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Multidimensional negative definite functions on the product of commutative hypergroups

Hossam A. Ghany[1,2]

[1]*Dept. of Mathematics, Taif University, Hawea (888), Taif, Saudi Arabia*

[2]*Dept. of Mathematics, Helwan University, Alameria (11282) Cairo, Egypt*

Email: h.abdelghany@yahoo.com

Abstract

The main aim of this paper is to give the integral representations for the so called multidimensional negative definite functions defined on the product of commutative hypergroups. Some necessary and sufficient conditions that guarantee, the product of multidimensional positive definite functions defined on the product of hypergroups is also multidimensional positive definite, are obtained. Moreover, many relations that governed the relations between the classes of multidimensional positive and negative definite functions are shown. Also, we prove that a continuous function with compact support ψ is multidimensional negative definite on the product of hypergroups, if and only if $exp(-t\psi)$ is multidimensional positive definite for each $t > 0$.

Keywords: Hypergroup; Lévy-Khinchin; negative definite; positive definite; Radon measure.

1. Introduction

A hypergroup (Bloom & Heyer, 1989; Dunkl, 1973; Jewett, 1975) is a locally compact Hausdorff space K with a certain convolution structure $*$ on the space of complex Radon measures on K, $M(K)$. The main problem in harmonic analysis in various setting is the existence of a product, usually called convolution, for functions and measures. Let δ_x be the Dirac measure at a point $x \in K$. Then the convolution $\delta_x * \delta_y$ of the two point measures δ_x and δ_y is a probability Radon measure on K with compact support and such that $(x, y) \mapsto supp(\delta_x * \delta_y)$ is a continuous mapping from $K \times K$ into the space of compact subsets of K. Unlike the case of groups, this convolution is not necessarily the point measure $\delta_{x.y}$ for a composition $x.y$ in K. A hypergroup $(K,*)$ is called commutative, if $(M(K),+,*)$ is a commutative algebra and hermitian, if the involution $^-$ is the identity map. Its easy to prove that every hermitian hypergroup is commutative. A locally bounded measurable function $\chi : K \to C$ is called a semicharacter, if $\chi(e) = 1$ and $\chi(x * y^-) = \chi(x)\overline{\chi(y)}$ for all $x, y \in K$. Every bounded semicharacter is called a character. If the character is not locally null then (Bloom & Heyer, 1995) it must be continuous. We will denote by $C(K), C_b(K), C_0(K)$ and $C_c(K)$ the spaces of continuous functions on K, that are bounded, vanish at infinity and that with compact support respectively. The dual K^* of K is just the set of continuous characters with the compact-open topology in which case K^* must be locally compact. In this paper we will be concerned with continuous characters on hypergroups. A locally bounded measurable function $\phi : K \to C$ is said to be positive definite if

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_i \overline{c_j} \phi(z_i * z_j^-) \geq 0$$

for all choice of $z_1, z_2, ..., z_n \in K, c_1, c_2, ..., c_n \in C$ and $n \in N$. We will denote by $K_1, K_2, ..., K_m$ hypergroups with convolutions $*_1, *_2, ..., *_m$ and involutions $^{-1}, ^{-2}, ..., ^{-m}$ respectively. A locally bounded measurable functional Φ defined on the product

$$\prod_{i=1}^{m} K_i := K_1 \times K_2 \times ... \times K_m$$

is called multidimensional positive definite on the product $\prod_{i=1}^{m} K_i$ (Ghany & Osaimi, 2014) if

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_i \overline{c_j} \Phi(\overrightarrow{x^i} * \overrightarrow{y^{-j}}) \geq 0$$

for all choice of $\vec{x}^1, \vec{x}^2, ..., \vec{x}^n, \vec{y}^1, \vec{y}^2, ..., \vec{y}^n \in \prod_{i=1}^{m} K_i, c_1, c_2, ..., c_n \in C$ and $m, n \in N$, where the convolution $*$ is defined on $\prod_{i=1}^{m} K_i$ by

$$\vec{x} * \vec{y} := (x_1 *_1 y_1, x_2 *_2 y_2, ..., x_m *_m y_m)$$

From now on, we will be denoted by $MP(\prod_{i=1}^{m} K_i)$, the set of multidimensional positive definite functions on $\prod_{i=1}^{m} K_i$. A locally bounded measurable functional Ψ defined on the product

$$\prod_{i=1}^{m} K_i := K_1 \times K_2 \times ... \times K_m$$

will be called multidimensional negative definite on the product $\prod_{i=1}^{m} K_i$ if

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_i \overline{c_j} \Psi(\overrightarrow{x^i} * \overrightarrow{y^{-j}}) \leq 0$$

for all choice of $\vec{x}^1, \vec{x}^2, ..., \vec{x}^n, \vec{y}^1, \vec{y}^2, ..., \vec{y}^n \in \prod_{i=1}^{m} K_i$ and for $c_1, c_2, ..., c_n \in C$ such that $\sum_{j=1}^{n} c_i = 0$. We will denoted by $MN(\prod_{i=1}^{m} K_i)$, the set of multidimensional negative definite functions on $\prod_{i=1}^{m} K_i$. The main aim of this paper is to give the integral representations for multidimensional negative definite functions and some of its related functions defined on the product of commutative hypergroups. We will arrive our aim as follows: In § 2, we will give the necessary and sufficient conditions guarantees that the product of two multidimensional positive definite functions defined on the product hypergroups $\prod_{i=1}^{m} K_i$ is also multidimensional positive definite on the product $\prod_{i=1}^{m} K_i$. § 3 is devoted to resume some properties of the set of multidimensional negative definite functions on the product $\prod_{i=1}^{m} K_i$ and to give some relations between the class of multidimensional positive definite functions and the class of the multidimensional negative definite functions. In § 4, we give the integral representations for multidimensional negative definite functions on the product hypergroups $\prod_{i=1}^{m} K_i$.

2. Multidimensional positive definite functions

As pointed out from (Okb El Bab & Ghany, 2010; Okb El Bab *et al.,*2011) the sum and the point-wise limit of positive definite functions on hypergroups are also positive definite, so we can easily prove that:

Lemma 2.1. Suppose that the functions $\phi_1, \phi_2, ..., \phi_m$ are positive definite functions, each of which belongs to $C_c(K_1), C_c(K_2), ..., C_c(K_m)$ respectively, then the sum

$$\Phi(\vec{x}) = \sum_{i=1}^{m} \alpha_i \phi_i(x_i)$$

is multidimensional positive definite on the product $\prod_{i=1}^{m} K_i$, for all $\alpha_i \geq 0$ and $\vec{x} = (x_1, x_2, ..., x_m) \in \prod_{i=1}^{m} K_i$.

Many properties of the convolution for Laplace-Stieltjes transform was established by Galue & Kalla (2013), so by virtue of the above lemma we have the following:

Theorem 2.2. Let $\phi_1, \phi_2, ..., \phi_m$ are bounded measurable positive definite functions belongs to $C_c(K_1), C_c(K_2), ..., C_c(K_m)$ respectively. Then the function

$$\Phi : \prod_{i=1}^{m} K_i \to C$$

defined by

$$\Phi(\vec{x}) = \phi_1(x_1).\phi_2(x_2)...\phi_n(x_m)$$

for all $\vec{x} = (x_1, x_2, ..., x_m) \in \prod_{i=1}^{m} K_i$ is a multidimensional positive definite on $\prod_{i=1}^{m} K_i$ and has an integral representation

$$\Phi(\vec{x}) = \int_{\prod_{i=1}^{m} K_i^*} \chi(\vec{x}) d\mu(\chi).$$

Proof. Recalling from Pederson, 1979 a bounded measurable function $\phi \in C_c(K)$ is positive definite if and only if there exists a ψ in $L^2(K)$ such that $\phi = \psi \bullet \tilde{\psi}$, where

$$f \bullet \tilde{g}(x) = \int_{K^*} f(x * y)\overline{g(y)} d\eta(y).$$

for all $f, g \in C_c(K)$, so without lose of generality, for the function $\Phi : K_1 \times K_2 \to C$ defined by

$$\Phi(x_1, x_2) = \phi_1(x_1).\phi_2(x_2)$$

there exists $\psi_i \in L^2(K_i), i = 1,2$ such that $\phi_1 = \psi_1 \bullet \tilde{\psi}_1$ and $\phi_2 = \psi_2 \bullet \tilde{\psi}_2$, so

$$\phi_1.\phi_2(\vec{x}) = (\psi_1 \bullet \tilde{\psi}_1(x_1)).(\psi_2 \bullet \tilde{\psi}_2(x_2))$$

$$= \int_{K_1^*} \psi_1(x_1 * y)\overline{\psi_1(y)} d\eta(y) \int_{K_2^*} \psi_2(x_2 * z)\overline{\psi_2(z)} d\eta(z)$$

$$= \int_{K_1^*}\int_{K_2^*} \psi_1(x_1 * y)\psi_2(x_2 * z)\overline{\psi_1(y)\psi_2(z)} d\eta(y) d\eta(z)$$

$$= \int_{K_1^*}\int_{K_2^*} \psi_1.\psi_2(x_1 * y, x_2 * z)\overline{\psi_1.\psi_2(y,z)} d\eta(y) d\eta(z)$$

$$= \int_{K_1^*}\int_{K_2^*} \psi_1.\psi_2((x_1, x_2) * (y,z))\overline{\psi_1.\psi_2(y,z)} d\eta(y) d\eta(z)$$

Applying Fubini's theorem to the right hand side we get

$$\phi_1.\phi_2(\vec{x}) = \int_{K_1^* \times K_2^*} \psi_1.\psi_2((x_1, x_2) * (y,z))\overline{\psi_1.\psi_2(y,z)} d\nu(y,z)$$

This implies

$$\phi_1.\phi_2(\vec{x}) = \psi_1.\psi_2 \bullet \psi_1.\psi_2(\vec{x}).$$

Theorem 2.3. Suppose that Φ_1 and Φ_2 are locally bounded measurable functions each of which belongs to $C_c(\prod_{i=1}^m K_i)$, then the product $\Phi_1.\Phi_2$ is a multidimensional positive definite function on $\prod_{i=1}^m K_i$ if and only if Φ_1 and Φ_2 are multidimensional positive definite function on $\prod_{i=1}^m K_i$

Proof. Obviously, we sufficiently need to prove that, a locally bounded measurable function $\Phi:\prod_{i=1}^m K_i \to C$ is multidimensional positive definite if and only if Φ has integral representation of the form

$$\Phi(\vec{x}) = \int_{\prod_{i=1}^m K_i^*} \chi(\vec{x})d\mu(\chi), \qquad \mu \in M_b(\prod_{i=1}^m K_i)$$

for all $\vec{x} \in \prod_{i=1}^m K_i$. In fact, for $\{\vec{x^1}, \vec{x^2}, ..., \vec{x^n}\} \subset \prod_{i=1}^m K_i$, $\{c_1, c_2, ..., c_n\} \subset C$, we can easily find that

$$\sum_{i=1}^n \sum_{j=1}^n c_i \overline{c_j} \Phi(\vec{x^i} + \overrightarrow{y^{-j}}) = \int_{\prod_{i=1}^m K_i^*} |\sum_{i=1}^n c_i \chi(\vec{x^i})|^2 \, d\mu(\chi) \geq 0$$

Moreover, Φ is exponentially bounded. To see this, we define an absolute value Λ_S associated with a locally compact subset $S \subset \prod_{i=1}^m K_i^*$ by

$$\Lambda_S := sup\{\chi(\vec{x}); \chi \in S\}$$

for $S = supp(\mu)$ we have

$$\Phi(\vec{x}) \leq \mu(S)\Lambda_S(\vec{x}) = \Phi(\vec{e})(\vec{x})$$

where $\vec{e} = (e_1, e_2, ..., e_m), e_i$ is the identity element in $K_i, i = 1,2,...,n$, so Φ is bounded with respect to Λ_S. This establish the "only if" part. On the other hand, suppose Φ is multidimensional positive definite, exponentially bounded and not identically zero, there exists an absolute value Λ such that $\Phi^{-1}(\vec{e})\Phi \in P_1^\Lambda(\prod_{i=1}^m K_i)$, the set of Λ-bounded multidimensional positive definite on $\prod_{i=1}^m K_i$ with $\Phi(\vec{e}) = 1$, by Berg *et al.* (1984) there exists a unique Radon probability measure ν on the locally compact set of Λ-bounded characters having $\Phi^{-1}(\vec{e})\Phi$ as barycenter. Using the continuous linear functionals $f \mapsto f(\vec{x})$ on $C^{\prod_{i=1}^m K_i}$, we get

$$\Phi(\vec{x}) = \int_{\prod_{i=1}^m K_i^*} \chi(\vec{x})d\mu(\chi)$$

where $\mu = \Phi(\vec{e})\nu$ belongs to the set of bounded Radon measure on the product $\prod_{i=1}^m K_i$.

Corollary 2.4. Let $\Phi \in C_c(\prod_{i=1}^m K_i)$ be multidimensional positive definite functions such that $|\Phi(\vec{x} * \overrightarrow{x^-})| < \chi$ for all $\vec{x} \in \prod_{i=1}^m K_i$. If $f(z) = \sum_{n=0}^\infty a_n z^n$ is holomorphic in $\{z \in C; z < \chi\}$ and $a_n \geq 0$ for all $n \geq 0$. Then the composed kernel $f \circ \phi$ is a gain multidimensional positive definite. In particular, if $\Phi \in C_c(\prod_{i=1}^m K_i)$ is multidimensional positive definite, then so is $exp(\Phi)$.

Remark. Some types of convolutions between analytic functions can be founded in (El-Ashwah & Aouf, 2014).

3. Multidimensional negative definite functions

One should be observe that, a function ψ is negative definite, if and only if $exp(-t\psi)$ is positive definite for each $t > 0$. While this result holds for all semigroups it is not clear how to prove the 'only if' part for hypergroups since the usual technique do not apply(the 'if' part always holds provided that Reψ is locally lower bounded). The problem is that except when x or y belong to the maximal subgroup of the hypergroup $exp(-t\psi(x*y))$ and $exp(-t\psi)(x*y)$ are usually not equal so that other methods have to be used to overcome this. Recalling the definition of multidimensional positive and negative definite functions the reader can easily prove the following lemmas:

Lemma 3.1. Suppose that $\psi_1, \psi_2, ..., \psi_m$ are negative definite functions on hypergroups $K_1, K_2, ..., K_m$ respectively, such that $\psi_i \in C_c(K_i), i = 1,2,...,m$. Then the functional sum

$$\Psi(\vec{x}) = \sum_{i=1}^m \alpha_i \psi_i(x_i)$$

is multidimensional negative definite on the product $\prod_{i=1}^m K_i$, for all $\alpha_i \geq 0$ and $\vec{x} = (x_1, x_2, ..., x_m) \in \prod_{i=1}^m K_i$.

Lemma 3.2. If Φ is multidimensional positive definite on the product $\prod_{i=1}^m K_i$, then $\Phi(\vec{e}) - \Phi$ is multidimensional negative definite on the product $\prod_{i=1}^m K_i$.

Lemma 3.3. Let $\Psi: \prod_{i=1}^m K_i \times \prod_{i=1}^m K_i \to C$. Put

$$\Phi(\vec{x}, \vec{y}) := \Psi(\vec{x}, \vec{x_0}) + \overline{\Psi(\vec{y}, \vec{x_0})} - \Psi(\vec{x}, \vec{y}) - \Psi(\vec{x_0}, \vec{x_0})$$

for fixed $\vec{x_0} \in \prod_{i=1}^m K_i$. Then Φ is multidimensional positive definite if and only if Ψ is multidimensional negative definite.

Theorem 3.4. A function $\Psi \in C_c(\prod_{i=1}^m K_i)$ is multidimensional negative definite on the product hypergroups $\prod_{i=1}^m K_i$ if and only if $exp(-t\psi)$ is multidimensional positive definite for each $t > 0$.

Proof. Suppose that Ψ is multidimensional negative definite. For obvious reasons we need only show that $exp(-t\psi)$ is multidimensional positive definite for $t = 1$. We choose $\vec{x}_0 \in \prod_{i=1}^{m} K_i$ and with Φ as in the above Lemma we have

$$-\Psi(\vec{x},\vec{y}) := \Phi(\vec{x},\vec{y}) - \overline{\Psi(\vec{y},\vec{x}_0)} - \Psi(\vec{x},\vec{x}_0) + \Psi(\vec{x}_0,\vec{x}_0)$$

where Φ is multidimensional positive definite. Hence

$$exp(-\Psi(\vec{x},\vec{y})) = exp(\Phi(\vec{x},\vec{y})).\overline{exp(-\Psi(\vec{y},\vec{x}_0))}.exp(-\Psi(\vec{x},\vec{x}_0)).exp(\Psi(\vec{x}_0,\vec{x}_0))$$

Since the product, $\overline{exp(-\Psi(\vec{y},\vec{x}_0))}.exp(-\Psi(\vec{x},\vec{x}_0))$, is multidimensional positive definite. From Theorem 2.3 we conclude that $exp(-t\psi)$ is multidimensional positive definite.

4. Lévy Khinchin type formula

A locally bounded measurable function q is called a quadratic form on the product hypergroups, if

$$q(\vec{x} * \vec{y}) + q(\vec{x} * \vec{y}^{\,-}) = 2q(\vec{x}) + 2q(\vec{y})$$

for all $\vec{x},\vec{y} \in \prod_{i=1}^{m} K_i$ and additive if $q(\vec{x} * \vec{y}) = 2q(\vec{x}) + 2q(\vec{y})$ for all $\vec{x},\vec{y} \in \prod_{i=1}^{m} K_i$. In the case $\prod_{i=1}^{m} K_i$ is hermitian, that when $\prod_{i=1}^{m} K_i$ carries the identity involution, then every quadratic form is an additive function and every multidimensional negative definite function is real.

Theorem 4.1. (Main Result). A continuous function $\Psi : \prod_{i=1}^{m} K_i \to R$ is multidimensional negative definite if and only if there exists an additive continuous function $h : \prod_{i=1}^{m} K_i \to R_+$ and a unique measure $\mu \in M_+(\prod_{i=1}^{m} \hat{K}_i \setminus \{\vec{1}\})$ such that for all $\vec{x} \in \prod_{i=1}^{m} K_i$

$$\Psi(\vec{x}) = \Psi(\vec{e}) + h(\vec{x}) + \int_{\prod_{i=1}^{m} \hat{K}_i \setminus \{\vec{1}\}} (1 - \chi(\vec{x}))d\mu(\chi)$$

Proof. It follows from the definition that Ψ is lower bounded by $\Psi(e)$. firstly, I assume that $\Psi(\vec{e}) = 0$. Let $S \supseteq \prod_{i=1}^{m} K_i$ be a minimal semigroup containing the product hypergroups $\prod_{i=1}^{m} K_i$ Introducing

$$\Delta_{\vec{y}}\Psi(\vec{x}) := \frac{1}{2}[\Psi(\vec{x} * \vec{y}) + \Psi(\vec{x} * \vec{y}^{\,-})] - \Psi(\vec{x}); \quad \vec{x},\vec{y} \in \prod_{i=1}^{m} K_i,$$

and as pointed of (Berg et al. 1984) $\Delta_{\vec{y}}\Psi$ is bounded and multidimensional positive definite on S. Therefore appealing to Bochner's theorem for hypergroups (Jewett, 1975)

$$\Delta_{\vec{y}}\Psi(\vec{x}) = \int_{\prod_{i=1}^{m} \hat{K}_i} \rho_\chi(\vec{x}) d\sigma_{\vec{y}}(\chi)$$

for some $\sigma_y \in M_+^b(\prod_{i=1}^{m} K_i)$, where we denote the canonical extension of $\chi \in \prod_{i=1}^{m} \hat{K}_i$ to a function on S by ρ_χ. A simple calculation implies

$$-\Delta_{\vec{z}}\Delta_{\vec{y}}\Psi(\vec{x}) = \int_{\prod_{i=1}^{m} \hat{K}_i} \rho_\chi(\vec{x})[1 - Re\rho_\chi(\vec{z})]d\sigma_{\vec{y}}(\chi)$$

$$= \int_{\prod_{i=1}^{m} \hat{K}_i} \rho_\chi(\vec{x})[1 - Re\rho_\chi(\vec{y})]d\sigma_{\vec{z}}(\chi)$$

for $\vec{x},\vec{y},\vec{z} \in \prod_{i=1}^{m} K_i$, implying

$$[1 - Re\rho_\chi(\vec{z})]d\sigma_{\vec{y}}(\chi) = [1 - Re\rho_\chi(\vec{y})]d\sigma_{\vec{z}}(\chi)$$

by the uniqueness of the Fourier transform. Noting that the $\{\chi \in \prod_{i=1}^{m} \hat{K}_i ; Re\chi(\vec{y}) \le 1\}$ are open sets in $\prod_{i=1}^{m} \hat{K}_i$ with union (over $\vec{y}$) given by $\prod_{i=1}^{m} \hat{K}_i \setminus \{\vec{e}\}$, we can find a unique Radon measure μ on $\prod_{i=1}^{m} \hat{K}_i \setminus \{\vec{e}\}$ such that for every $\vec{y} \in S$

$$[1 - Re\rho_\chi(\vec{y})]d\mu(\chi) = d\sigma_{\vec{y}}(\chi), \quad on \quad \prod_{i=1}^{m} \hat{K}_i$$

The set $\hat{S}$ of all bounded semigroup characters on the semigroup S is a compact Hausdorff space with respect to the topology of point wise convergence. The canonical mapping $\zeta : \prod_{i=1}^{m} K_i \to \hat{S}, \zeta(\chi) := \rho_\chi$ is continuous, and obviously $\Delta_{\vec{y}}\psi$ is the Laplace transform of $\sigma_{\vec{y}}^{\zeta}$ (the image measure of $\sigma_{\vec{y}}$ under ζ) for each $y \in S$ hence from Berg *et al.*(1984), there exists an additive continuous function $h : \prod_{i=1}^{m} K_i \to R_+$ on S such that for all $\vec{x} \in S$

$$\Psi(\vec{x}) = h(\vec{x}) + \int_{\hat{S} \setminus \{\vec{e}\}} (1 - \rho(\vec{x}))d\mu^{\zeta}(\rho)$$

$$= h(\vec{x}) + \int_{\prod_{i=1}^{m} \hat{K}_i \setminus \{\vec{e}\}} (1 - \chi(\vec{x}))d\mu(\chi)$$

where $\mu \in M_+(\prod_{i=1}^{m} \hat{K}_i \setminus \{\vec{e}\})$

5. Conclusion

In this paper, I have given Lèvy-Khinchin type formula for multidimensional negative definite functions, defined on the product of commutative hypergroups. Many aspects that govern the relations between the classes of multidimensional positive and negative definite functions are shown. Also, I have proved that a continuous function with compact support ψ is multi-dimensional negative definite on the product of hypergroups, if and only if $exp(-t\psi)$ is multidimensional positive definite for each $t > 0$.

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دوال متعددة الأبعاد سالبة بالتحديد على جداء فوزمر إبدالية

حسام عبد الجاني[1,2]
[1,2]قسم الرياضيات، جامعة الطائف، حوا 888، الطائف، المملكة العربية السعودية.
قسم الرياضيات، جامعة حلوان، الاميرية 11282 القاهرة، مصر
المؤلف : h.abdelghany@yahoo.com

خلاصة

الهدف الأساسي لهذا البحث هو إعطاء تمثيلات تكاملية لما يسمى بالدوال متعددة الأبعاد السالبة بالتحديد و المعرفة على جداء فوزمر إبدالية. ونحصل على شروط ضرورية و كافية تضمن بأن تكون الدوال متعددة الأبعاد الموجبة بالتحديد و المعرفة على جداء فوزمر هي أيضاً متعددة أبعاد موجبة بالتحديد. بل أكثر من ذلك ، نثبت العديد من العلاقات التي تربط الدوال السالبة بالتحديد بتلك الموجبة بالتحديد. كما نثبت أيضاً بأن الدالة المتصلة التي لها حامل متراص ψ هي متعددة الأبعاد سالبة بالتحديد على جداء الفوزمر إذا و فقط إذا كانت $exp(-t\psi)$ دالة متعددة الأبعاد موجبة بالتحديد لكل قيم $t > 0$.