إجراء تجربة مونت كارلو للمقارنة بين أداء المقدرات. وتم استخدام متوسط مربعات الأخطاء (MSE) كمعيار للأداء. وتم ايضاح فوائد المقدرات الجديدة باستخدام مجموعة بيانات حقيقية. ووفقا لكل من نتائج المحاكاة والتطبيق، فإن مقدراتنا الجديدة تتمتع بأداء أفضل من ناحية متوسط مربعات الأخطاء (MSE) في معظم الحالات.